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Testing QCD Scaling violations in the HERA Energy Range

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1. Introduction
2. Kinematical Range
3. Radiative Corrections
4. Structurefunctions
5. QCD Analysis
   a) statistical precision for Λ
   b) systematic effects
   c) $\alpha_s(Q^2)$
6. Conclusions

BASED ON WORK DONE PARTLY IN COLLABORATION WITH: G. INGELMAN, M. KLEIN, T. NAUMANN & R. RÜCKL.

1. INTRODUCTION

DEEP INELASTIC $\binom{\overset{(-)}{\nu}}{\ell^\pm}$ N - SCATTERING IS ONE OF THE CLEANEST POSSIBILITIES TO TEST SCALING VIOLATIONS OF STRUCTUREFUNCTIONS.

PAST & PRESENT :	eN	SLAC-Exp.
	μN	EMC, BCDMS, Exps. at FNAL NMC, ...
	νN	BEBC, 15 ft., CDHS, CHARM, ...

$1 < Q^2 \lesssim 200$ GeV^2

$.01 < x < .9$

END OF 1990 : HERA

$\longrightarrow x \sim 10^{-4}$

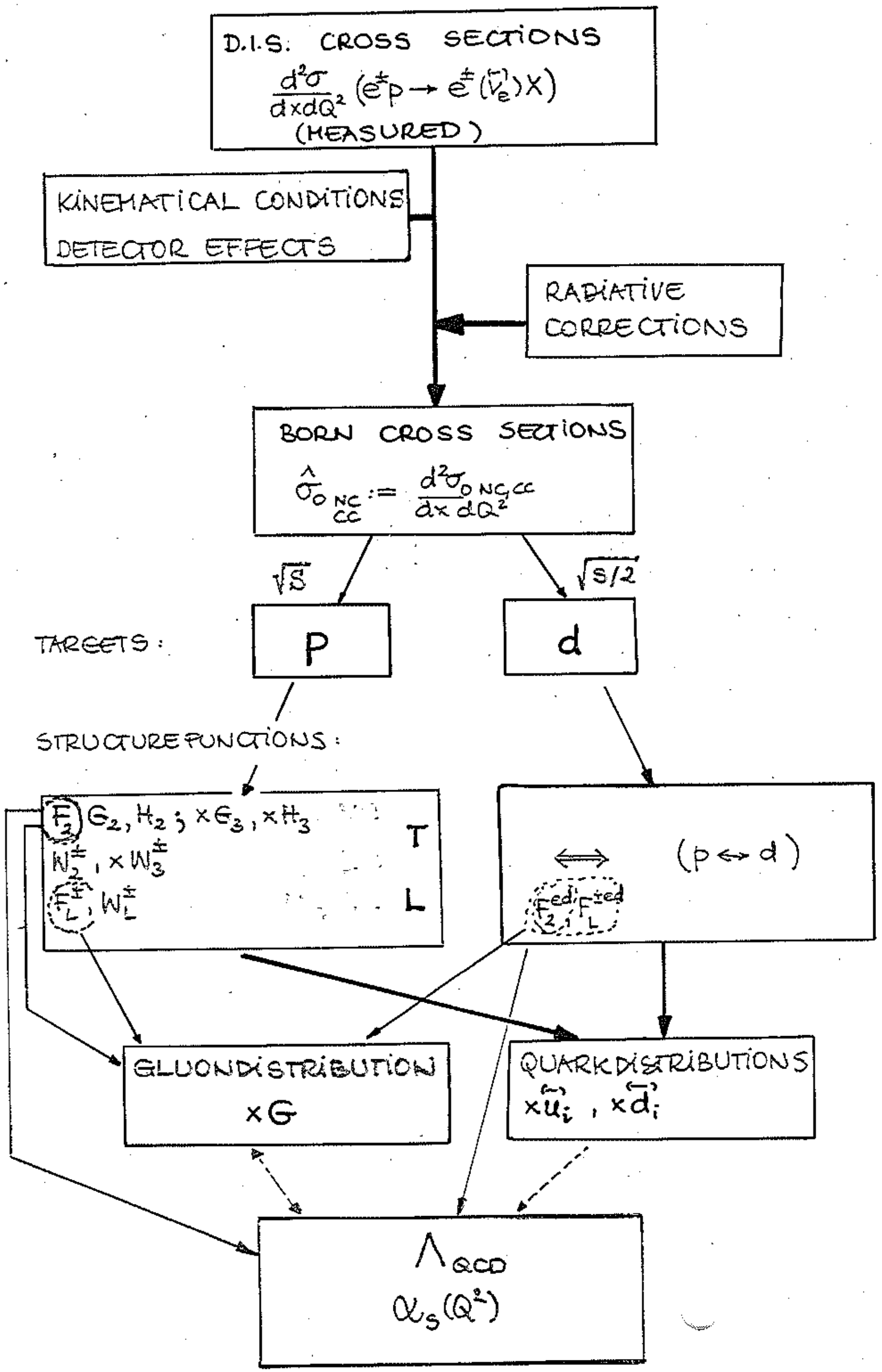
$\longrightarrow Q^2 \sim O(10^4\,\text{GeV}^2)$

