Twist-4 Gluon Recombination Corrections for Deep Inelastic Structure Functions

Johannes Blümlein

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WITH V. RAVINDRAN, J. RUAN & W. ZHU

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1. Introduction

- TWIST EXPANSION, LCE — WILSON 1969; BRANDT, PREPARATA, FRISHMAN ~ 1970
- TWIST: $d - n \cdot s$ — GROSS, TREIMAN 1971
- LO ANOMALOUS DIM. TWIST 2. — GROSS, WILCZEK; GEORGI, POLITZER 1973, 74

⟶ UV SINGULARITY ≡ MASS SINGULARITY

TWIST 2

ALTARELLI, PARISI 1977

CAN THE GROTH OF PARTON DENSITIES BE DAMPED ?

1ST SIMPLE INTUITIVE MODEL (POMERON BASED)

GRIBOV, LEVIN, RYSKIN 1981

A. MÜLLER, QIU, 1986

DLA ⟶ AGK CUTTING RULES

ABRAMOVSKII, GRIBOV, KANCHELI 1974

BARTELS, RYSKIN 1997

- ARE THESE CONTRIBUTIONS QCD-BASED ?
- DLA USUALLY DEPARTS SIGNIFICANTLY FROM REAL QCD !
- ENERGY-MOMENTUM CONSERVATION ! (NOT OBEYED)
- WHAT IS RESUMMED ?

NUMERICAL RESULTS:

BARTELS, JB, SCHULER 1990/91
COLLINS, KWIECINSKI 1990
KWIECINSKI 1985

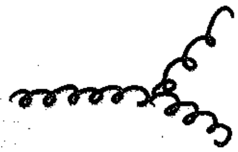
- DOES THE NON-LINEAR GLR RESUMMATION EXIST ?

NO. BARTELS 1992/93

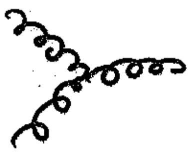
- MORE ATTEMPTS IN THE GLR DIRECTION

LEVIN et al. 1990ies , GAY-DUCATI et al.

RECOMBINATION CORRECTIONS



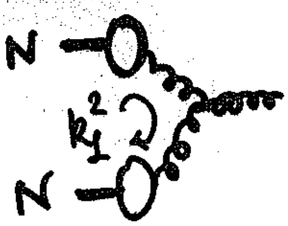
GLUON GROTH



GLUON DEPLETION ?

L. DURAND 1984

NOT SUCCESSFUL



CLOSE, QIU, ROBERTS 1989

⟶ TWO NUCLEONS NEEDED.

RECOMBINATION CORRECTIONS :

$O\left(\frac{dp_\perp^2}{p_\perp^4}\right)$

W. ZHU 1999

RECALCULATION & IMPROVEMENT

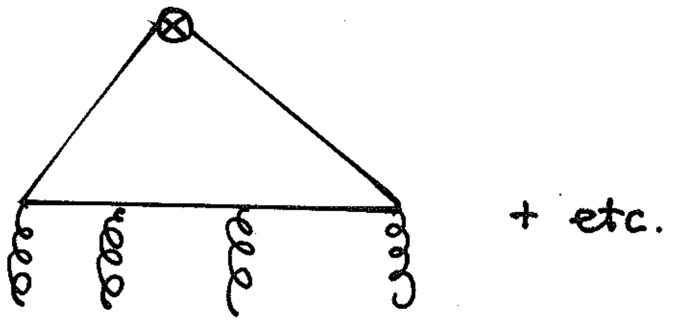
JB, V. RAVINDRAN & W. ZHU 2000.

⟶

VALID FOR ALL X
NO DLA
ENERGY MOMENTUM CONSERVATION

NO AGK RULES NEEDED.

2. 4–Gluon Operator Matrix Elements



$$G_{(n)}(x_1, \ldots, x_n) = A_n x_1^{\alpha_1} \ldots x_n^{\alpha_n} (1 - x_1 - \ldots - x_n)^\beta$$

assume: $\forall \alpha_i \equiv \alpha$

$$\hat{G}_2(x_1, x_2; x_1', x_2') = G_2(x_1, x_2; x_1', x_2') \, \delta(x_1 + x_2 - x_1' - x_2')$$

$$\hat{G}_2(x_1, \ldots, x_4) = G_1(x_1) \, G_1(x_2) \, \chi(x_1, x_2, x_3)$$

↑ correlator.

assume : $\chi \equiv 1$.

