TABLE 1
Comparison of one-loop and two-loop kernels at small-x

	one-loop	two-loop
$P(x)^{\mathrm{F}}{}_{\mathrm{F}}$	$C_{\mathrm{F}}\frac{1+x^2}{1-x}+\ldots$	$2N_{\mathrm{f}}T_{\mathrm{R}}C_{\mathrm{F}}\frac{20}{9}\frac{1}{\underline{x}}+\ldots$
$P(x)^{\mathrm{F}}{}_{\mathrm{G}}$	$2N_{\mathrm{f}}T_{\mathrm{R}}[x^2+(1-x^2)]$	$2N_{\mathrm{f}}T_{\mathrm{R}}C_{\mathrm{G}}\frac{20}{9}\frac{1}{\underline{x}}+\ldots$
$P(x)^{\mathrm{G}}{}_{\mathrm{F}}$	$C_{\mathrm{F}}\frac{1}{x}[1+(1-x)^2]$	$2N_{\mathrm{f}}T_{\mathrm{R}}C_{\mathrm{F}}\left(-\frac{20}{9}\frac{1}{\underline{x}}\right)+C_{\mathrm{F}}C_{\mathrm{G}}\frac{1}{x}+\ldots$
$P(x)^{\mathrm{G}}{}_{\mathrm{G}}$	$2C_{\mathrm{G}}\left[\frac{1}{x}+\frac{1}{1-x}-2+x-x^2\right]+\ldots$	$\frac{2N_{\mathrm{f}}T_{\mathrm{R}}}{\underline{x}}\left[-\frac{23}{9}C_G+\frac{2}{3}C_F\right]+\ldots$

TENDENCY AT LOW X: $\sim \frac{1}{x}$

TUNG '89

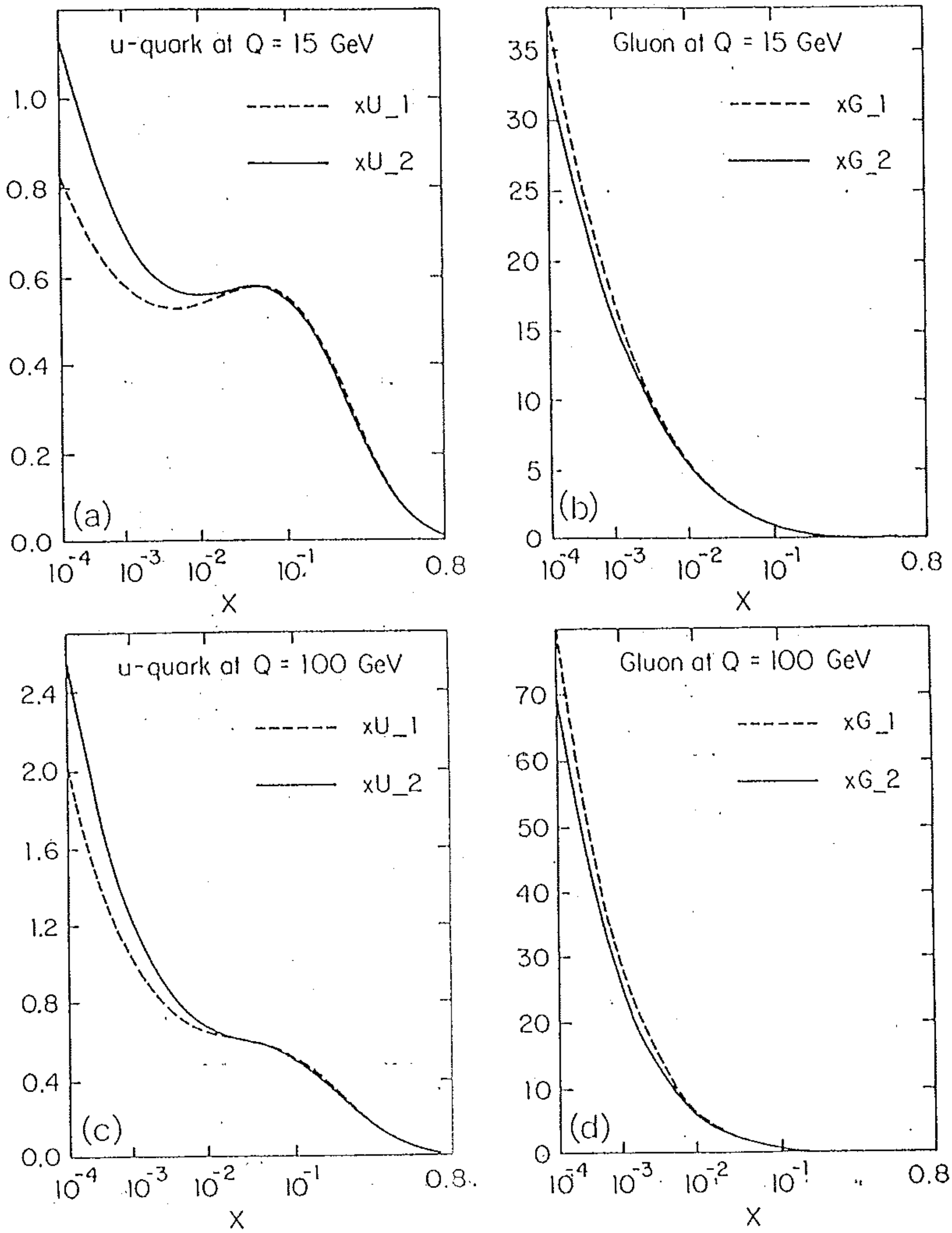


Fig. 1. Comparison of first- and second-order evolved parton distributions. Plotted are x times the probability distributions. Parton species and Q-values are as labeled. Initial distributions at $Q = 4.0$ GeV are taken from EHLQ set 1.

THE TREATMENT OF CONSTANT TERMS :

LOOP CORRECTIONS : log - SINGULARITIES

$\sim m_q^2 \to 0$

+ FINITE TERMS

DIFFERENT FACTORIZATION SCHEMES :

1) **DIS**: PRESERVE THE SHAPE OF

$q(x, Q^2) + \bar{q}(x, Q^2)$

TO ALL ORDERS, I.E. NO ADD. CONST. TERMS OCCUR.

$\longrightarrow$ $F_2 = \sum_i e_i^2 (q + \bar{q})$ remains uncorrected in shape.

2) $\overline{MS}$: UV + IR DIVERGENCES TREATED VIA MS-BAR cf. SECT. 3.

5-19

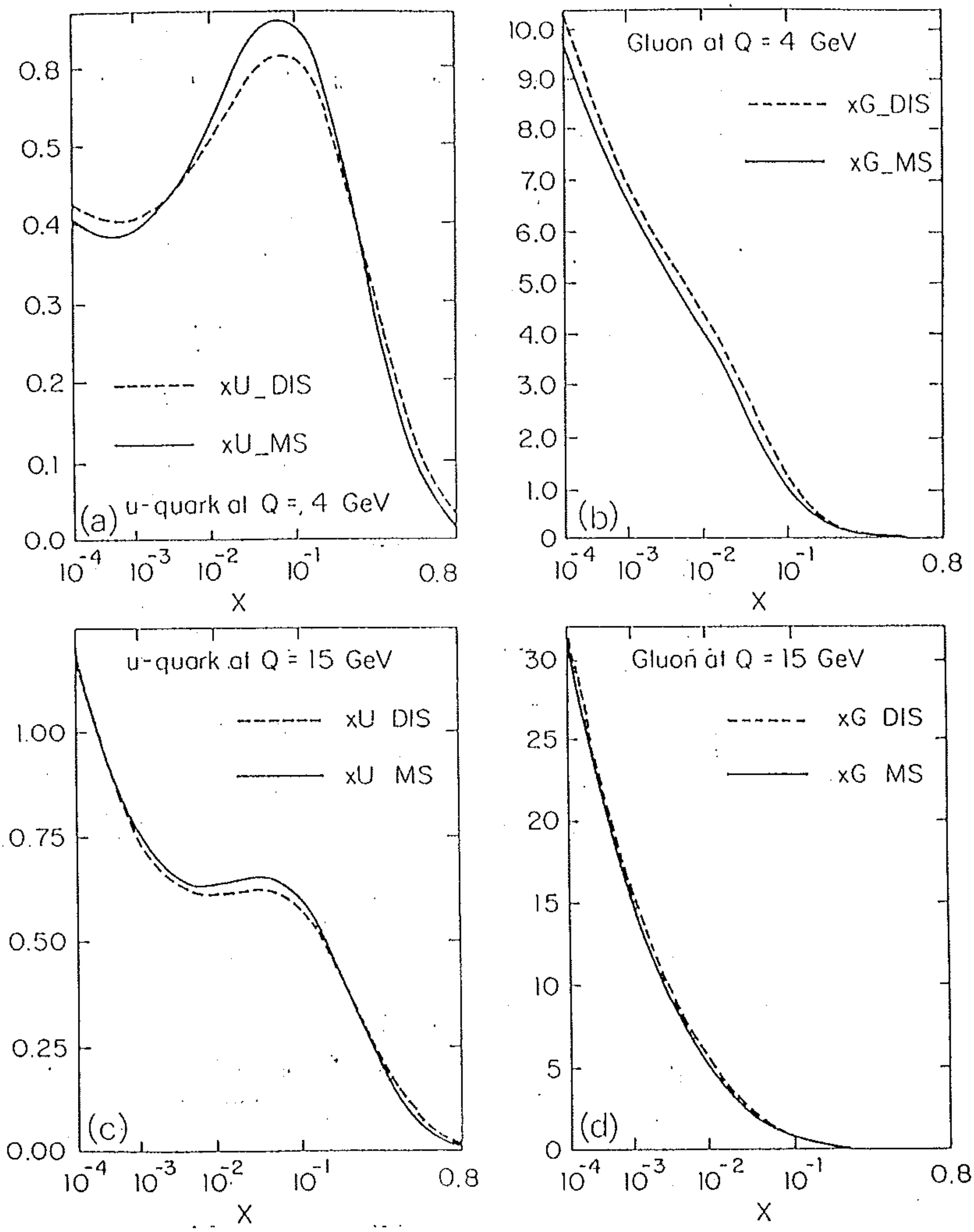


Fig. 3. Comparison of DIS-scheme and MS-bar scheme parton distributions. Plotted are x times the probability distributions. Parton species and Q-values are as labeled. Initial distributions at $Q = 4.0$ GeV are taken from EHLQ set 1.

F_2 AND F_L : TWIST 2

$$F_2(x,Q^2) = \mathscr{F}(x,Q^2) + \frac{\alpha_s}{4\pi}\int_x^1 \frac{dy}{y} f_q^2(x/y)\mathscr{F}(y,Q^2)$$

$$+ \frac{\alpha_s}{4\pi}\int_x^1 \frac{dy}{y} f_G^2(x/y)\delta_\psi^2 \mathscr{G}(y,Q^2), \qquad (3)$$

$\overline{\text{MS}}$

Sanchez-G. 1990

where f_q^2 and f_G^2 are the finite parts using the universal scheme in the one-loop calculation of F_2 [6]*,

$$f_q^2(x) = 2C_F x\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\frac{1}{(1-x)_+}\right.$$

$$\left. - \frac{1+x^2}{1-x}\ln x + 3 + 2x - \left(\frac{9}{2} + \frac{\pi^2}{3}\right)\delta(1-x)\right],$$

$$f_G^2(x) = 2n_f x\left[(x^2 + (1-x)^2)\left(\ln\left(\frac{1-x}{x}\right) - 1\right) + 6x(1-x)\right]. \qquad (4)$$

cf. DIS TRANSLAT. PRESCR. → FACTORIZ.

$$F_L(x,Q^2) = \int_x^1 \frac{dy}{y} K^{NS}(y,Q^2)\mathscr{F}(x/y,Q^2)$$

$$+ \int_x^1 \frac{dy}{y} K^S(y,Q^2)\mathscr{F}^S(x/y,Q^2) + \int_x^1 \frac{dy}{y} K^G(y,Q^2)\mathscr{G}(x/y,Q^2), \qquad (1)$$

where

$$\mathscr{F}(x,Q^2) = \sum_{i=1}^{n_f} e_i^2 x\big(q_i(x,Q^2) + \bar{q}_i(x,Q^2)\big),$$

$$\mathscr{F}^S(x,Q^2) = \delta_\psi^2 \sum_{i=1}^{n_f} x\big(q_i(x,Q^2) + \bar{q}_i(x,Q^2)\big),$$

$$\mathscr{G}(x,Q^2) = xG(x,Q^2), \qquad (2)$$

$$K^{NS}(x,Q^2) = \frac{\alpha_s}{4\pi} f_{L,q}^{(1)}(x) + \left(\frac{\alpha_s}{4\pi}\right)^2 f_{L,q}^{NS(2)}(x),$$

$$K^S(x,Q^2) = \left(\frac{\alpha_s}{4\pi}\right)^2 f_{L,q}^{S(2)}(x),$$

$$K^G(x,Q^2) = \frac{\alpha_s}{4\pi}\delta_\psi^2 f_{L,G}^{(1)}(x) + \left(\frac{\alpha_s}{4\pi}\right)^2 \delta_\psi^2 f_{L,G}^{(2)}(x), \qquad (5)$$

