Die Struktur des Protons bei kurzen Abständen

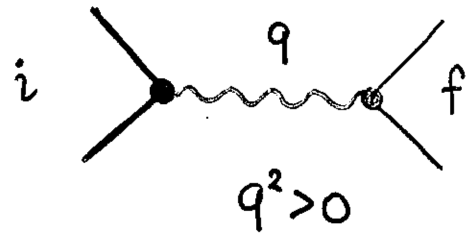
Johannes Blümlein

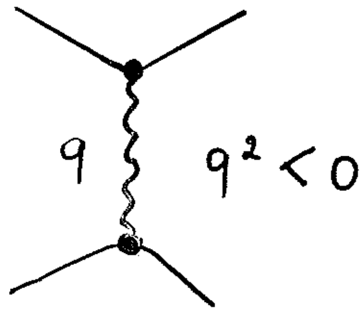
DESY

1. Einleitung: Von E. Rutherford zu HERA
2. Lichtkegel-Entwicklung und Parton Modell
3. Scaling und seine Verletzung
4. Stand der QCD Störungstheorie
5. Polarisierte tief–inelastische Streuung
6. Der Bereich kleiner Bjorken-x
7. Zukünftige Entwicklungen

1. Introduction

TWO BASIC SCATTERING PROCESSES IN HIGH ENERGY PHYSICS:





TIMELIKE

SPACELIKE

e.g. e^+e^--ANNIHILATION

e.g. D.I.S.

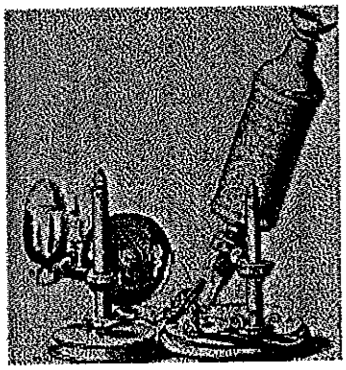
$q^2 < 0 \longrightarrow$ MICROSCOPE !

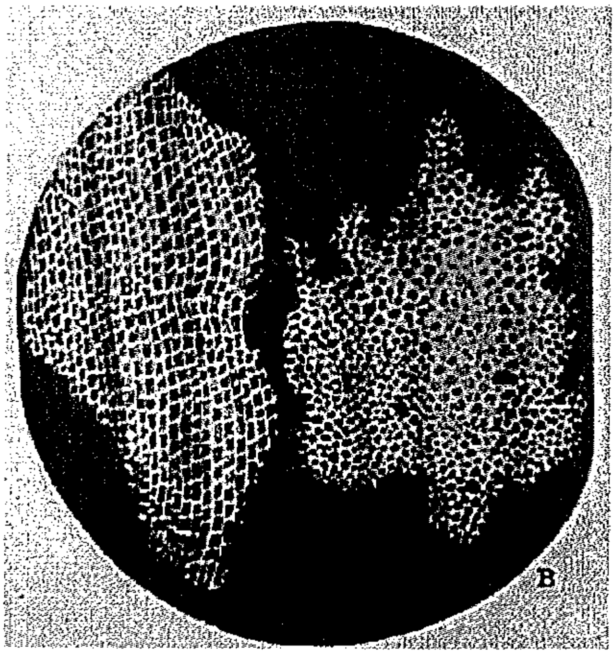
SEARCH FOR THE SPACELIKE STRUCTURE OF NUCLEONS.

1. Introduction

THE DOOR TO THE VERY SMALL
IS OPENED BY MICROSCOPES.

Robert Hooke (1635-1703)

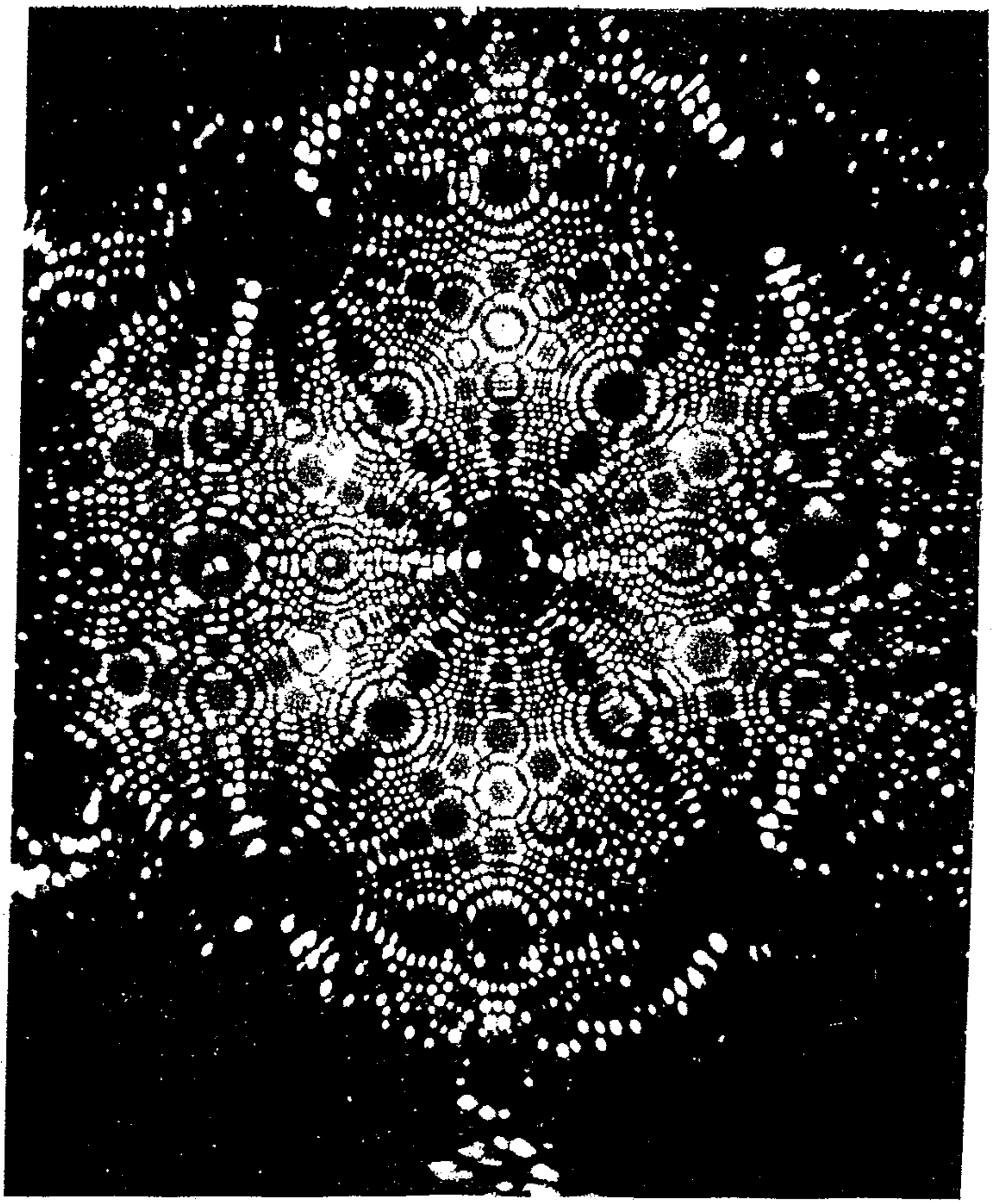




Perhaps his most famous microscopical observation was his study of thin slices of cork. He wrote:

> **. . . I could exceedingly plainly perceive it to be all perforated and porous. . . these pores, or cells, . . . were indeed the first *microscopical* pores I ever saw, and perhaps, that were ever seen, for I had not met with any Writer or Person, that had made any mention of them before this.**

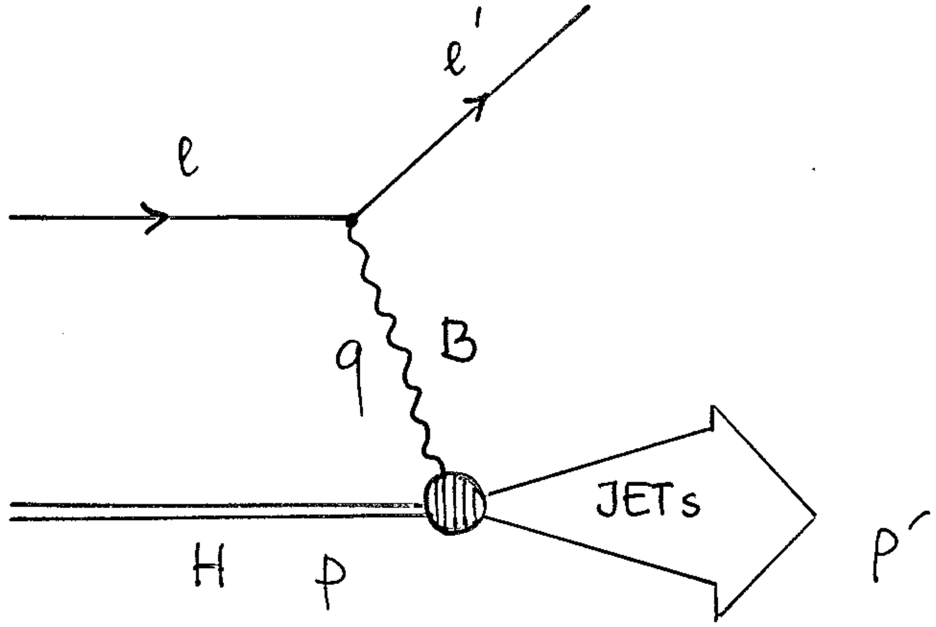
Hooke had discovered plant cells -- more precisely, In August 1955, **Erwin Mueller** became the first person to see an atom and thus visually validate a theory of matter propounded since ancient times. His field-ion microscope - later refined as the atom probe field-ion microscope - created a new research field and contributed to the understanding of the structure of metallic substances.



Tafel 36

Wolframkristall-Halbkugel von 320 Å Radius (Aufnahme mit dem Feldemissions-Elektronenmikroskop von Prof. Dr. E. W. MÜLLER, The Pennsylvania State University, USA) (4.4.5.)

THE PROCESS:



ℓ: $e^{\pm}, \mu^{\pm}$ ℓ': ~

$\overset{(-)}{\nu}_e, \overset{(-)}{\nu}_\mu, \overset{(-)}{\nu}_\tau$

B: $\gamma, W^{\pm}, Z^0$ (γ, Z^0: NC; $W^{\pm}$: CC)

H: p, n, d, A $(\gamma\,(Q^2 \approx 0), \pi^{\pm,0}, K^{\pm})$

KINEMATIC VARIABLES:

$$Q^2 = -q^2 = (\ell - \ell')^2 \gtrsim 0$$

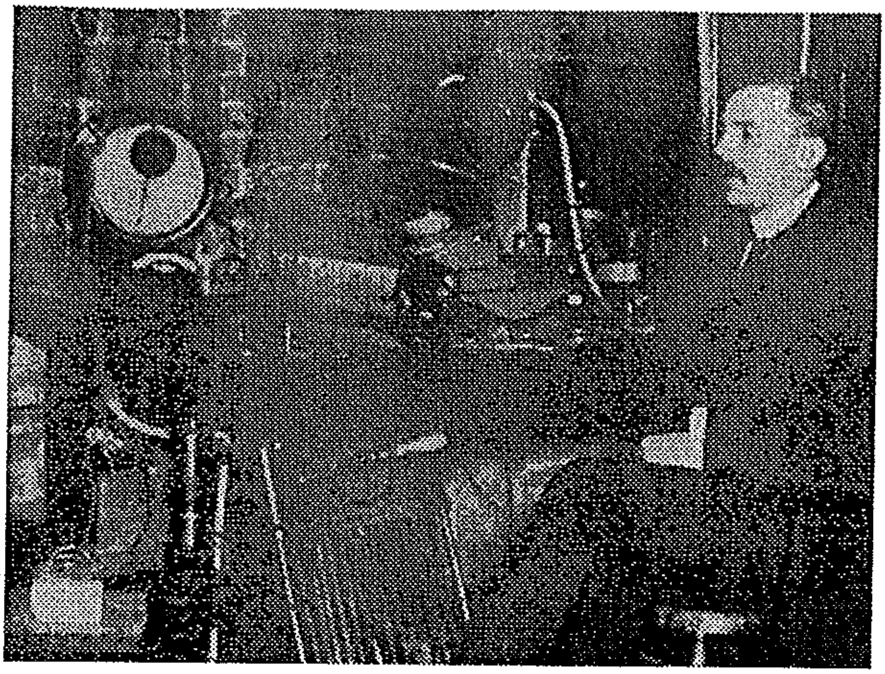
$$W^2 = (p+q)^2 = M_p^2 + 2pq - q^2 \geq M_p^2$$

$$x = \frac{Q^2}{2pq}, \quad y = \frac{2pq}{2p\ell} = \frac{2pq}{S}$$

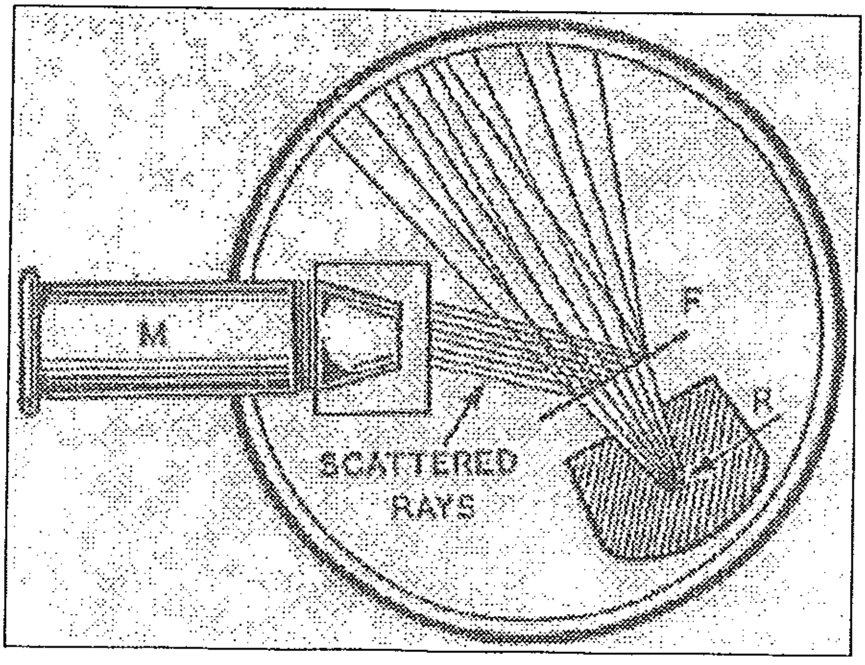
$$= \frac{Q^2}{Sy} \qquad 0 \leq x, y \leq 1.$$

Study of the Nucleon Structure

Year	Discovery	
1911	Atomic Nuclei	E. Rutherford
1933	anomalous magnetic moment of p	R. Frisch, O. Stern
1933 1940	anomalous magnetic moment of n	R. Bacher L. Alvarz, F. Bloch
late 1950ies	charge distribution inside p and n	R. Hofstadter et al.
1969	scaling of structure functions	E.D. Bloom et al. M. Breidenbach et al.
1970ies	scaling violations	νN and μN experiments
$\sim$ 1975	1st extraction of quark and gluon distributions from DIS data	νN experiments



Ernest Rutherford in his Laboratory at McGill University ca. 1903.



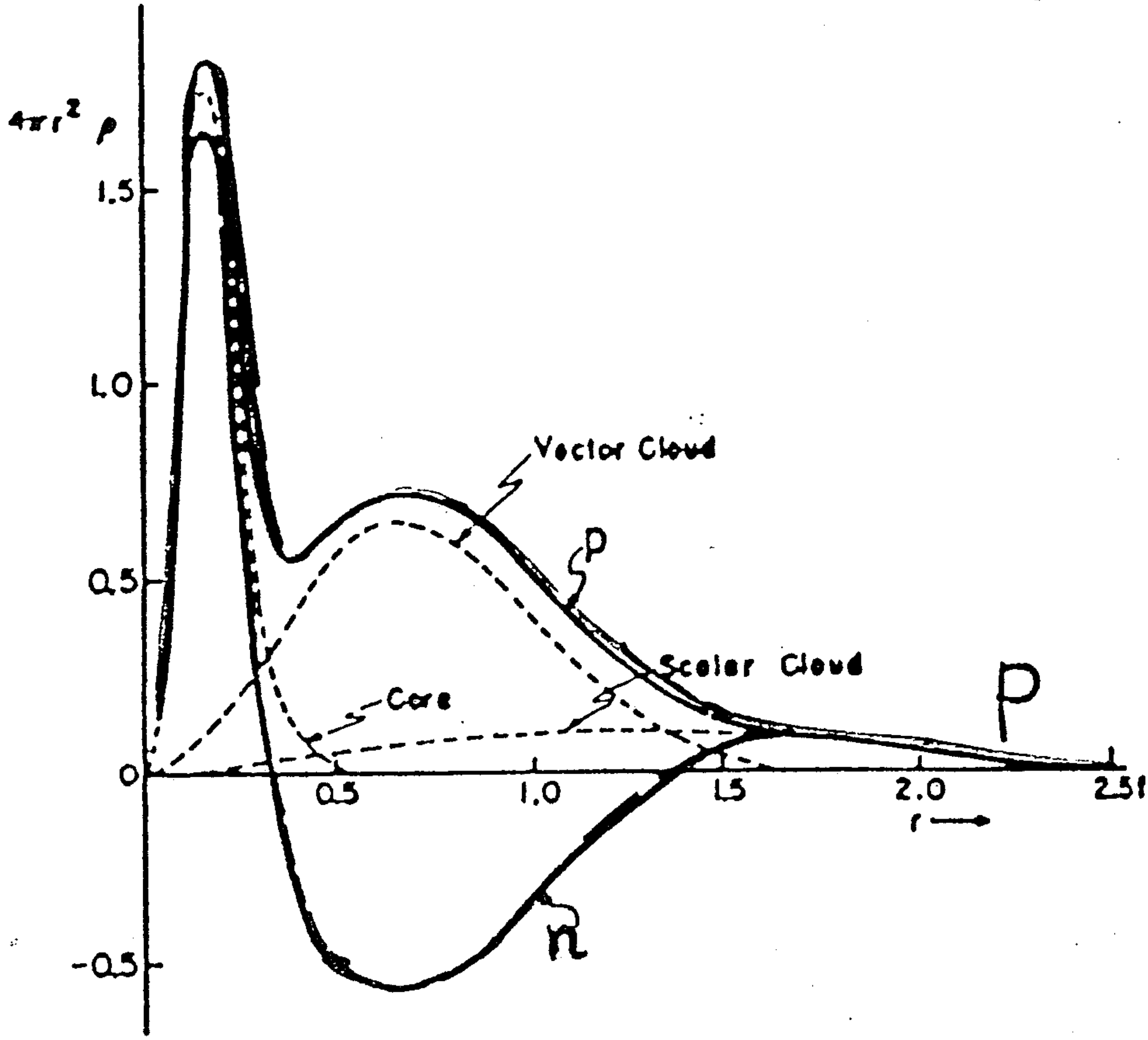


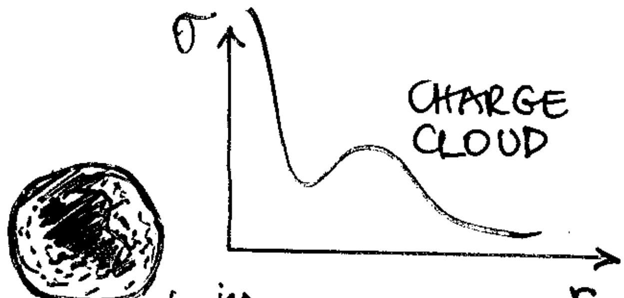
FIG. 4. Charge distribution for the proton and the neutron implied by the form factors shown for the fit (b) in Fig. 2(b).

THE RESOLUTION OF THE 'MICROSCOPE':

$$\Delta x \sim \frac{1}{|Q|} = \frac{1}{\sqrt{-q^2}}$$

EXAMPLES:

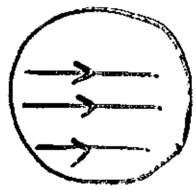
$Q^2 \sim 0.5\, M_p^2$



'50ies HOFSTADTER et al.

'69 MIT - SLAC.

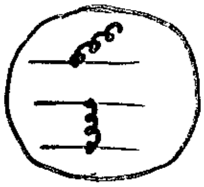
$Q^2 \sim 3\, M_p^2$



QUARKS

GLUON RADIATION

$Q^2 \sim 10\, M_p^2 \ldots 500\, M_p^2$



∼ '74

HERA.

⋮

IF THERE ARE NEW COMPOSITENESS SCALES ONE MAY FIND THEM IN THE FUTURE

$Q^2 > 10^4\ \mathrm{GeV}^2$, $1\,\mathrm{GeV}^2 \sim 1\, M_p^2$

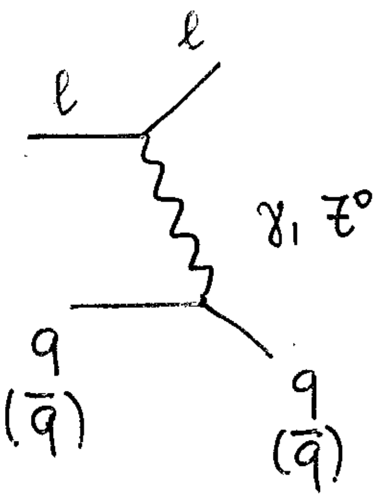
BREAKING UP THE PROTON :

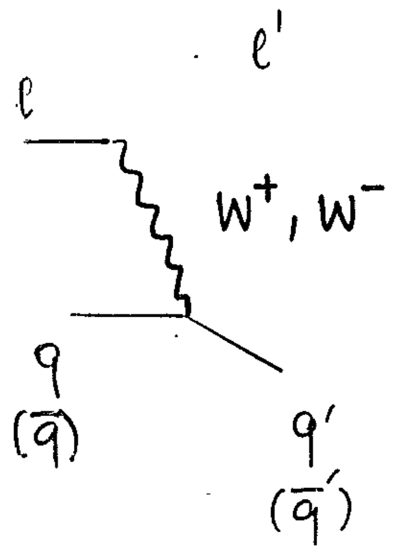
RESOLVING QUARKS & GLUONS

HERE : PARTONS

(FEYNMAN)

THE SCATTERING PROCESSES ARE (BORN LEVEL)