$$G_2(x_1) = \frac{Q_0^2}{Q^2} \, G_1^2(x_1).$$

or:

$$G_2(x_1) = \frac{9}{8} Q_0^2 \, x_1^2 \, G_1^2(x_1)$$

3. Coefficient Function

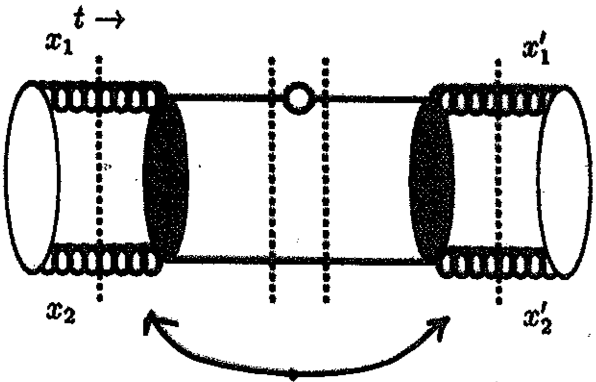


Figure 1: Direct diagrams contributing to (5). The grey oval symbolizes the set of diagrams in Figure 3. Orthogonal dashed lines stand for the time ordering. The separated white ovals symbolize the two parts of the non-perturbative 4–gluon density. x_1, x_2, x_1' and x_2' are the longitudinal momentum fractions, with $x_1 + x_2 - x_1' - x_2' = 0$. The circle stands for the forward subprocess $\gamma^* + q \rightarrow \gamma^* + q$ through which the virtual photon couples to the amplitude.

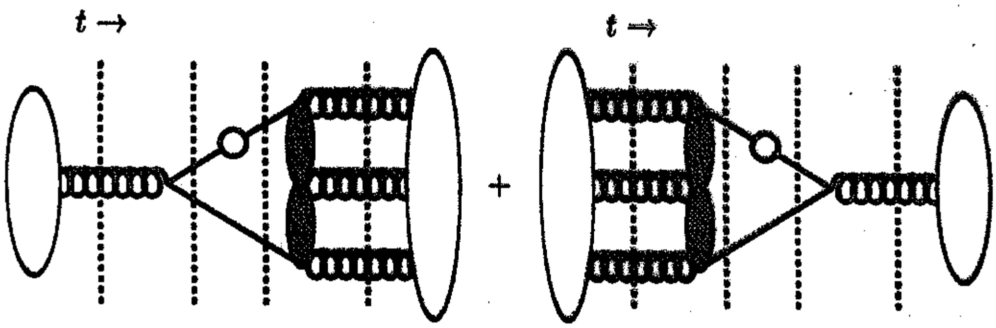


Figure 2: Interference diagrams associated to the process in Figure 1.

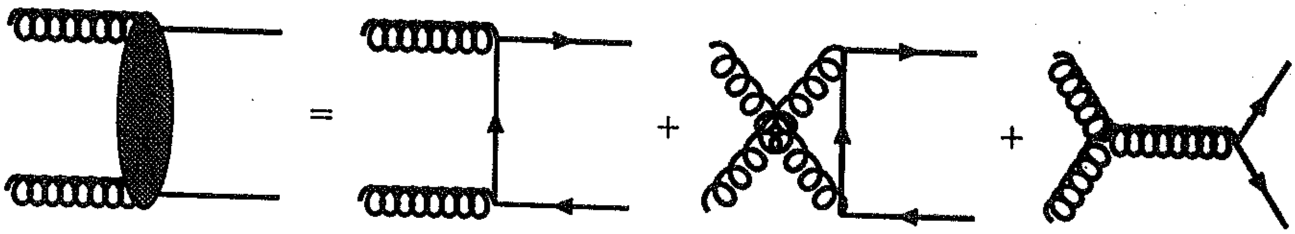


Figure 3: Diagrams symbolized by grey ovals in Figure 1 and 2.

IN THE SYMMETRIC CASE: $x_1 = x_2$ DGR. 3 VANISHES.

THE $4G \to 2Q$ COEFFICIENT FUNCTION:

$$\propto \frac{dp_\perp^2}{(p_\perp^2+m^2)^2} :$$

$\to$ TOPT.

$$C_{G\to q}^{4\to 2,\, 2\to 2}(x_1; x) = \frac{1}{96}\,\frac{(2x_1 - x)^2}{x_1^5}\left[14x^2 - 3x\,x_1 + 18x_1^2\right]$$

ONE MAY SHOW THAT:

$$\boxed{C_{G,q}^{4\to 2,\, 2\to 2} = -\,C_{G,q}^{4,2;\,1\to 3} = -\,C_{G,q}^{4,2;\,3\to 1}}$$

AT ALL x !