5/29

$$f^{(1)}_{L,q}(x) = 4C_F x^2, \tag{6}$$

$$f^{(1)}_{L,G}(x) = 8n_f x^2(1-x), \tag{7}$$

$$
\begin{aligned}
f^{NS(2)}_{L,q}(x) = {} & 4C_F(C_A - 2C_F)x^2 \\
& \times \Bigg[4\frac{6 - 3x + 47x^2 - 9x^3}{15x^2}\ln x - 4\,\mathrm{Li}_2(-x)(\ln x - 2\ln(1+x)) - 8\zeta(3) \\
& \quad - 2\ln^2 x\ln(1-x^2) + 4\ln x\ln^2(1+x) - 4\ln x\,\mathrm{Li}_2(x) \\
& \quad + \tfrac{2}{5}(5 - 3x^2)\ln^2 x - 4\frac{2 + 10x^2 + 5x^3 - 3x^5}{5x^3} \\
& \quad \times (\mathrm{Li}_2(-x) + \ln x\ln(1+x)) + 4\zeta(2)\left(\ln(1-x^2) - \frac{5 - 3x^2}{5}\right) \\
& \quad + 8S_{1,2}(-x) + 4\,\mathrm{Li}_3(x) + 4\,\mathrm{Li}_3(-x) - \tfrac{23}{3}\ln(1-x) \\
& \quad - \frac{144 + 294x - 1729x^2 + 216x^3}{90x^2}\Bigg] \\
& + 8C_F^2 x^2\left[\mathrm{Li}_2(x) + \ln^2\left(\frac{x}{1-x}\right) - 3\zeta(2) - \frac{3 - 22x}{3x}\ln x \right. \\
& \left. + \frac{6 - 25x}{6x}\ln(1-x) - \frac{78 - 355x}{36x}\right] - \tfrac{8}{3}C_F n_f x^2\left[\ln\left(\frac{x^2}{1-x}\right) - \frac{6 - 25x}{6x}\right],
\end{aligned}
$$

$$
\begin{aligned}
f^{S(2)}_{L,q}(x) = {} & \tfrac{16}{9}C_F n_f\Big[3(1 - 2x - 2x^2)(1-x)\ln(1-x) \\
& + 9x^2(\mathrm{Li}_2(x) + \ln^2(x) - \zeta(2)) + 9x(1 - x - 2x^2)\ln x \\
& - 9x^2(1-x) - (1-x)^3\Big],
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
f^{(2)}_{L,G}(x) = {} & 16C_A n_f x^2\left[\frac{1 - 3x - 27x^2 + 29x^3}{3x^2}\ln(1-x) - 2(1-x)\ln x\ln(1-x) \right. \\
& + 2(1+x)\mathrm{Li}_2(-x) + 4\,\mathrm{Li}_2(x) + 3\ln^2 x + 2(x-2)\zeta(2) \\
& + (1-x)\ln^2(1-x) + 2(1+x)\ln x\ln(1+x) \\
& \left. + \frac{24 + 192x - 317x^2}{24x}\ln x + \frac{-8 + 24x + 501x^2 - 517x^3}{72x^2}\right] \\
& 16C_F n_f x^2\left[\mathrm{Li}_2(x) + 2\frac{5 + 3x^2}{15}\ln^2(x) - \frac{1 + 3x - 4x^2}{2x}\ln(1-x) \right. \\
& + \frac{-2 + 10x^3 - 12x^5}{15x^3}(\mathrm{Li}_2(-x) + \ln x\ln(1+x)) \\
& - \frac{5 + 12x^2}{15}\zeta(2) + \frac{4 + 13x + 78x^2 - 36x^3}{30x^2}\ln x \\
& \left. - \frac{4 - 16x - 213x^2 + 225x^3}{30x^2}\right].
\end{aligned}
\tag{10}
$$

CORRECTED
BY VAN NEERVEN
& ZIJLSTRA
PHYS. LETT. B 273 (1991) 476
Dec 26 1991.

VAN NEERVEN/
ZIJLSTRA.

Since we disagree with the result for the longitudinal gluonic Wilson coefficient $xC_L^{(2),G}$ given in eq. (10) of ref. [14], it is appropriate to give our result below. In the $\overline{\rm MS}$ scheme it reads

$$\begin{aligned}
C_L^{(2),G}(x,1) = n_f C_F \Big[& 16x[\mathrm{Li}_2(1-x) + \ln x \ln(1-x)] + \left(-\tfrac{32}{3}x + \tfrac{64}{3}x^3 + \frac{32}{15x^2}\right)[\mathrm{Li}_2(-x) + \ln x \ln(1+x)] \\
& + (8 + 24x - 32x^2)\ln(1-x) - (\tfrac{32}{3}x + \tfrac{32}{3}x^3)\ln^2 x + \frac{1}{15}\left(-104 - 624x + 288x^2 - \frac{32}{x}\right)\ln x \\
& + \left(-\tfrac{32}{3}x + \tfrac{64}{3}x^3\right)\zeta(2) - \tfrac{128}{15} - \tfrac{304}{5}x + \tfrac{336}{5}x^2 + \frac{32}{15x}\Big] \\
+ n_f C_A \Big[& -64x\,\mathrm{Li}_2(1-x) + (32x + 32x^2)[\mathrm{Li}_2(-x) + \ln x \ln(1+x)] + (16x - 16x^2)\ln^2(1-x) \\
& + (-96x + 32x^2)\ln x \ln(1-x) + \left(-16 - 144x + \tfrac{464}{3}x^2 + \frac{16}{3x}\right)\ln(1-x) + 48x\ln^2 x \\
& + (16 + 128x - 208x^2)\ln x + 32x^2\zeta(2) + \tfrac{16}{3} + \tfrac{272}{3}x - \tfrac{848}{9}x^2 - \frac{16}{9x}\Big]. \qquad (9)
\end{aligned}$$

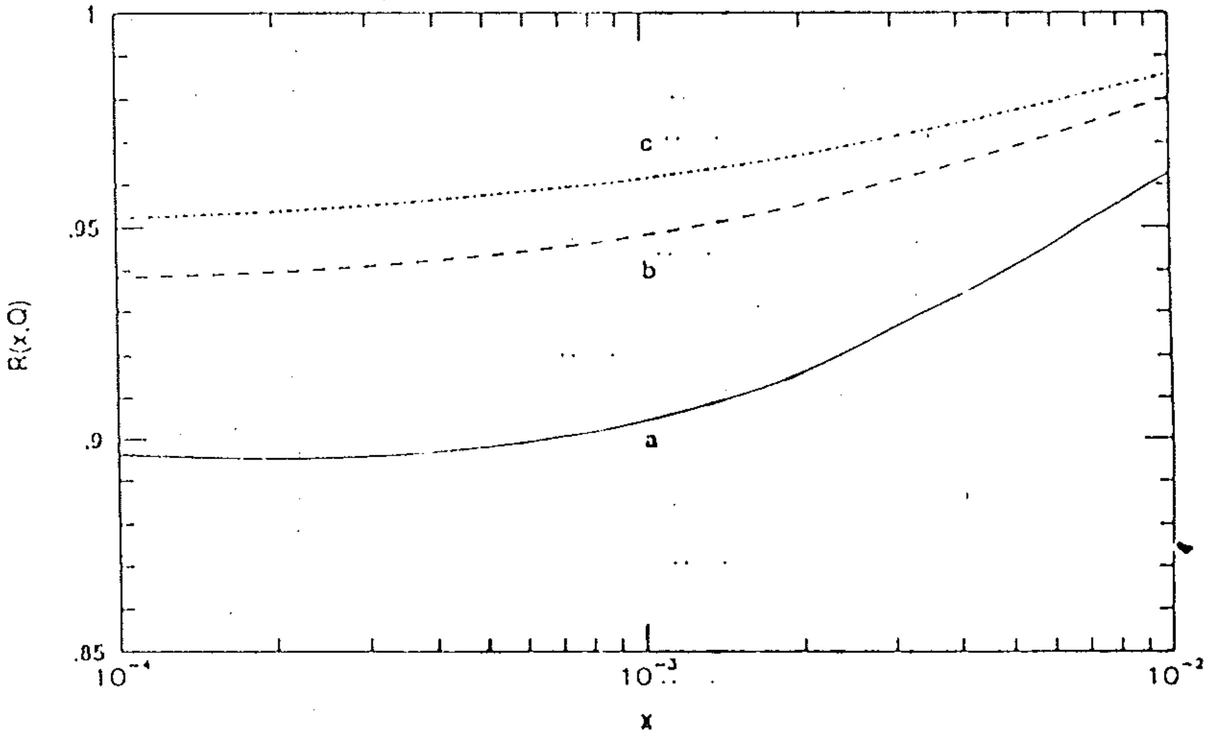


Fig. 2. Correction R due to the second order Wilson coefficient in the region $10^{-4} \leq x \leq 10^{-2}$. (a) $Q=5$ GeV, (b) $Q=10$ GeV, (c) $Q=15$ GeV.

EXTRACTING THE GLUON DISTRIBUTION FROM F_L:

COOPER-SARKAR *et al.* 1988:

LO:

$$F_L(x, Q^2) = \frac{\alpha_s(Q^2)}{\pi} \left\{ \frac{4}{3} \int_x^1 \frac{dy}{y} \left(\frac{x}{y}\right)^2 F_2(y, Q^2) \right.$$
$$\left. + 2 \sum_q e_q^2 \int_x^1 \frac{dy}{y} \left(\frac{x}{y}\right)^2 \left(1-\frac{x}{y}\right) y\, G(y, Q^2) \right\}$$

$$= \frac{4}{3} \frac{\alpha_s(Q^2)}{\pi} \left[I_F(x, Q^2) + \frac{5}{3} I_G(x, Q^2) \right]$$

$$I_F = \int_0^{1-x} dz\, (1-z)\, F_2\left(\frac{x}{1-z}, Q^2\right) \simeq \frac{1}{2} F_2(2x)$$

$$I_G = \frac{1}{\theta}\, xG\left(\frac{x}{\xi}\right), \quad \xi = \frac{1+\delta}{3+\delta}, \quad \theta = (3+\delta)(2+\delta) * \xi^\delta$$

$\theta \simeq 5.85$, $\xi = 0.4$, δ – low x power of xG.

$$\boxed{xG(x, Q^2) \simeq \frac{3}{5} \cdot 5.85 \left\{ \frac{3\pi}{4\alpha_s(Q^2)} F_L(0.4x, Q^2) - \frac{1}{2} F_2(0.8x, Q^2) \right\}.}$$

Similar relations: NTLO

+ incl. low x, but more involved.

TARGET MASS EFFECTS AND HIGHER TWISTS

i) TARGET MASS EFFECTS:

$$\mathcal{O}\left(\frac{4x^2 m_p^2}{Q^2}\right)$$

MASSLESS CASE: ONLY OPERATORS OF SPIN n CONTRIBUTE TO M_n

$$\langle p| O^{NS}_{\mu\nu, n=2} |p\rangle = \frac{i}{2!} \langle p| \bar{q} \frac{\lambda^a}{2} \gamma_\mu D^\nu q - \frac{1}{4} g_{\mu\nu} \bar{q} q |p\rangle$$

$$\hookrightarrow -\frac{1}{4} M^2 g_{\mu\nu}$$

$\longrightarrow$ ADDITIONAL TERMS $\sim \left(\frac{M^2}{Q^2}\right)^n$; many spins contribute.

HOW TO GET THE SPIN $= n$ CONTRIBUTION FROM M_n ?

$$T(j(x)\,j(0)) \longrightarrow \sum_n (-x^2+i0)^{\frac{1}{2}(d_n-n-2d_j)}\, x^{\mu_1}\cdots x^{\mu_n}\, O^{(d_n)}_{\mu_1\cdots\mu_n}$$

$$T(\nu,Q^2) = \frac{i}{\pi} p_0 V \int dx\, e^{iqx} \langle p|T(j(x)\,j(0))|p\rangle$$

$\nu = p\cdot q,\quad Q^2 = -q^2.$
$m_N = 1$

$$= \frac{i}{\pi}\sum_n \{p_{\mu_1}\cdots p_{\mu_n} - \text{trace terms}\}$$

$$\times\left(\frac{\partial}{i\partial q_{\mu_1}}\right)\cdots\left(\frac{\partial}{i\partial q_{\mu_n}}\right)\sum_\alpha \langle p|O_n^{(\alpha)}|p\rangle \int dx\, e^{iqx}(-x^2+i0)^{\alpha'}$$

EXPAND $T(\nu,Q^2)$ IN FOUR DIMENSIONAL SPHERICAL HARMONICS :

$$T(\nu,Q^2) = \frac{1}{\pi}\sum_{n\ \text{even}} C_n^1\left(\frac{i\nu}{Q}\right)\sin\frac{\pi}{2}(n+1)\,\frac{1}{Q^n}\,\mu_n(Q^2)$$

$$\mu_n(Q^2) = \frac{i\,Q^{2n}}{\sin\frac{\pi}{2}(n+1)}\left(\frac{\partial}{i\partial Q^2}\right)^n \int dx\, e^{iqx}\sum_\alpha (-x^2+i0)^{\alpha'}\langle p|O_n^{(\alpha)}|p\rangle$$

$C_n^1(\xi)$ GEGENBAUER POLYNOMIAL

NACHTMANN '73, '74.