NEUTRAL CURRENT

CHARGED CURRENT

$\therefore\ e^- + u \rightarrow \nu_e\, d$

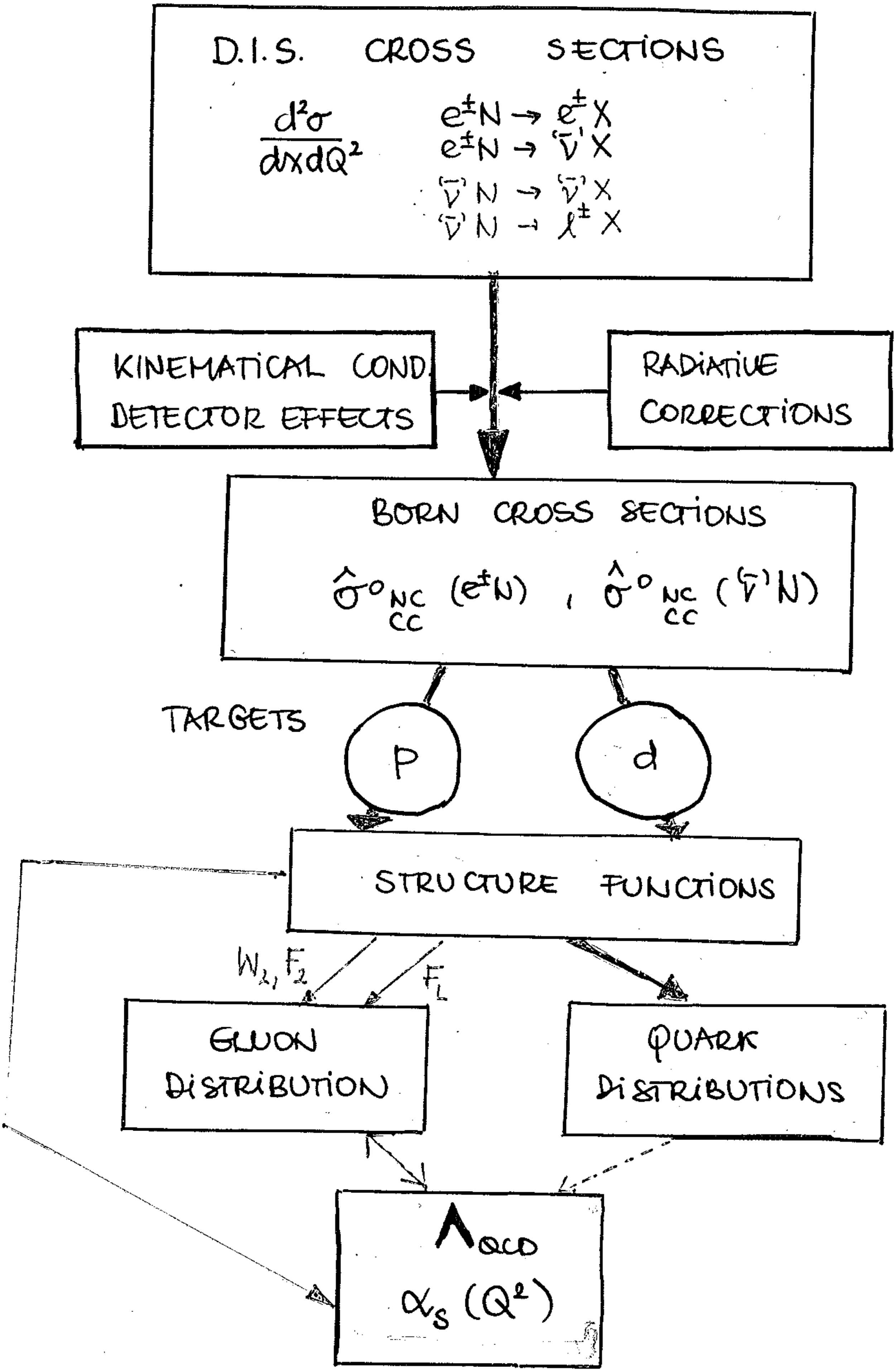
$e^- \rightarrow W^- \nu_e$

$W^- + u \rightarrow d$

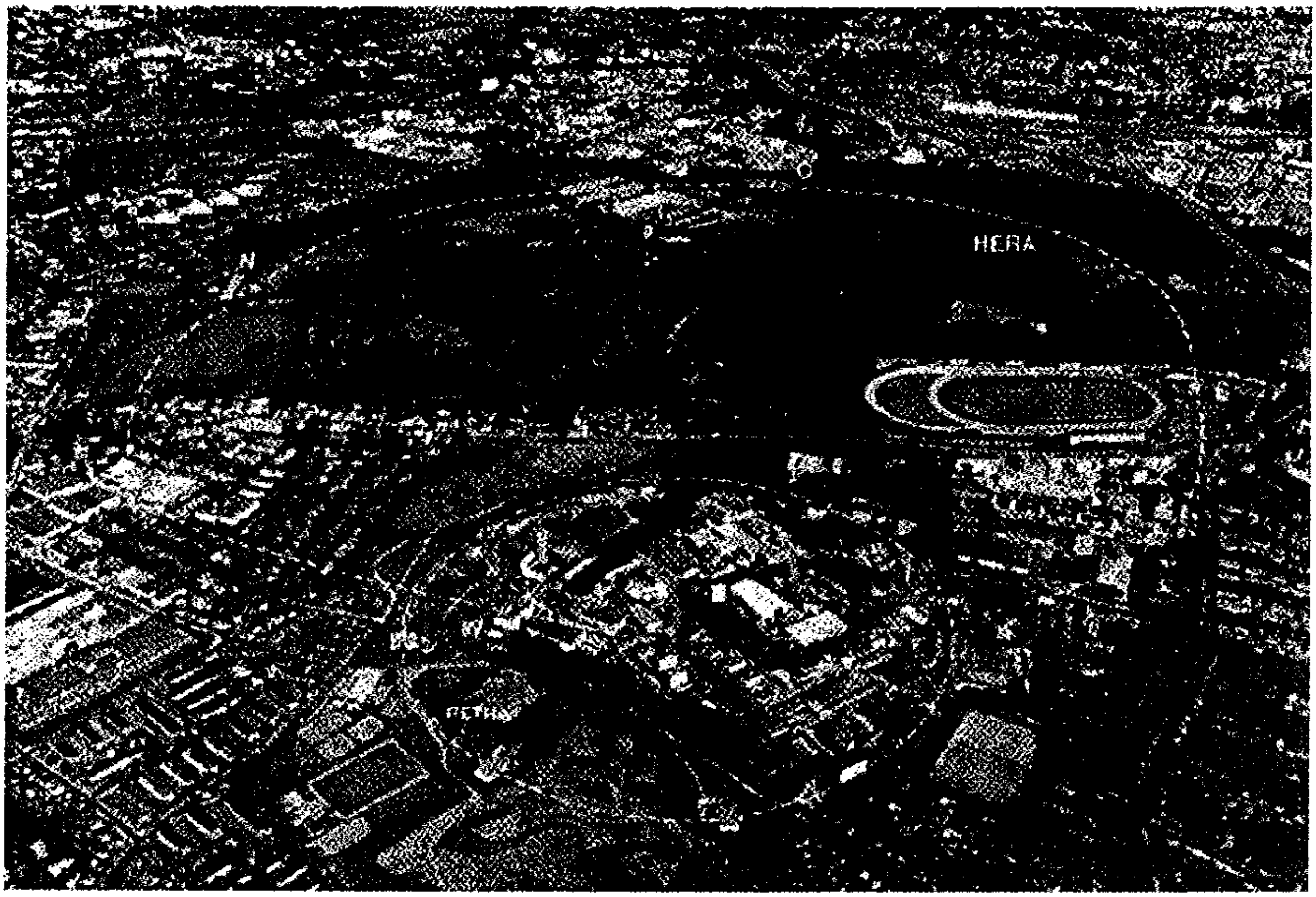
$-1 + \frac{2}{3} \rightarrow -\frac{1}{3}$

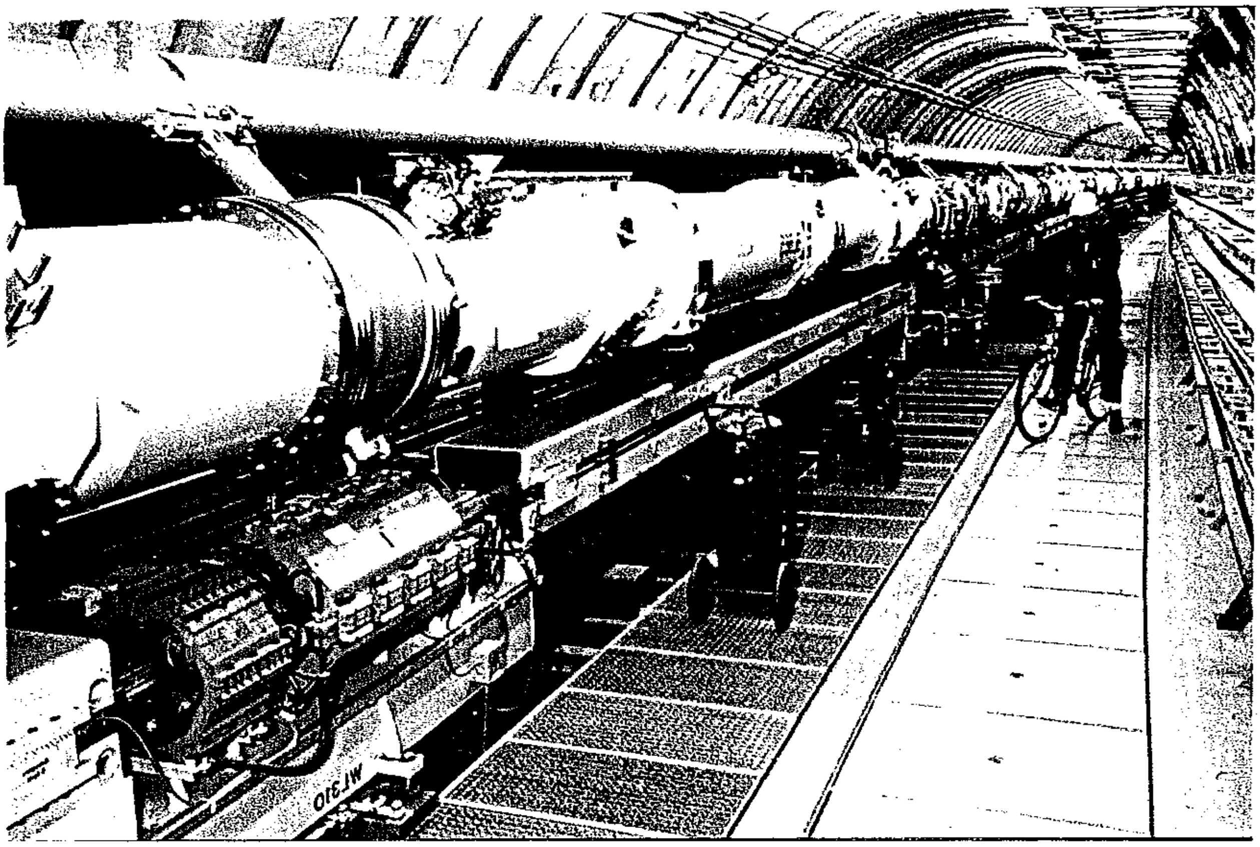
etc.

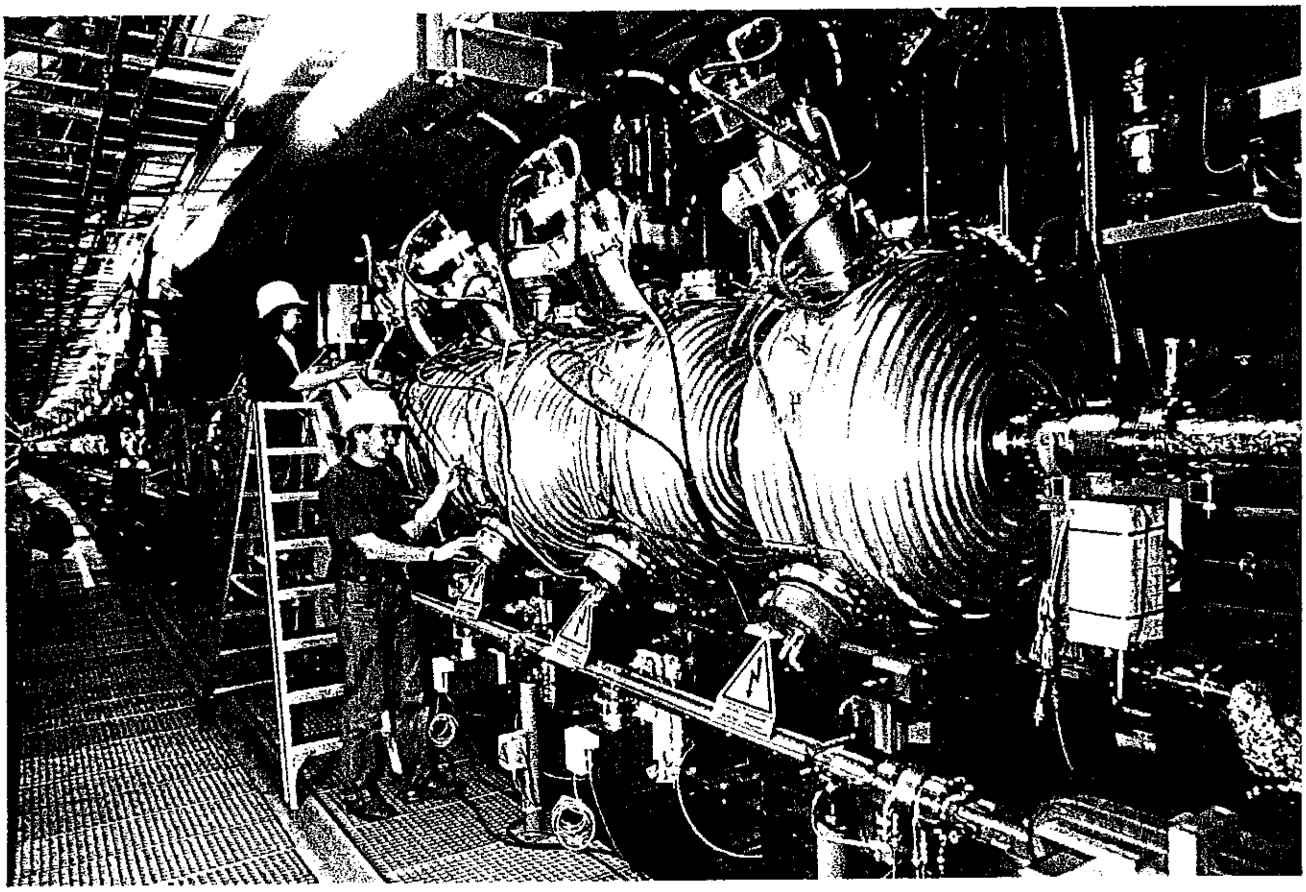
FLAVOUR SENSITIVITY , e_q^2 ; g_{Aq}, g_{Vq}.

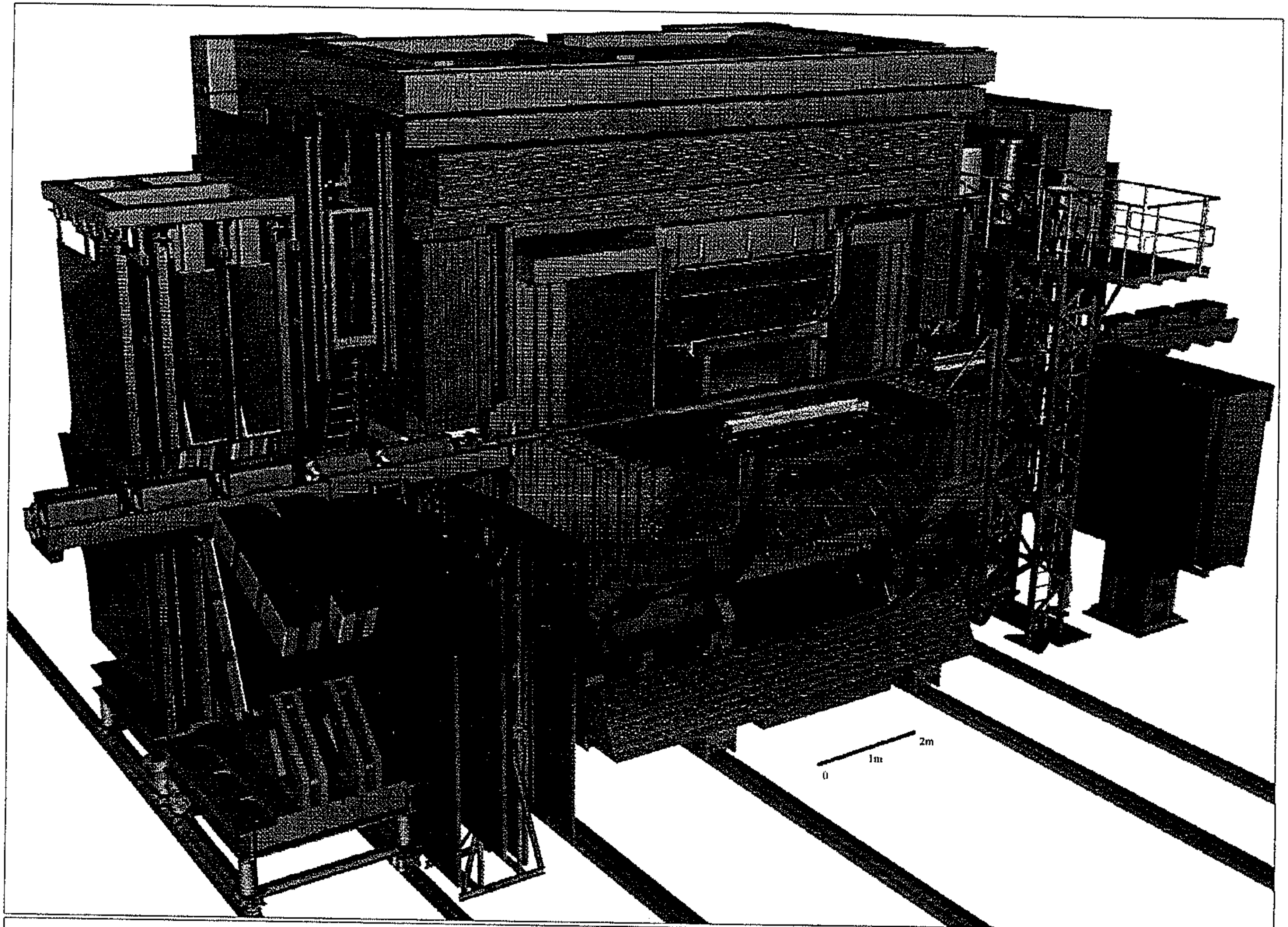


D.I.S. CROSS SECTIONS
$\frac{d^2\sigma}{dx\,dQ^2}$
$e^{\pm}N \to e^{\pm}X$
$e^{\pm}N \to \overset{(-)}{\nu}X$
$\overset{(-)}{\nu}N \to \overset{(-)}{\nu}X$
$\overset{(-)}{\nu}N \to \ell^{\pm}X$
KINEMATICAL COND.
DETECTOR EFFECTS
RADIATIVE
CORRECTIONS
BORN CROSS SECTIONS
$\hat{\sigma}^0_{NC,CC}(e^{\pm}N)$, $\hat{\sigma}^0_{NC,CC}(\overset{(-)}{\nu}N)$
TARGETS
p
d
STRUCTURE FUNCTIONS
W_2, F_2
F_L
GLUON
DISTRIBUTION
QUARK
DISTRIBUTIONS
Λ_{QCD}
$\alpha_s(Q^2)$


HERA
N

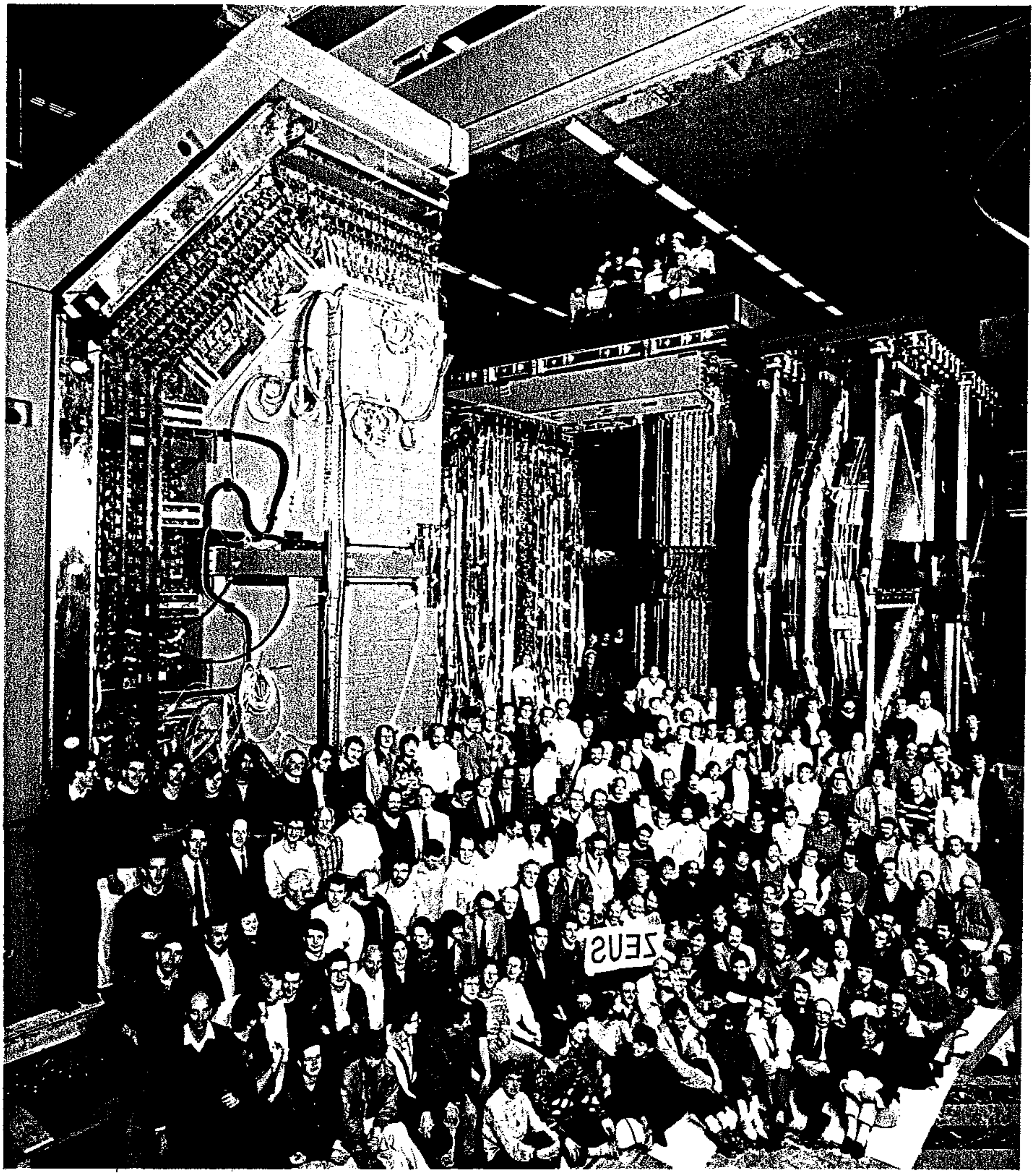






ZEUS (HERA)

Software :SDRC-IDEAS level VI.i
Performed by : Carsten Hartmann
Status : October 1993

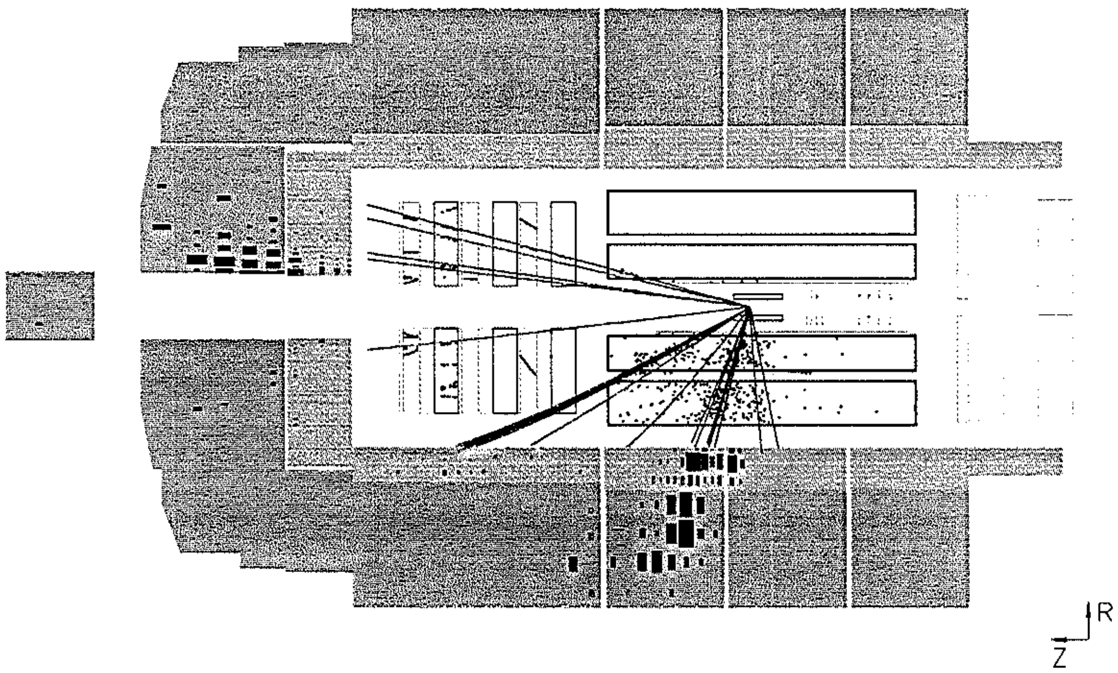

ZEUS

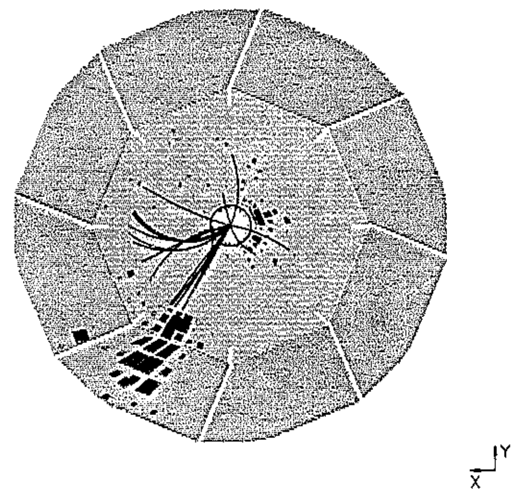
Run 241350 Event 112678 Class: 4 5 6 7 8 10 16 19 24 25 26 28 29 Run Date 23/4/99

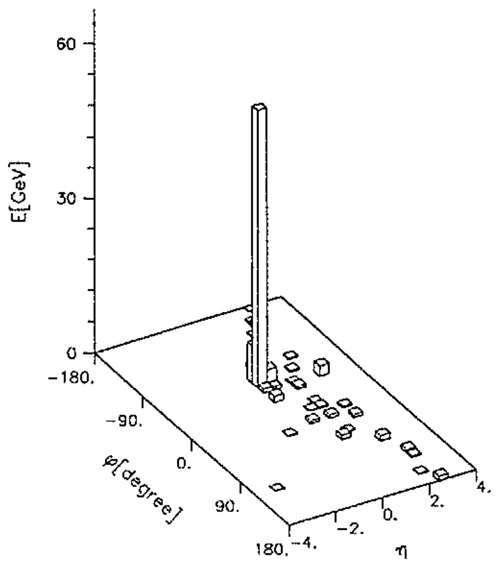
in due time to DIS '99 (Zeuthen)

CC : P_t = 65 GeV/c y = 0.96

$Q^2 \sim 70.000 - 100.000\ GeV^2$
highest ever seen!