4. Numerical Results

PREDICTION FOR THE SLOPE OF F_2.

$$\frac{\partial F_2(x,Q^2)}{\partial \log Q^2} = \frac{\partial F_2^{\tau=2}}{\partial \log Q^2}$$

ANTISCREENING $\longrightarrow$

$$+\left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{Q^2} \int_{x/2}^{1/2} dx_1 \left(\frac{x}{x_1}\right) C^{4\to 2,2\to 2}(x_1,x) \cdot G_2(x_1)$$

SCREENING $\longrightarrow$

$$-2\left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{Q^2} \int_{x}^{1/2} dx_1 \left(\frac{x}{x_1}\right) C^{4\to 2,2\to 2}(x_1,x) \cdot G_2(x_1)$$

$$\frac{\partial F_2^{\tau=2}}{\partial \log Q^2} = \left(\frac{\alpha_s}{2\pi}\right) \times \left\{ \sum_q e_q^2 \left[P_{qq} \otimes (q+\bar{q})\right](x) + \left[\sum_q e_q^2\right]\left[P_{qG} \otimes G_1\right](x) \right\}$$

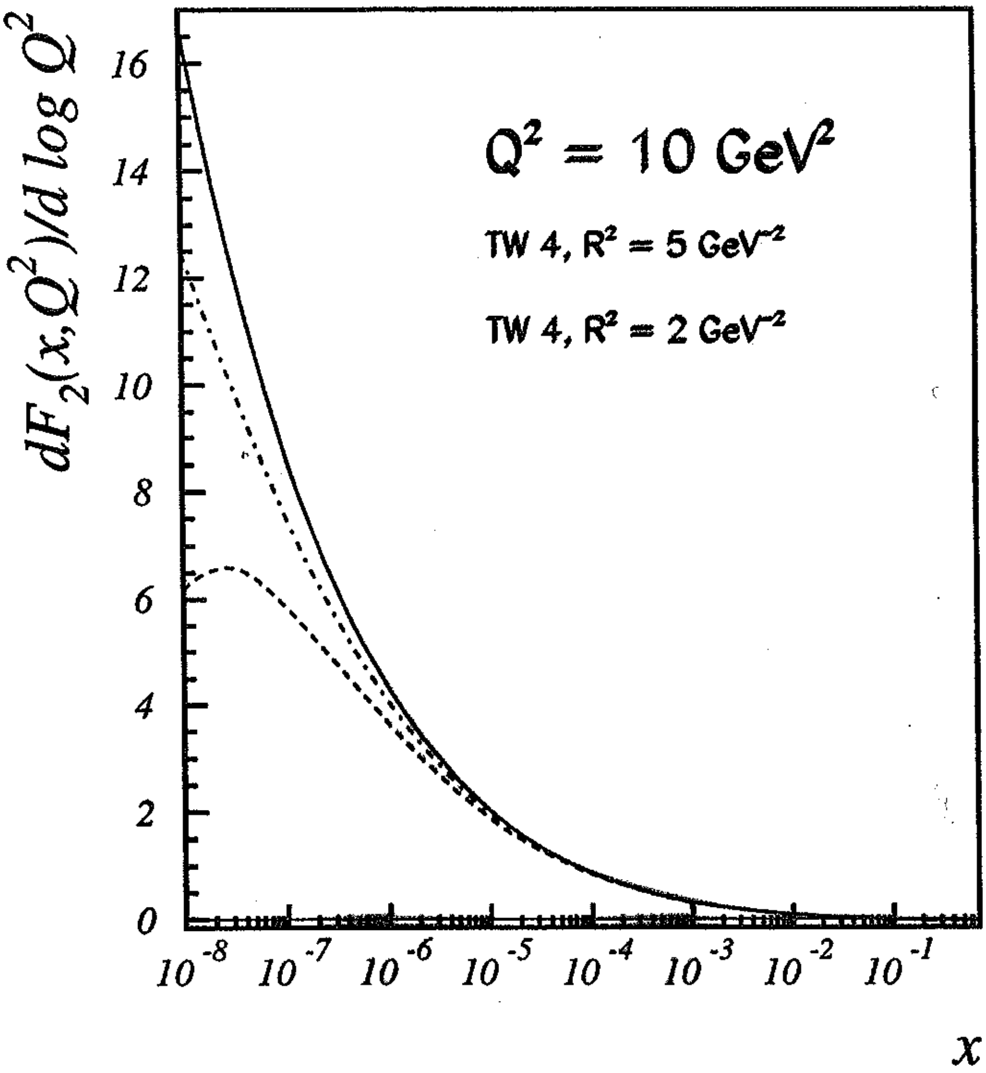


Figure 4b: Same as figure 1a for $Q^2 = 10\,\text{GeV}^2$.