$\mu_n(Q^2)$ HAS <u>ONLY</u> SPIN n CONTRIBUTIONS.

$$\mu_n^{(2)}(Q^2) = \int_0^1 dx \, \frac{\xi^{n+1}}{x^3} K^{(2)}(n,x,Q^2) \, F_2(x,Q^2)$$

$$\mu_n^{(3)}(Q^2) = \int_0^1 dx \, \frac{\xi^{n+1}}{x^2} K^{(3)}(n,x,Q^2) \, F_3(x,Q^2)$$

$$K_{(2)}(n,x,Q^2) = \left[n^2 + 2n + 3 + 3(n+1)\left(\frac{2x}{\xi} - 1\right) + n(n+2)\left(-\frac{4x}{\xi}\right)\left(1 - \frac{x}{\xi}\right)\right]$$

$$/ (n+2)(n+3)$$

$$K_{(3)}(n,x,Q^2) = \frac{1 + (n+1)\left(\frac{2x}{\xi} - 1\right)}{(n+2)} \qquad \xi = \frac{2x}{1 + \sqrt{1 + \frac{4M^2x^2}{Q^2}}} = \frac{2x}{1+r}$$

OTHER REPRESENTATION: FINITE m_p-EFFECT:
GEORGI, POLITZER & DE RUJULA

$$M^{(2)}(n,Q^2) \underset{\substack{\text{TWIST 2} \\ \text{CONTR. OPE}}}{\overset{Df}{:=}} \int_0^1 dx \, x^n \, \mathcal{F}_{2,3}(x,Q^2)$$

WHAT IS F_2, F_1, xF_3 IN TERMS OF $\mathcal{F}_2$?

$$F_2(x,Q^2) = \frac{x^2}{r^3} F(\xi) + 6\frac{M^2}{Q^2} \frac{x^3}{r^2} \int_\xi^1 d\xi' F(\xi') + 12 \frac{M^4}{Q^4} \frac{x^4}{r^5} \int_\xi^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi'')$$

$$2xF_1(x,Q^2) = \frac{x}{r} F(\xi) + 2\frac{M^2}{Q^2} \frac{x^2}{r^2} \int_\xi^1 d\xi' F(\xi') + 2\frac{M^4}{Q^4} \frac{x^3}{r^2} \int_\xi^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi'')$$

$$xF_3(x,Q^2) = \frac{x}{r} F(\xi) + \frac{4M^2}{Q^2} \frac{x^2}{r^5} \int_\xi^1 d\xi' F(\xi')$$

NOT EQUIVALENT TO THE NACHTMANN PROJECTION.

REASON: ONE CAN NOT APPROXIMATE:

$$M_n(Q^2) = \sum_{i,\tau} \left(\frac{1}{Q^2}\right)^{\frac{\tau}{2}-1} \bar{C}^i_{\tau,n}(Q^2)\ \bar{O}^i_{\tau,n}$$

BY: $$M^{(2)}(n,Q^2) = \sum_i \bar{C}^i_{2,n}(Q^2)\ \bar{O}^i_{2,n} \quad :$$

HIGHER TWISTS ARE NEEDED.

CF. BITAR, JOHNSON, TUNG,
ROBERTS

ii) REMARKS ON HIGHER TWIST TERMS

TWIST 2:

OPERATORS OF SPIN n:

$$\bar{q}\gamma^{\mu_1} D^{\mu_2} \dots D^{\mu_n} q$$

$$G^{\mu_1\nu} D^{\mu_2} \dots D^{\mu_{n-1}} G_{\nu}^{\mu_n}.$$

TWIST 4:

n OPERATORS:

$$\bar{q}\gamma^{\mu} D^{\mu_1} \dots G_{\mu\mu_i} \dots D^{\mu_n} q$$

n^2 OPERATORS:

$$\bar{q}\gamma^{\mu_1} D^{\mu_2} \dots D^{\mu_i} q\bar{q}\gamma^{\mu_{i+1}} \dots D^{\mu_n} q$$

AT LEAST:
n times more CONTRIBUTING OPERATORS:

$$M_n(Q^2) \sim M_n^{\tau=2}(Q^2)\left[1 + cn\frac{\Lambda^2}{Q^2}\right]$$

$$\updownarrow \frac{d}{dx}$$

X - SPACE:

$$F_2(x,Q^2) = F_2^{\tau=2}(x,Q^2)\left[1 + \frac{\Lambda^2}{Q^2}\frac{c}{1-x}\right]$$

(EDUCATED GUESS).

HIGHER TWIST INFLUENCE : $x \to 1$

LOW Q^2.

→ WORK BY : POLITZER 1980

R.K. ELLIS, FURMANSKI, PETRONZIO 1981/83

BUCHOVOSTOV, KURAEV, LIPATOV, 1983

FURTHER EFFORTS NEEDED

→ LOW x

HEAVY FLAVOURS AND QCD EVOLUTION

1) QUARK MASS EFFECTS ON α_s:

$\beta_i = \beta_i(N_f^{active}) \longrightarrow N_f = N_f(Q^2)$

$\longrightarrow$ MATCHING BETWEEN FLAVOUR THRESHOLDS REQUIRED.

NAIVE WAY:

$$\alpha^{-1}_{s\,(5)}(Q^2) = \alpha_s^{-1}(Q^2, 5) + \alpha_s^{-1}(m_b^2, 4) - \alpha_s^{-1}(m_b^2, 5)$$

$$\alpha^{-1}_{s\,(3)}(Q^2) = \alpha_s^{-1}(Q^2, 3) + \alpha_s^{-1}(m_c^2, 4) - \alpha_s^{-1}(m_c^2, 3)$$

PROBLEM: NO SMOOTH $\alpha_s(Q^2)$. Fig

THE MATCHING PROCEDURE IS AN EFFECTIVE CHANGE OF THE RENORMALIZATION SCHEME.

MARCIANO 1984:

$$\Lambda_{(3)}/\Lambda_{(4)} = \left(\frac{m_c}{\Lambda_{(4)}}\right)^{2/27}\left[\ln\left(\frac{m_c^2}{\Lambda_{(4)}^2}\right)\right]^{\frac{107}{2025}}$$

$$\Lambda_{(4)}/\Lambda_{(5)} = \left(\frac{m_b}{\Lambda_{(4)}}\right)^{\frac{2}{23}}\left[\ln\left(\frac{m_b^2}{\Lambda_{(4)}^2}\right)\right]^{\frac{963}{13225}}$$

I.E. $\Lambda_{(4)} = 200$ MeV; $m_c = 1.5$ GeV, $m_b = 4.5$ GeV:

$$\Lambda_{(3)} : \Lambda_{(4)} : \Lambda_{(5)} = 250 : 200 : 133 \text{ MeV}$$

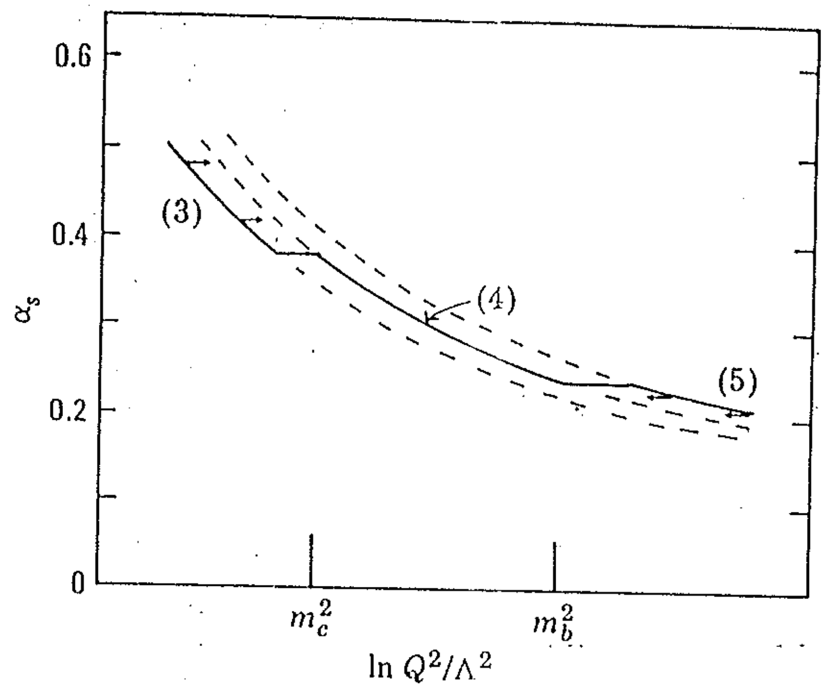


Fig. 5.11 Schematic representation of α_s for $N_f = 3, 4, 5$ showing how the three curves are modified into a single function by shifts along the Q^2 axis.

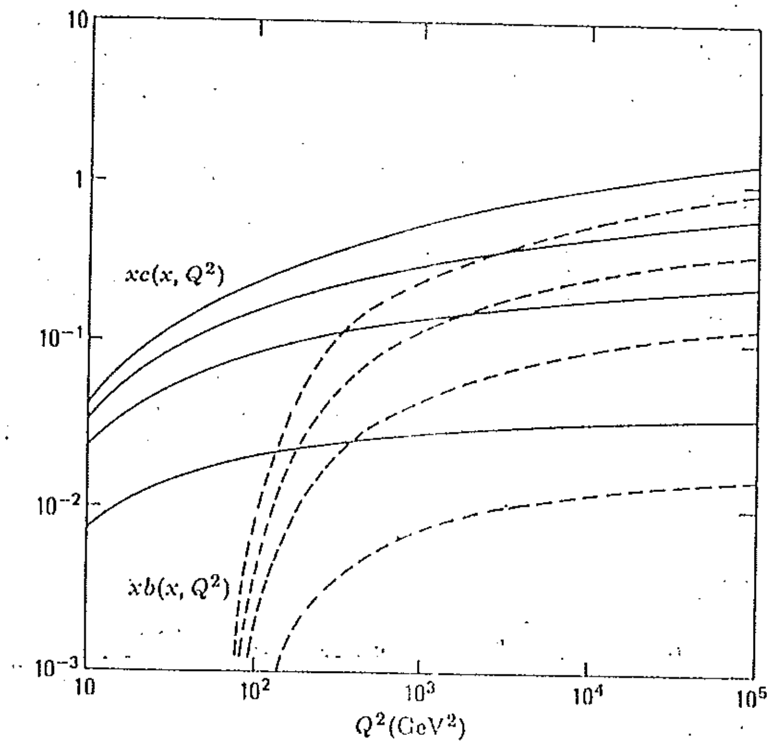


Fig. 7.11 Charm and bottom quark distributions versus Q^2 at fixed x values of 0.1, 0.01, 0.001 and 0.0001.

CONTINOUS MATCHING: GEORGI / POLITZER, 1976

$$\beta_g = -\frac{g^3}{16\pi^2}\left\{11 - \frac{2}{3}\underbrace{\sum_{\text{quarks}}\left[1 - 6\frac{m_i^2}{Q^2} + \frac{12 m_i^4/Q^4}{(1+4m_i^2/Q^2)^{1/2}} \times \ln\frac{(1+4m_i^2/Q^2)^{1/2}+1}{(1+4m_i^2/Q^2)^{1/2}-1}\right]}_{N_f(Q^2)}\right\}$$

$$\boxed{N_f(Q^2) \simeq \sum_{\text{quarks}}{}' \frac{1}{1 + 5\frac{m_i^2}{Q^2}}}$$

NTLO : YOSHINO, HAGIWARA 1984

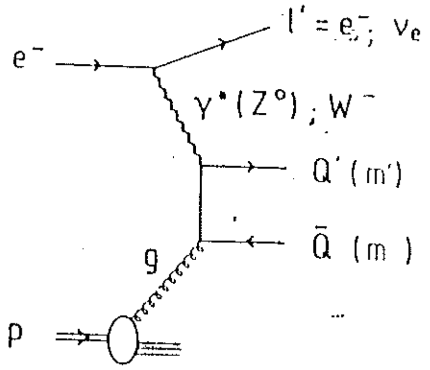
MODIFICATION OF THE SCALING VARIABE x:

$$x' = \frac{1}{2}x\left[1 + \frac{m_f^2 - m_i^2}{Q^2} + \sqrt{1 + \frac{2(m_f^2+m_i^2)}{Q^2} + \frac{(m_f^2-m_i^2)^2}{Q^4}}\right]$$

$$x'_{nc} = \frac{1}{2}x\left[1 + \sqrt{1 + \frac{4m_q^2}{Q^2}}\right] \qquad m_f \equiv m_i$$

$$x'_{cc} = \frac{1}{2}x\left[2 + \frac{2m_f^2}{Q^2}\right] = x\left[1 + \frac{m_f^2}{Q^2}\right]; \quad m_i \equiv 0$$

2) PARTON DISTRIBUTIONS e.g. +4



+ crossed

GLÜCK, GODBOLE, REYA 1988; SCHULER 1988 an ref. theesin

Fig. 1. Lowest order ('Bethe–Heitler') QCD process for leptoproduction of heavy quarks

$$\begin{aligned}
f_1(z,Q^2)=\frac{\alpha_s}{\pi}\frac{q_+}{4}\Bigg\{&-v\bar{v}\left[(1-2z)^2+\frac{q_-}{q_+}\frac{mm'}{Q^2}4z(1-z)\right]\\
&+\Bigg[\tfrac{1}{2}-z(1-z)+\frac{m^2-m'^2}{Q^2}z(1-2z)\\
&+\frac{(m^2-m'^2)^2}{Q^4}z^2+\frac{q_-}{q_+}\frac{mm'}{Q^2}\\
&\cdot 2z\left(1-z-z\frac{m^2+m'^2}{Q^2}\right)\Bigg]L+[m\leftrightarrow m']\tilde{L}\Bigg\}\\
f_2(z,Q^2)=\frac{\alpha_s}{\pi}q_+z\Bigg\{&v\bar{v}\Bigg[-\tfrac{1}{2}+4z(1-z)\\
&-\left(\frac{m^2+m'^2}{Q^2}-\frac{(m^2-m'^2)^2}{Q^4}\right)z(1-z)\Bigg]\\
&+\tfrac{1}{4}\Bigg[1-2z(1-z)+\frac{m^2}{Q^2}(1+8z-18z^2)\\
&+\frac{m'^2}{Q^2}(1-4z+6z^2)-\frac{q_-}{q_+}\frac{2mm'}{Q^2}\\
&-\frac{m^4+m'^4}{Q^4}2z(1-3z)+\frac{m^2m'^2}{Q^4}4z(1-5z)\\
&+\frac{m^6-m^4m'^2-m^2m'^4+m'^6}{Q^6}\,2z^2\Bigg]L\\
&+\tfrac{1}{4}[m\leftrightarrow m']\tilde{L}\Bigg\}\\
f_3(z,Q^2)=\frac{\alpha_s}{\pi}\Bigg\{&v\bar{v}\frac{m^2-m'^2}{Q^2}2z(1-z)\\
&-\Bigg[\tfrac{1}{2}-z(1-z)+\frac{m^2-m'^2}{Q^2}z(1-2z)\\
&-\frac{m^4-m'^4}{Q^4}z^2\Bigg]L+[m\leftrightarrow m']\tilde{L}\Bigg\}
\end{aligned}\tag{2.1}$$

where

$$L=\ln\frac{1+\frac{m^2-m'^2}{Q^2}\frac{z}{1-z}+v\bar{v}}{1+\frac{m^2-m'^2}{Q^2}\frac{z}{1-z}-v\bar{v}},\qquad \tilde{L}=L(m\leftrightarrow m')$$

$$v^2=1-\frac{(m+m')^2}{Q^2}\frac{z}{1-z},\qquad \bar{v}^2=1-\frac{(m-m')^2}{Q^2}\frac{z}{1-z}.$$

finite F_L !