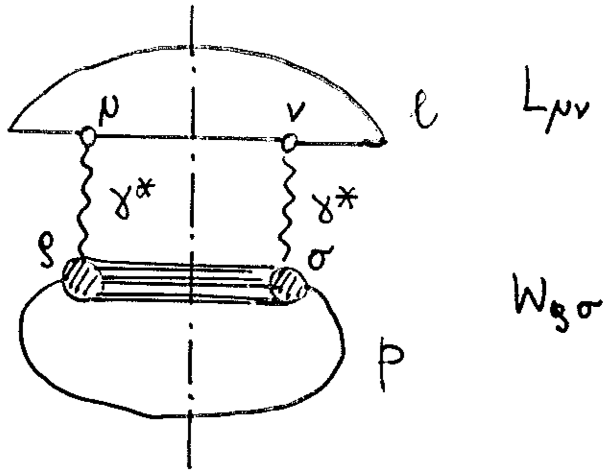






2. Light Cone Expansion and Parton Model

$$\frac{d^2\sigma^{DIS}}{dx\,dy} \underset{\gamma^*}{\propto} \sum_{S'} \overline{|M|^2} = \frac{1}{Q^4} \overline{L_{\mu\nu}\, g^{\mu\varrho}\, W_{\varrho\sigma}\, g^{\nu\sigma}}$$



$L_{\mu\nu}$ CALCULABLE: $L_{\mu\nu} = 2(\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu - \ell\ell' g_{\mu\nu}) \equiv L_{\nu\mu}$

$W_{\mu\nu}$ · UNCALCULABLE!

$$L_{\mu\nu} = L_{\nu\mu} \quad \xrightarrow[\gamma]{} \quad W_{\mu\nu} = W_{\nu\mu}$$

CURRENT CONSERVATION: $q_\mu W^{\mu\nu} = W^{\mu\nu} q_\nu \equiv 0.$

↓ DRELL, WALECKA 1964

$$W_{\mu\nu}^{\gamma^*} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2; \nu) + \frac{1}{M^2}\left(p_\mu - \frac{p.q}{q^2} q_\mu\right)\left(p_\nu - \frac{p.q}{q^2} q_\nu\right) \cdot W_2(q^2, \nu)$$

$2M\nu = S y_B$

NON-PERTURBATIVE STRUCTURE FUNCTIONS.

BEHAVIOUR FOR: $Q^2 \to \infty$, $\nu \to \infty$

$$x = \frac{Q^2}{2M\nu} = \frac{Q^2}{Sy} \equiv \text{fix}$$

BJORKEN LIMIT.

[illegible]: BJORKEN 68/9

$$\left.\begin{aligned} MW_1(\nu, Q^2) &\longrightarrow F_1\left(x = \frac{Q^2}{2M\nu}\right) \\ \nu W_2(\nu, Q^2) &\longrightarrow F_2\left(x = \frac{Q^2}{2M\nu}\right) \end{aligned}\right\} \text{SCALING.}$$

LIGHT CONE EXPANSION

$$W_{\mu\nu} = \int d^4x \, e^{iqx} \langle p | [j_\mu(x), j_\nu(0)] | p \rangle$$

$$T[j_\mu(x), j_\nu(0)] = \frac{x^2 g_{\mu\nu} - 2x_\mu x_\nu}{\pi^4 (x^2 - i\varepsilon)^4} - \frac{i x^\lambda \sigma_{\mu\lambda\nu\varrho} O_V^\varrho(x,0)}{2\pi^2 (x^2 - i\varepsilon)^2}$$

$$- \frac{x^\lambda \varepsilon_{\mu\lambda\nu\varrho} O_A^\varrho(x,0)}{2\pi^2 (x^2 - i\varepsilon)^2} O_A^\varrho(x,0) + O_{\mu\nu}(x,0)$$

disc. graph

less singular

$$O_V^\mu(x,y) = :\bar{\psi}(x)\gamma_\mu\psi(y) - \bar{\psi}(y)\gamma^\mu\psi(y):$$

$$O_A^\mu(x,y) = :\bar{\psi}(x)\gamma_\mu\gamma_5\psi(y) + \bar{\psi}(y)\gamma_\mu\gamma_5\psi(x):$$

$$O_{\mu\nu}(x,y) = :\bar{\psi}(x)\gamma_\mu\psi(x)\bar{\psi}(y)\gamma_\nu\psi(y):$$

$$\psi(x) = \psi(0) + x^{\mu}\left[\partial_\mu \psi(x)\right]_{x=0} + \frac{1}{2!}x^{\mu_1}x^{\mu_2}\left[\partial_{\mu_1}\partial_{\mu_2}\psi(x)\right]_{x=0} + \dots$$

$$O^{\mu}_{V,A}(x,0) = \sum_{n=0}^{\infty} \frac{1}{n!} x^{\mu_1} \dots x^{\mu_n} O^{\mu}_{V,A\,\mu_1\dots\mu_n}(0)$$

EXPECTATION VALUES:

$$\langle p | O^{\mu}_{V}(0)_{\mu_1\dots\mu_n} | p \rangle = a_n p^{\mu} p_{\mu_1}\dots p_{\mu_n} + \text{trace terms}$$

$$\langle p | O^{\mu}_{A}(0)_{\mu_1\dots\mu_n} | p \rangle = a^5_n p^{\mu} p_{\mu_1}\dots p_{\mu_n} + \text{trace terms.}$$

a_n and a^5_n ARE MELLIN MOMENTS OF THE FUNCTIONS $f(z)$ AND $f_5(z)$, RESP.

$$f_{(5)}(z) = \sum_{n=0}^{\infty} a_n^{(5)} \frac{z^n}{n!} = \int_{-\infty}^{+\infty} d\xi \, e^{iz\xi} \tilde{f}(\xi)$$

UNPOL. CASE: O_A ABSENT.

$$W_{\mu\nu} = \frac{1}{2\pi^2 i} \sigma_{\mu\lambda\nu\rho} p^{\rho} \frac{\partial}{\partial q_\lambda} \int_{-\infty}^{+\infty} d\xi \, \tilde{f}(\xi) \int d^4x \, e^{i(q+\xi p)} \, \varepsilon(x_0) \, \delta'(x^2)$$

BJORKEN LIMIT! $Q^2 \to \infty$, $\nu \to \infty$, $\frac{Q^2}{2M\nu} = \text{fixed}$.

$$W_{\mu\nu} = \frac{1}{2}\tilde{f}(\xi)\left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right] + \frac{1}{pq} \xi \tilde{f}(\xi) \left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right)$$

$$W_1(\nu, Q^2) = \frac{1}{2}\tilde{f}(\xi) \quad \underset{\text{Def}}{=}: F_1(\xi) \qquad \xi = \frac{Q^2}{2p\cdot q} \equiv x_B$$

$$\nu W_2(\nu, Q^2)/2M = \frac{1}{2}\tilde{f}(\xi)\cdot\xi =: \frac{1}{2}F_2(\xi)$$

PARTON MODEL

FEYNMAN 1969

ANSATZ: $W_i(\nu, Q^2)$ IS OBTAINED AS AN INTEGRAL OVER THE MOMENTUM DISTRIBUTIONS OF LOCAL SUBCOMPONENTS: PARTONS.

$$W_2(\nu, Q^2) = \sum_i \int_0^1 dx_i \, f(x_i) \, \underbrace{x_i e_i^2 \, \delta\left(\frac{qp_i}{M} - \frac{Q^2}{2M}\right)}_{e_i^2 \, \delta\left(\nu - \frac{Q^2}{2Mx_i}\right)}$$

↑ NUMBER DENSITY OF PARTONS

$qp_i = x_i \, qp \quad , \quad 2qP = \frac{Q^2}{x} \quad , \quad M\nu = qp$

$\sum_i x_i = 1$

$$\nu W_2(\nu, Q^2) = \sum_i e_i^2 x_i f(x_i) \equiv F_2(x)$$

$$\delta(\ldots) \Rightarrow x_i = \frac{Q^2}{2M\nu} \equiv x_B$$

MOMENTUM FRACTION — BJORKEN x.

IMPLIED! (FORWARD, SC. & TWIST 2!)

3. Scaling and Scaling Violation

- SCALING IS AN ASYMPTOTIC (IDEALIZED) PROPERTY OF STRUCTURE FUNCTIONS IN THE BJORKEN LIMIT

- α_s CORRECTIONS — HIGHER LOOP ORDERS
- $\frac{\Lambda^{2N}}{[Q^2]^N}$ CORRECTIONS — HIGHER TWIST OPERATORS
- $\left(\frac{m_q^2}{Q^2}\right)^k$ CORRECTIONS — QUARK MASS TERMS
- $\left(\frac{M^2}{Q^2}\right)^\ell$ CORRECTIONS — TARGET MASS TERMS.

VIOLATE SCALING.

⟹ LEADING TWIST, m_q^2/Q^2 & $M^2/Q^2 \to 0$:

SCALING VIOLATIONS ARE DUE TO

$$\underline{\alpha_s(Q^2)}.$$

⟹ POSSIBILITY FOR A PRECISE MEASUREMENT OF α_s!

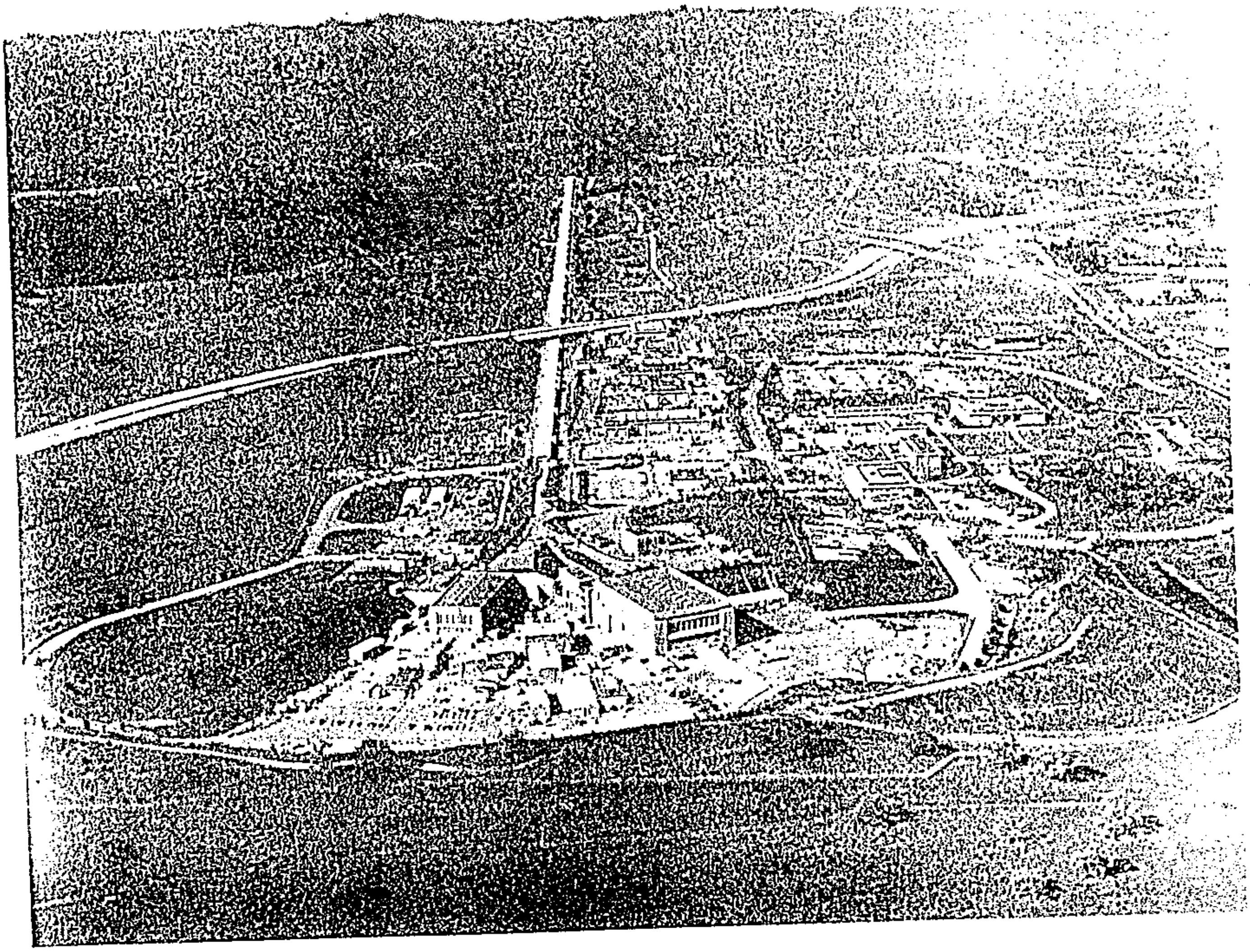


FIG. 1. View of the Stanford Linear Accelerator. The electron injector is at the top, the experimental area in lower center. The deep inelastic scattering studies were carried out in End Station A, the largest of the buildings in the experimental area.

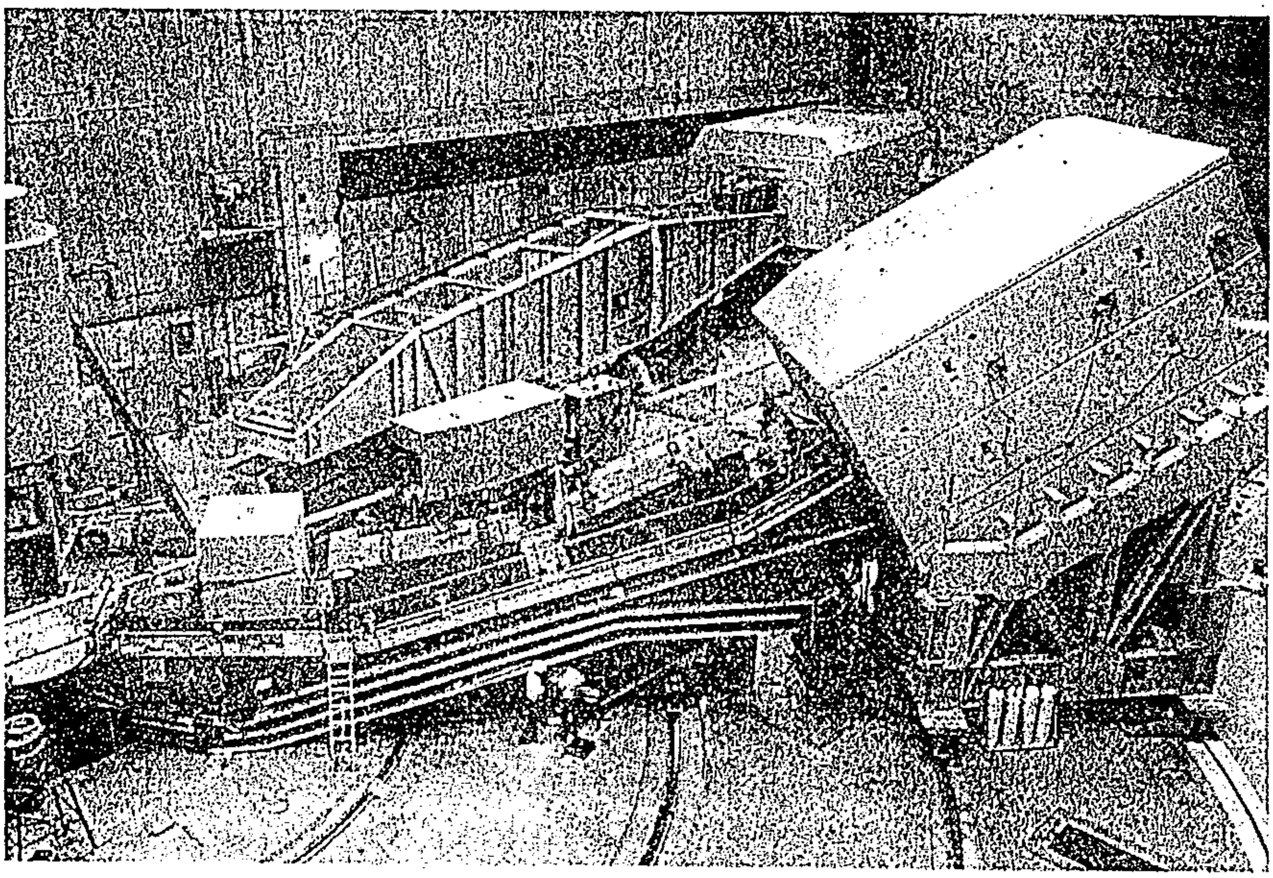
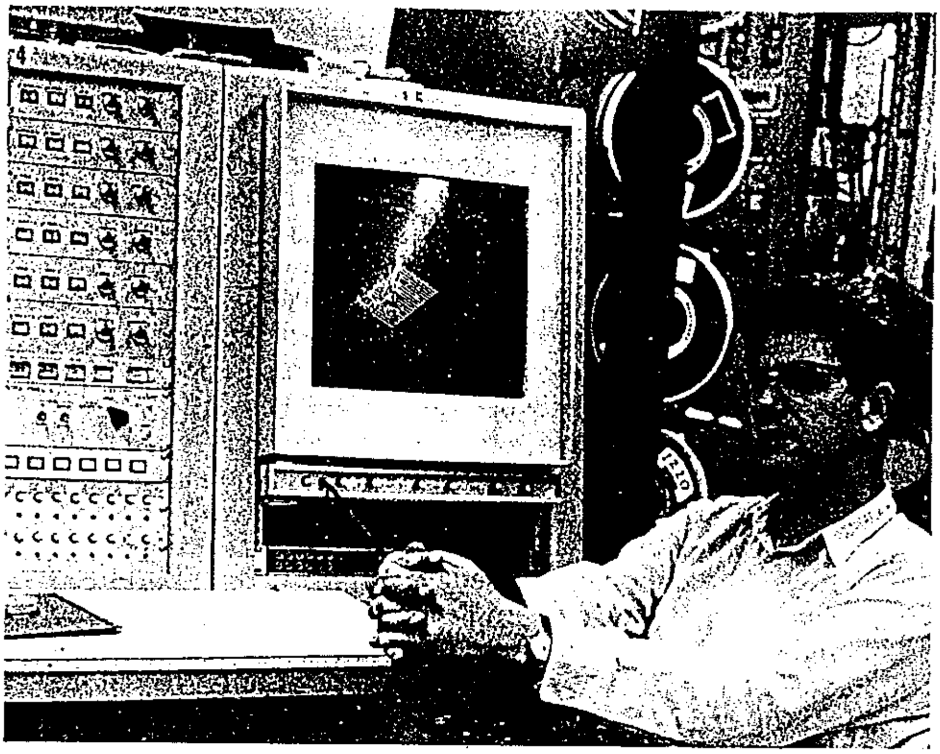
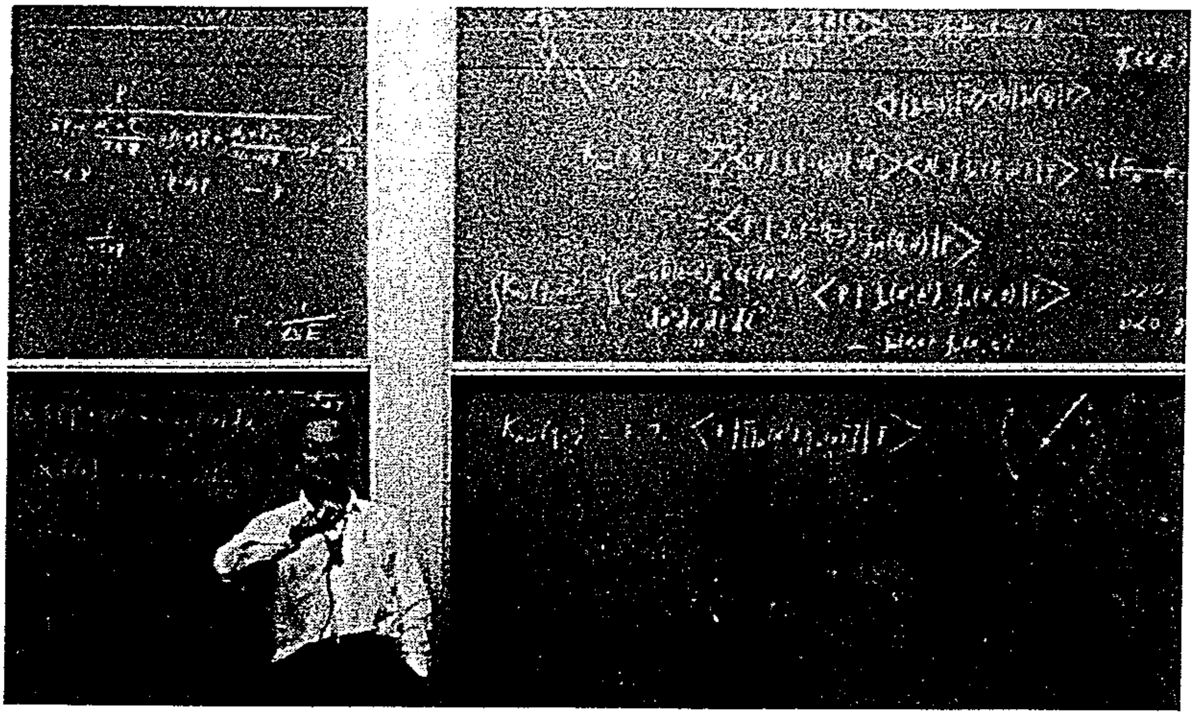


FIG. 20. Photograph of the 8 and 20 GeV spectrometers in End Station A.

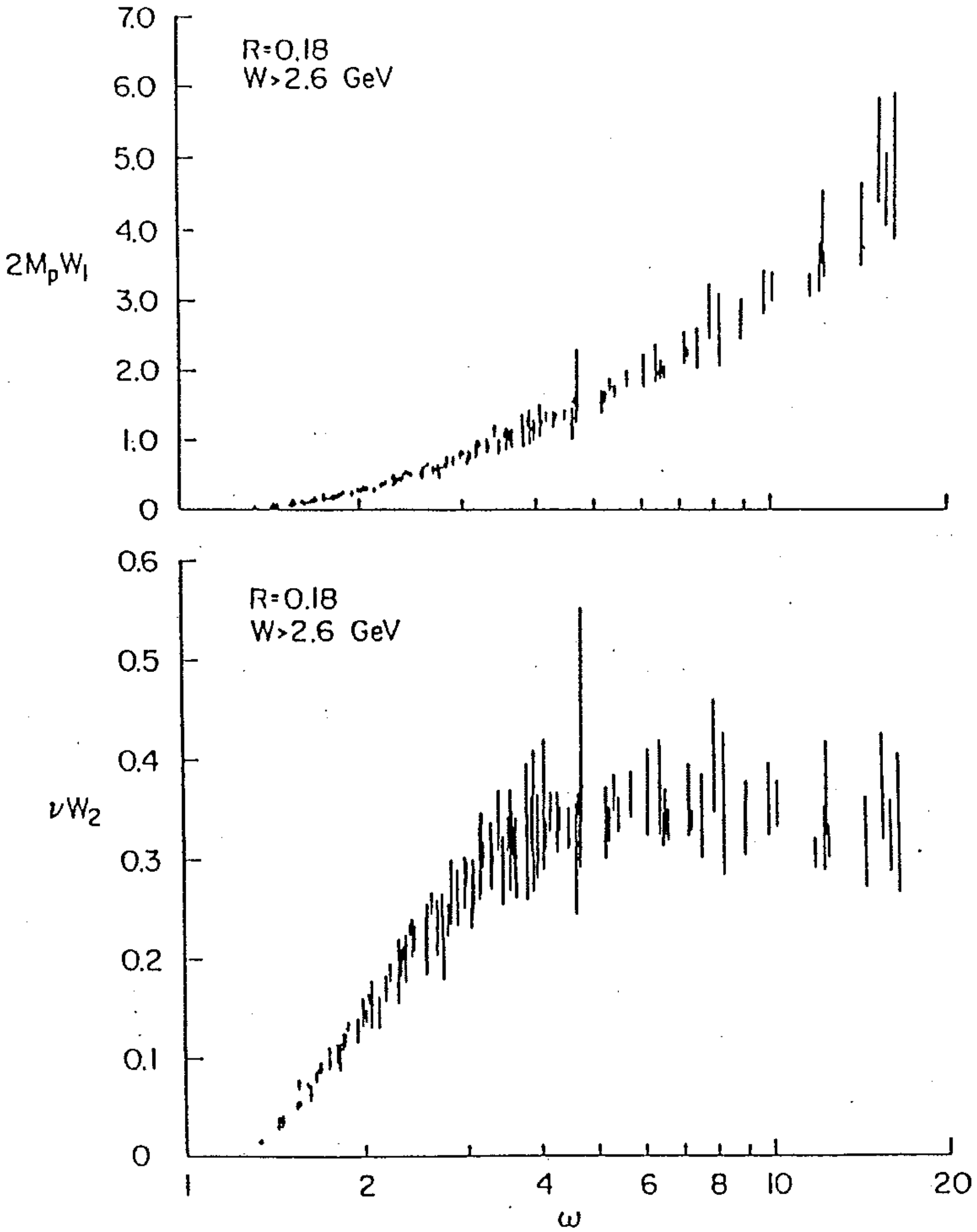


Richard Taylor relaxing during the first End Station A experiment as elastic electron-proton scattering events accumulate in the computer display behind him.