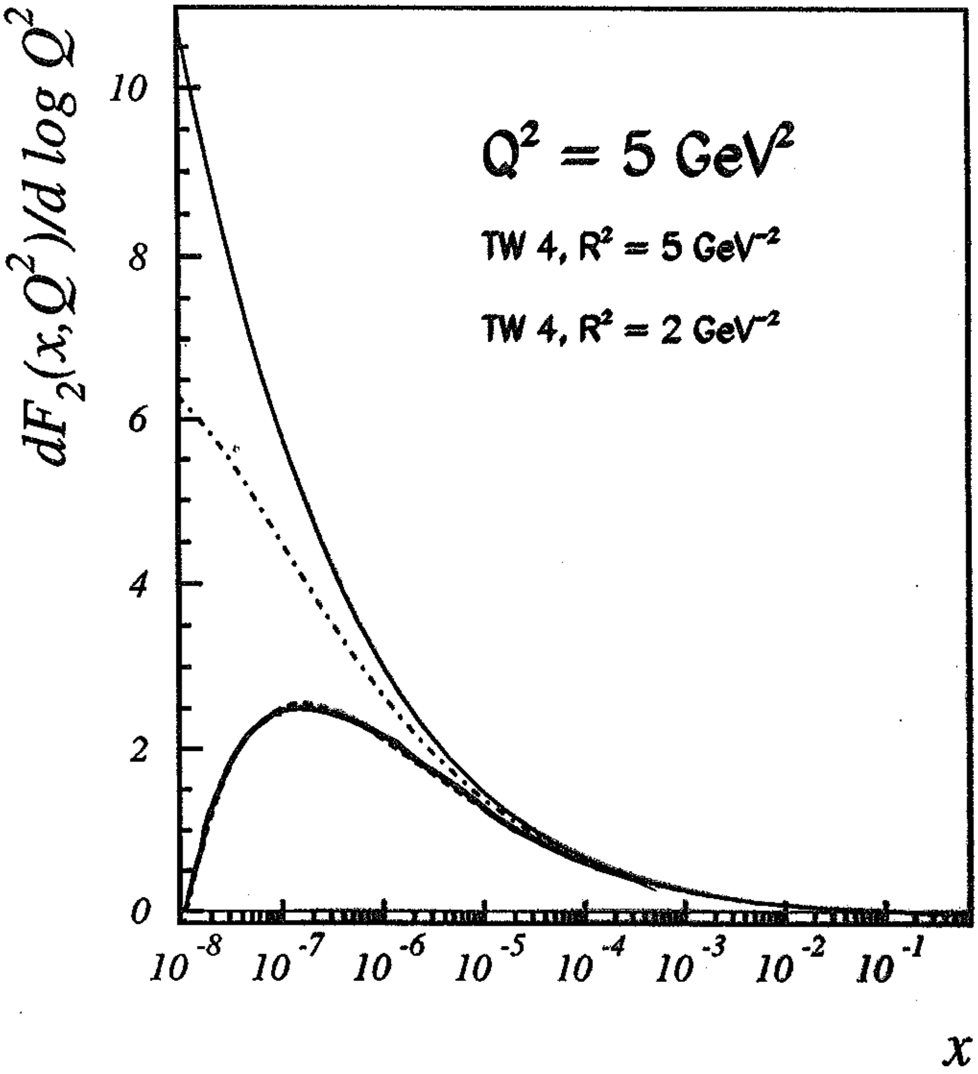


Figure 4a: The slope $dF_2(x,Q^2)/d\log Q^2$ at $Q^2 = 5\,\mathrm{GeV}^2$. Full line: leading order twist–2 contributions (parameterization Ref. [24]). Dash-dotted line: Eq. (7) with twist–4 mass scale $R^2 = 5\,\mathrm{GeV}^{-2}$, and dashed line: $R^2 = 2\,\mathrm{GeV}^{-2}$.

"LEADING" POLE EXPANSION FAILS.