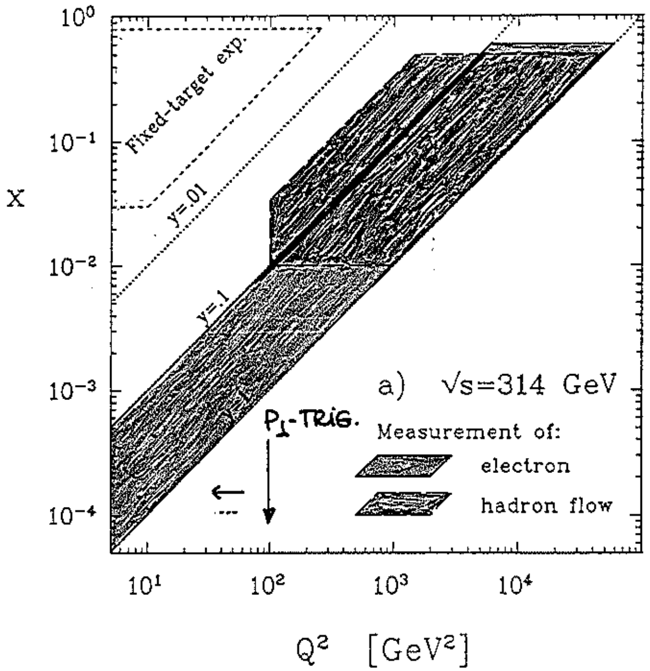
- WIDE Q^2 RANGE
- PRECISE STRUCTUREFUNCTION-MEASUREMENT AT VERY LOW x

} NEW POSSIBILITIES TO TEST QCD

- $e^\pm p \to \overset{(-)}{\nu}_e X$ MEASURABLE AT HIGH Q^2 – PARTON DISTRIBUTIONS AT HIGH Q^2

2. KINEMATICAL RANGE FOR DIS AT HERA

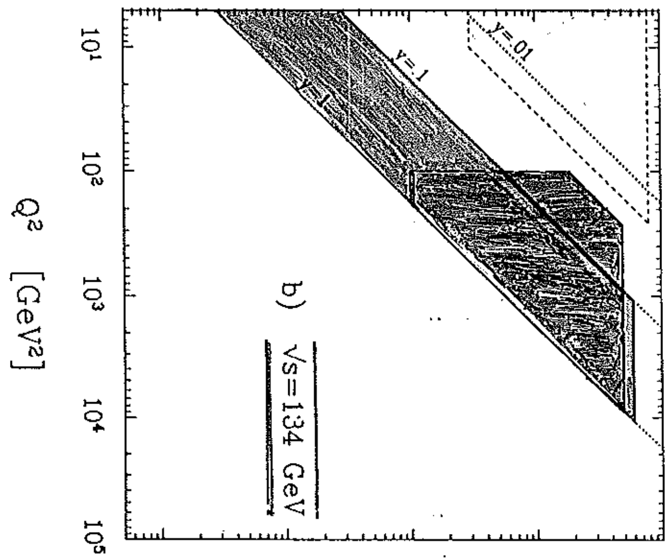




SHEARING ⊕ SYSTEMATIC EFFECT
$< 10\%$
J.PELTESSE

$\mathcal{L}_0 = 1.5 \cdot 10^{31}\ cm^{-2}\,s^{-1}$

$\Rightarrow$ $\int dt\,\mathcal{L} = 100\ pb^{-1} \equiv$ 75 fully efficient days



$$\mathcal{L} \rightarrow \mathcal{L}_0 \cdot \frac{E_p}{820\,\mathrm{GeV}}$$

5 ORDERS OF MAGNITUDE IN Q^2

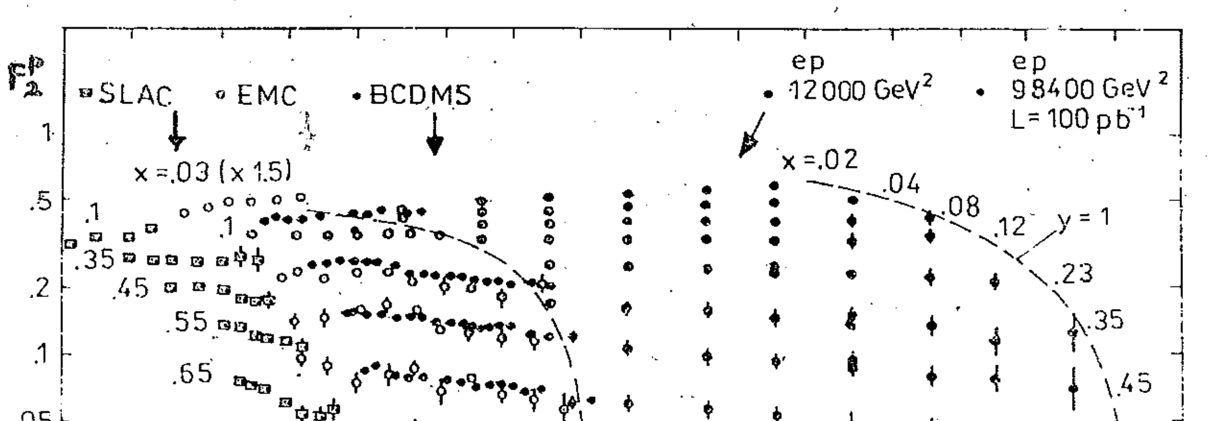
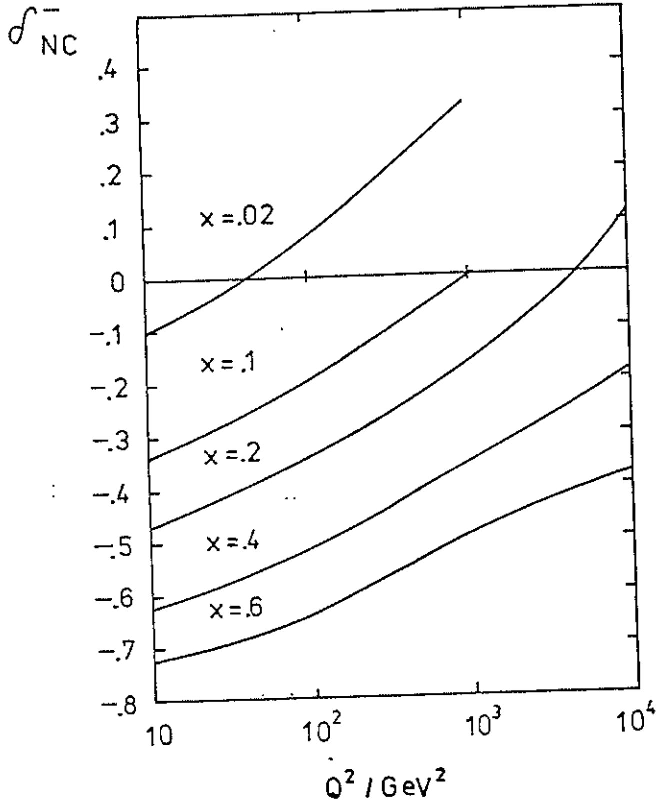