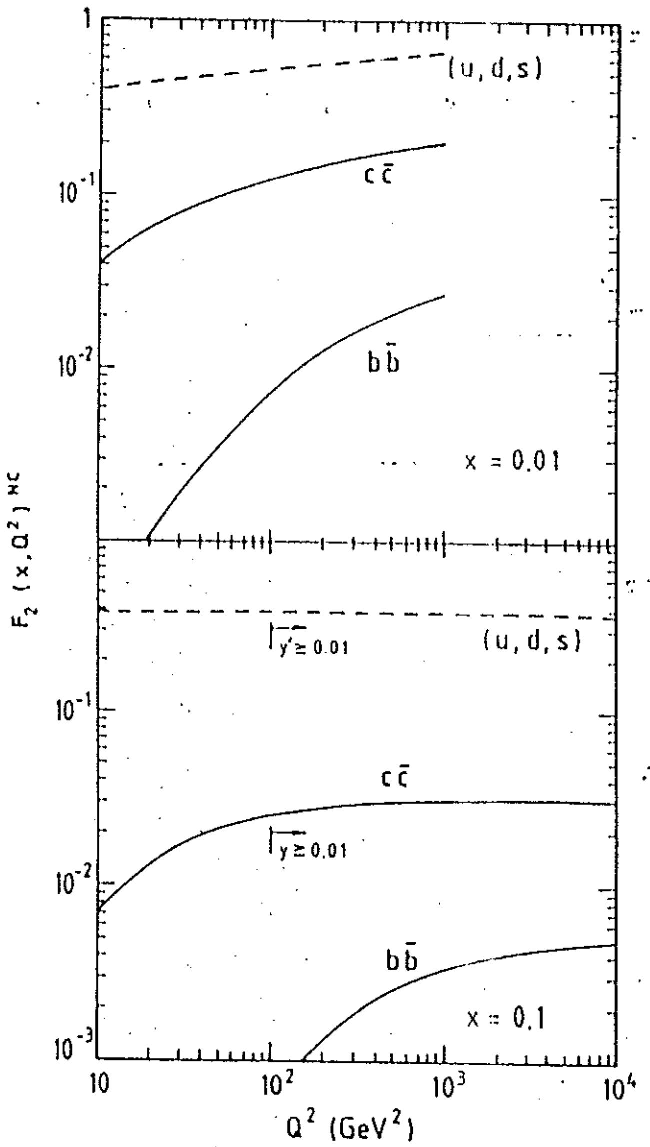
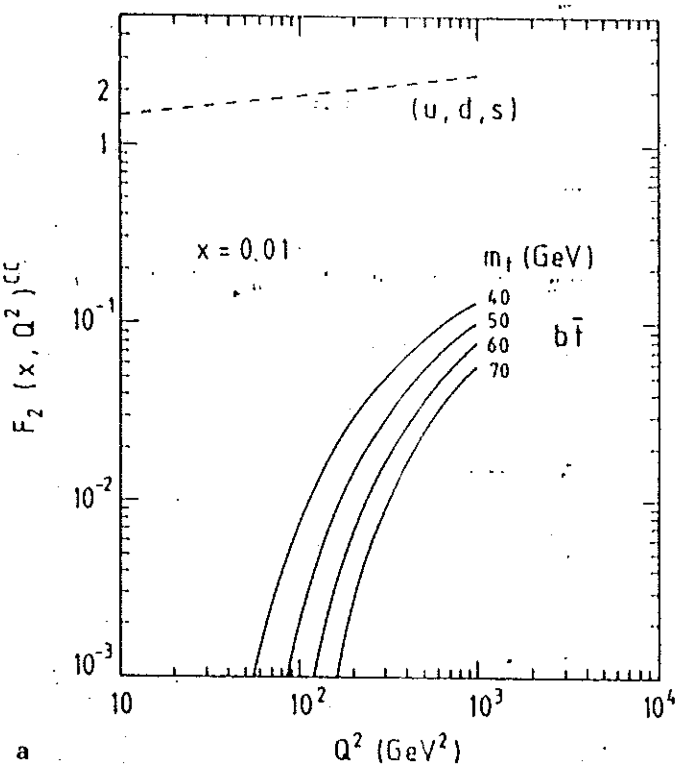


Fig. 4. Predicted Q^2-dependence of $F_2(x, Q^2)$ for NC events (only γ^* exchange is considered) according to (2.1) and (2.2) for $x = 0.01$ and 0.1 using $m_c = 1.3$ GeV and $m_b = 4.5$ GeV. The standard leading $\ln Q^2$ scaling violating contributions of the light quarks (u, d, s) to F_2 are shown by the dashed curves. The parton distributions are taken from [8]



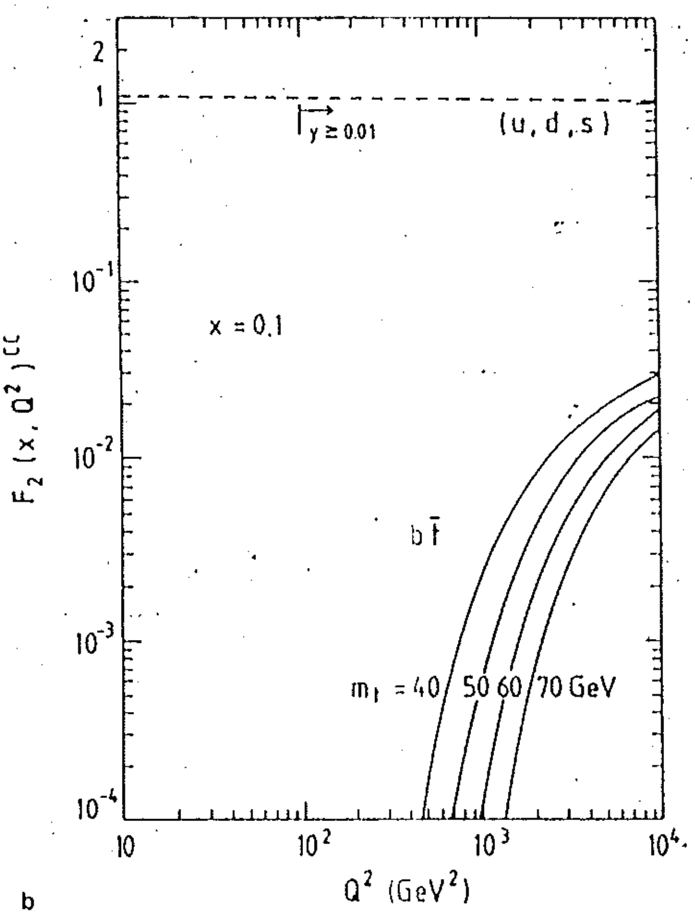


Fig. 5. **a** The same as in Fig. 4 but for CC events at $x = 0.01$. **b** The same as in **a** but at $x = 0.1$

SUMMARY OF LECTURE 2

1) EQUIVALENCE OF THE OPE AND THE ALTARELLI/PARISI METHOD TO DETERMINE THE EVOLUTION KERNELS OF THE STRUCTURE FUNCTIONS

2) THERE ARE NON NEGLIGABLE CONTRIBUTIONS IN NEXT TO LEADING ORDER:

a) $F_2(x, Q^2)$

b) $F_L(x, Q^2)$ PARTICULARLY ALSO AT SMALL x.

- LOWERING OF $xG(x, Q^2)$
- RISING e.g. $xu(x, Q^2)$.
- LOWERING F_L !

3) MASS EFFECTS:

$(M_n^2/Q^2)^n \longrightarrow$ NACHTMANN VARIABLES

$(m_{q_i}^2/Q^2)^n \longrightarrow$ REQU. LO, ✓, NTLO-CORRECT.

RELEVANT AT SMALL x

- $\alpha_s^{HF}(Q^2)$.

4) DYNAMICAL HIGHER TWIST OPERATORS:

STILL MANY OPEN QUESTIONS: RELEVANCE AT SMALL x !

WE WILL DISCUSS IN THE FOLLOWING:

- EFFECTS AT SMALL x

  a) THE FADIN-KURAEV-LIPATOV EQUATION.

  b) AP-EVOLUTION IN THE SMALL x LIMIT

  c) THE SUMMATION OF FAN DIAGRAMS: A CONCEPT FOR SATURATION.

- TECHNICAL ASPECTS: EVOLUTION PROGRAMS.

- OUTLOOK: PROBLEMS & FURTHER DEVELOPMENT.

EVOLUTION AT SMALL X : FADIN, KURAEV, LIPATOV EQU.

RANGE: $\alpha_s(Q_0^2)\ln(Q^2/Q_0^2) \ll 1 \qquad \alpha_s(Q_0^2)\ln\frac{1}{x} \sim 1$

SUM: ORDERS $\sim \left(\alpha_s \ln\frac{1}{x}\right)^n$

FADIN, KURAEV, LIPATOV 1975, 1977

BALITZKIJ, LIPATOV 1978

1) α_s = FIXED.

$$f(n,k^2) = \frac{1}{n-1} f_0(n,k^2) + \frac{3\alpha_s}{\pi(n-1)} L_1 \otimes f$$

$$L_1 \otimes f = \int_0^\infty \frac{dk'^2}{k'^2} \left\{ \frac{k^2}{|k'^2-k^2|}\left[f(n,k'^2) - f(n,k^2)\right] + \frac{f(n,k^2)}{\sqrt{k^4+4k'^4}} \right\}$$

L_1 IS HOMOGENEOUS IN k^2 & k'^2, I.E. ITS EIGENFUNCTIONS ARE THE SIMPLE POWERS.

EIGENVALUE - EQUATION:

$$L_1 \otimes f_\omega = K(\omega) \otimes f_\omega$$

$$f_\omega = (k^2)^{\omega+\frac{1}{2}}$$

$$K(\omega) = -2\gamma - \psi\left(\tfrac{1}{2}+\omega\right) - \psi\left(\tfrac{1}{2}-\omega\right).$$

$$f_0(n, k^2) = \int \frac{d\omega}{2\pi i} (k^2)^{\omega + \frac{1}{2}} e_0(n, \omega)$$

$$f(n, k^2) = \int \frac{d\omega}{2\pi i} (k^2)^{\omega + \frac{1}{2}} e(n, \omega)$$

$$\Longrightarrow \quad e(n, \omega) = \frac{e^0(n, \omega)}{n - 1 - \frac{3\alpha_s}{\pi} K(\omega)}$$

BRANCH POINT SINGULARITY AT:

$$n_0 = 1 + \frac{3\alpha_s}{\pi} K(0) = 1 + 2.64\,\alpha_s$$

BEHVIOUR OF: $G(x)_{x \ll 1} \sim \frac{1}{x^{n_0}}$

2) RUNNING α_s:

$$f(n, k^2) = \frac{f_0(n, k^2, k_0^2)}{n-1} + \frac{1}{n-1} L_2(k_0^2, k^2) \otimes f$$

$$L_2 \otimes f = \frac{3\alpha_s(k^2) k^2}{\pi} \int_{k_0^2}^{\infty} \frac{dk'^2}{k'^2} \left[\frac{f(n, k'^2) - f(n, k^2)}{|k'^2 - k^2|} + \frac{f(n, k^2)}{\sqrt{k^4 + 4k'^4}} \right]$$

DISCRETE SPECTRUM, BOUND FROM BELOW.