Richard Feynman lecturing at SLAC in October 1968. This was his first formal presentation of the parton model, the talk most SLAC physicists recall igniting their interest in partons.

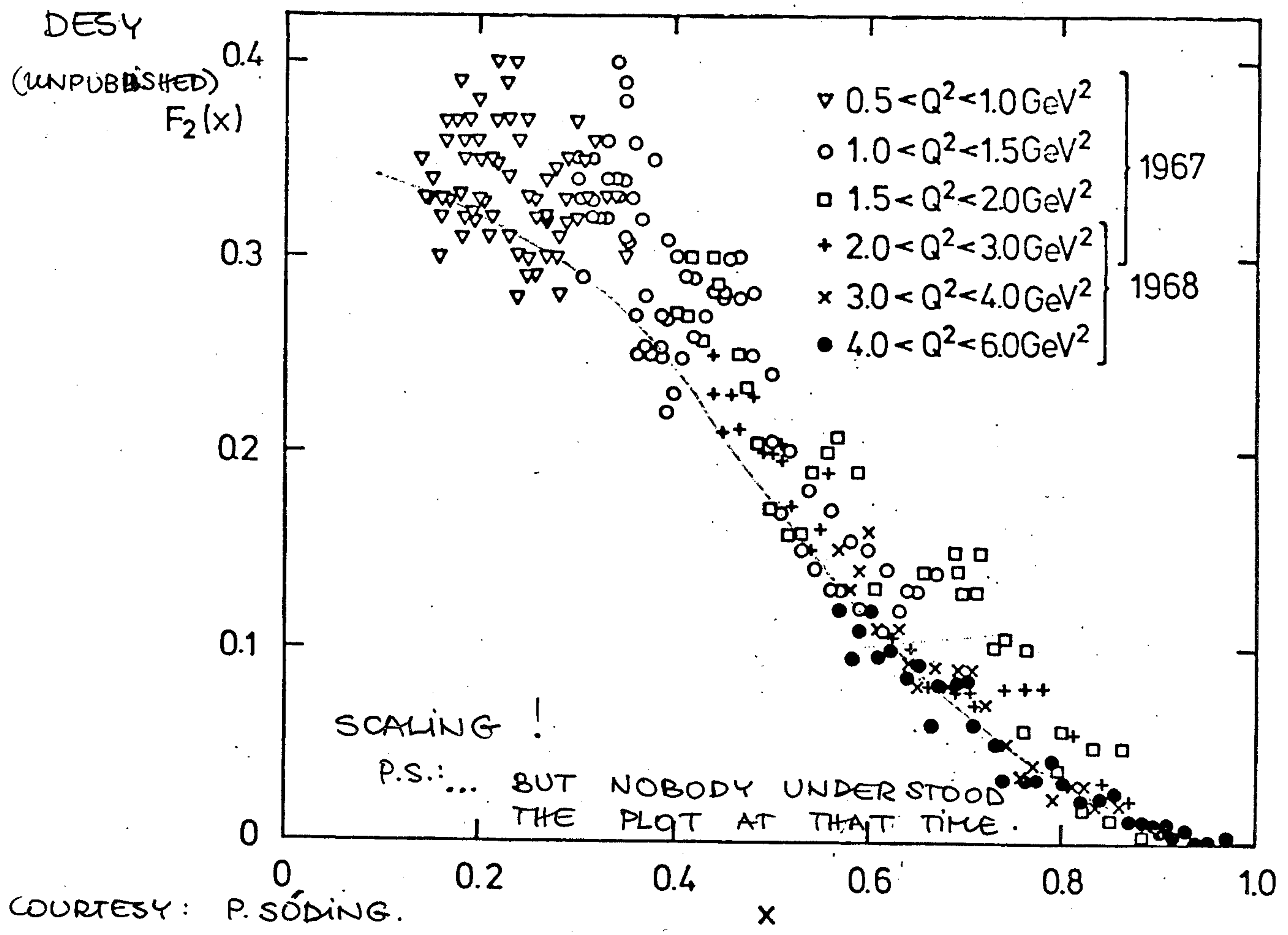
Bjorken had had most of the essential ideas well before his SLAC visit that month; Feynman's only new contribution was

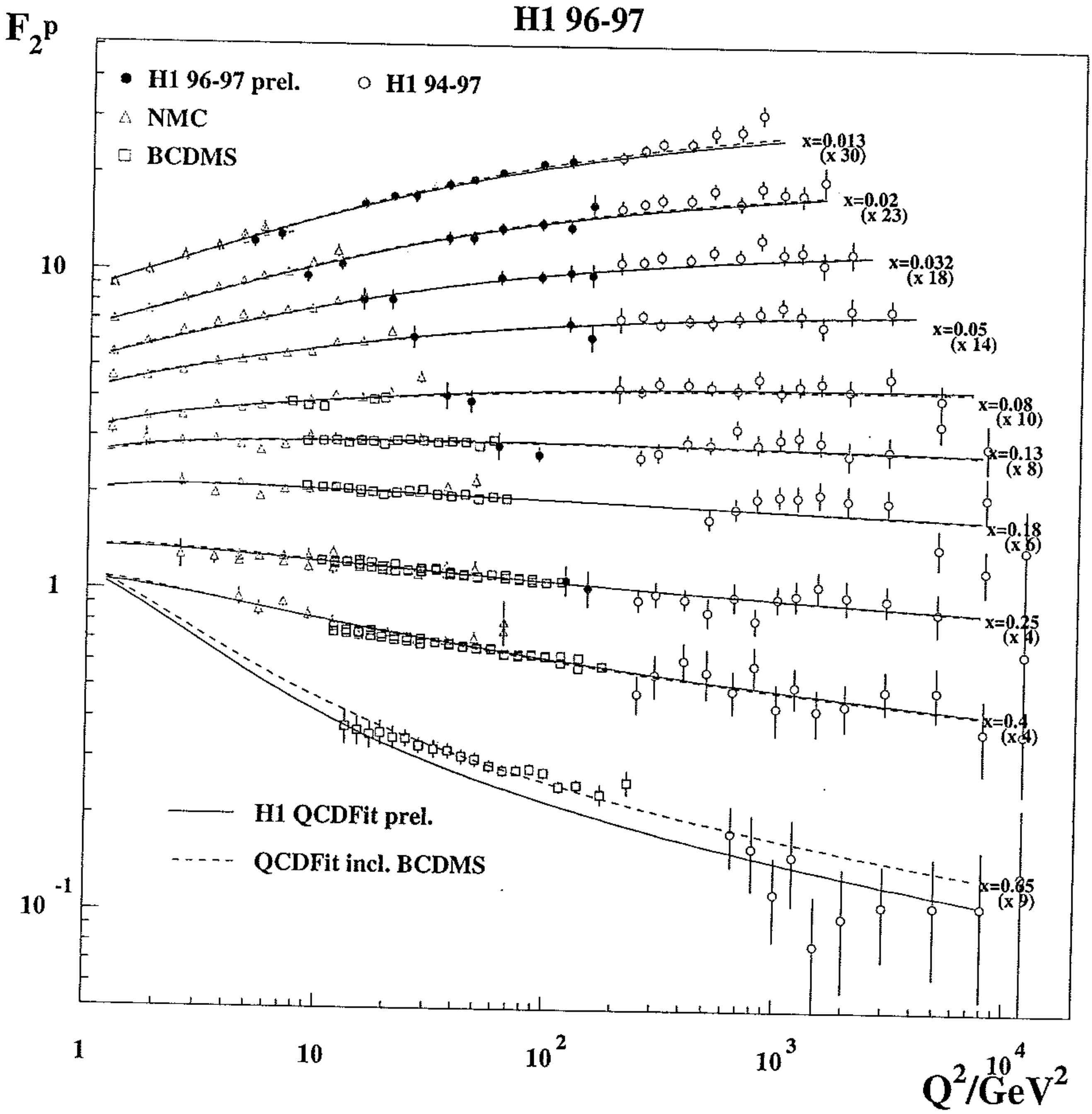


$$\omega = \frac{2pq}{Q^2} \equiv \frac{1}{x_B}$$

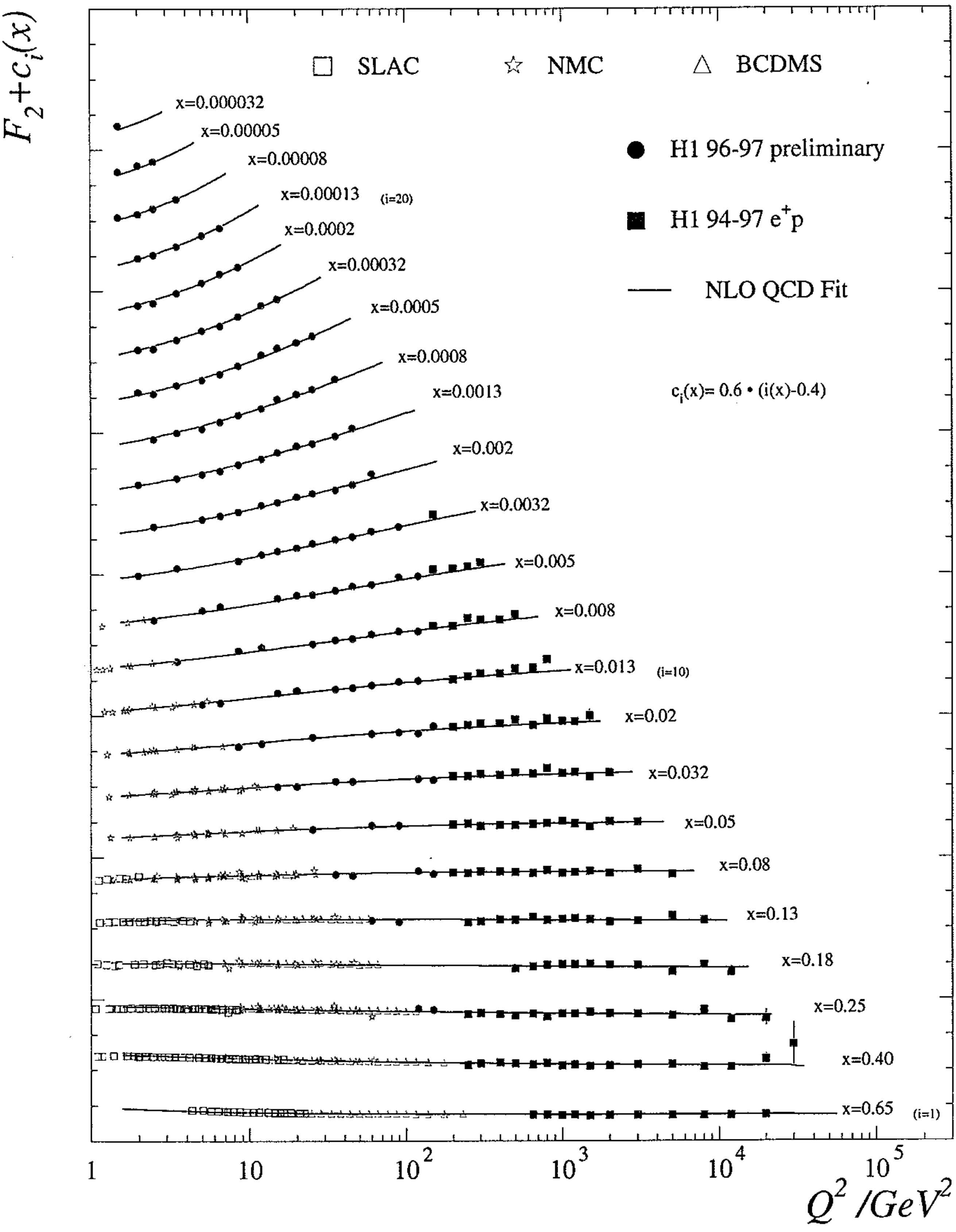
FIG. 2. $2MW_1$ and νW_2 for the proton as functions of ω for $W > 2.6$ GeV, $q^2 > 1$ (GeV/c^2), and $R = 0.18$. Data from Miller *et al.* (1972). The quantity R is discussed in the section of this paper entitled *Non-Constituent Models*

SLAC - MIT


DESY
(UNPUBLISHED)
$F_2(x)$
0.4
0.3
0.2
0.1
0
0.2
0.4
0.6
0.8
1.0
x
▿ $0.5 < Q^2 < 1.0\,GeV^2$
○ $1.0 < Q^2 < 1.5\,GeV^2$
□ $1.5 < Q^2 < 2.0\,GeV^2$
+ $2.0 < Q^2 < 3.0\,GeV^2$
× $3.0 < Q^2 < 4.0\,GeV^2$
● $4.0 < Q^2 < 6.0\,GeV^2$
1967
1968
SCALING !
P.S.: ... BUT NOBODY UNDERSTOOD THE PLOT AT THAT TIME.
COURTESY: P. SÖDING.



H1 96-97
F_2^p
H1 96-97 prel.
H1 94-97
NMC
BCDMS
x=0.013 (x 30)
x=0.02 (x 23)
x=0.032 (x 18)
x=0.05 (x 14)
x=0.08 (x 10)
x=0.13 (x 8)
x=0.18 (x 6)
x=0.25 (x 4)
x=0.4 (x 4)
x=0.65 (x 9)
H1 QCDFit prel.
QCDFit incl. BCDMS
10
1
10^{-1}
1
10
10^2
10^3
10^4
Q^2/GeV^2



SLAC
NMC
BCDMS
H1 96-97 preliminary
H1 94-97 e+p
NLO QCD Fit
c_i(x)= 0.6 • (i(x)-0.4)
x=0.000032
x=0.00005
x=0.00008
x=0.00013 (i=20)
x=0.0002
x=0.00032
x=0.0005
x=0.0008
x=0.0013
x=0.002
x=0.0032
x=0.005
x=0.008
x=0.013 (i=10)
x=0.02
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.40
x=0.65 (i=1)
F_2+c_i(x)
Q^2 /GeV^2
1
10
10^2
10^3
10^4
10^5

4.1. The Running Coupling Constant

RGE FOR THE STRONG COUPLING CONSTANT:

$a_s = g_s^2 / 16\pi^2$

$$\frac{\partial a_s}{\partial \log \mu^2} = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 + O(a_s^6)$$

COLOR FACTORS: (QCD: $N_c = 3$, $N_A = 8$)

$\beta_0, \beta_1, \beta_2$ } $C_A = N_c$

$C_F = \frac{N_c^2 - 1}{2N_c}$, $T_F = \frac{1}{2}$

β_3 —"—

firstly at 4-loops.

$$\frac{d_A^{abcd} d_A^{abcd}}{N_A} = \frac{N_c^2 (N_c^2 + 36)}{24}$$

$$\frac{d_A^{abcd} d_F^{abcd}}{N_A} = \frac{N_c (N_c^2 + 6)}{48}$$

$$\frac{d_F^{abcd} d_F^{abcd}}{N_A} = \frac{N_c^4 - 6N_c^2 + 18}{96 N_c^2}$$

$$\beta_0 = 11 - \frac{2}{3} N_f$$

> 0

GROSS, WILCZEK 1973

POLITZER 1973

T' HOOFT

DISCOVERY OF ASYMPTOTIC FREEDOM

$$\beta_1 = 102 - \frac{38}{3} N_f$$

CASWELL 1974

JONES 1974

FIG

$$\beta_2 = \frac{2857}{2} - \frac{5033}{18} N_f + \frac{325}{54} N_f^2$$

TARASOV, VLADIMIROV, ZHARKOV 1980

LARIN, VERMASEREN 1993

$$\beta_3 = \left(\frac{149753}{6} + 3564\,\zeta_3\right) - \left(\frac{1078361}{162} + \frac{6508}{27}\zeta_3\right) N_f$$

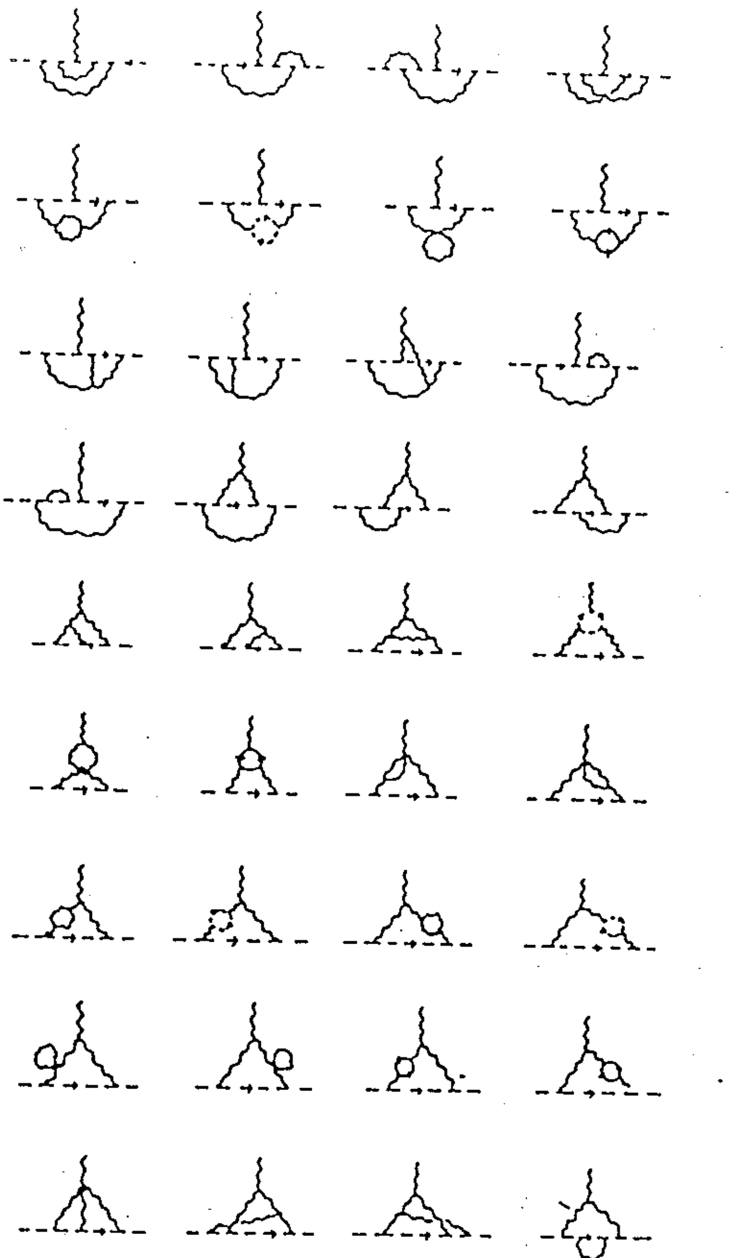
$$+ \left(\frac{50065}{162} + \frac{6472}{81}\zeta_3\right) N_f^2 + \frac{1093}{729} N_f^3$$

VAN RITBERGEN, VERMASERN, LARIN 1997.

1st ORDER:

CASWELL, 1974
JONES 1974

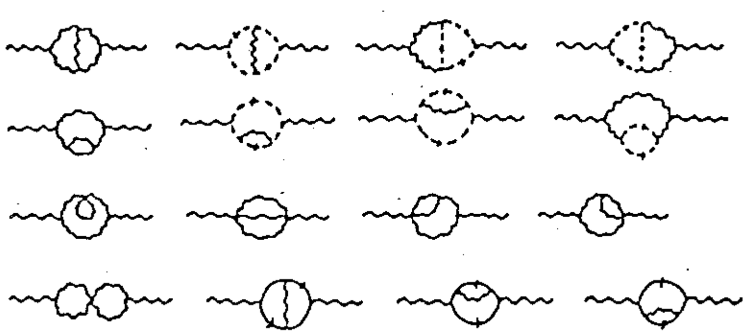
BELAVIN, MIGDAL 1974
(and diff. authors later)



$V_{G\tilde{g}\tilde{g}}$

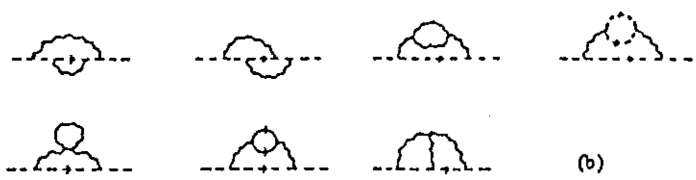
(c)

Σ_G



(a)

$\Sigma_{\tilde{g}}$



(b)

59 diagrams

$$\beta_1 = -\frac{34}{3} C_2^2(G) + \frac{20}{3} C_2(G) T(R) N_f + 4 C_2(R) T(R) N_f$$

$$= -102 + \frac{38}{3} N_f .$$

SOLUTION OF THE RGE :

$$\frac{1}{\alpha_s(Q^2)} = \frac{1}{\alpha_s(Q_0^2)} + \frac{\beta_0}{4\pi} \log\left(\frac{Q^2}{Q_0^2}\right)$$

$$+ \phi^{(n)}(\alpha_s(Q^2);\beta_i) - \phi^{(n)}(\alpha_s(Q_0^2);\beta_i)$$

$$\alpha_s^{LO}(Q^2) = \frac{\alpha_s^{LO}(Q_0^2)}{1 + \frac{\beta_0}{4\pi}\alpha_s(Q_0^2)\log\left(\frac{Q^2}{Q_0^2}\right)}$$

$$\phi^{(2)}(x;\beta_i) = -\frac{\beta_1}{8\pi\beta_0}\log\left|\frac{16\pi^2x^2}{16\pi^2\beta_0 + 4\beta_1\pi x + \beta_2 x^2}\right|$$

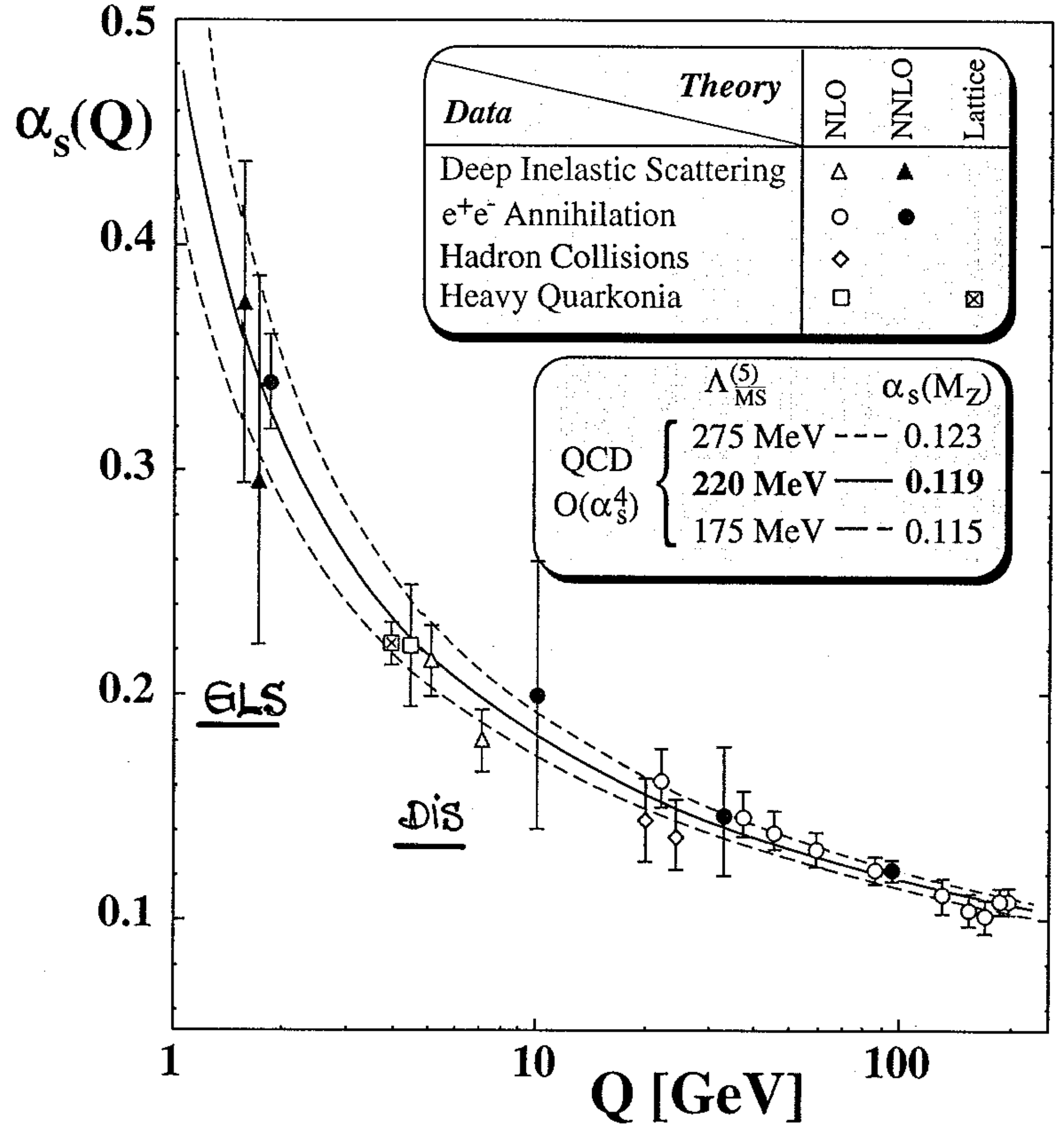
$$+ \frac{\beta_1^2 - 2\beta_0\beta_2}{8\pi\beta_0\sqrt{4\beta_2\beta_0 - \beta_1^2}} \arctan\left(\frac{2\pi\beta_1 + \beta_2 x}{2\pi\sqrt{4\beta_0\beta_2 - \beta_1^2}}\right)$$

... etc.