Fig. 6

3. RADIATIVE CORRECTIONS



BARDIN et al.

$$\delta^-_{NC} = \left(\frac{d^2\sigma_0}{dx\,dQ^2} + \alpha \frac{d^2\hat{\sigma}_1}{dx\,dQ^2} \right) \Big/ \frac{d^2\sigma_0}{dx\,dQ^2} - 1 \quad : e^-p$$

$O(\alpha)$ CALCULATIONS : $\frac{d^2\sigma}{dx\,dy}$

NC : BÓHM, SPIESBERGER 1987
BARDIN et al. 1981, '87, '88

CC : BARDIN et al. 1988
SPIESBERGER 1989

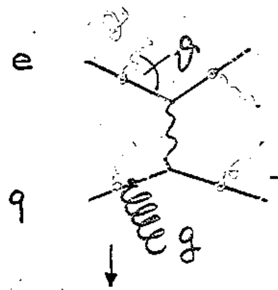
THE MAJOR CONTRIBUTIONS CAN BE DERIVED IN THE LEADING LOG APPROXIMATION ALREADY !

W. VAN NEERVEN $|\gamma|^2$ 1989 PHE 89-08, 89-10.
J.B. NC & CC

a) 

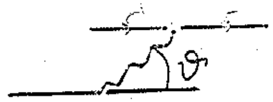
$\otimes \quad \delta(1-x) + \left[\frac{3}{2}\delta(1-x) + \frac{1+x^2}{1-x}\theta(1-x) \right] \cdot \frac{\alpha}{2\pi} \ln\left(\frac{Q^2}{m_f^2}\right) e_f^2$

$\vartheta \ll 1$



$\longrightarrow 0$ (KLN)

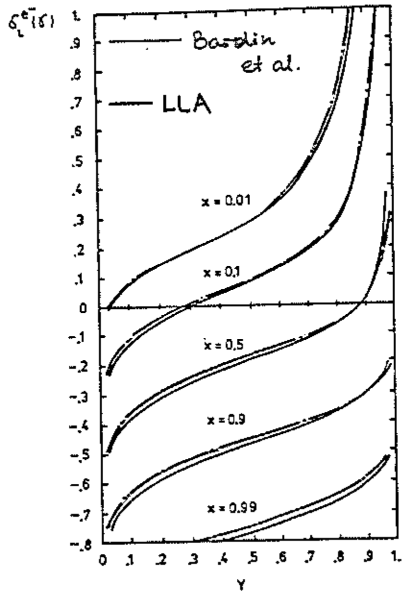
$O(\alpha)$ CORR. TO SCALING VIOLATIONS

b) 

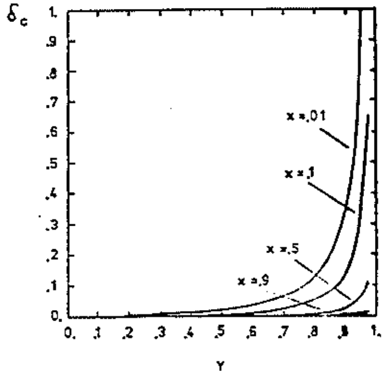
$\propto \frac{\alpha}{2\pi} \ln\left(\frac{Q^2}{m_q^2}\right) \longrightarrow \frac{\alpha}{2\pi} \ln\left(\frac{Q^2}{\Lambda^2}\right)$

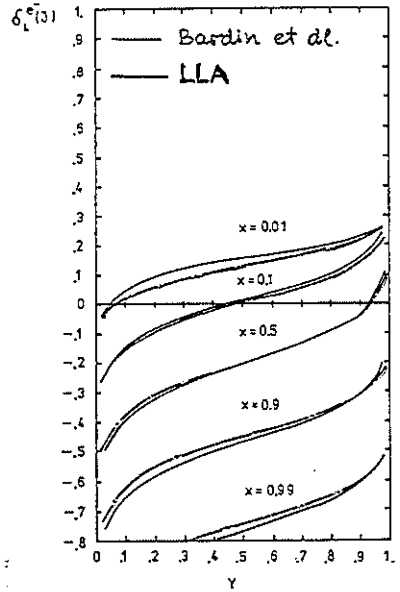
LARGE : AT $y \to 1$, SMALLER x.