COLLINS, KWIECINSKI
1989

$$1 + \frac{3.6}{\pi} \alpha_s(k_0^2) \leq n_0 \leq 1 + 4 \ln 2 \left(\frac{3}{\pi}\right) \alpha_s(k_0^2)$$

$\therefore$ $Q^2 = 10\ \mathrm{GeV}^2$
$\Lambda = 200\ \mathrm{MeV}$
$N_f = 4$

$$1.31 \leq n_0 \leq 1.72$$

MARCHESINI, WEBBER 1991

$$N = 1 + \bar{\alpha}_s \left[-2\gamma_E - \psi(\gamma_N) - \psi(1-\gamma_N) \right]$$

$$\bar{\alpha}_s = \frac{3}{\pi} \alpha_s$$

CALCULATE NOW: $\gamma_N = \gamma_N(\alpha_s)$

THIS ANOMALOUS DIMENSION MAY BE GIVEN IN ALL ORDERS:

$$\gamma_N(\bar{\alpha}_s) = \frac{\bar{\alpha}_s}{N-1} + 2\zeta_3 \left(\frac{\bar{\alpha}_s}{N-1} \right)^4 + 2\zeta_5 \left(\frac{\bar{\alpha}_s}{N-1} \right)^6 + \dots$$

NO CONTRIBUTION IN $O(\alpha_s^2, \alpha_s^3, \alpha_s^5)$.

SO FAR, ONLY THE GLUONIC EVOLUTION WAS CONSIDERED, I.E.

$$f(n,k^2) = \frac{d\,g(n,k^2)}{d\ln k^2}.$$

A MORE REALISTIC PICTURE INCLUDES THE QUARK DEGREES AS WELL, HOWEVER IN THE AP - APPROXIMATION:

KWIECINSKI, 1985

THE TRANSFORMATION OF THE $\frac{1}{n-1}$ TERM INTO X - SPACE YIELDS

$$\frac{f(n,k^2)}{n-1} \longrightarrow \frac{\partial f(x,k^2)}{\partial \ln(1/x)}$$

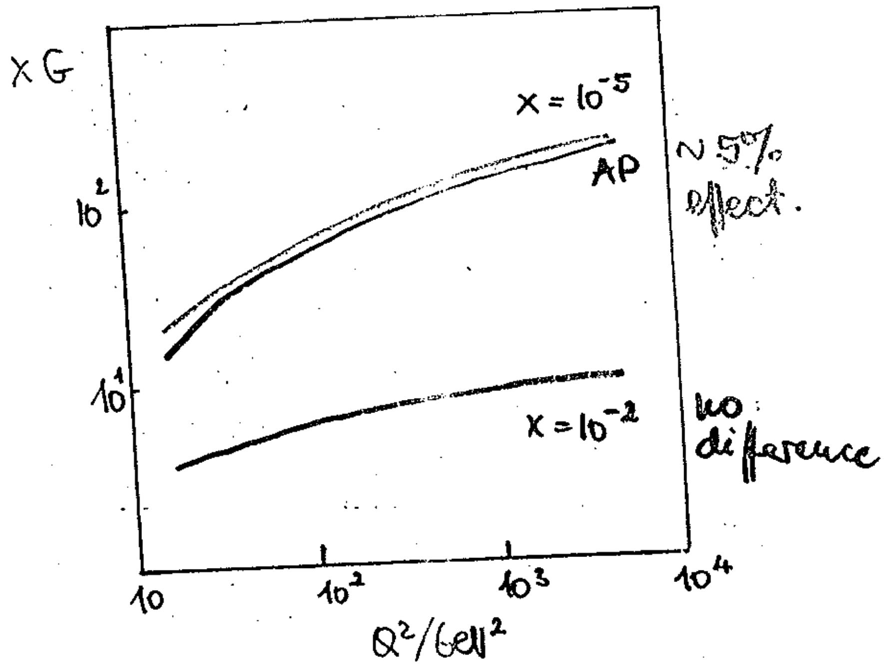
THIS IS IN SIMILARITY OF THE AP $\frac{\partial}{\partial \ln Q^2}$, HOWEVER, FOR GLUONS <u>ONLY</u>!

$$f(n,k^2) = f_0(n,k^2) + \frac{3\alpha_s(k^2)}{\pi}\left\{\int_{k_0^2}^{\infty}\frac{dk'^2}{k'^2}\left[\frac{f(n,k'^2)-f(n,k^2)}{|k'^2-k^2|} + \frac{f(n,k^2)}{\sqrt{4k'^4+k^4}}\right] + \tilde{A}_{gg}(n)\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}f(n,k'^2) + 2N_f A_{gq}(n)\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}h(n,k'^2)\right\}$$

$$\tilde{A}_{gg}(n) = \frac{4}{9}\int_0^1 dz\, z^{n-1} P_{gg}(z) - \frac{1}{n-1}\,; \quad \text{else: } A_{ab}(n) = \frac{4}{9}\int_0^1 dz\, z^{n-1} P_{ab}(z)$$

$$h(n,k^2) = h^0(n,k^2) + 3\,\frac{\alpha_s(k^2)}{\pi}\left[A_{qq}(n) \int_{k_0^2}^{k^2} \frac{dk'^2}{k'^2}\, h(n,k^2) + A_{qg}(n) \int_{k_0^2}^{k^2} \frac{dk'^2}{k'^2}\, f(n,k^2) \right]$$

f_0 & h_0 are related to g, q at Q_0^2.



SIMILAR RESUTS :

MARCHESINI & WEBBER 1991 , MC.

LEVIN, MARCHESINI, RYSKIN, WEBBER 1991

ALTARELLI - PARISI KERNELS AT SMALL x

$$\frac{d q_a(x,Q^2)}{d \ln Q^2} = \mathbb{P}\left(x, \frac{\alpha_s(Q^2)}{2\pi}\right)_{ab} \otimes q_b(x,Q^2)$$

$$\mathbb{P}\left(x, \frac{\alpha_s}{\pi}\right) = \frac{\alpha_s}{2\pi} P^0(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 P^1(x) + \dots$$

LO NTLO

$$A(x) \otimes B(x) = \int_0^1 dx_1 \int_0^1 dx_2 \, \delta(x - x_1 x_2) \, A(x_1) B(x_2)$$

CONSIDER NOW: $x \ll 1$

MOST RELEVANT PART (LO) GLUONS.

$$P^0_{GG}(x) = \overbrace{2C_G}^{6} \left\{ \frac{1}{x} + \frac{1}{1-x} - 2 + x - x^2 \right\} + \dots$$

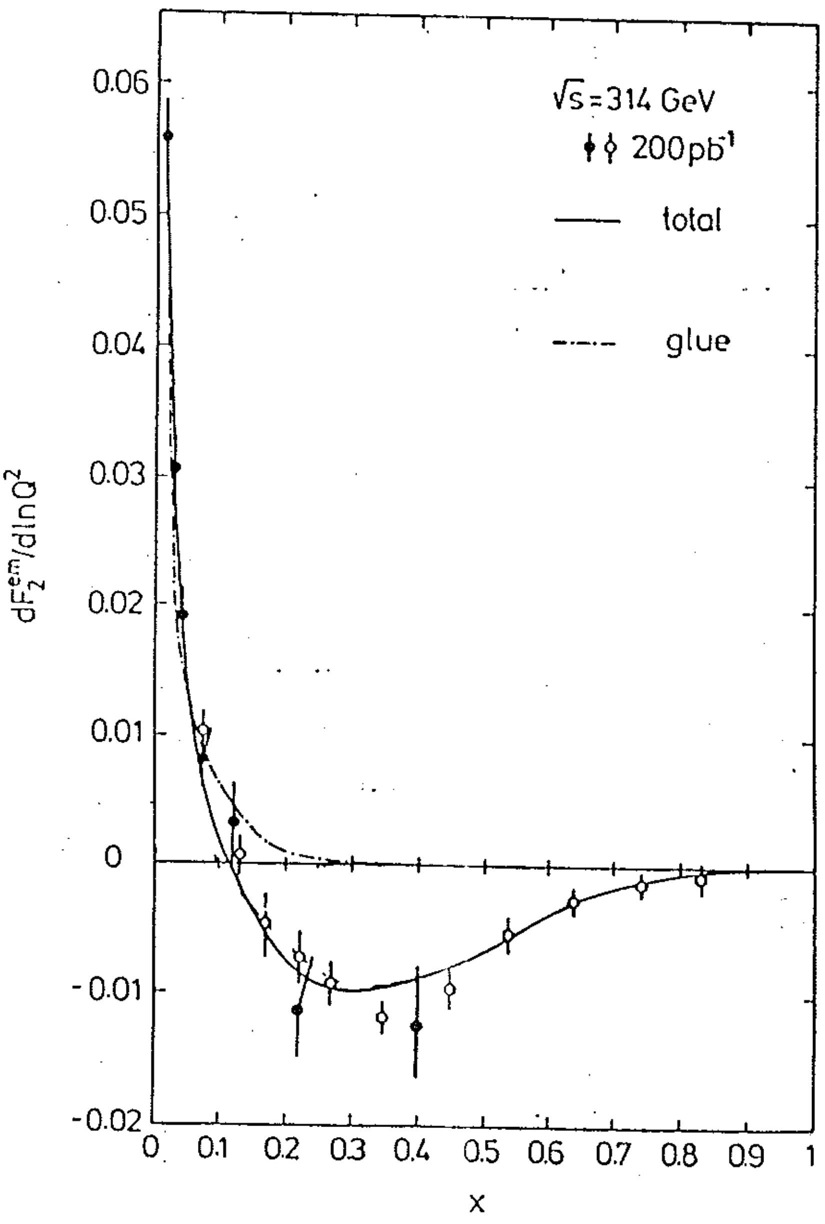
$$P^1_{GG}(x) = 2N_f T_R \left\{ -\frac{23}{9} C_G + \frac{2}{3} C_F \right\} \frac{1}{x} + \dots$$

FURMANSKI, PETRONZIO
1980

BAULIEU, KOUNNAS
1979/80

WU-KI TUNG '80

Df: $\overline{G}(x,Q^2) = x\, G(x,Q^2)$



$\sqrt{s}$=314 GeV
200pb^{-1}
total
glue
$dF_2^{em}/d\ln Q^2$
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x

$$\frac{d\overline{G}}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dx'}{x'} \left[6 - \frac{61}{9} N_f \frac{\alpha_s(Q^2)}{2\pi} \right] \frac{x^2}{x'^2} \overline{G}(x')$$

most
singular part

NEW VARIABLES:

$$y = \frac{8N_c}{\beta_0} \ln \frac{1}{x} \qquad \xi = \ln \ln \frac{Q^2}{\Lambda^2}$$

SOLUTION:

1 LOOP:

$$\frac{\partial^2 \overline{G}(y,\xi)}{\partial y \partial \xi} = \frac{1}{2} \overline{G}(y,\xi)$$

2 LOOP:

$$\frac{\partial^2 \overline{G}(y,\hat{\xi})}{\partial y \partial \hat{\xi}} = \frac{1}{2} \overline{G}(y,\hat{\xi})$$

$$\hat{\xi} = \xi + f(\xi)$$

$$\overline{G}(x,Q^2) \Rightarrow \overline{G}(y,\overset{(\wedge)}{\xi}) = \sum_{\nu=0}^{\infty} \left\{ A_\nu \left(\frac{2\xi}{y} \right)^{\nu/2} + B_\nu \left(\frac{y}{2\xi} \right)^{\nu/2} \right\} \times I_\nu\left(\sqrt{2\xi y}\right)$$

$$\boxed{\text{UNITARITY - VIOLATING AS} \quad y \to \infty, \quad x \to 0}$$

GLAP - EQUATION CAN NOT HOLD FOR ARBITRARY LOW x.

NTLO CORRECTIONS DO NOT ! CURE THIS.

ARE TWIST 2 OPERATORS UNITARITY VIOLATING AS $x \to 0$? (LO, NTLO, ... ?)

(OPEN QUESTION).

FAN - DIAGRAMS

IF HIGHER ORDER TWIST- 2 TERMS FAIL,

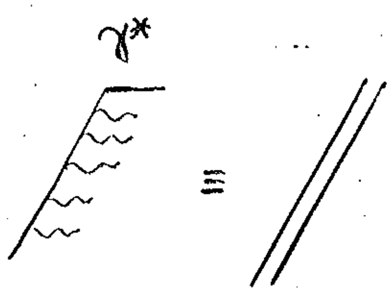
MAY LO (....) HIGHER TWIST TERMS HELP ?!

WORD OF CAUTION : FIRST THEORETICAL STEPS, NO RIGOUROSE THEORY YET.

1980 : ST. PETERSBURG / LENINGRAD - GROUP

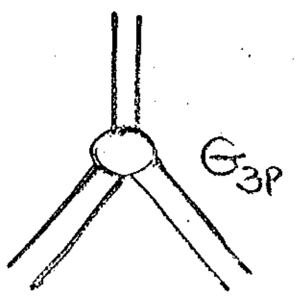
INTUITIVE PICTURE OF PARTONRECOMBINATION, IF THE PARTONDENSITIES ARE GETTING LARGE.

a) ASYMPTOTIC FREEDOM:



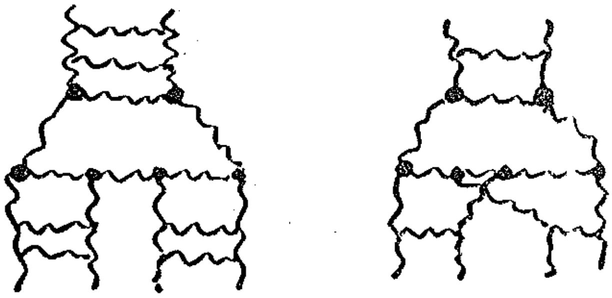
SINGLE LADDERS

INCREASE OF PARTONIC DENSITY:



GLR : STATE, THAT ALL LEADING TERMS ARE 2 → 1 LADDER RECOMBINATIONS

ONE HAS TO CONSIDER ALL POSSIBLE CUTS FOR:



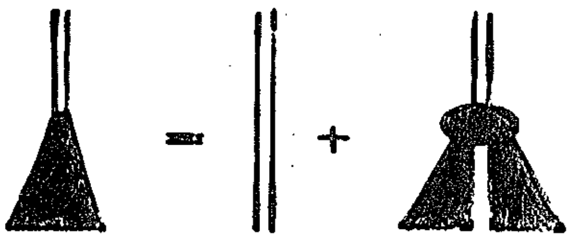
etc.