$N_f \leq 5 \qquad 4\beta_0\beta_2 - \beta_1^2 > 0$

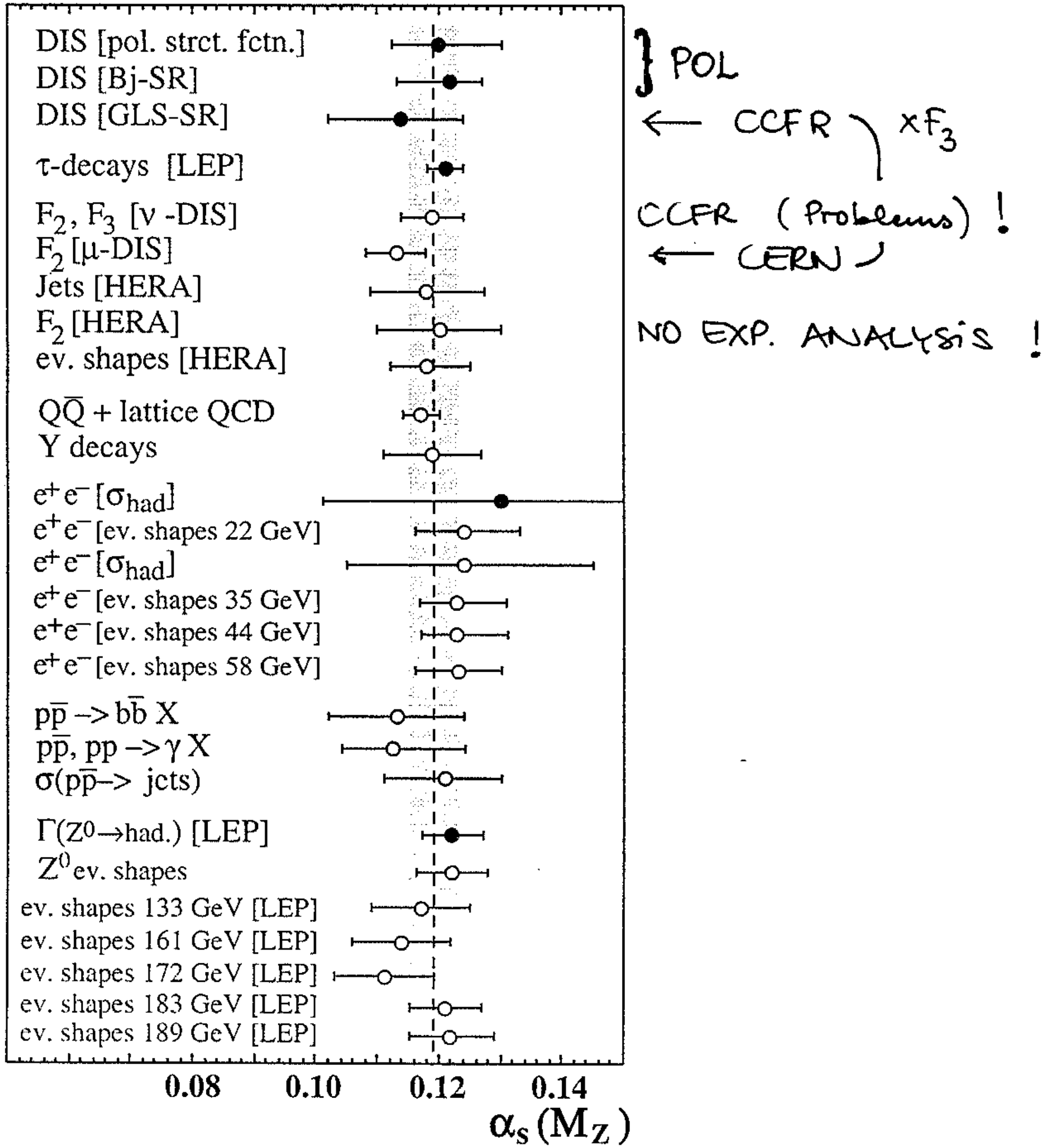
$N_f = 6 \qquad 4\beta_0\beta_2 - \beta_1^2 < 0.$

ITERATIVE SOLUTION.



0.5
0.4
0.3
0.2
0.1
$\alpha_s(Q)$
Theory
Data
NLO
NNLO
Lattice
Deep Inelastic Scattering
e+e- Annihilation
Hadron Collisions
Heavy Quarkonia
$\Lambda^{(5)}_{\overline{MS}}$
$\alpha_s(M_Z)$
QCD
$O(\alpha_s^4)$
275 MeV 0.123
220 MeV 0.119
175 MeV 0.115
GLS
DIS
1
10
100
Q [GeV]

S. BETHKE


DIS [pol. strct. fctn.]
DIS [Bj-SR]
DIS [GLS-SR]
τ-decays [LEP]
F2, F3 [ν -DIS]
F2 [μ-DIS]
Jets [HERA]
F2 [HERA]
ev. shapes [HERA]
QQ̄ + lattice QCD
Υ decays
e+e− [σhad]
e+e− [ev. shapes 22 GeV]
e+e− [σhad]
e+e− [ev. shapes 35 GeV]
e+e− [ev. shapes 44 GeV]
e+e− [ev. shapes 58 GeV]
pp̄ −> bb̄ X
pp̄, pp −> γ X
σ(pp̄−> jets)
Γ(Z0→had.) [LEP]
Z0 ev. shapes
ev. shapes 133 GeV [LEP]
ev. shapes 161 GeV [LEP]
ev. shapes 172 GeV [LEP]
ev. shapes 183 GeV [LEP]
ev. shapes 189 GeV [LEP]
0.08
0.10
0.12
0.14
αs (MZ)
} POL
← CCFR
xF3
CCFR (Problems) !
← CERN
NO EXP. ANALYSIS !

S. BETHKE

Process	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	Theory
DIS [pol. strct. fctn.]	0.7 - 8		$0.120^{+0.010}_{-0.008}$	$^{+0.004}_{-0.005}$	$^{+0.009}_{-0.006}$	NLO
DIS [Bj-SR]	1.58	$0.375^{+0.062}_{-0.081}$	$0.121^{+0.005}_{-0.009}$	–	–	NNLO
DIS [GLS-SR]	1.73	$0.295^{+0.092}_{-0.073}$	$0.114^{+0.010}_{-0.012}$	$^{+0.005}_{-0.006}$	$^{+0.009}_{-0.010}$	NNLO
τ-decays	1.78	0.339 ± 0.021	0.121 ± 0.003	0.001	0.003	NNLO
DIS [ν; F_2 and F_3]	5.0	0.215 ± 0.016	0.119 ± 0.005	0.002	0.004	NLO
DIS [μ; F_2]	7.1	0.180 ± 0.014	0.113 ± 0.005	0.003	0.004	NLO
DIS [HERA; F_2]	2 - 10		0.120 ± 0.010	0.005	0.009	NLO
DIS [HERA; jets]	10 - 100		0.118 ± 0.009	0.003	0.008	NLO
DIS [HERA; ev.shps.]	7 - 100		$0.118^{+0.007}_{-0.006}$	0.001	$^{+0.007}_{-0.006}$	NLO
$Q\overline{Q}$ states	4.1	0.223 ± 0.009	0.117 ± 0.003	0.000	0.003	LGT
Υ decays	4.13	0.220 ± 0.027	0.119 ± 0.008	0.001	0.008	NLO
e^+e^- [σ_{had}]	10.52	0.20 ± 0.06	$0.130^{+0.021}_{-0.029}$	$^{+0.021}_{-0.029}$	–	NNLO
e^+e^- [ev. shapes]	22.0	$0.161^{+0.016}_{-0.011}$	$0.124^{+0.009}_{-0.006}$	0.005	$^{+0.008}_{-0.003}$	resum
e^+e^- [σ_{had}]	34.0	$0.146^{+0.031}_{-0.026}$	$0.123^{+0.021}_{-0.019}$	$^{+0.021}_{-0.019}$	–	NLO
e^+e^- [ev. shapes]	35.0	$0.145^{+0.012}_{-0.007}$	$0.123^{+0.008}_{-0.006}$	0.002	$^{+0.008}_{-0.005}$	resum
e^+e^- [ev. shapes]	44.0	$0.139^{+0.010}_{-0.007}$	$0.123^{+0.008}_{-0.006}$	0.003	$^{+0.007}_{-0.005}$	resum
e^+e^- [ev. shapes]	58.0	0.132 ± 0.008	0.123 ± 0.007	0.003	0.007	resum
$p\bar{p} \to b\bar{b}X$	20.0	$0.145^{+0.018}_{-0.019}$	0.113 ± 0.011	$^{+0.007}_{-0.006}$	$^{+0.008}_{-0.009}$	NLO
$p\bar{p}$, $pp \to \gamma X$	24.2	$0.137^{+0.017}_{-0.014}$	$0.111^{+0.012}_{-0.008}$	0.006	$^{+0.010}_{-0.005}$	NLO
$\sigma(p\bar{p} \to$ jets)	30 - 500		0.121 ± 0.009	0.001	0.009	NLO
e^+e^- [$\Gamma(Z^0 \to$ had.)]	91.2	0.122 ± 0.005	0.122 ± 0.005	0.004	0.003	NNLO
e^+e^- [ev. shapes]	91.2	0.122 ± 0.006	0.122 ± 0.006	0.001	0.006	resum
e^+e^- [ev. shapes]	133.0	0.111 ± 0.008	0.117 ± 0.008	0.004	0.007	resum
e^+e^- [ev. shapes]	161.0	0.105 ± 0.007	0.114 ± 0.008	0.004	0.007	resum
e^+e^- [ev. shapes]	172.0	0.102 ± 0.007	0.111 ± 0.008	0.004	0.007	resum

S. BETHKE

4.2. The Splitting Functions and Evolution Equations

$O(\alpha_s)$: LO UNPOLARIZED

$$P_{NS}^{(0)}(z) \equiv P_{qq}^{(0)}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\,\delta(1-z) \right]$$

$$P_{qg}^{(0)}(z) = T_f \left[(1-z)^2 + z^2 \right]$$

$$P_{gq}^{(0)}(z) = C_F \; \frac{1+(1-z)^2}{z}$$

$$P_{gg}^{(0)}(z) = 2\,C_G \left[\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z) + \frac{1}{2}\beta_0\,\delta(1-z) \right]$$

GROSS, WILCZEK 1973

GEORGI, POLITZER 1973

LIPATOV 1975

DOKSHITSER 1977

ALTARELLI, PARISI 1977

KIM, SCHILCHER 1977, 78

$$\int_0^1 dz\; z^{N-1}\, P_{ab}^{(0)}(z) = -\frac{1}{4}\,\gamma_{ab}^{(0)}(N)$$

CONNECTION TO ANOMALOUS DIMENSIONS.

$O(\alpha_s^2)$: NLO UNPOLARIZED

FLORATOS, D ROSS, SACHRAIDA 1977-79

CURCI, FURMANSKI, PETRONZIO 1980

FURMANSKI, PETRONZIO 1980

GONZALEZ-ARROYO, LOPEZ, YNDURAIN 1979, 80

FLORATOS, KOUNNAS, LACAZE 1981abc

VAN NEERVEN, HAMBERG 1992

∴ NS :

$$P_{qq}^{(1)}(z) = \underline{C_F^2}\, P_F(z) + \underline{\tfrac{1}{2} C_F C_A}\, P_G(z) + \underline{C_F N_f T_f}\, P_{N_f}(z)$$

$$P_{q\bar{q}}^{(1)}(z) = \underline{\left[C_F^2 - \tfrac{1}{2} C_F C_A\right]}\, P_A(z)$$

$$P_F(z) = -2\left(\frac{1+z^2}{1-z}\right)\ln z \ln(1-z) - \left(\frac{3}{1-z} + 2z\right)\ln z - \frac{1}{2}(1+z)\ln^2 z$$

$$P_G(z) = \left(\frac{1+z^2}{1-z}\right)\left[\ln^2 z + \frac{11}{3}\ln z + \frac{67}{9} - \frac{1}{3}\pi^2\right] + 2(1+z)\ln z + \frac{40}{3}(1-z)$$

$$P_{N_f}(z) = -\frac{2}{3}\left[\frac{1+z^2}{1-z}\left(\ln z + \frac{5}{3}\right) + 2(1-z)\right]$$

$$P_A(z) = 2\,\frac{1+z^2}{1-z}\int_{z/(1+z)}^{1/(1+z)} \frac{du}{u}\ln\left(\frac{1-u}{u}\right) + 2(1+z)\ln z + 4(1-z)$$

$O(\alpha_s)$: LO POLARIZED

$$\hat{P}^{(0)}_{NS,qq}(z) = \hat{P}^{(0)}_{qq,S}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

$$\hat{P}^{(0)}_{qg}(z) = T_f\left[4z - 2\right]$$

$$\hat{P}^{(0)}_{gq}(z) = C_F\left[2 - z\right]$$

$$\hat{P}^{(0)}_{gg}(z) = C_A\left[\left(\frac{2}{1-z}\right)_+ + 2 - 4z + \frac{1}{2}\beta_0\,\delta(1-z)\right]$$

ITO 1975

K. SASAKI 1975

~~AHMED~~, G. ROSS 1975, 76

ALTARLLI, PARISI 1977

NO TERMS $\propto \frac{1}{z}$.

TABLE 1

Detailed contribution of various diagrams to $\Gamma_{qq}(x,\alpha,1/\epsilon)$

	$\Gamma_{qq}(x,\alpha,1/\epsilon)$	C_F^2						$\frac{1}{2}C_F C_G$					$\frac{1}{3}N_F C_F$
							S.H					S.H	
A	$\frac{1+x^2}{1-x}$	$7-\frac{2}{3}\pi^2$	$-7+\frac{2}{3}\pi^2$	0	0	0	0	$7-\frac{2}{3}\pi^2$	0	$-11+\pi^2$	$\frac{103}{9}-\frac{2}{3}\pi^2$	$\frac{67}{9}-\frac{1}{3}\pi^2$	$-\frac{10}{9}$
A	$\frac{1+x^2}{1-x}\ln^2 x$	-2	1	-1	2	0	0	-1	1	1	0	1	0
A	$(1+x)\ln x$	0	$-\frac{7}{2}$	2	-1	0	$-\frac{5}{2}$	$\frac{7}{2}$	-2	$\frac{1}{2}$	0	2	0
A	$1-x$	3	-11	0	3	0	-5	11	0	-1	$\frac{10}{3}$	$\frac{40}{3}$	$-\frac{4}{3}$
B	$(1+x)\ln^2 x$	0	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	0	0	0	0
B	$\frac{1+x^2}{1-x}\ln^2(1-x)$	0	0	0	0	0	0	0	0	2	-2	0	0
B	$\frac{1+x^2}{1-x}\ln x\ln(1-x)$	-4	2	0	0	0	-2	-2	0	6	-4	0	0
B	$\frac{1+x^2}{1-x}\ln x$	0	$-\frac{3}{2}$	0	0	0	$-\frac{3}{2}$	$\frac{3}{2}$	0	$-\frac{3}{2}$	$\frac{11}{3}$	$\frac{11}{3}$	$-\frac{2}{3}$
B	$\frac{1+x^2}{1-x}\ln(1-x)$	3	-3	4	-4	0	0	3	-4	5	-4	0	0
B	$(1-x)\ln x$	-4	2	0	3	0	1	-2	0	2	0	0	0
B	$(1-x)\ln(1-x)$	0	0	0	0	0	0	0	0	4	-4	0	0
B	$I_1\frac{1+x^2}{1-x}$	4	-4	0	0	0	0	4	0	-8	4	0	0
B	$I_0\frac{1+x^2}{1-x}$	0	0	4	-4	0	0	0	-4	8	-4	0	0
B	$I_0\frac{1+x^2}{1-x}(\ln x+\ln(1-x))$	-4	4	0	0	0	0	-4	0	8	-4	0	0
B	$I_0(1-x)$	-4	4	0	0	0	0	-4	0	8	-4	0	0

Appropriate colour factors are shown in the first line. Terms of type A satisfy the Gribov-Lipatov relation while those of type B break it.

SINGLET:

$P_{ij}^{(1)}(x)$:

$$\hat{P}_{FF}^{(1,S)} = C_F^2[-1 + x + (\tfrac{1}{2} - \tfrac{3}{2}x)\ln x - \tfrac{1}{2}(1+x)\ln^2 x - (\tfrac{3}{2}\ln x + 2\ln x \ln(1-x))p_{FF}(x) + 2p_{FF}(-x)S_2(x)]$$
$$+ C_F C_G[\tfrac{14}{3}(1-x) + (\tfrac{11}{6}\ln x + \tfrac{1}{2}\ln^2 x + \tfrac{67}{18} - \tfrac{1}{6}\pi^2)p_{FF}(x) - p_{FF}(-x)S_2(x)]$$
$$+ C_F T_R N_F[-\tfrac{16}{3} + \tfrac{40}{3}x + (10x + \tfrac{16}{3}x^2 + 2)\ln x - \tfrac{112}{9}x^2 + \tfrac{40}{9}x^{-1} - 2(1+x)\ln^2 x - (\tfrac{10}{9} + \tfrac{2}{3}\ln x)p_{FF}(x)] ,$$

$$\hat{P}_{FG}^{(1,S)} = C_F^2[-\tfrac{5}{2} - \tfrac{7}{2}x + (2 + \tfrac{7}{2}x)\ln x + (-1 + \tfrac{1}{2}x)\ln^2 x - 2x\ln(1-x) + (-3\ln(1-x) - \ln^2(1-x))p_{FG}(x)]$$
$$+ C_F C_G[\tfrac{28}{9} + \tfrac{65}{18}x + \tfrac{44}{9}x^2 + (-12 - 5x - \tfrac{8}{3}x^2)\ln x + (4 + x)\ln^2 x + 2x\ln(1-x) + (-2\ln x\ln(1-x)$$
$$+ \tfrac{1}{2}\ln^2 x + \tfrac{11}{3}\ln(1-x) + \ln^2(1-x) - \tfrac{1}{6}\pi^2 + \tfrac{1}{2})p_{FG}(x) + p_{FG}(-x)S_2(x)]$$
$$+ C_F T_R N_F[-\tfrac{4}{3}x - (\tfrac{20}{9} + \tfrac{4}{3}\ln(1-x))p_{FG}(x)] ,$$

$$\hat{P}_{GF}^{(1,S)} = C_F T_R N_F[4 - 9x + (-1 + 4x)\ln x + (-1 + 2x)\ln^2 x + 4\ln(1-x)$$
$$+ (-4\ln x\ln(1-x) + 4\ln x + 2\ln^2 x - 4\ln(1-x) + 2\ln^2(1-x) - \tfrac{2}{3}\pi^2 + 10)p_{GF}(x)]$$
$$+ C_G T_R N_F[\tfrac{182}{9} + \tfrac{14}{9}x + \tfrac{40}{9}x^{-1} + (\tfrac{136}{3}x - \tfrac{38}{3})\ln x - 4\ln(1-x) - (2 + 8x)\ln^2 x + (-\ln^2 x$$
$$+ \tfrac{44}{3}\ln x - 2\ln^2(1-x) + 4\ln(1-x) + \tfrac{1}{3}\pi^2 - \tfrac{218}{9})p_{GF}(x) + 2p_{GF}(-x)S_2(x)] ,$$

$$\hat{P}_{GG}^{(1,S)} = C_F T_R N_F[-16 + 8x + \tfrac{20}{3}x^2 + \tfrac{4}{3}x^{-1} + (-6 - 10x)\ln x + (-2 - 2x)\ln^2 x]$$
$$+ C_G T_R N_F[2 - 2x + \tfrac{26}{9}x^2 - \tfrac{26}{9}x^{-1} - \tfrac{4}{3}(1+x)\ln x - \tfrac{20}{9}p_{GG}(x)]$$
$$+ C_G^2[\tfrac{27}{2}(1-x) + \tfrac{67}{9}(x^2 - x^{-1}) + (-\tfrac{25}{3} + \tfrac{11}{3}x - \tfrac{44}{3}x^2)\ln x + 4(1+x)\ln^2 x + (\tfrac{67}{9} - 4\ln x\ln(1-x)$$
$$+ \ln^2 x - \tfrac{1}{3}\pi^2)p_{GG}(x) + 2p_{GG}(-x)S_2(x)] .$$

$$S_2(x) \equiv \int_{(1+x)/x}^{1/(1+x)} \frac{dz}{z}\ln\left(\frac{1-z}{z}\right); \quad S_1(x) \equiv \int_0^{1-x} \frac{dz}{z}\ln(1-z) .$$

$O(\alpha_s^2)$: NLO POLARIZED

ZIJLSTRA, VAN NEERVEN 1994 $\hat{P}^{(1)}_{qq}$, $\hat{P}^{(1)}_{qG}$

MERTIG, VAN NEERVEN 1995

VOGELSANG 1995

$O(\alpha_s^3)$: NNLO

COMPLETE RESULTS ARE NOT YET AVAILABLE BOTH FOR THE POLARIZED AND UNPOLARIZED CASE.

→ NEEDED TO CONTROL $\Delta\alpha_s^{THY} \longrightarrow \pm 0.002$ @ $Q = M_Z$.
(SCHEME DEPENDENCE)

→ HERA : $\delta\alpha_s^{stat+syst} = \pm 0.002$ CAN BE ACHIEVED.

NON-SINGLET: γ_{10}

$$
\begin{aligned}
\gamma_{10}^{\rm ns} = {} & a_s C_F \left(\tfrac{12055}{1386}\right) \\
& + a_s^2 \left[C_F C_A \left(\tfrac{19524247733}{523908000}\right) + C_F^2 \left(-\tfrac{9579051036701}{1331250228000}\right) + n_f C_F \left(-\tfrac{2451995507}{288149400}\right)\right] \\
& + a_s^3 \left[C_F C_A^2 \left(\tfrac{9409156857976645 3}{435681892800000} + \tfrac{151796299}{8004150}\zeta_3\right)\right. \\
& + C_F^2 C_A \left(-\tfrac{16389982059548833}{465937579800000} - \tfrac{151796299}{2668050}\zeta_3\right) \\
& + C_F^3 \left(-\tfrac{22077113008087364056 87}{1278663181493544 00000} + \tfrac{151796299}{4002075}\zeta_3\right) \\
& + n_f C_F C_A \left(-\tfrac{9007773127403}{389001690000} - \tfrac{48220}{693}\zeta_3\right) + n_f C_F^2 \left(-\tfrac{755220732104711 27}{1230075210672000} + \tfrac{48220}{693}\zeta_3\right) \\
& \left. + n_f^2 C_F \left(-\tfrac{27995901056887}{11981252052000}\right)\right]
\end{aligned}
$$

IMPROVEMENTS NEEDED:

- CALCULUS OF FINITE (IFINITE) MULTIPLE HARMONIC SUS
- EULER-ZAGIER VALUES

BROADHURST, KREIMER
VERMASEREN, VAN RITBERGEN
BLÜMLEIN

4.3. Coefficient Functions

PROCESS-DEPENDENT QUANTITIES.