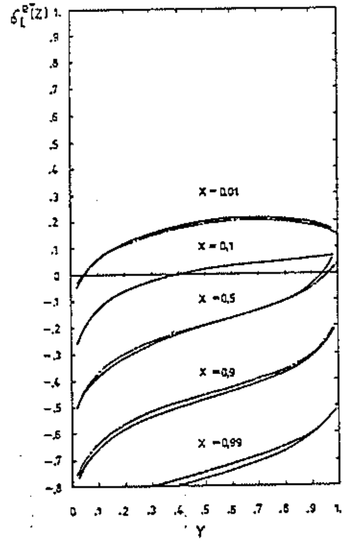
NC



$|\gamma|^2$





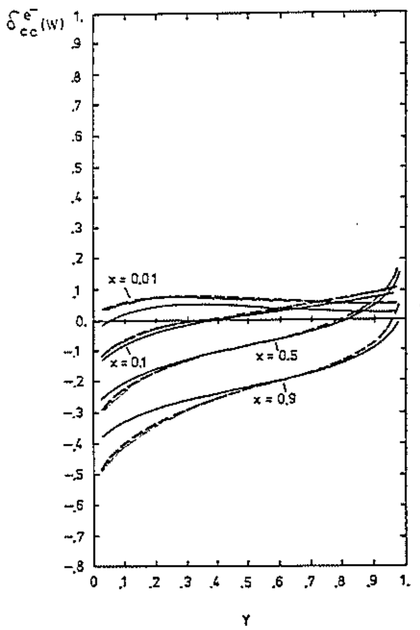


$\sim \ln\left(\frac{S}{M_p^2}\right)$

$\ln\left(\frac{Q^2}{\Lambda^2}\right)$

$\sim 2\mathrm{Re}\,\gamma Z^*$

$\sim |Z|^2$

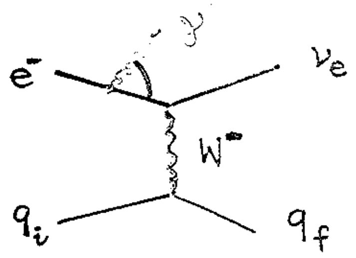


CC

← CROSSING OF x = CONST. LINES

M_W - FINITE.

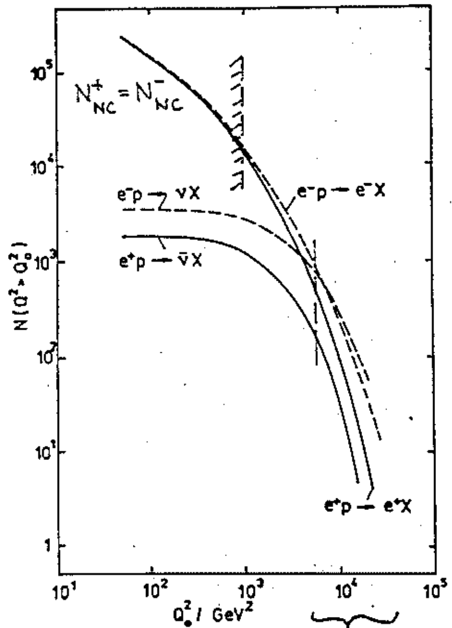
$\sim |W|^2$



VERY RECENTLY:

NUMERICAL AGREEMENT BETWEEN THE RESULTS OF THE DUBNA-ZEUTHEN GROUP AND THE WÜRZBURG GROUP IS OF $O(1\%)$ OR BETTER!

4. STRUCTURE FUNCTIONS



EVENTRATES FOR $Q^2 > Q_0^2$:

$x > .01$, $y > .03$

$\mathcal{L} = 100\, pb^{-1}$

UNFOLDING OF QUARK DISTRIBUTIONS

$N_{cc} \ll N_{NC}$

F_2^{em}

↓

QCD - TESTS

NEUTRAL CURRENT

$e^{\pm}p$

$$\hat{\sigma}_{\pm} = \frac{d^2\sigma^{\pm}}{dx\,dQ^2}\cdot\frac{Q^4 x}{2\pi\alpha^2} = Y_+\Big[F_2 + \kappa(Q^2)(-v \mp \lambda a)\,G_2 + \kappa^2(Q^2)(v^2+a^2 \pm 2va\lambda)\,H_2\Big]$$

$$+\,Y_-\Big[\kappa(Q^2)(\pm a + \lambda v)\times G_3 + \kappa^2(Q^2)\big[(-v^2-a^2)\lambda \mp 2va\big]\times H_3\Big]$$

$$\kappa(Q^2) = \frac{1}{4 s_\theta^2 c_\theta^2}\cdot\frac{Q^2}{Q^2+M_Z^2}$$

$$B_+(\lambda) = \frac{1}{2Y_+}\left[\hat{\sigma}_+(-\lambda) + \hat{\sigma}_-(+\lambda)\right]$$

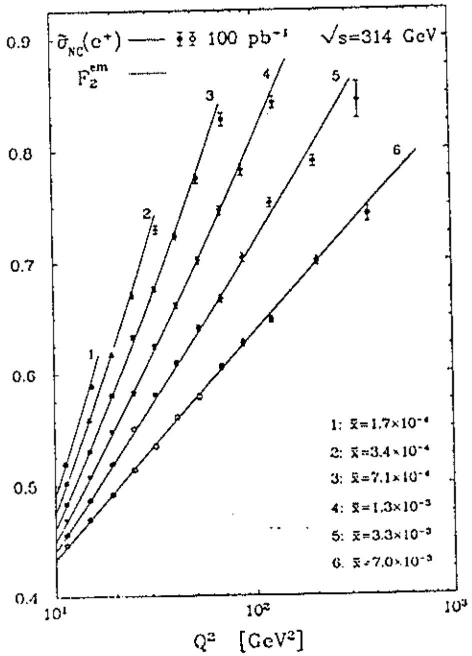
$$= F_2 + \underbrace{(-v+\lambda a)}_{\approx 0,\ \lambda=0}\,\kappa\, G_2 + (v^2+a^2-2va\lambda)\,\kappa^2 H_2$$