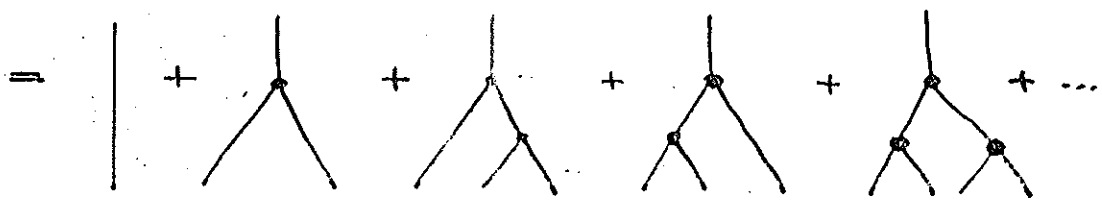
MÜLLER & QIU 1985:

$$G_{3P} = \frac{3}{4}\pi^2 \frac{1}{\beta_0} e^{e^{\xi_0}} \exp\left[-(e^{\xi}+\xi)\right]$$

$$\xi_0 = \ln\ln\left(\frac{Q_0^2}{\Lambda^2}\right), \quad \xi = \ln\ln\left(\frac{Q^2}{\Lambda^2}\right)$$

THE FAN DIAGRAM EQUATION





$$\frac{\partial^2 F(\xi,y)}{\partial \xi \, \partial y} = \frac{1}{2} F(\xi,y) - C \exp\left[-(e^{\xi}+\xi)\right] F^2(\xi,y)$$

$$C = \frac{3}{4} \pi^2 \frac{1}{\beta_0} \frac{Q_0^2}{\Lambda^2}$$

$$\exp\left[-(e^{\xi}+\xi)\right] = \frac{1}{\ln\left(\frac{Q^2}{\Lambda^2}\right)} \cdot \frac{\Lambda^2}{Q^2} \ldots$$ HIGHER TWIST

$\uparrow$!

$F(\xi,y) \triangleq x\,G(x,Q^2)$ <u>ASSUMING</u>, THAT THE MULTIGLUONDISTRIBUTIONS

$$G_n(x_1,\ldots,x_n) \simeq G(x_1)\ldots G(x_n)$$

FACTORIZE.

RECENT IMPROVEMENT: J. BARTELS, J.B., G. SCHULER:
ARBITRARY $G(x)$ INITIAL
CONDITIONS.

$$F(\xi,y) = \overline{G}(\xi_0,y) + \int_{\xi_0}^{\xi} d\xi' \int_0^{y} dy' \, F(\xi',y') \left[\frac{1}{2} - C \exp\left(-(e^{\xi'}+\xi')\right) \times F(\xi',y')\right]$$

$\uparrow$

INPUT AT Q_0^2.

PROPERTIES OF THE SOLUTION

- THE SOLUTIONS ARE BOUNDED $F \leq F_0^{DLA}$.
- $\lim\limits_{y \to \infty} F(y, \xi) = \frac{1}{2C} \exp[e^{\xi} + \xi] = \mathrm{const}_{\xi}$.

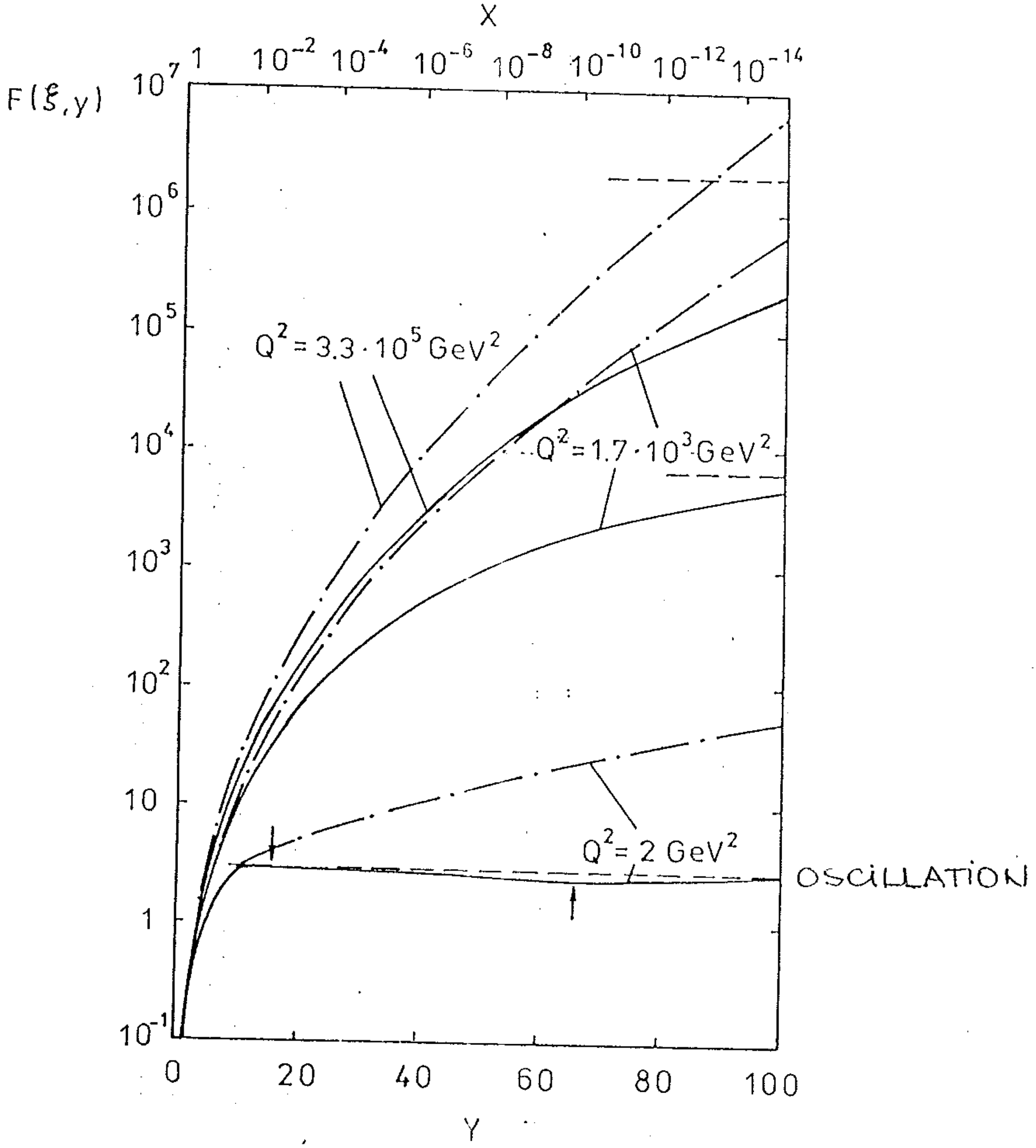
  THIS LIMIT IS INDEPENDENT OF THE INPUT AT Q_0^2 !
- $\exists!\ F(\xi, y)$

TECHNICAL SOLUTION: QUADR. EQU. AT A FINE GRID IN (ξ, y).

FIG.

⟶ DAMPING THE GROTH OF $xG(x, Q^2)$

SIMILAR RESULTS: KWIECINSKI + DURHAM GROUP, ROBERTS.



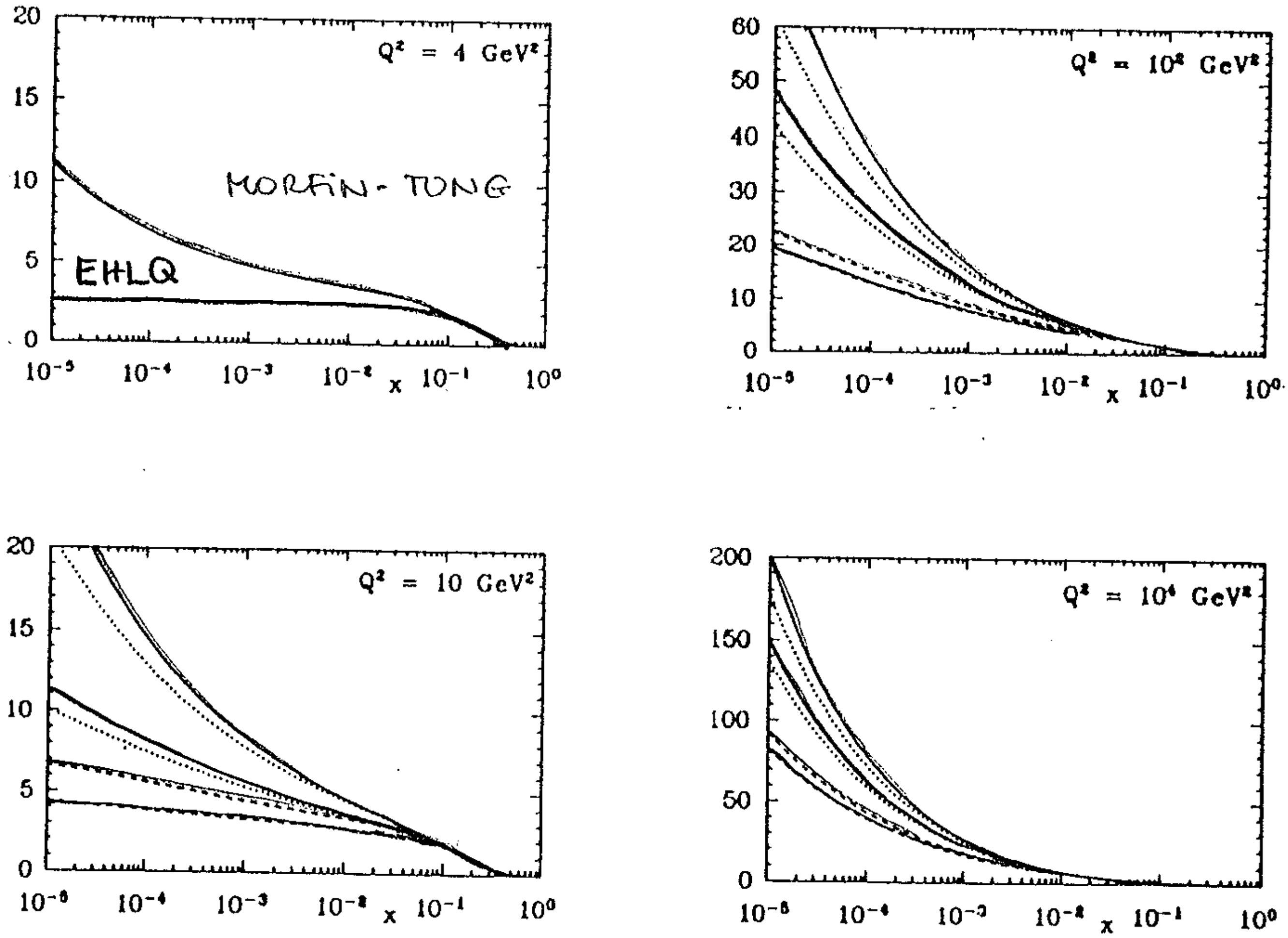
$G(y) \sim 3(1-x)^5$

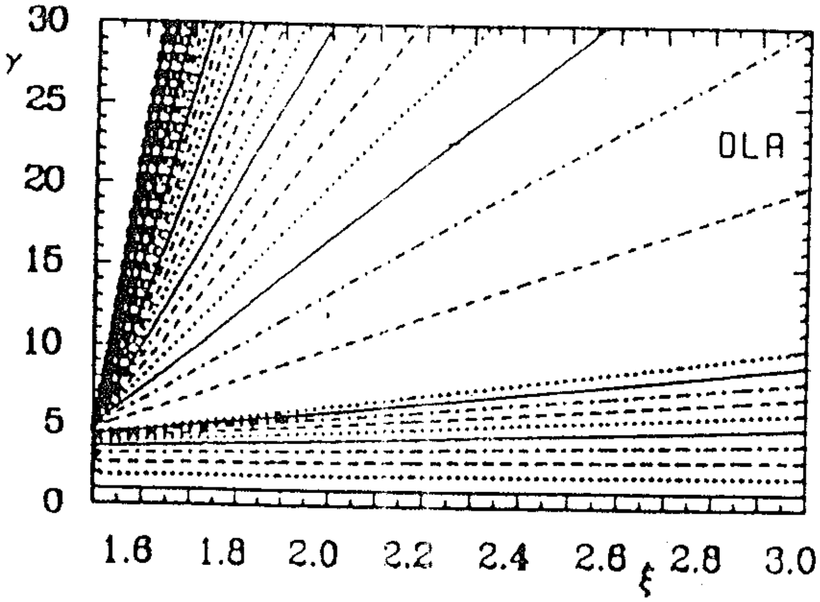
$Q_0^2 = \Lambda^2$, $\Lambda = 200$ MeV

Fig. 2

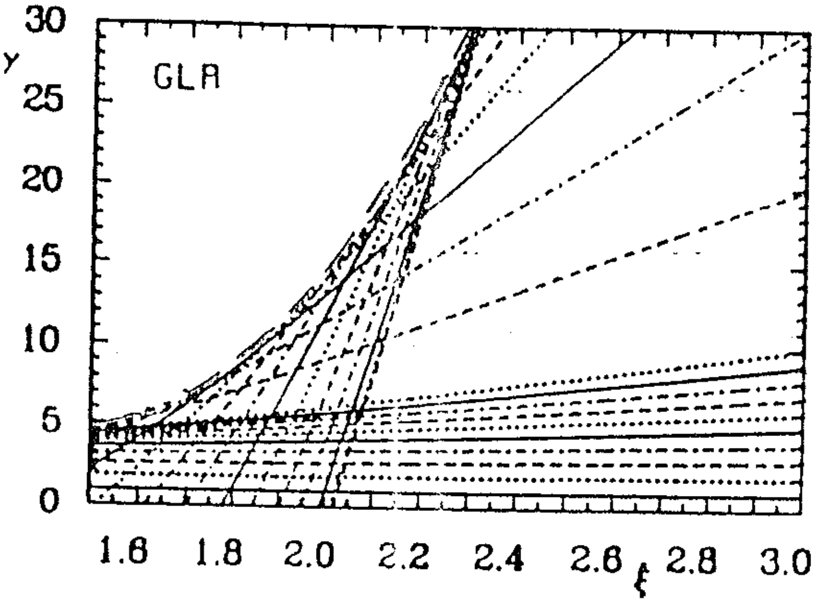
J. BARTELS, J.B., G. SCHULER

$Q_0^2 = 4\ GeV^2 \longrightarrow$ NONLINEARITY.