$O(\alpha_s)$: UNPOLARIZED

$$C^{(1)}_{F_2 q}(z) = C_F \left[\frac{1+z^2}{1-z}\left[\log\left(\frac{1-z}{z}\right) - \frac{3}{4}\right] + \frac{1}{4}(9+5z)\right]_+$$

$$C^{(1)}_{F_2 g}(z) = 2N_f T_f \left\{ [z^2 + (1-z)^2] \log\left(\frac{1-z}{z}\right) - 1 + 8z(1-z) \right\}$$

$$C^{(1)}_{F_1 q}(z) = C^{(1)}_{F_2 q} - C_F \cdot 2z$$

$$C^{(1)}_{F_1 g}(z) = C^{(1)}_{F_2 g} - 8N_f T_f z(1-z)$$

$$C^{(1)}_{F_3 q}(z) = C^{(1)}_{F_2}(z) - C_F(1+z).$$

FURMANSKI, PETRONZIO 1982 (AND VARIOUS AUTHORS BEFORE (ERRORS, SOMETIMES)).

$O(\alpha_s)$: POLARIZED

$$C^{(1)}_{g_1 q} = C^{(1)}_{F_1 q}$$

$$C^{(1)}_{g_1 g} = 4N_f T_f \left\{ (2z-1)\log\left(\frac{1-z}{z}\right) + 3 - 4z \right\}.$$

ALTARELLI, ELLIS, MARTINELLI 1979
HUMPERT, VAN NEERVEN 1981; BODWIN, QIU 1990

$O(\alpha_s^2)$: ZIJLSTRA, VAN NEERVEN 1992 (UNPOL.) ; 1994 POLARIZED.

The coefficient functions $C_i(x, Q^2)$ read

$$\begin{aligned} C_{\rm NS}(z,Q^2) &= a_s c_{L,q}^{(1)}(z) + a_s^2 c_{L,q}^{(2),NS}(z) \\ C_{\rm S}(z,Q^2) &= a_s^2 c_{L,q}^{(2),PS}(z) \\ C_{\rm g}(z,Q^2) &= a_s c_{L,g}^{(1)}(z) + a_s^2 c_{L,g}^{(2)}(z) \ , \end{aligned} \tag{25}$$

where $a_s = \alpha_s(Q^2)/(4\pi)$. The leading order coefficient functions are given by [38]

C_L :

$$c_{L,q}^{(1)}(z) = 4C_F z \tag{26}$$

$$c_{L,g}^{(1)}(z) = 8N_f z(1-z) \ . \tag{27}$$

In the $\overline{\rm MS}$ scheme the NLO coefficient functions read [21, 22] [6]

$$\begin{aligned} c_{L,q}^{(2),NS}(z) &= 4C_F(C_A - 2C_F)z\Big\{4\frac{6-3z+47z^2-9z^3}{15z^2}\ln z \\ &-4{\rm Li}_2(-z)[\ln z - 2\ln(1+z)] - 8\zeta(3) - 2\ln^2 z[\ln(1+z)+\ln(1-z)] \\ &+4\ln z\ln^2(1+z) - 4\ln z{\rm Li}_2(z) \ + \frac{2}{5}(5-3z^2)\ln^2 z \\ &-4\frac{2+10z^2+5z^3-3z^5}{5z^3}[{\rm Li}_2(-z) + \ln z\ln(1+z)] \\ &+4\zeta(2)\left[\ln(1+z)+\ln(1-z) - \frac{5-3z^2}{5}\right] + 8{\rm S}_{1,2}(-z) + 4{\rm Li}_3(z) \\ &+4{\rm Li}_3(-z) - \frac{23}{3}\ln(1-z) - \frac{144+294z-1729z^2+216z^3}{90z^2}\Big\} \\ &+8C_F^2 z\Big\{{\rm Li}_2(z) + \ln^2 z - 2\ln z\ln(1-z) + \ln^2(1-z) - 3\zeta(2) \\ &-\frac{3-22z}{3z}\ln z + \frac{6-25z}{6z}\ln(1-z) - \frac{78-355z}{36z}\Big\} \\ &-\frac{8}{3}C_F N_f z\left\{2\ln z - \ln(1-z) - \frac{6-25z}{6z}\right\}, \end{aligned} \tag{28}$$

$$\begin{aligned} c_{L,q}^{(2),PS}(z) &= \frac{16}{9z}C_F N_f\Big\{3(1-2z-2z^2)(1-z)\ln(1-z) + 9z^2[{\rm Li}_2(z)+\ln^2 z - \zeta(2)] \\ &\quad +9z(1-z-2z^2)\ln z - 9z^2(1-z) - (1-z)^3\Big\}, \end{aligned} \tag{29}$$

$$\begin{aligned} c_{L,g}^{(2)}(z) &= C_F N_f\Big\{16z[{\rm Li}_2(1-z) + \ln z\ln(1-z)] \\ &+\left(-\frac{32}{3}z + \frac{64}{5}z^3 + \frac{32}{15z^2}\right)[{\rm Li}_2(-z) + \ln z\ln(1+z)] + (8+24z-32z^2)\ln(1-z) \\ &-\left(\frac{32}{3}z + \frac{32}{5}z^3\right)\ln^2 z + \frac{1}{15}\left(-104 - 624z + 288z^2 - \frac{32}{z}\right)\ln z \\ &+\left(-\frac{32}{3}z + \frac{64}{5}z^3\right)\zeta(2) - \frac{128}{15} - \frac{304}{5}z + \frac{336}{5}z^2 + \frac{32}{15z}\Big\} \\ &+C_A N_f\Big\{-64{\rm Li}_2(1-z) + (32z+32z^2)[{\rm Li}_2(-z) + \ln z\ln(1+z)] \\ &+(16z-16z^2)\ln^2(1-z) + (-96z+32z^2)\ln z\ln(1-z) \\ &+\left(-16 - 144z + \frac{464}{3}z^2 + \frac{16}{3z}\right)\ln(1-z) + 48z\ln^2 z + (16+128z-208z^2)\ln z \end{aligned}$$

[6] Previous calculations [39, 40] turned out to be partly incorrect, whereas agreement was shown between Refs. [21, 22] and [41]. Refs. [40] were later corrected in Ref. [42].

$$
\begin{aligned}
C^{G}_{L,8} = {} & a_s n_f \left(\tfrac{4}{45}\right) + a_s^2 n_f C_F \left(-\tfrac{51097}{51030}\right) + a_s^2 n_f C_A \left(\tfrac{7712869}{2551500}\right) \\
& + a_s^3 \mathrm{fl}^g_{11} n_f^2 \frac{d^{abc}d^{abc}}{N_A} \left(\tfrac{3665714041}{285768000} + \tfrac{77209}{1575}\zeta_3 - \tfrac{208}{3}\zeta_5\right) \\
& + a_s^3 n_f C_F C_A \left(-\tfrac{520855237960033}{7129340064000} - \tfrac{6119609}{519750}\zeta_3 + \tfrac{128}{3}\zeta_5\right) \\
& + a_s^3 n_f C_F^2 \left(\tfrac{2384408424295187}{71293400640000} + \tfrac{7723411}{779625}\zeta_3 - \tfrac{128}{3}\zeta_5\right) \\
& + a_s^3 n_f C_A^2 \left(\tfrac{27404278602137}{289340100000} + \tfrac{20438}{23625}\zeta_3 - \tfrac{32}{3}\zeta_5\right) \\
& + a_s^3 n_f^2 C_F \left(\tfrac{124374980290567}{35646700320000} - \tfrac{608}{1485}\zeta_3\right) + a_s^3 n_f^2 C_A \left(-\tfrac{11324757281}{1377810000} - \tfrac{8}{45}\zeta_3\right) \\
= {} & a_s \left(0.0888888889 n_f\right) + a_s^2 \left(7.7335451042 n_f\right) \\
& + a_s^3 \left(-0.2322211886 n_f^2 \mathrm{fl}^g_{11} + 592.3307972098 n_f - 21.3033368117 n_f^2\right) .
\end{aligned}
$$

$$
\begin{aligned}
C^{\mathrm{ns}}_{L,10} = {} & a_s C_F \left(\tfrac{4}{11}\right) + a_s^2 n_f C_F \left(-\tfrac{163679}{114345}\right) + a_s^2 C_F C_A \left(\tfrac{89670761}{8731800} - \tfrac{48}{11}\zeta_3\right) \\
& + a_s^2 C_F^2 \left(-\tfrac{1999510607}{528273900} + \tfrac{96}{11}\zeta_3\right) \\
& + a_s^3 \mathrm{fl}_{11} n_f \frac{d^{abc}d^{abc}}{n} \left(-\tfrac{5073093424963}{528099264000} - \tfrac{1820773}{363825}\zeta_3 + \tfrac{160}{11}\zeta_5\right) \\
& + a_s^3 n_f C_F C_A \left(-\tfrac{176183576988227323}{1699159381920000} + \tfrac{55485434}{1216215}\zeta_3\right) \\
& + a_s^3 n_f C_F^2 \left(\tfrac{9048874326307637}{190368782604000} - \tfrac{1174256}{15015}\zeta_3\right) + a_s^3 n_f^2 C_F \left(\tfrac{63272639}{11320155}\right) \\
& + a_s^3 C_F C_A^2 \left(\tfrac{2366034921481985137}{6796637527680000} - \tfrac{95022195887}{187297110}\zeta_3 + \tfrac{3760}{11}\zeta_5\right) \\
& + a_s^3 C_F^2 C_A \left(-\tfrac{323139848004267269}{3354750574560000} + \tfrac{22904191}{17325}\zeta_3 - \tfrac{14240}{11}\zeta_5\right) \\
& + a_s^3 C_F^3 \left(-\tfrac{8875623866986459673S3}{316621359226972S000} - \tfrac{357031607224}{468242775}\zeta_3 + \tfrac{13440}{11}\zeta_5\right) \\
= {} & a_s \left(0.4848484848\right) + a_s^2 \left(32.0176594698 - 1.9085982480 n_f\right) \\
& + a_s^3 \left(-2.3976416945 n_f \mathrm{fl}_{11} + 2081.2132221274\right.
\end{aligned}
$$

$O(\alpha_s^3)$: SO FAR THE FIRST MOMENTS

1996 : LARIN, VERMASEREN, VAN RITBERGEN

$$
\begin{aligned}
C^{\rm ns}_{2,10} = {} & 1 + a_s C_F \left(\tfrac{2006299}{138600}\right) + a_s^2 n_f C_F \left(-\tfrac{561457267429757}{15975002736000}\right) \\
& + a_s^2 C_F C_A \left(\tfrac{6124093193824187}{29045459520000} - \tfrac{104674}{1155}\zeta_3\right) \\
& + a_s^2 C_F^2 \left(\tfrac{5587087999873240 13}{14760902528064000} + \tfrac{88798}{1155}\zeta_3\right) \\
& + a_s^3 \mathrm{fl}_{11} n_f \frac{d^{abc}d^{abc}}{n} \left(\tfrac{3753913187503}{352066176000} + \tfrac{81388}{606375}\zeta_3 - \tfrac{448}{33}\zeta_5\right) \\
& + a_s^3 n_f C_F C_A \left(-\tfrac{216642449260393572149 87}{23550349033411200000} + \tfrac{10519793104}{42567525}\zeta_3 - \tfrac{24110}{693}\zeta_4\right) \\
& + a_s^3 n_f C_F^2 \left(-\tfrac{1521387460036994061010049}{27200653133589936000 00} - \tfrac{3997754476}{42567525}\zeta_3 + \tfrac{24110}{693}\zeta_4\right) \\
& + a_s^3 n_f^2 C_F \left(\tfrac{5708442804785155191 1}{996360920644320000} + \tfrac{48220}{18711}\zeta_3\right) \\
& + a_s^3 C_F C_A^2 \left(\tfrac{7092211199654579390952 37}{2355034903341120000 00} - \tfrac{14713925739913}{6243237000}\zeta_3 \right. \\
& \left. + \tfrac{151796299}{16008300}\zeta_4 + \tfrac{190858}{231}\zeta_5\right) \\
& + a_s^3 C_F^2 C_A \left(\tfrac{1635000930492693338960 8829}{836943173341228800000 0} + \tfrac{1430215936081}{6163195500}\zeta_3 \right. \\
& \left. - \tfrac{151796299}{5336100}\zeta_4 - \tfrac{22658}{99}\zeta_5\right) \\
& + a_s^3 C_F^3 \left(-\tfrac{32477795323709206237706 10131}{9215581281660270316800 0000} + \tfrac{2182208825245282}{1622461215375}\zeta_3 \right. \\
& \left. + \tfrac{151796299}{8004150}\zeta_4 - \tfrac{75212}{99}\zeta_5\right) \\
= {} & 1 + a_s (19.3006156806) + a_s^2 (639.2106629599 - 46.8613184159 n_f) \\
& + a_s^3 (-14.4587445075 n_f \mathrm{fl}_{11} + 24953.1349702005 \\
& - 3770.1021201303 n_f + 80.5209797251 n_f^2),
\end{aligned}
$$

UP TO

C_2^{NS}: $n=10$

$C_2^{L\,q\,NS}$: –"–

OTHERS $N = 8$.

4.4. Some Sum Rules

- KEY ROLE OF CERTAIN INTEGRALS OVER STR. FUNCTIONS

⟶ MEASURE : α_s.

1) GROSS - LLEWELLYN SMITH SR:

$$\int_0^1 dx\, F_3^{\bar{\nu}p+\nu p}(x) = 6\Big[1-\frac{\alpha_s}{\pi}+\left(\frac{\alpha_s}{\pi}\right)^2\left[-\frac{55}{12}+\frac{1}{3}N_f\right] + \left(\frac{\alpha_s}{\pi}\right)^3\Big[-\frac{13841}{216}-\frac{44}{9}\zeta_3+\frac{55}{2}\zeta_5 + \left(\frac{10009}{1296}+\frac{91}{54}\zeta_3-\frac{5}{3}\zeta_5\right)N_f-\frac{115}{648}N_f^2\Big]\Big]$$

2) POL. BJORKEN SR:

$$\int_0^1 dx\, g_1^{ep-en}(x) = \frac{1}{3}\left|\frac{g_V}{g_A}\right|\Big[\ldots\ldots + \left(\frac{\alpha_s}{\pi}\right)^3\Big[\ldots \left(\frac{10339}{1296}+\frac{61}{54}\zeta_3\ldots\right)N_f\ldots\Big]\Big]$$

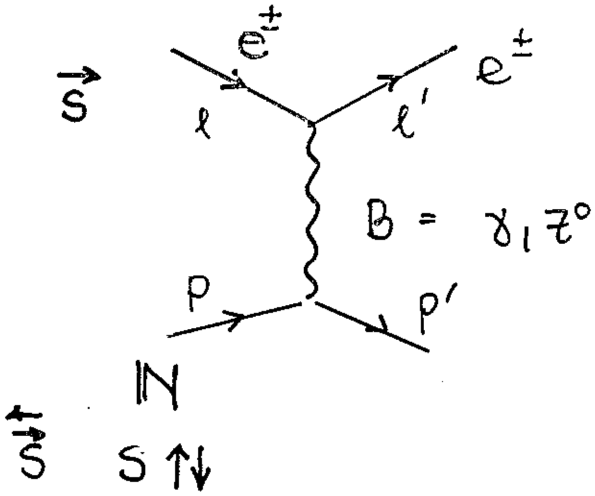
ALMOST SIMILAR TO (1)! LARIN, VERMASEREN 1991

3) UNPOL. BJORKEN SR:

$$\int_0^1 dx\, F_1^{\bar{\nu}p-\nu p}(x) = 1-\frac{2}{3}\left(\frac{\alpha_s}{\pi}\right)+\left(\frac{\alpha_s}{\pi}\right)^2\left(-\frac{23}{6}+\frac{8}{27}N_f\right) + \left(\frac{\alpha_s}{\pi}\right)^3\Big[-\frac{4075}{108}+\frac{622}{27}\zeta_3-\frac{680}{27}\zeta_5 + \left(\frac{3565}{648}-\frac{59}{27}\zeta_3+\frac{10}{3}\zeta_5\right)N_f-\frac{155}{372}N_f^2\Big]$$

LARIN, VERMASEREN, TKACHOV 1991.

5. Polarized DIS and the Next Twist



LONGITUDINAL TRANSVERSE
POLARIZATION.

$$\frac{d^2\sigma^L}{dx\,dy} \propto \frac{1}{|\gamma^*|^2}\left[-2y\left(2-y-\frac{2xyM^2}{S}\right)x\,g_1(x,Q^2) + \frac{8yx^2M^2}{S}\,g_2(x,Q^2)\right]$$

$$\frac{d^2\sigma^T}{dx\,dy} \propto \frac{1}{|\gamma^*|^2}\sqrt{\frac{M^2}{S}}\sqrt{xy\left(1-y-\frac{xyM^2}{S}\right)}\left[-2y\,x\,g_1(x,Q^2) + 4x\,g_2(x,Q^2)\right]$$

(MORE TERMS FOR ELECTROWEAK CURRENTS)

UNFOLD : g_1 & g_2 EXPERIMENTALLY.

WHAT IS THEIR STRUCTURE ACCORDING TO THE

LIGHT CONE EXPANSION ?

LOWEST ORDER QCD:

- TWO OPERATOR EXPECTATION VALUES

a_n , d_n n = SPIN

↑   ↑

TWIST 2   TWIST 3   + HIGHER TWISTS $\sim 1/Q^{2k}$

$|\gamma^*|^2$ 2 SF'S , $\Delta q + \Delta\bar{q}$ → 1 RELATION

$|\gamma + Z^0|^2$ 5 SF'S , $\Delta q \pm \Delta\bar{q}$ → 3 RELATIONS

MORE PRECISELY: ONE REL. PER LOWEST TWIST

$|\gamma^*|^2$:

$$g_2^{II}(x,Q^2) = -g_1^{II}(x,Q^2) + \int_x^1 \frac{dy}{y} g_1^{II}(y,Q^2)$$

WANDZURA, WILCZEK 1977

$$g_1^{III}(x,Q^2) = \frac{4M^2x^2}{Q^2}\left[g_2^{III}(x,Q^2) - 2\int_x^1 \frac{dy}{y} g_2^{III}(y,Q^2)\right]$$

JB, TKABLADZE 1998

(ALL MASSES NEED TO BE RESUMMED)

$|\gamma + Z^0|^2$: DICUS 1972 } TWIST 2   2 REL.
JB, KOCHELEV 1996 }
JB, TKABLADZE 1998   TWIST 3   OTHER 2 REL

EXPERIMENTAL STUDY POSSIBLE AT SLAC SOON.

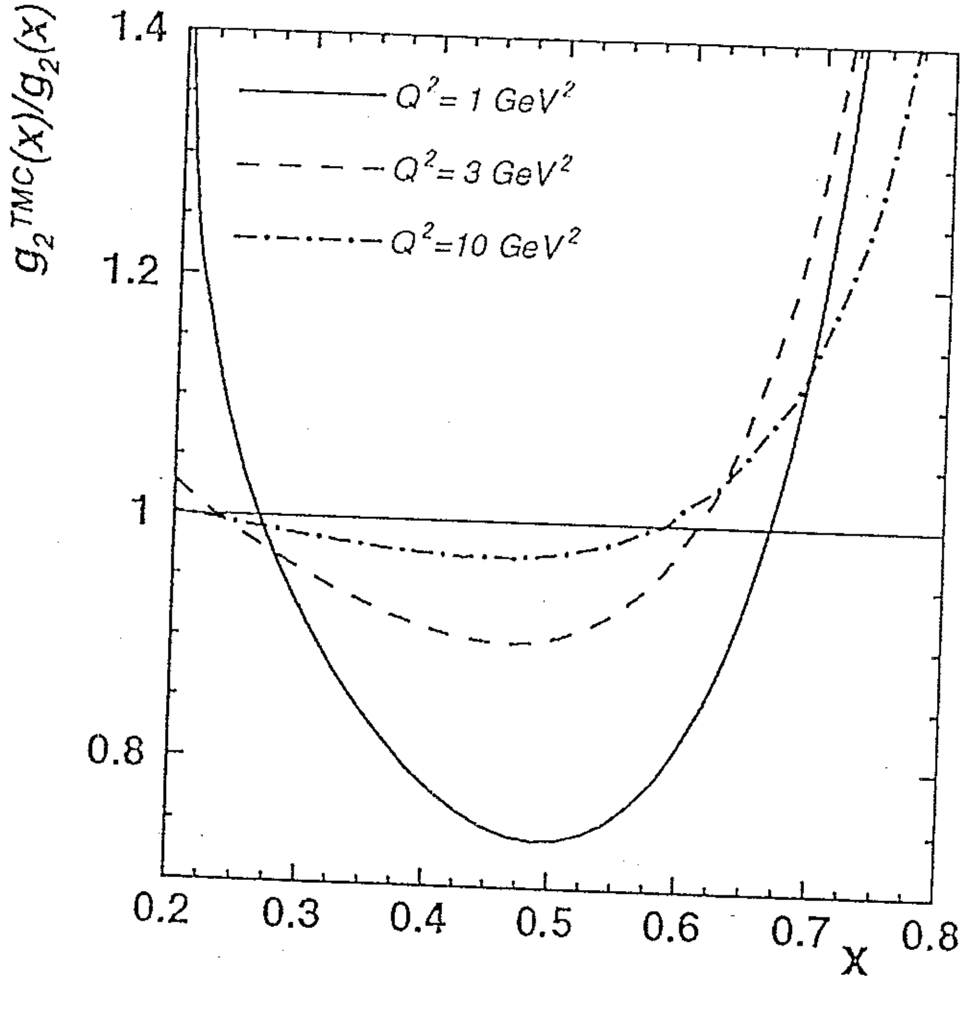


Figure 2. The ratio $g_2^{TMC}(x)/g_2(x)$ versus x.

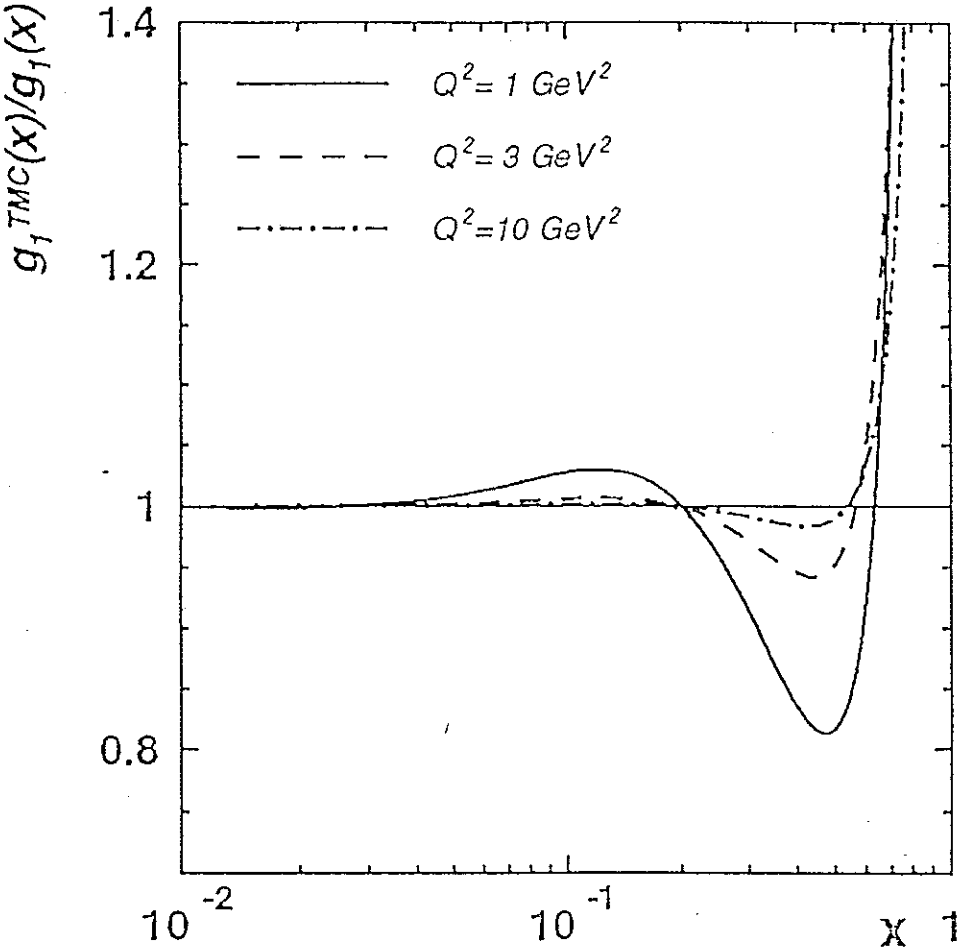


Figure 1. The ratio $g_1^{TMC}(x)/g_1(x)$ versus x.