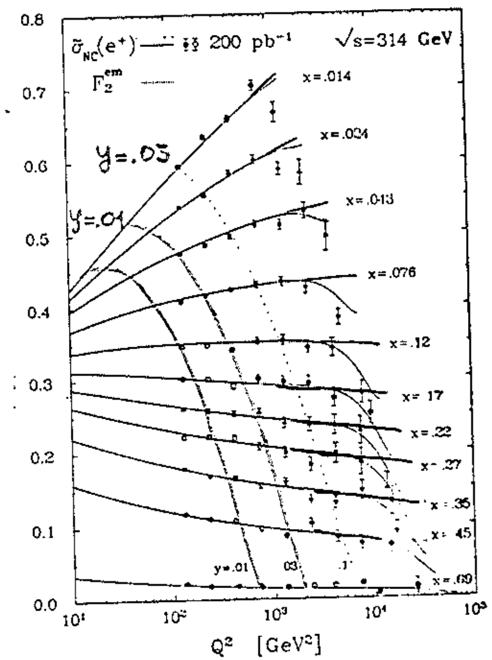
small at lower Q^2

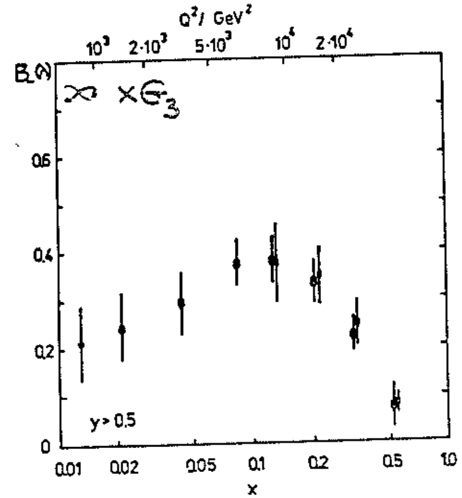
$$B_-(\lambda) = \frac{1}{2Y_-}\left[\hat{\sigma}_+(-\lambda) - \hat{\sigma}_-(+\lambda)\right]$$

$$= \underbrace{(a-\lambda v)}_{=1/2}\,\kappa \times G_3 + (-2va + \lambda(v^2+a^2))\,\kappa^2 \times H_3$$

small! $\lambda = 0$







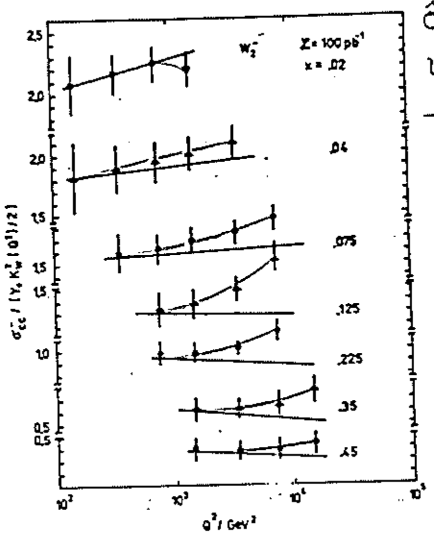
100 pb^{-1} / per beam

CHARGED CURRENT

$e^{\pm}p$

$$\hat{\sigma}_{\pm} = \frac{d^2\sigma}{dx\,dQ^2}\frac{Q^4 x}{2\pi\alpha^2} = \kappa_W^2(Q^2)\,\frac{1\pm\lambda}{2}\left[Y_+ W_2^{\pm} \mp Y_- x W_3^{\pm}\right]$$

$$\kappa_W = Q^2/(Q^2+M_W^2)/4s_\theta^2$$



$\hat{\sigma}_{cc}^{-} / (Y_+ \kappa_W^2/2)$, $\lambda = 0$.
vs. x & Q^2
—— $W_2^-(x, Q^2)$

$e^{\pm}d$: – DIFFERENT STRUCTURE FUNCTIONS
$\longrightarrow$ $W_2 = \Sigma$ from: $\frac{1}{2}(\hat{\sigma}_{cc}^{+} + \hat{\sigma}_{cc}^{-})$

– NO SIMPLIFICATION IN THE NC CASE.

COMBINATIONS OF PARTONDENSITIES UNFOLDED FROM DIS CROSS SECTIONS:

- $u_v, d_v, u_v \pm d_v, e_u u_v - e_d d_v$, — NS
- Σ — S
- $u_v + S/2$, $d_v + S/2$, $\Sigma_i(u_i + \bar{u}_i)$, $\Sigma_i(d_i + \bar{d}_i)$, etc. — NS $\oplus$ S

5. QCD ANALYSIS

$$F(x,Q^2) = \sum_i \left[\alpha_i q_i(x,Q^2) + \beta_i \bar{q}_i(x,Q^2)\right].$$

$$F(x,Q^2) = aF_{NS}(x,Q^2) + bF_S(x,Q^2)$$

$$F_{NS}(x,Q^2) = \sum_{i,j}\alpha_{ij}\left[q_i(x,Q^2) - \bar{q}_j(x,Q^2)\right]$$

$$F_S(x,Q^2) = \sum_i \left[q_i(x,Q^2) + \bar{q}_i(x,Q^2)\right].$$

$$\frac{\partial}{\partial t}F_{NS}(x,t) = P_{qq}(x)\otimes F_{NS}(x,t)$$

$$\frac{\partial}{\partial t}\begin{pmatrix} F_S(x,t) \\ G(x,t)\end{pmatrix} = \begin{pmatrix} P_{qq}(x) & 2N_f P_{qG}(x) \\ P_{Gq}(x) & P_{GG}(x)\end{pmatrix} \otimes \begin{pmatrix} F_S(x,t) \\ G(x,t)\end{pmatrix}$$

$$A(x)\otimes B(x) = \int_0^1 dx_1 \int_0^1 dx_2\, \delta(x - x_1 x_2) A(x_1) B(x_2)$$

$$t = (2/\beta_0) ln[ln(Q^2/\Lambda^2)/ln(Q_0^2/\Lambda^2)]$$

$$\beta_0 = 11 - \tfrac{2}{3}N_f$$

DIFFERENT NUMERICAL APPROACHES:

ABBOTT, ATWOOD, BARNETT	1980 ←	MINUIT BASED
K. KATO, SHIMIZU, YAMAMOTO	1980	
GÒNZALES-ARROYO, LOPEZ	1980	
DEVOTO et al.	1983	
FURMANSKI, PETRONZIO	1982 ←	ALGEBRA OF LAGUERRE POLYNOM.
VIRCHAUX, ORAOU	1987	
KRIVOKHIZHIN	1987	
WU KI TUNG		
⋮		

$$\sum_i L_i(\mathcal{G})$$
$$\mathcal{G} = \ln \frac{1}{x}$$

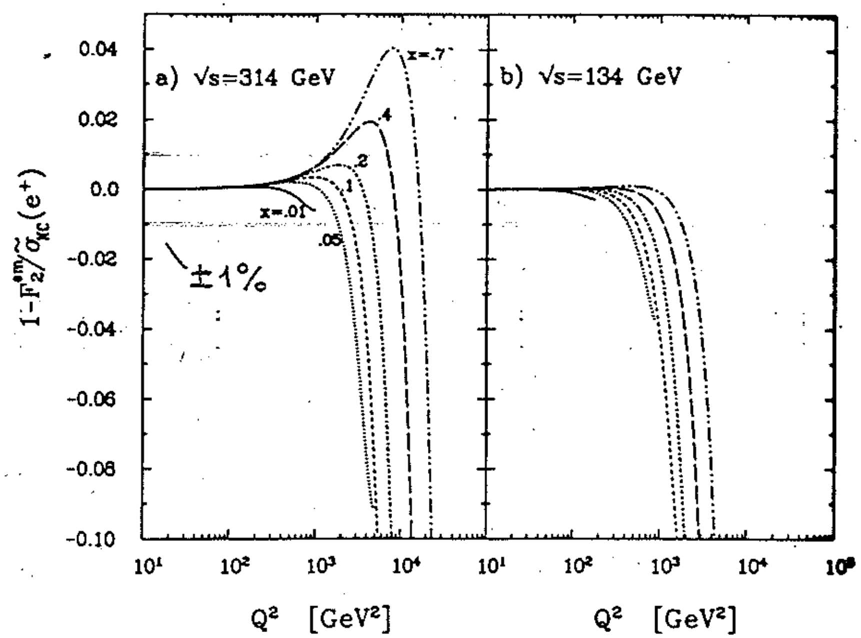
$$\chi^2 = \sum_{\text{bins}} \left(\frac{F_2^{exp}(x,Q^2) - F_2^{THY}(x,Q^2)}{\delta F_2} \right)^2 \rightarrow \text{MIN.}$$

QCD ANALYSIS : PARTON DISTRIBUTIONS & STRUCTURE FUNCTIONS EXCEPT OF F_2

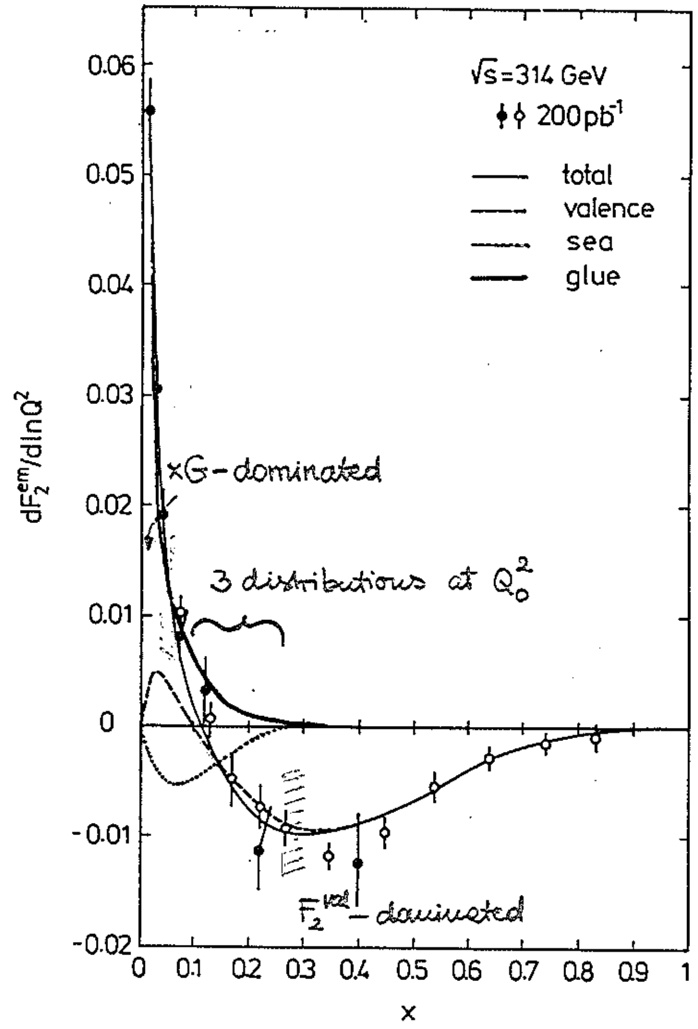
$$\delta\Lambda_{Stat} \gtrsim 300 \text{ MeV}$$

NO QCD-TEST POSSIBLE.