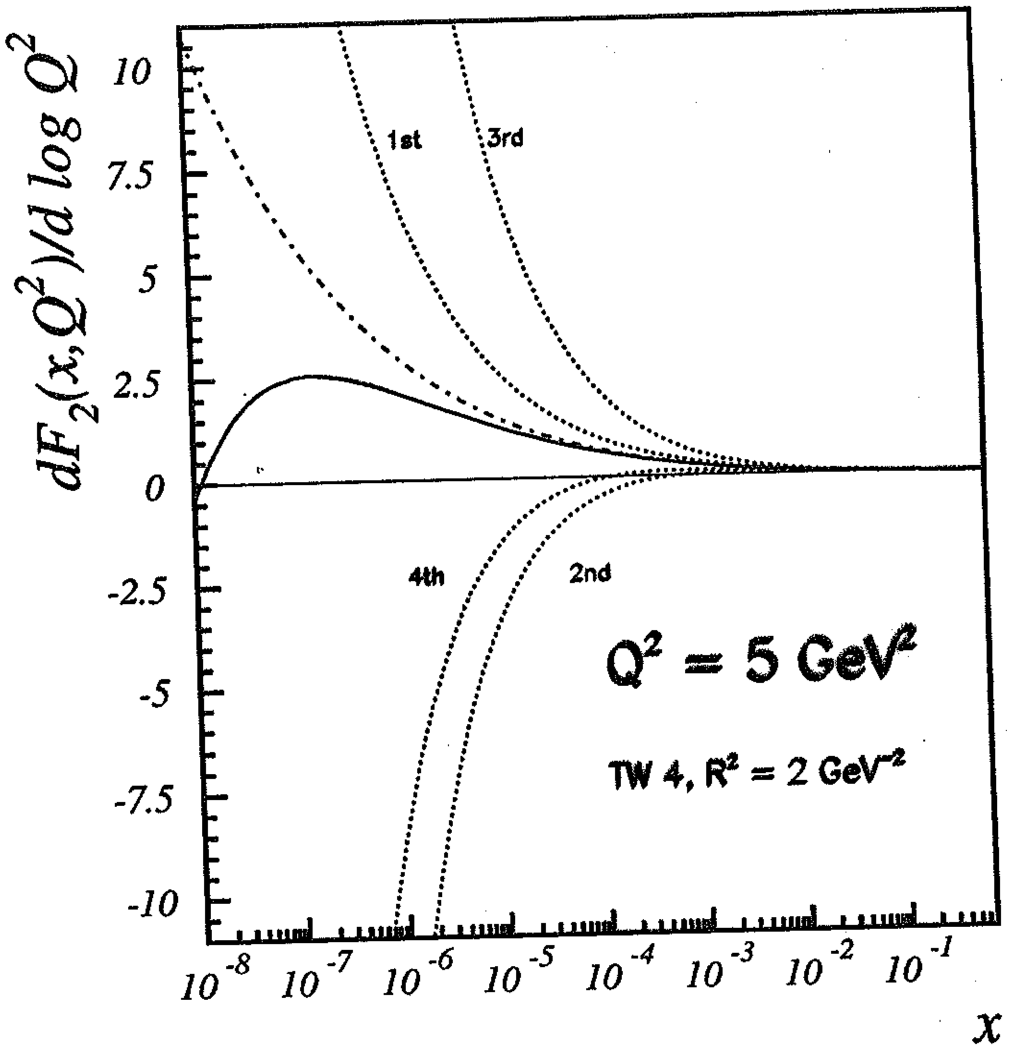


Figure 5: Comparison of the slope $dF_2(x,Q^2)/d\log Q^2$ at $Q^2 = 5\,\mathrm{GeV}^2$ and twist–4 mass scale $R^2 = 2\,\mathrm{GeV}^{-2}$, Eq. (7) (full line) with the corresponding results obtained approximating the coefficient function Eq. (5) by the sequence of contributing powers. 1st: z^0, 2nd: z etc. (dotted lines). Dash-dotted line: twist–2 contribution.

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5. Conclusions

- A study has been performed to the slope of F_2 due to twist–4 coefficient functions for the process $4G \rightarrow 2q$ within time–ordered perturbation theory and limited to $dp_\perp^2/p_\perp^4$ accuracy.
- Numerical results were DERIVED choosing a special ansatz for the non–perturbative two–particle gluon density $\mathbf{G_2}$.
- The new contributions contain anti–screening and screening terms, the latter of which dominate for smaller values of x and diminish the growth of the slope due to the twist–2 contributions.
- The numerical results show that this effect is of significant size only below $x \sim 2 \cdot 10^{-6}$ for $Q^2 \sim 5\,\mathrm{GeV}^2$. This region is yet beyond the kinematic range which can be probed at the ep collider HERA but may be investigated at future lepton–hadron facilities operating at larger cms energies.