6. The Domain of Small x

STRUCTURE FUNCTIONS RISE AT SMALL x.

FIG.

EVEN AT RATHER LOW Q^2!

REGGE THEORIE PREDICTED A FLAT BEHAVIOUR.

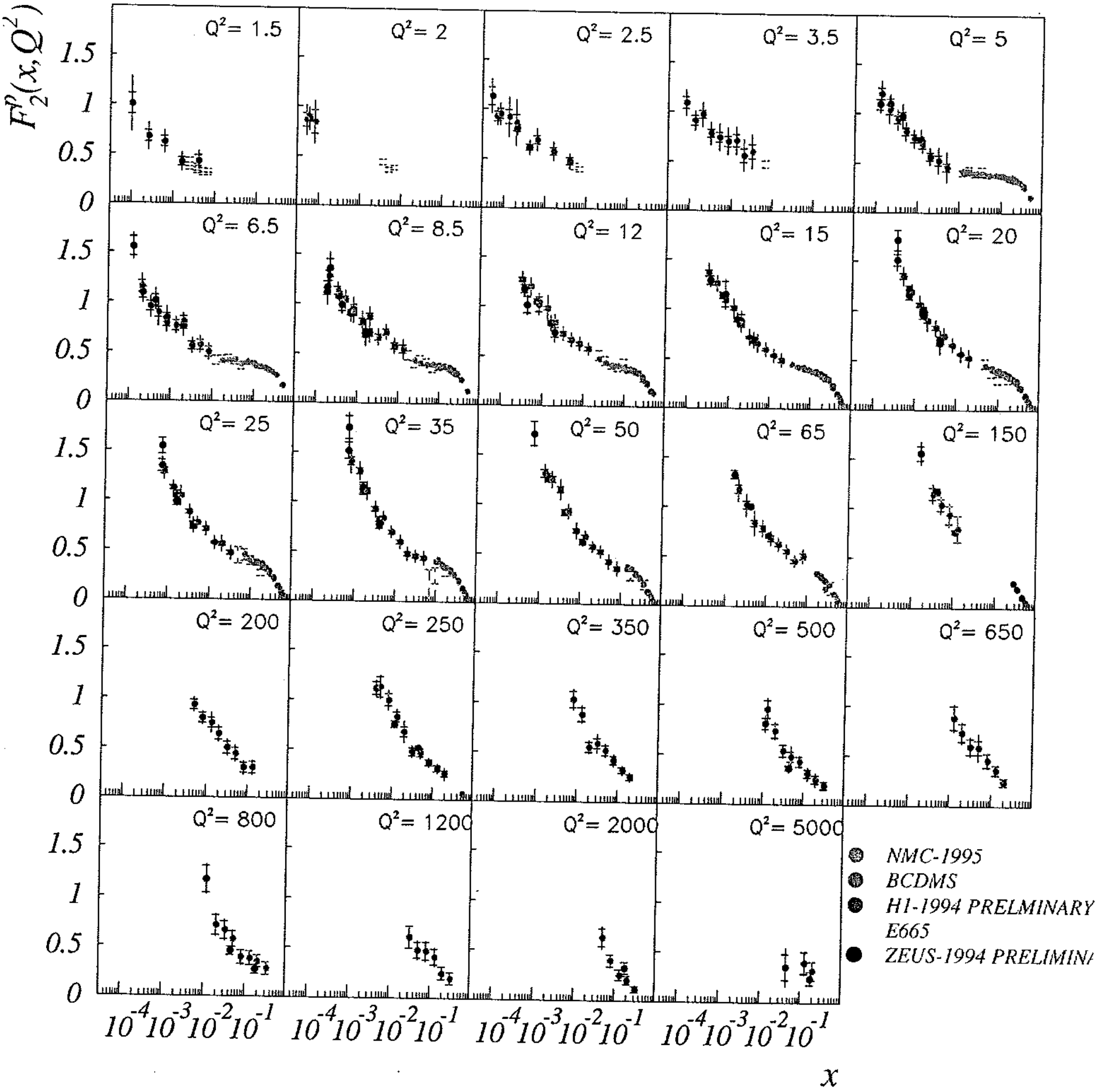
⟶ WHAT IS DESCRIBED BY PERTURBATIVE QCD?

⟶ BFKL - RESUMMATION (ALLOWED ?)

- SINCE $F_2(x, Q^2)$ RISES ALREADY AT LOW Q^2 PART OF THE RISE IS NON-PERTURBATIVE AND CANNOT BE DESCRIBED BY RESUMMATIONS AT $\alpha_s \ll 1$.

- RESUMMATIONS ARE ONLY USEFUL IF THEY ARE DOMINANT.

⟶ THIS IS NOT THE CASE FOR BFKL IN INCLUSIVE QUANTITIES.



$F_2^p(x,Q^2)$
Q²= 1.5
Q²= 2
Q²= 2.5
Q²= 3.5
Q²= 5
Q²= 6.5
Q²= 8.5
Q²= 12
Q²= 15
Q²= 20
Q²= 25
Q²= 35
Q²= 50
Q²= 65
Q²= 150
Q²= 200
Q²= 250
Q²= 350
Q²= 500
Q²= 650
Q²= 800
Q²= 1200
Q²= 2000
Q²= 5000
NMC-1995
BCDMS
H1-1994 PRELMINARY
E665
ZEUS-1994 PRELIMIN/
x

SINGULARITY STRUCTURE OF SMALL X EXPANSIONS

a) PERTURBATIVE: POLES.

SCALARS: $\Gamma \sim \sum_k C_k^S (1+N) \left[\frac{\alpha_s}{(N+1)^2}\right]^k$ $\quad N = -1$

FERMIONS: $\Gamma \sim \sum_k C_k^F N \left(\frac{\alpha_s}{N^2}\right)^k$ $\quad N = 0$

VECTORS: $\Gamma \sim \sum_k C_k^V \left(\frac{\alpha_s}{N-1}\right)^k$ $\quad N = 1$

BALITZKI, KURAEV } 76–86
FADIN LIPATOV
98 –

b) AFTER RESUMMATION:

ALL POLES DISAPPEAR TO BRANCH POINTS !

SCALARS: $\Gamma = (N+1)\left[\sqrt{1 - \frac{4a_s}{(N+1)^2}} - 1\right]$ LOVELACE 75, JB, VAN NEERVEN 92

$N = -1 \pm \sqrt{4a_s}$

FERMIONS: $\Gamma^+ = N\left[\sqrt{1 - \frac{8a_s C_F}{N^2}} - 1\right]$ $\quad NS^+$ ('83)

KIRSCHNER, LIPATOV '83

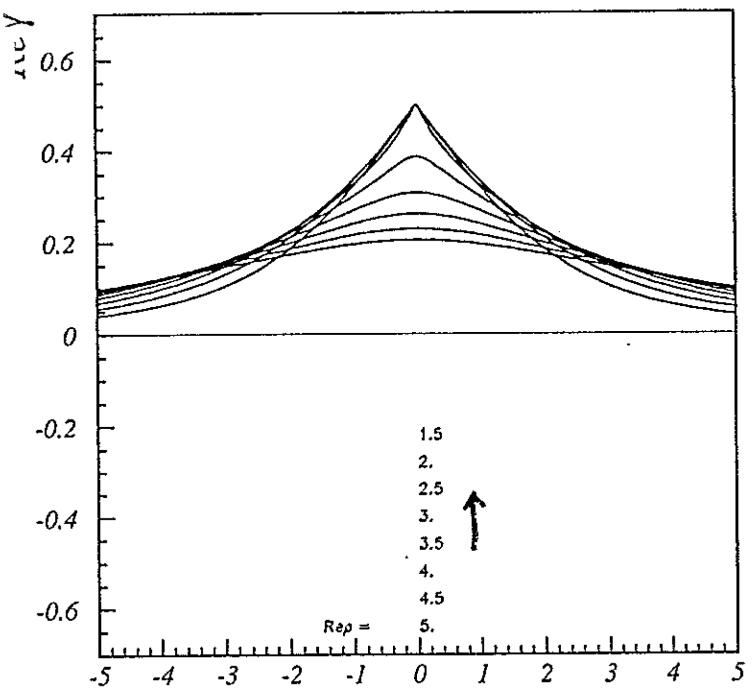
$\Gamma^- = -N\left\{1 - \sqrt{1 - \frac{8a_s C_F}{N^2}\left[1 - \frac{8a_s N_c}{N}\frac{d}{dN}\phi(z)\right]}\right\}, \quad NS^-$

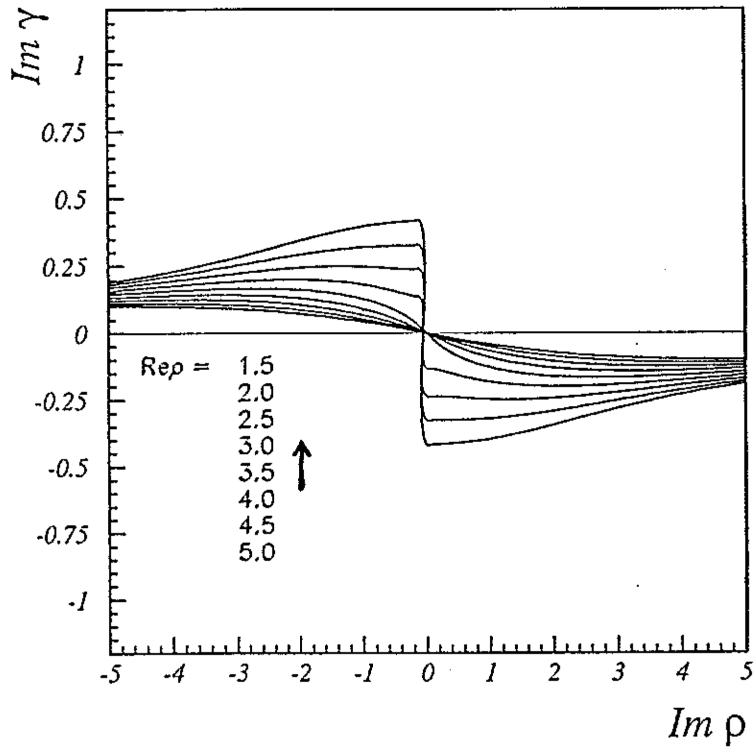
$\phi(z) = \log\left[e^{z^2/4} D_{-\frac{1}{2N_c^2}}(z)\right], \quad z = N/\sqrt{2N_c a_s}$

LO:

The behaviour of $\gamma_c(\rho)$ for $\rho \in C$

$Re\rho \geq 1.5$ J.B. ('94)

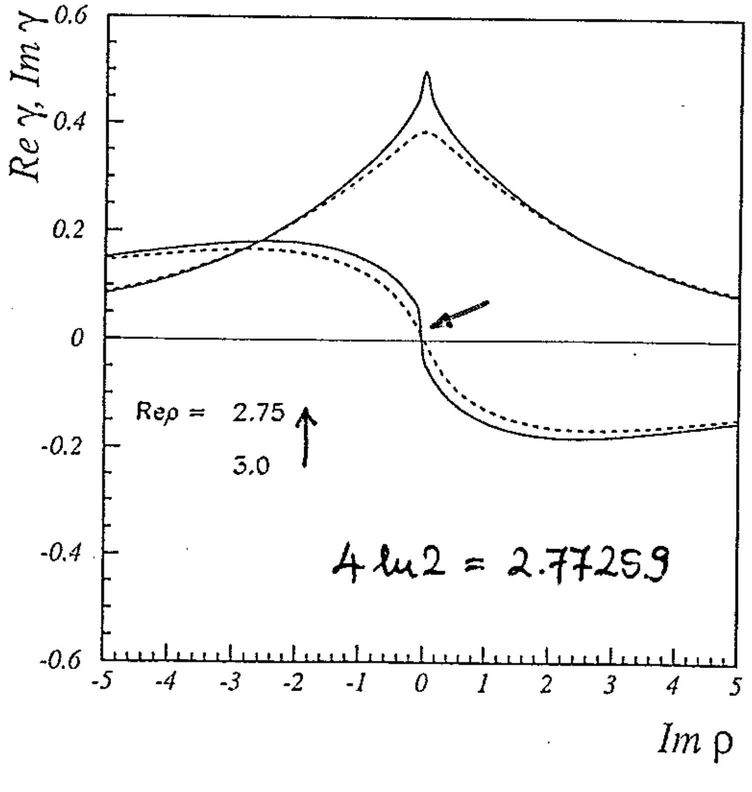




(USE:
ADAPTIVE
NEWTON
ALGORITHM).

$$\rho = \frac{\bar{N}\pi}{\alpha_s N_c}$$

$\bar{N} = N-1$



LOCATION OF THE BRANCH POINTS

$$\rho = \frac{\ell - 1}{\bar{\alpha}_s} = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma).$$

$$1 = \left[-\psi'(\gamma) + \psi'(1-\gamma)\right] \frac{\partial \gamma}{\partial \rho}$$

$$\frac{1}{\partial \gamma / \partial \rho} = \psi'(1-\gamma) - \psi'(\gamma) = 0$$

$$\boxed{\psi'(z) - \frac{\pi^2}{2} \frac{1}{\sin^2 \pi z} = 0}$$

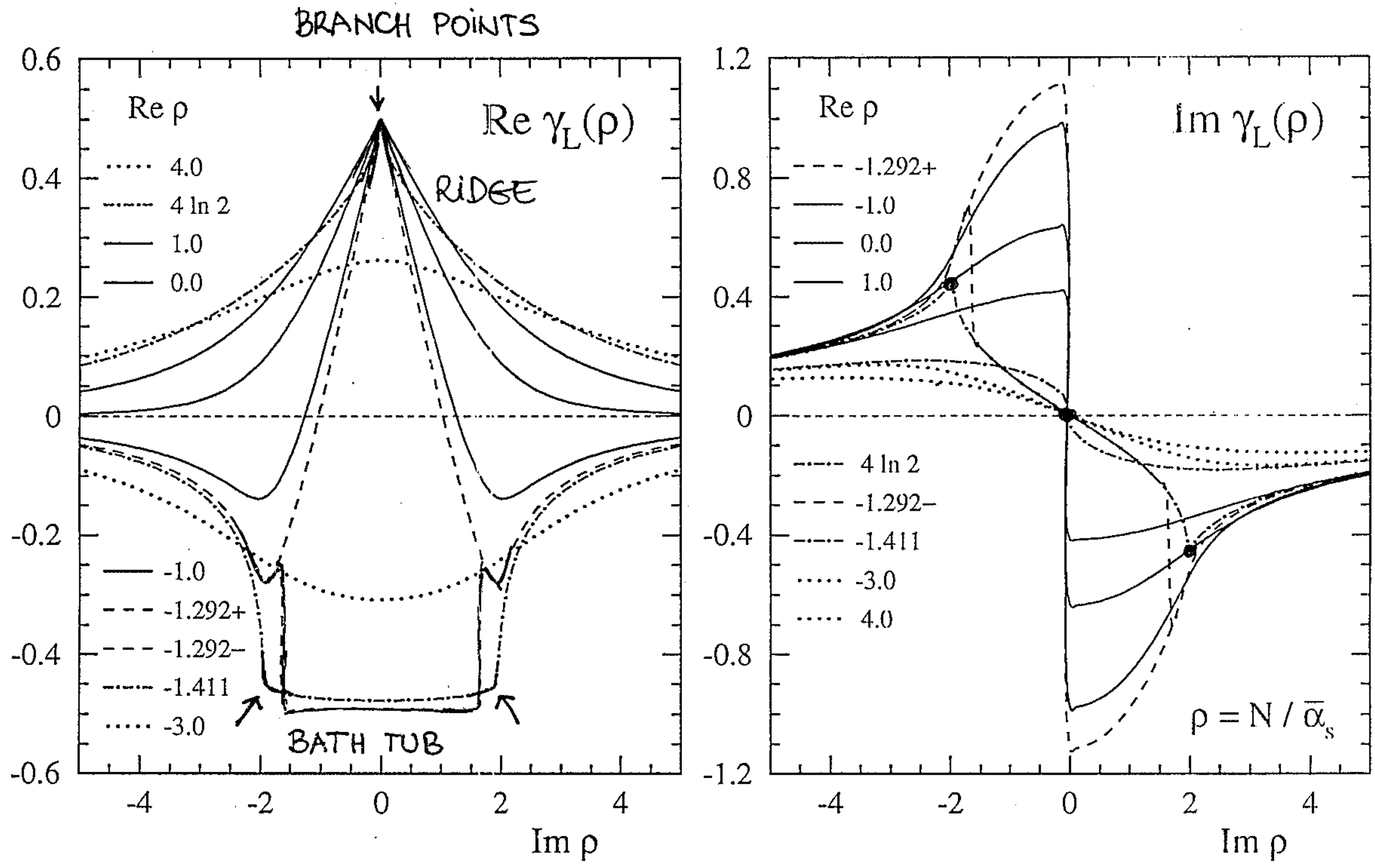
$$\gamma_1 = \frac{1}{2} + 0i \qquad \rho_1 = 4 \ln 2$$

$$\gamma_{2,3} = -0.425214 \pm i\, 0.473898$$

$$\rho_{2,3} = -1.4105 \pm i\, 1.9721.$$

K. ELLIS, HAUTMANN, WEBBER '95

ƎB '95


BRANCH POINTS
Re γ_L(ρ)
RIDGE
BATH TUB
Re ρ
4.0
4 ln 2
1.0
0.0
-1.0
-1.292+
-1.292−
-1.411
-3.0
Im ρ
Im γ_L(ρ)
ρ = N / ᾱ_s

COMPLEX N-PLANE

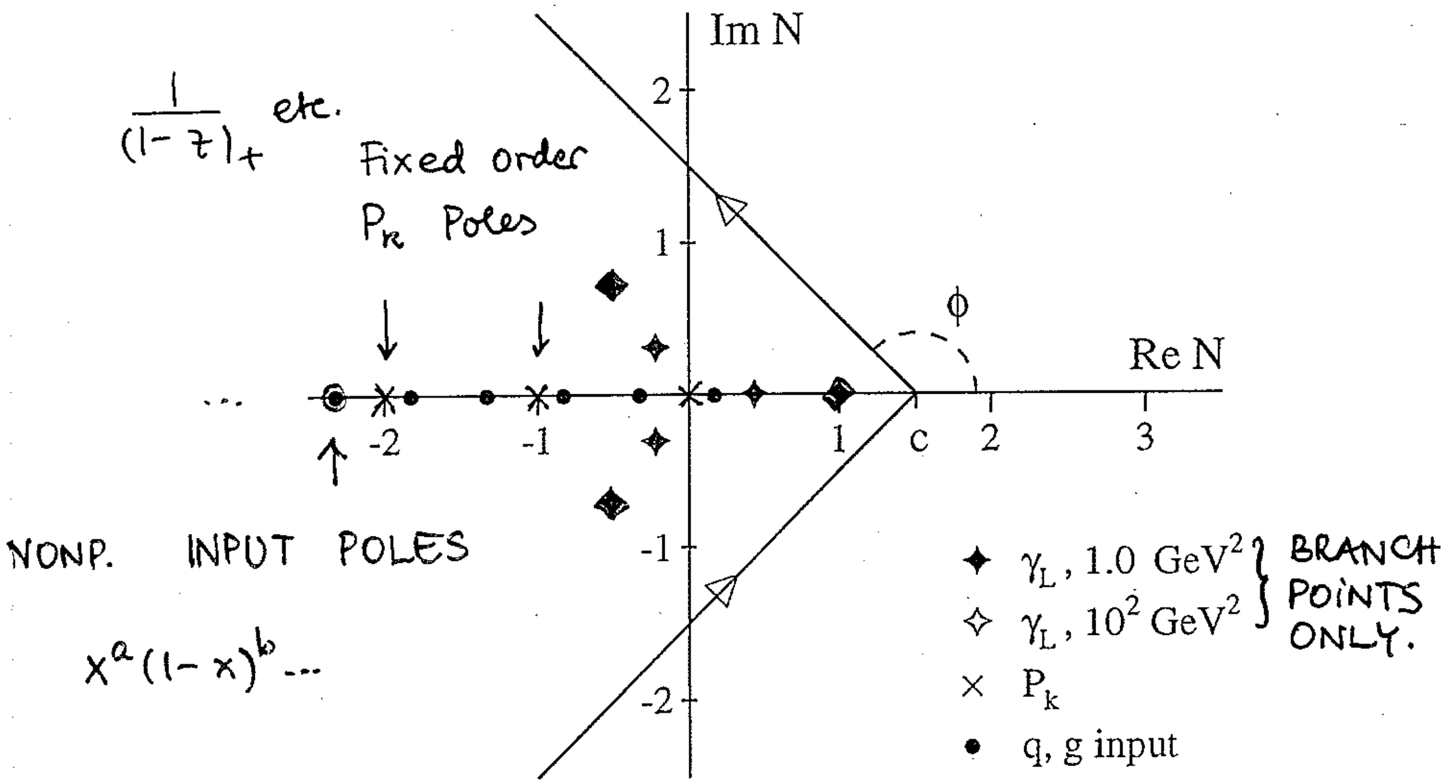
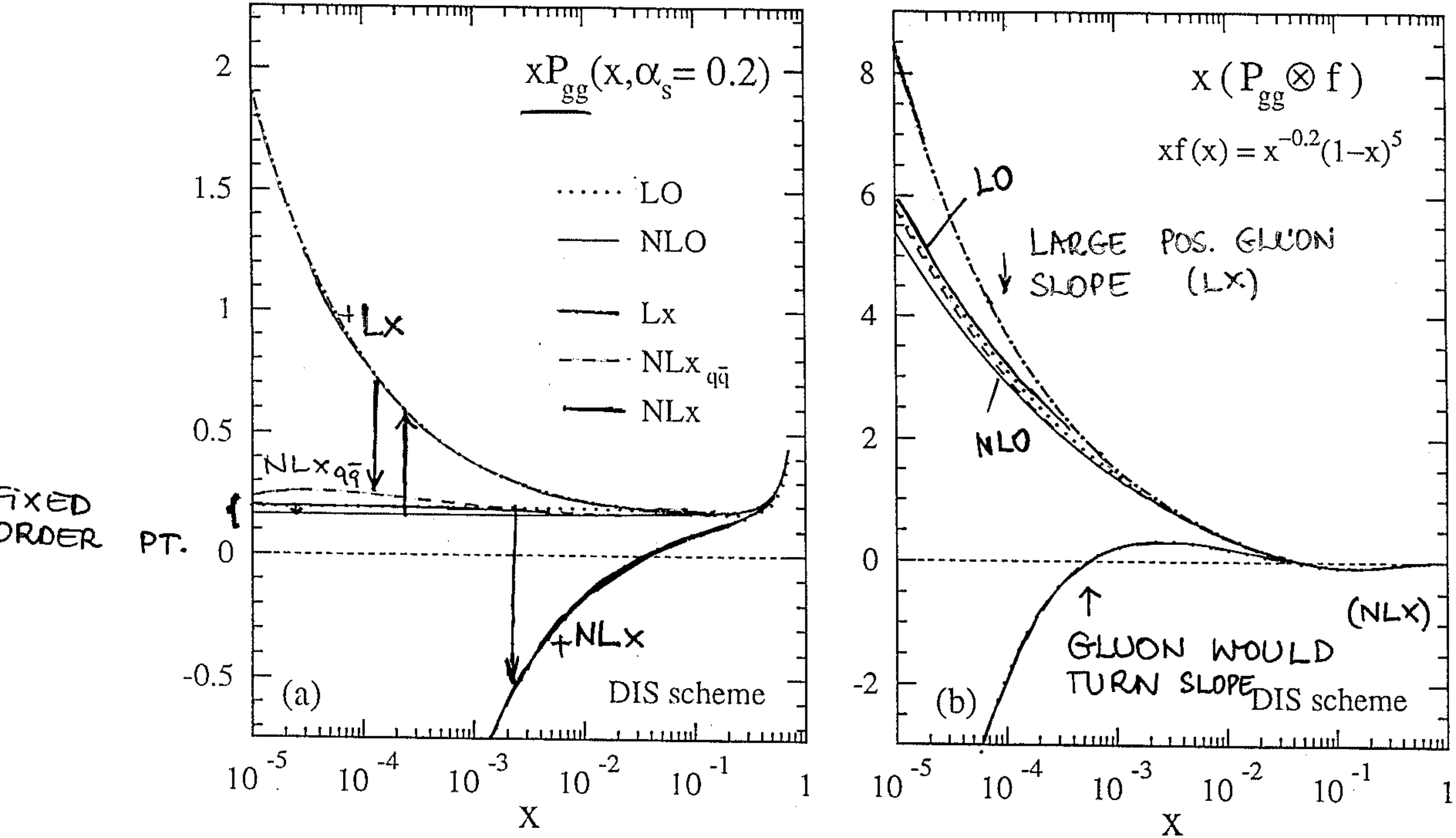


Fig. 4

- SOLUTION OF THE RGE'S IN MELLIN SPACE
- EXACT ACCOUNT FOR ALL COMMUTATION RELATIONS
$[P_{ij}^{\ell}, P_{ij}^{m}] \neq 0$ FOR $\ell \neq m$.