QCD-ANALYSIS OF F_2



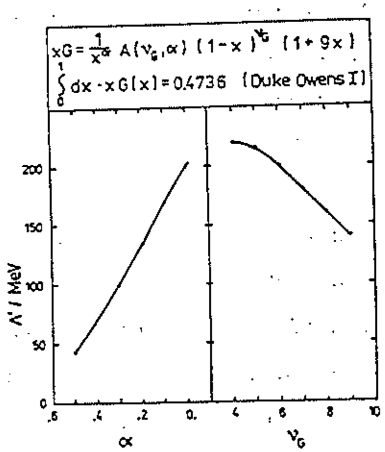
$$\frac{|1 - F_2^{em}/\tilde{\sigma}_{NC}|}{\delta_{Stat}} < \varepsilon \cong 1$$



a)

Table 2 Statistical precision on Λ from QCD fits to $\bar{\sigma}_{NC}(e^+) \approx F_2^{em}$

x-range	type of fit	full y range Λ [MeV]	restrictions according to eqs. (5,6) Λ [MeV]
a) $\sqrt{s} = 314$ GeV, $Q^2 \geq 100$ GeV^2, $\int \mathcal{L}\, dt = 200\ pb^{-1}$			
$x \geq 0.25$	non singlet (11)	145 ± 48	175 ± 176
$x \geq 10^{-2}$	eqs. (11,12)	297 ± 76	177 ± 135
$x \geq 10^{-2}$	eqs. (11,12), xG fix	215 ± 16	201 ± 25
b) $\sqrt{s} = 314$ GeV, $Q^2 \geq 10$ GeV^2, $\int \mathcal{L}\, dt = 100\ pb^{-1}$			
$x \geq 10^{-4}$	eqs. (11,12)	196 ± 5	225 ± 25
c) $\sqrt{s} = 134$ GeV, $Q^2 \geq 18$ GeV^2, $\int \mathcal{L}\, dt = 100\ pb^{-1}$			
$x \geq 0.25$	non singlet (11)	200 ± 47	460 ± 263
$x \geq 10^{-2}$	eqs. (11,12)	211 ± 27	227 ± 58



$\delta\Lambda_{TH} = -20$ MeV : for $R_{QCD} \to R=$

$x \geq 10^{-2}$

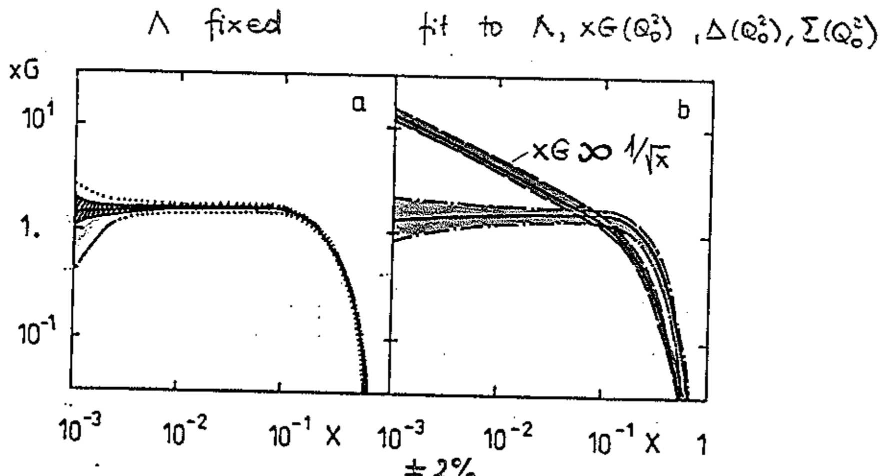


Fig. 4 a) xG(stat) and (stat.⊕ syst.) for $\delta\Lambda = 0$
b) xG(stat) from joint fit of Λ and xG with xG(DO) and $xG \sim 1/\sqrt{x}$.

b) SYSTEMATIC EFFECTS

- RESTRICTED PHASE SPACE : SMEARING etc. < 10%
  J. FELTESSE
- CALIBRATION UNCERTAINTY OF THE CALORIMETERS

e & h

$$\hat{E}_{e'} = E_e(1+\varepsilon_e) \quad ; \quad \hat{E}_h = E_h(1+\varepsilon_h)$$

$$(x, y, Q^2) \to (\hat{x}_e, \hat{y}_e, \hat{Q}_e^2)$$
$$(x, y, Q^2) \to (\hat{x}_h, \hat{y}_h, \hat{Q}_h^2)$$

$$\frac{d\hat{\sigma}(\hat{x},\hat{Q}^2)}{d\hat{x}d\hat{Q}^2}\, d\hat{x}d\hat{Q}^2 = \frac{d\sigma(x,Q^2)}{dxdQ^2}\, dxdQ^2.$$

$$\hat{\tilde{\sigma}}(\hat{x},\hat{Q}^2) = \frac{f(x,Q^2)}{f(\hat{x},\ \hat{Q}^2)}\left(\frac{dxdQ^2}{d\hat{x}d\hat{Q}^2}\right)\tilde{\sigma}(x,Q^2),$$

WE CONSIDER : a) $x \geq 10^{-2}$, $Q^2 > 10^2\ GeV^2$ JET MEASUREMENT
b) $x \leq 10^{-2}$, $Q^2 \leq 10^2\ GeV^2$ ELECTRON MEASUREMENT

$$\varepsilon_{e,h} = \pm 1\%$$
$$\downarrow$$

a) $\Delta\Lambda = \pm 70\ MeV$
b) $\Delta\Lambda = \mp 40\ MeV$

SCALES LINEARLY IF $\varepsilon_{e,h} \ll 1$

c) $\alpha_s(Q^2)$

$$\alpha_s(Q^2) = \frac{12\pi}{33-4N_f} \cdot \frac{1}{\ln\left(\frac{Q^2}{\Lambda^2}\right)} \qquad N_f = 4$$