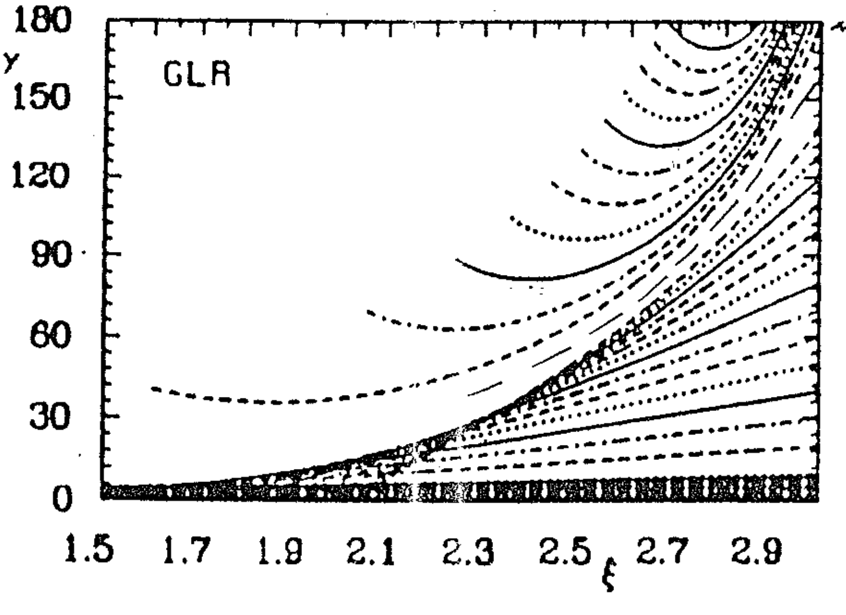


AP



TRAJECTORIES:

STARTING BELOW



ABOVE THE CRITICAL LINE

SEMICLASSICAL SOLUTIONS

GLR : $F = e^{S}$

$$\boxed{S_{1y} S_{1\xi} = \frac{1}{2} - C \exp\left[S - e^{\xi} - \xi\right]}$$

FOR :

$S_{1y} S_{1\xi} \gg S_{1y\xi}$

$y \xi \gg 1$, !

LOOK FOR TRAJECTORIES IN (ξ, y).

BARTELS, JB, SCHULER ; KWIECINSKI, COLLINS

$$\dot{y} = S_{1\xi}$$

$$\dot{\xi} = S_{1y}$$

$$\dot{S} = 2 S_{1y} S_{1\xi}$$

$$\dot{S}_{1y} = -C \exp\left(S - e^{\xi} - \xi\right) S_{1y}$$

$$\dot{S}_{1\xi} = -C \exp\left(S - e^{\xi} - \xi\right)\left(S_{1\xi} - 1 - e^{\xi}\right)$$

GLAP: $C = 0 \quad \wedge \quad \ddot{y} = \ddot{\xi} = 0 \longrightarrow$ STRAIGHT LINES

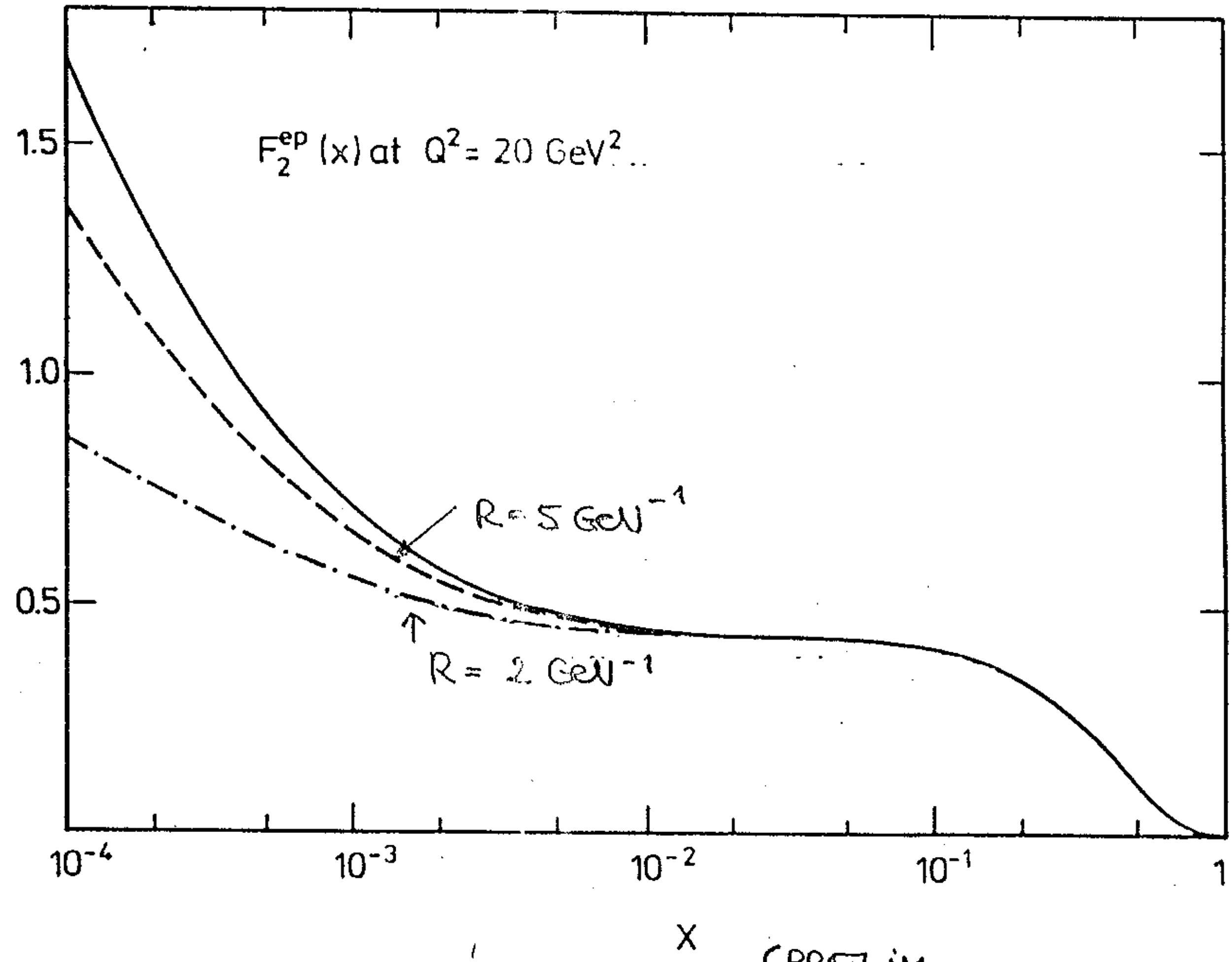
FIG.

$C \neq 0$ DEPENDING ON THE STARTING VALUE : CURL FROM ABOVE/BELOW 'CRITICAL LINE'

$\Rightarrow$

ABOVE : CLEARLY : NONLINEAR – NONPERT. RANGE.

KWIECINSKI, MARTIN, ROBERTS, STIRLING



(PRELIM., IMPROVED RELATION TO xG REQUIRED.)

SUMMARY : LOW X

1) FOR $x \to 0$ THE LO + NTLO TWIST 2 TERMS VIOLATE UNITARITY.

2) FAN DIAGRAMS RESTORE UNITARITY. THIS CONCEPT USES SUMMATION OVER INFINITE ORDERS OF HIGHER TWIST CONTRIBUTIONS.

3) QUESTION: ARE THESE DIAGRAMS THE <u>ONLY</u> RELEVANT ONES?
(PROOF LACKING : TWIST 4, ...)

4) SOLUTIONS FOR THE GLR-EQU. WITH ARBITRARY SINGLE GLUON INITIAL CONDITION DERIVED.

5) QUESTION: HOW RELEVANT ARE 2-, 3-, 4- GLUON INITIAL STATES AND WHERE?

6) THE SEMICLASSICAL ANALYSIS INDICATES THE RANGE, WHERE THE GLR APPROACH BREAKS DOWN (NONPERTURBATIVE RANGE).

TECHNICAL ASPECTS OF A QCD ANALYSIS

i) METHODS :

- 'NUMERICAL SOLUTIONS' DIFF. EQU.
- DETERMINATION OF EVOLUTION KERNELS + CONVOLUTION
- INVERSE MELLIN TRANSFORM OF CORNWALL-NORTON MOMENTS
- POLYNOMIAL EXPANSIONS $\rightarrow$ ALGEBRAIZATION

ii) APPLICATION OF THE χ^2-METHOD

$$\chi^2(\Lambda, \alpha_i(Q_0^2)) = \sum_{\text{BINS}} \left[\frac{F_{TH}(\Lambda, \alpha_i(Q_0^2), x, Q^2) - F_{EXP}(x, Q^2)}{\delta F_{EXP}} \right]^2$$

MIN! $\chi^2 \; |\Lambda, \alpha_i|_{i=1}^{N}$

NUMERICAL SOLUTION OF THE INTEGRO-DIFFERENTIAL EQUATIONS

- DISCRETIZE THE DIFFERENTIALS & INTEGRALS
- SOLVE ALGEBRAIC EQUATIONS
- OPTIMIZE STEP SIZES

MORE RECENTLY : USE OF VECTOR-PROCESSORS

— METHOD WITH SIMPLE MATHEMATICAL STRUCTURE

— RATHER TIME CONSUMING UNDER AT FIT OF THE PARAMETERS Λ, $\alpha_i \Big|_{i=1}^{N} = \alpha_i(Q_0^2)$.

- ABBOTT, ATWOOD, BARNET 1980
- VIRCHAUX, ORAOU 1987.

INVERSE MELLIN TRANSFORM OF THE KERNEL

DEVOTO, DUKE, OWENS, ROBERTS 1983
GROSS 1974; GONZALEZ-ARROYO et al. 1979, 1980

e.g. $$F^{NS}(x, Q^2) = \int_x^1 dy\, b(x, y, Q^2, Q_0^2)\, F^{NS}(x, Q_0^2)$$

$$b(x,y;Q^2,Q_0^2) = \frac{1}{2\pi i} \frac{x}{y^2} \int dn\, e^{-\gamma^0_{n+2} \tilde{S}/2\beta_0} \left\{ 1 + \frac{\alpha_S(Q^2) - \alpha_S(Q_0^2)}{4\pi} \times \right.$$

$$\left. \times \left[\frac{\gamma^{(1)}_{N+2}}{2\beta_0} - \frac{\gamma^{(0)}_{n+2}}{2\beta_0^2} + C^{(1)}_{n+2} \right] \right\}$$

SIMILAR: SINGLET CASE.