NO LESS SINGULAR TERMS CONSIDERED.

JB, VOGT '98

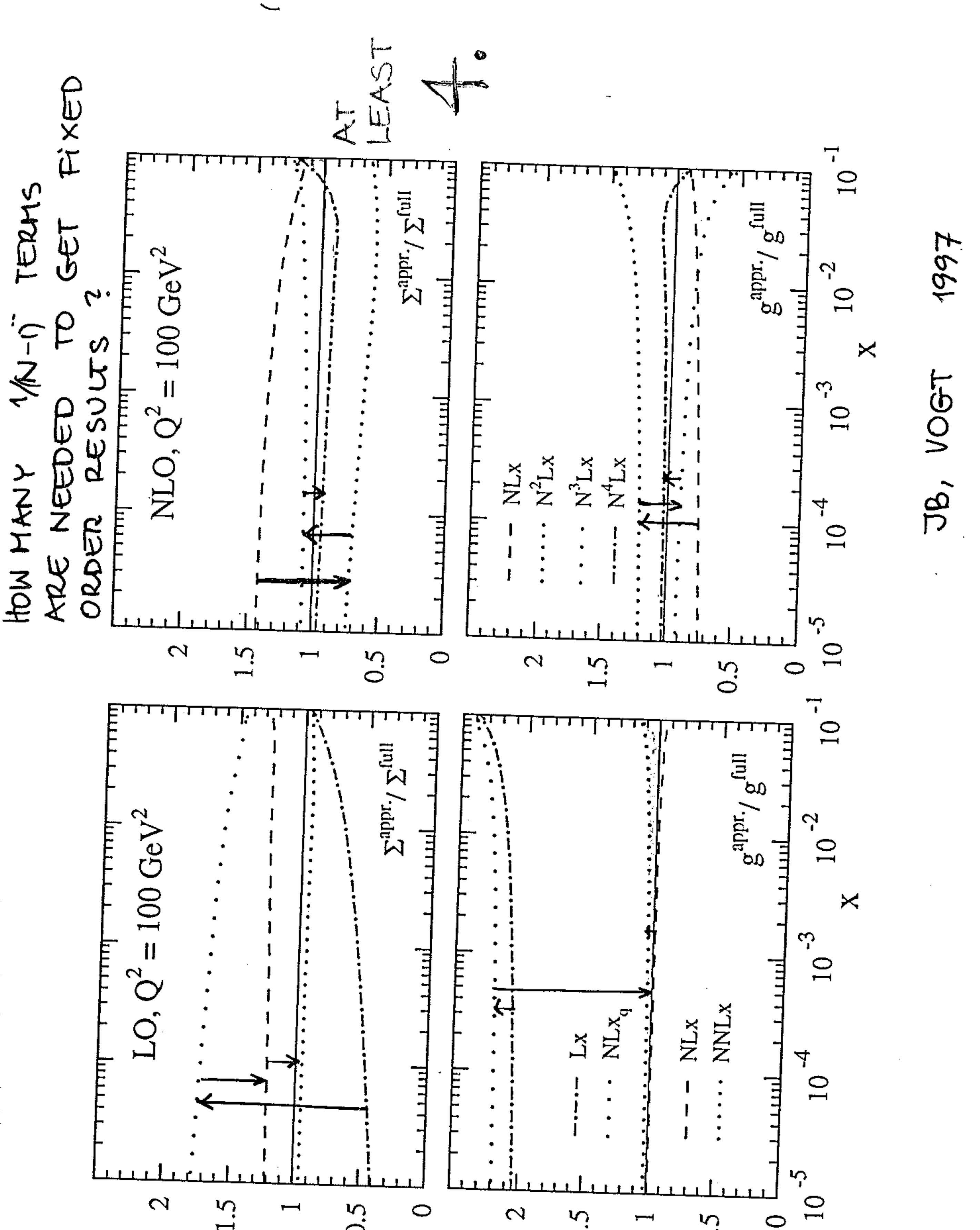
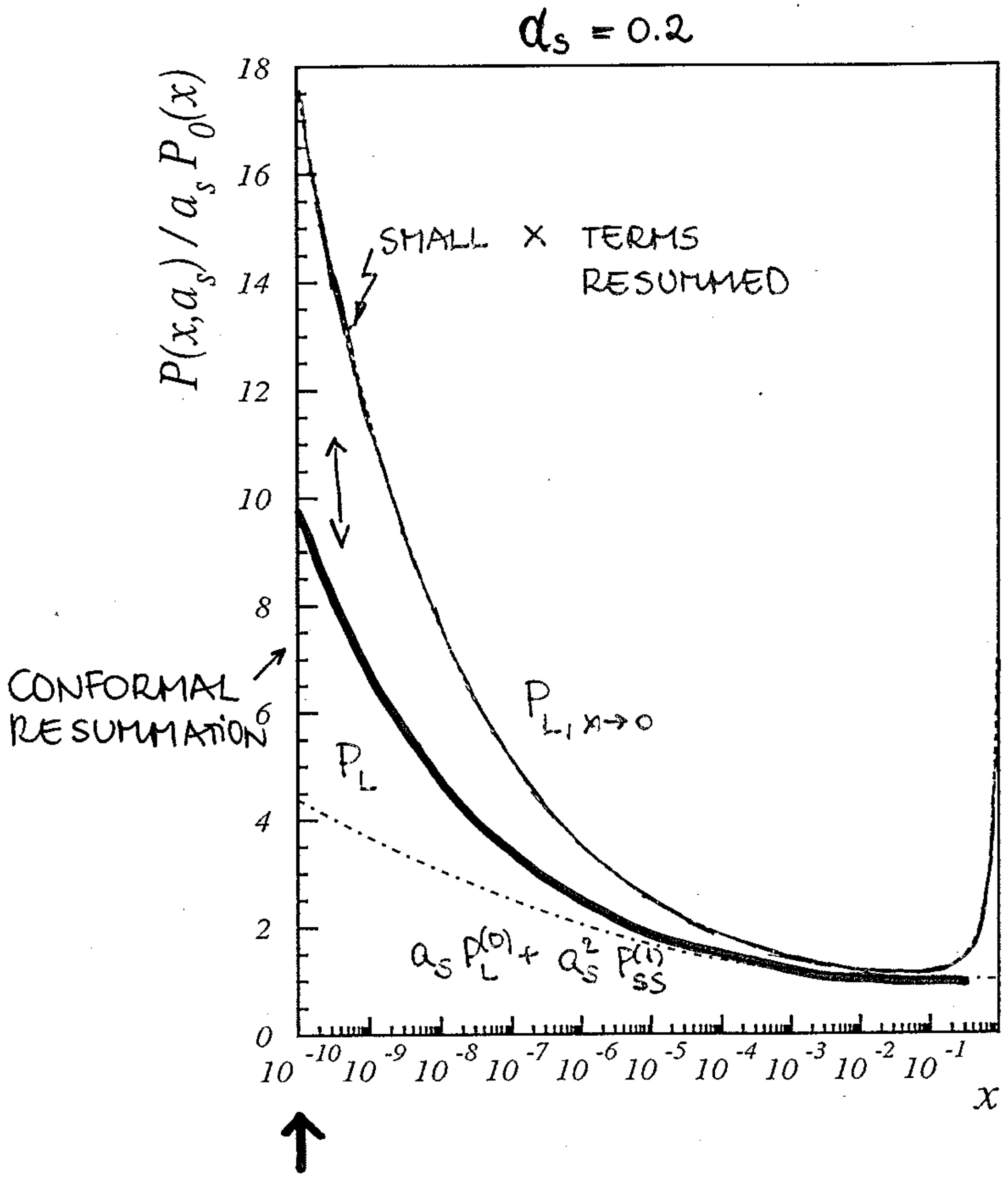

HOW MANY 1/(N-1)^i TERMS ARE NEEDED TO GET FIXED ORDER RESULTS ?
AT LEAST 4.
NLO, Q^2 = 100 GeV^2
LO, Q^2 = 100 GeV^2
Σ^appr./Σ^full
g^appr./g^full
NLx
N^2Lx
N^3Lx
N^4Lx
Lx
NLx_q
NNLx
x
JB, VOGT 1997

Fig. 6

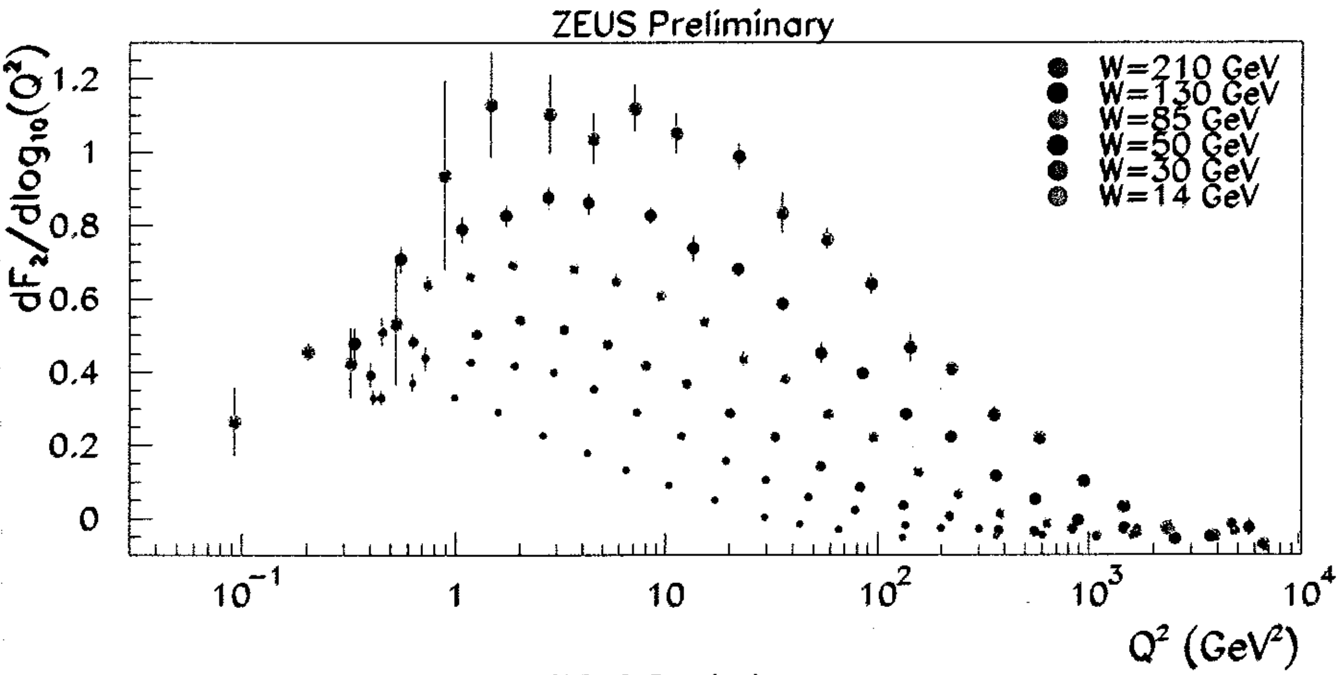


$\alpha_s = 4\pi \cdot a_s$

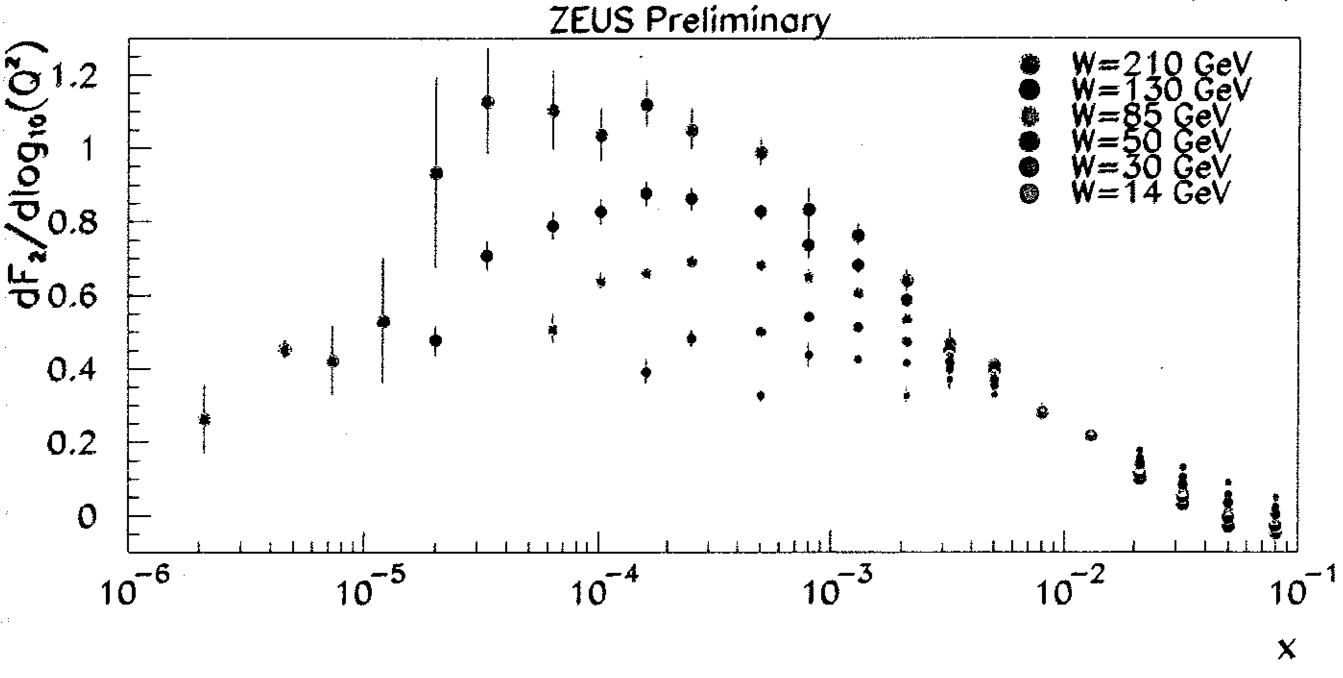
Figure 1: Fixed–order and resummed splitting functions $P(x, a_s)$ normalized to $a_s P_{SS}^{(0)}(x)$ for $\alpha_s = 0.2$. Dash-dotted line : $P = a_s P_L^{(0)} + a_s^2 P_{SS}^{(1)}$ Eqs. (7), (8). Solid line : $P = P_L$, Eq. (3). Dashed line : $P = P_{L,x\to 0}$, Eq. (11).

JB, VAN NEERVEN '98

$\phi^3_{D=6}$.



ZEUS Preliminary
W=210 GeV
W=130 GeV
W=85 GeV
W=50 GeV
W=30 GeV
W=14 GeV
$dF_2/dlog_{10}(Q^2)$
Q^2 (GeV^2)



ZEUS Preliminary
W=210 GeV
W=130 GeV
W=85 GeV
W=50 GeV
W=30 GeV
W=14 GeV
$dF_2/dlog_{10}(Q^2)$
x

7. Future Developments

EXPERIMENT: HIGH LUMINOSITY RUNS AT HERA
(UNPOL.) 2001–2006

→ PROBE SHORTEST DISTANCES
NEW STRUCTURES ?

LATER: POSSIBLY TESLA ⊗ HERA,
WE HOPE.

2006 :

$$\boxed{\Delta\alpha_s = \pm 2\%}$$

- PRECISE QUARK & GLUON DENSITIES.

THEORY: 3 LOOP ANOMALOUS DIMENSIONS

↷ $\Delta\alpha_s^{THY} = 1..2\%$

EXPERIMENT (POL):

$\Delta G(x, Q^2)$ MORE PRECISE
$g_2(x, Q^2)$ VERY PRECISE

↳ CLEAR WINDOW TO TWIST 3

FIELD THEORY:

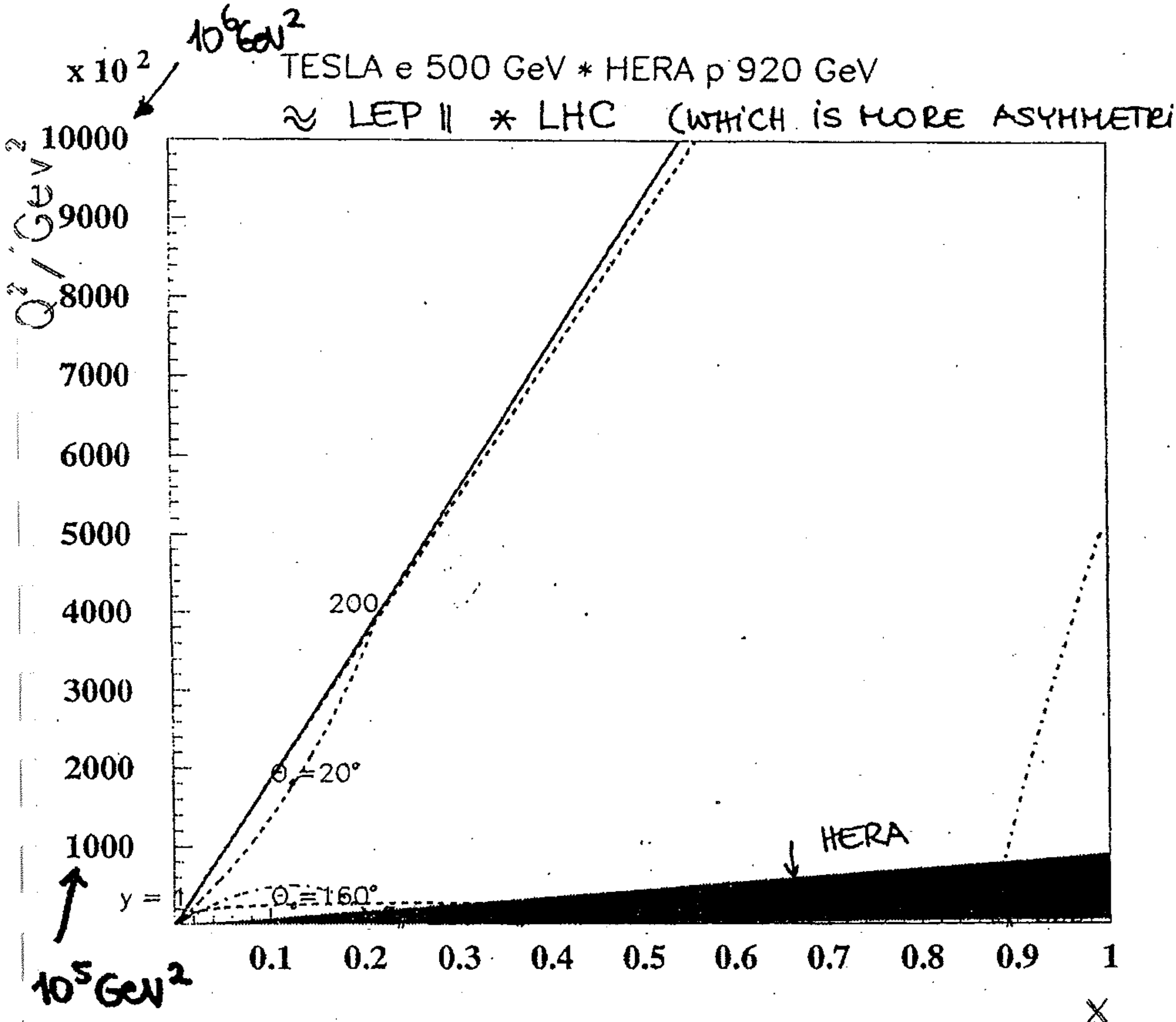
- NEW TECHNOLOGIES FOR HIGHER ORDERS
  MULTIPLE HARMONIC SUMS, EULER-ZAGIER ζ VALUES.
  MASSLESS, MASSIVE CONTRIBUTIONS.

- RECOMBINATION CORRECTIONS

- HIGHER TWIST TERMS

- NON-FORWARD SCATTERING

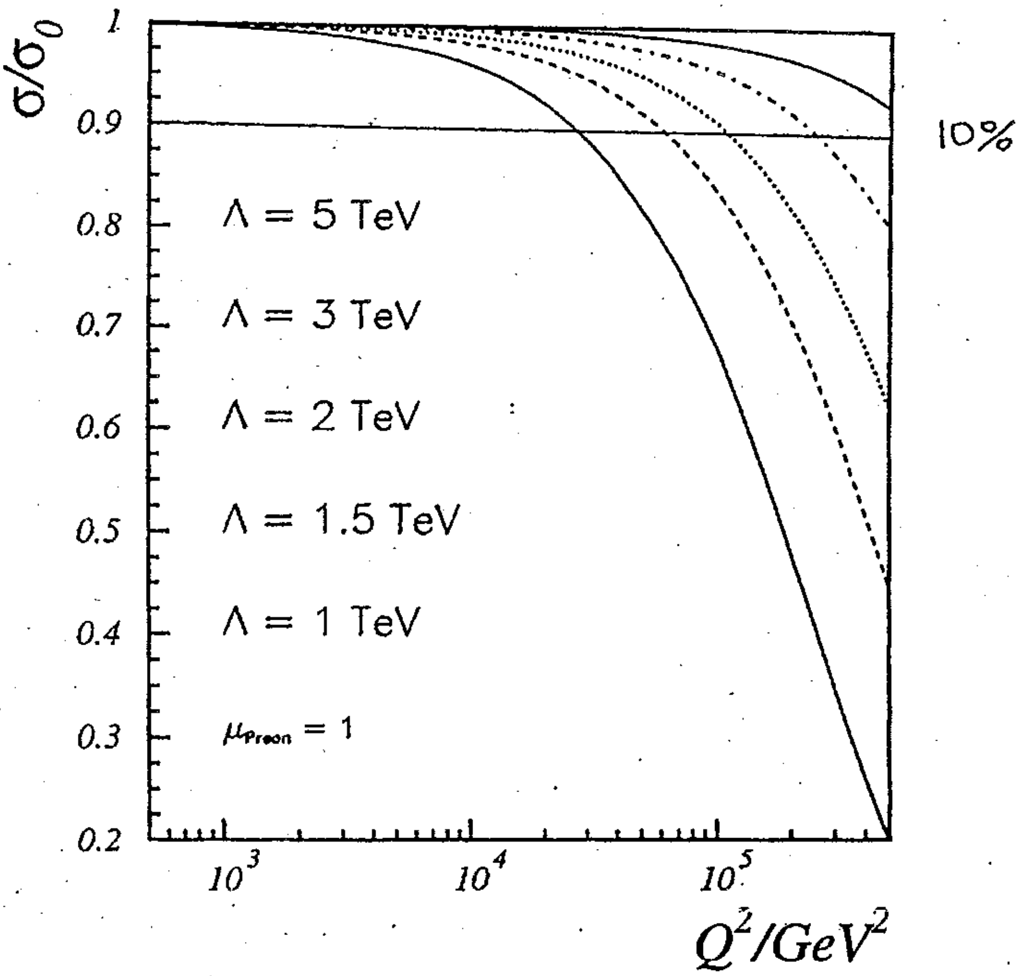
- LGT : OME'S FOR STRUCTURE FUNCTIONS



M. KLEIN

Q^2 RISES BY x 100 !

HERA×TESLA, LEP×LHC : $\sqrt{s} \sim 1.8\,\mathrm{TeV}$; $Q^2_{\max} \approx 5 \times 10^5\,\mathrm{GeV}^2$.



Still higher scales :

- $Q^2 \gtrsim \Lambda^2_{\mathrm{Preon}}$: one may expect also resonant contributions depending on the yet unknown binding force
- $Q^2 \gg \Lambda^2_{\mathrm{Preon}}$: New Scaling: elastic lepton–preon scattering. if both the leptons and the photon remain pointlike