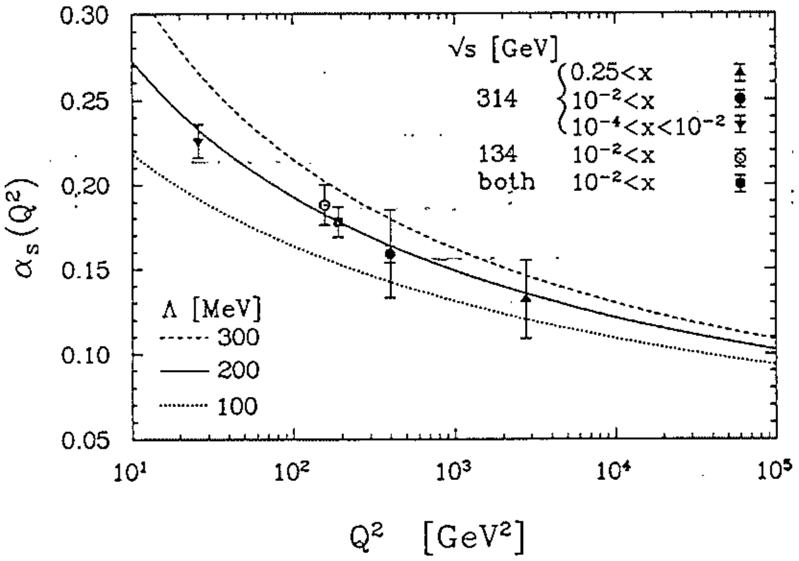
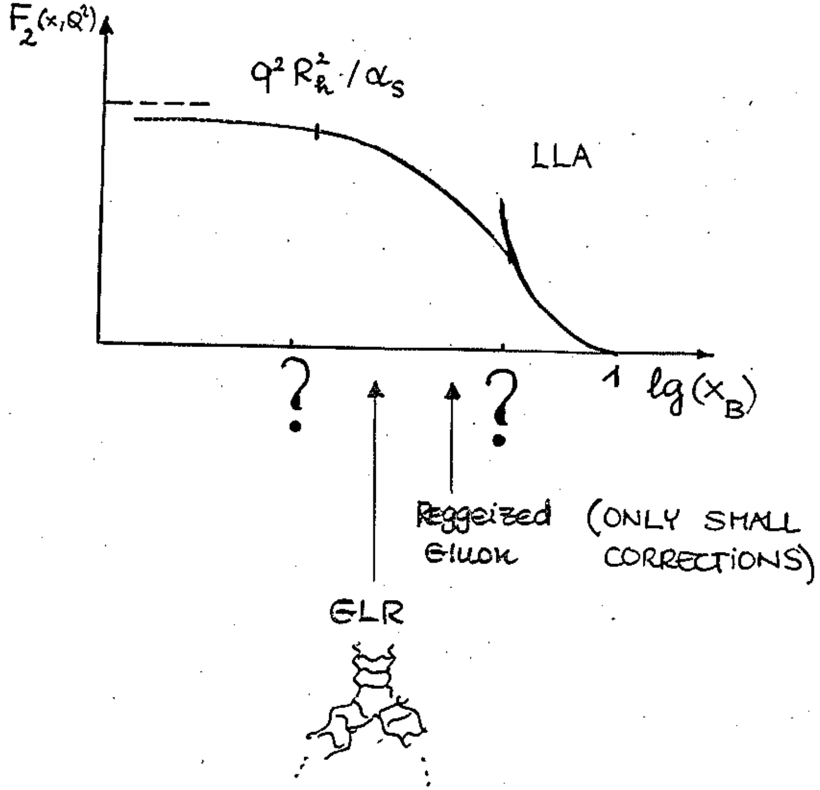


Fig. 8

F_2 AT LOW X



6. Conclusions

1. HERA will probe the proton structure via deep inelastic scattering

- up to $Q^2 \sim 10^4\ GeV^2$
- down to $x \sim 10^{-4}$

i.e. extend the kinematical range in x and Q^2 explored so far by two orders of magnitude.

2. The $\mathcal{O}(\alpha)$ electroweak radiative corrections to deep inelastic scattering are calculated by different groups and agree within $\mathcal{O}(1\%)$. To a very wide extent they can be described by the leading log approximation (QED) already in the HERA energy range.

3. Among the various structure functions and combinations of parton distributions which can be determined at HERA from both neutral and charged current scattering only F_2^{em} can be measured with sufficient statistical precision in x and Q^2 to be used for the QCD analysis.

4. The statistical precision on Λ_{QCD} for $\sqrt{s} = 314\ GeV^2$, L $= 200\ pb^{-1}$, $x > 0.01$ and $Q^2 > 100 GeV^2$ is about 100 MeV.
It can be improved :

- using constraints on $xG(x, Q_o^2)$
- including data at lower y ($y > 0.01$)
- running also at lower $\sqrt{s}$ ($= 134 GeV^2$)
- extending the analysis to data at low x ($x \sim 10^{-3...4}$)

5. A major systematic effect on Λ_{QCD} is caused by the calibration uncertainties of the electromagnetic and hadronic calorimeter. These effects deserve further study.

6. The low x range $10^{-4} < x < 10^{-2}$ and $10 < Q^2 < 10^3\ GeV^2$ could become a new testing ground for QCD. F_2^{em} can be measured with very high statistical precision there.

However, there are still more efforts needed to work out the theory of QCD evolution in this range.

7. The measurements of the above quantities are long term tasks and require the control of systematics at the per cent level.