K. KATO, Y. SHIMIZU, H. YAMAMOTO : 1979, 1980, 1982

$$F_{NS}(x, Q^2) = \int_x^1 \frac{dy}{y} K_{NS}(y, \tilde{s}) \, F_{NS}\left(\frac{x}{y}, Q_0^2\right)$$

$$K_{NS}(y, \tilde{s}) = 1 + \ln\frac{1}{y}\left[c_1 + c_2\sqrt{u} + c_3 u + c_4 u\sqrt{u} + c_5 y + c_6 y^2\right]$$

$$c_k = a_{0k} + a_{1k} s' + a_{2k} s'^2 + a_{3k} s'^3 \quad , \quad s' = \tilde{s}/2$$

$$u = \frac{1}{10} \ln\frac{1}{y}$$

RESULT OF A FIT!

$$\begin{pmatrix} \Sigma(x, Q^2) \\ xG(x, Q^2) \end{pmatrix} = \begin{pmatrix} K_{qq} & K_{qG} \\ K_{Gq} & K_{GG} \end{pmatrix} (x, \tilde{s}) \otimes \begin{pmatrix} \Sigma(x, Q_0^2) \\ xG(x, Q_0^2) \end{pmatrix}$$

$$K_{ij}(x, \tilde{s}) = \frac{1}{x} \tilde{s}^N \left(\ln\frac{1}{x}\right)^{b_0 + b_1 \tilde{s}} \left[c_1 + c_2\sqrt{u} + c_3 u + c_4 u\sqrt{u} + c_5 x + c_6 x^2\right]$$

$N = 2 \qquad K_{qq}, K_{GG}$

$N = 1 \qquad K_{qG}, K_{Gq} \qquad \ldots\ldots$

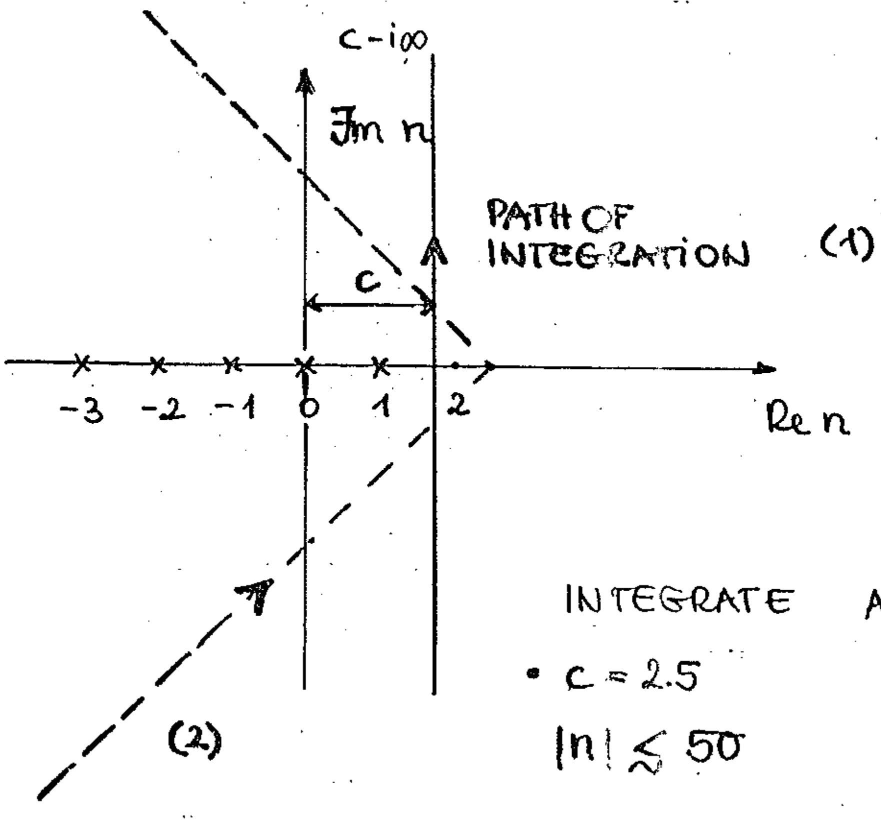
INVERSION OF MELLIN-MOMENTS

e.g. GLÜCK, REYA, VOGT

- CALCULATE THE MOMENTS OF THE SPLITTING FUNCTIONS
- FORMULATE THE ANALYTICAL CONTINUATION

$$n \in \mathbb{Z} \longrightarrow z \in \mathbb{C}$$!

$$F(x,Q^2) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dn\, x^{-n+1} \langle F(Q^2)\rangle_n$$



INTEGRATE ALONG (2)

- $c = 2.5$

$|n| \lesssim 50$ $\quad \sqrt{|\mathrm{Re}\, z|^2 + |\mathrm{Im}\, z|^2} \lesssim 50$

$$\frac{1}{n+1} \longrightarrow \frac{1}{z+1}$$

$$\sum_{k=1}^{n-1} \frac{1}{k} = \psi(n) + \gamma_E \Rightarrow \psi(z) + \gamma_E, \text{ etc.}$$

POLYNOMIAL EXPANSIONS

ORTHOGONAL DECOMPOSITIONS OF EVOLUTION EQUATIONS

- ANY FUNCTION IN $\mathcal{L}_2[0,1]$ MAY BE REPRESENTED BY THE ELEMENTS OF THIS HILBERTSPACE:

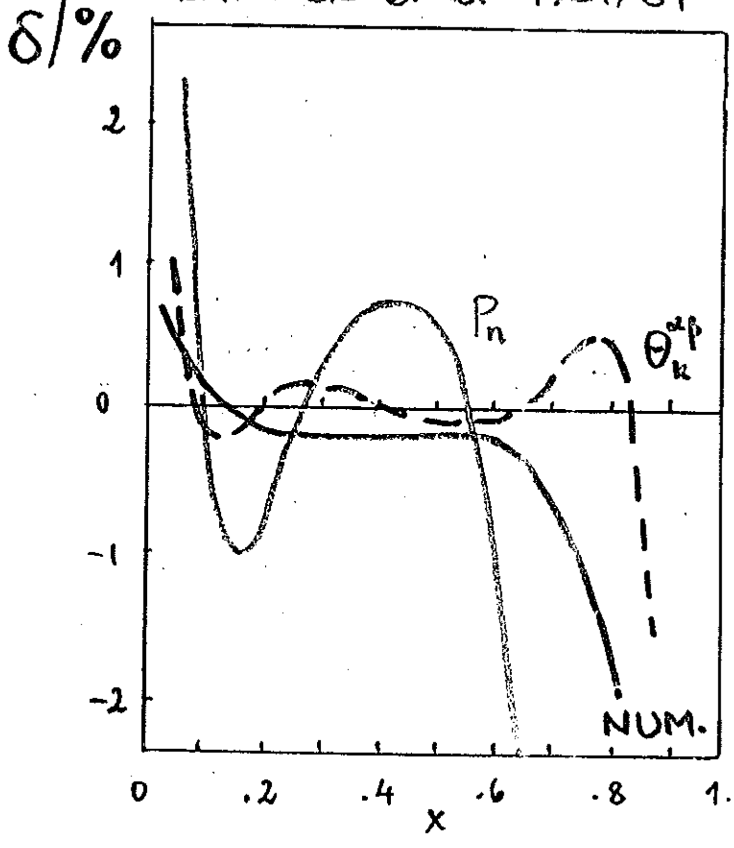
$$f \equiv |f\rangle = \sum_n \langle c_n | f \rangle \, |c_n\rangle = \sum_n a_n(f) |c_n\rangle$$

IN THIS WAY ONE MAY ALGEABRIZE THE EVOLUTION EQUATIONS.

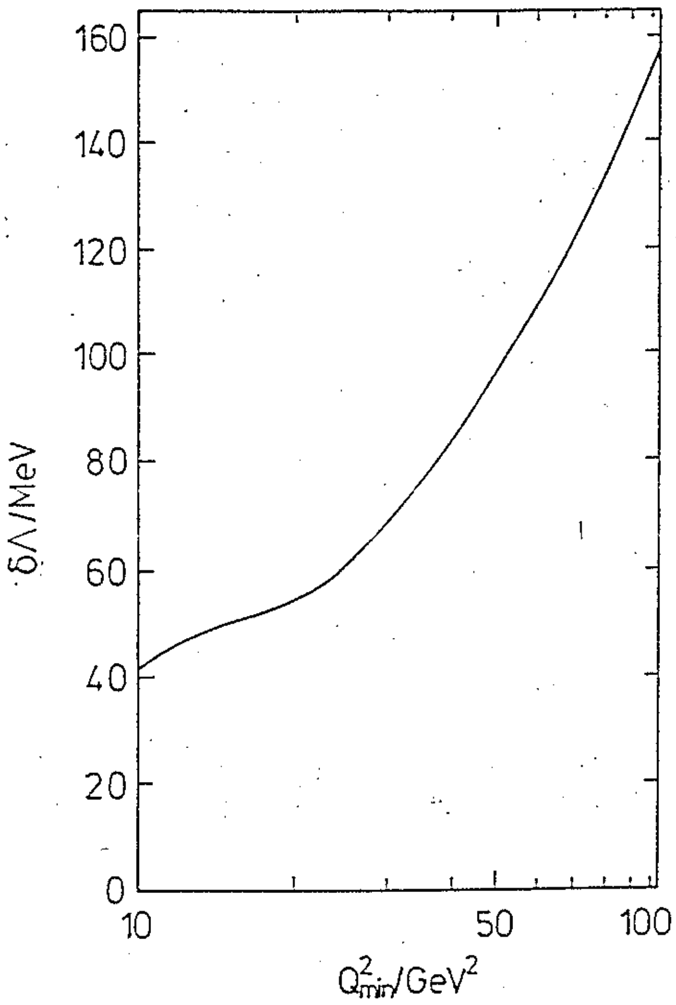
- ONE CAN FIND $a_n(f)$ ANALYTICALLY.

			TYPE
FURMANSKI, PETRONZIO	1982	$L_n(\ln \frac{1}{x})$	LAGUERRE
YNDURAIN	1978	$B_n(x)$	BERNSTEIN
CHÝLA, RAMEŠ	1986	$P_n(x)$	LEGENDRE
KRIVOKHIZHIN et al.	1987	$\Theta_k^{\alpha\beta}(x)$	JACOBI
PARISI, SOURLAS	1979		

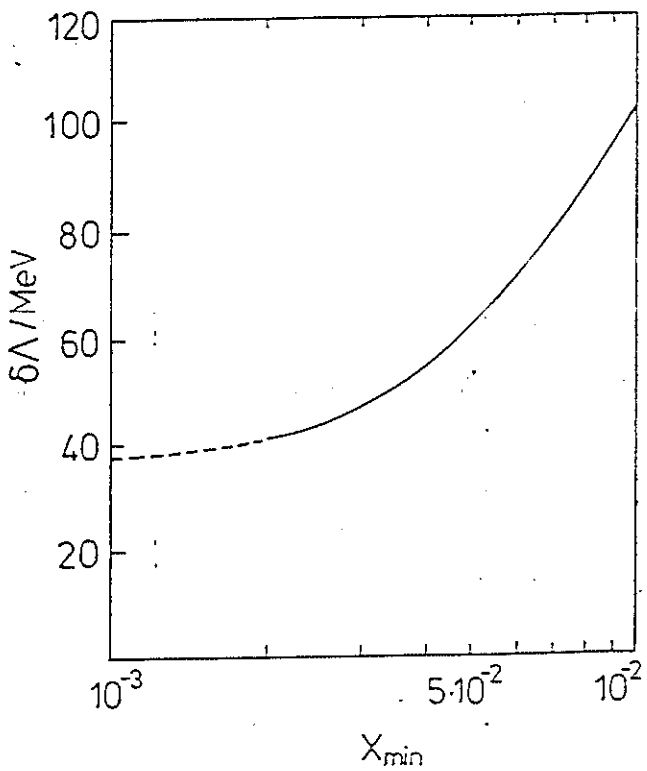
BARKER et al. 1981/84



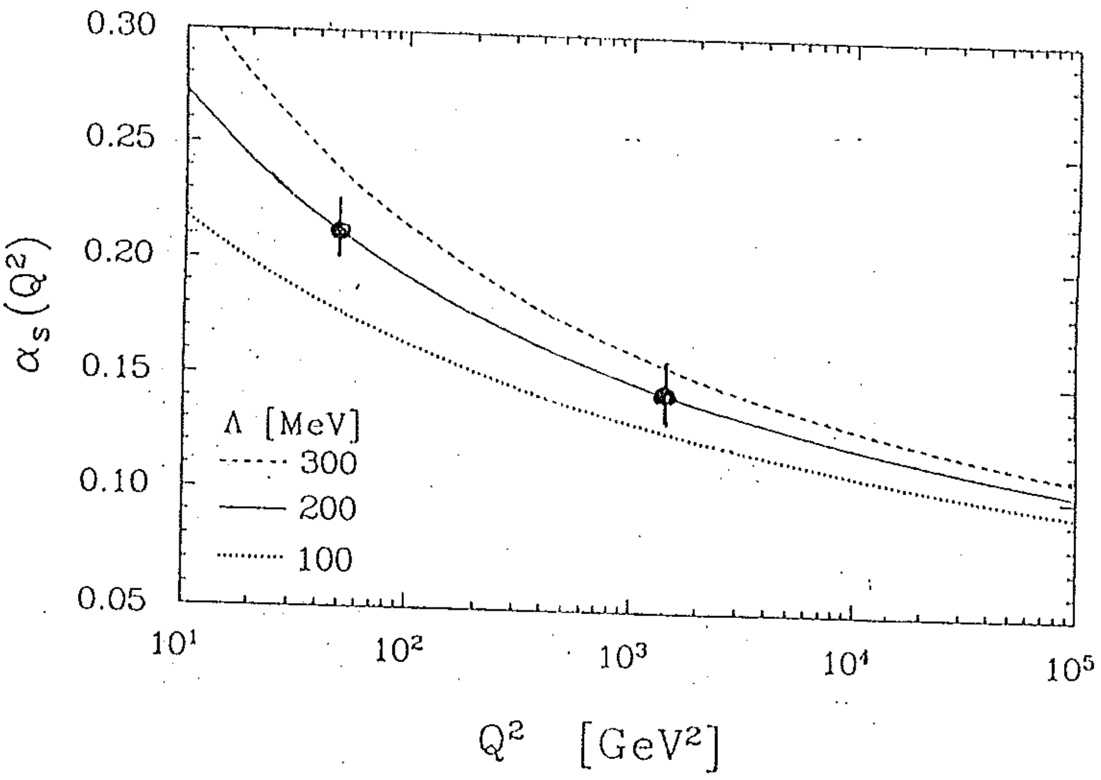
CHÝLA, RAMEŠ, 1986



δΛ/MeV
0
20
40
60
80
100
120
140
160
10
50
100
Q^2_{min}/GeV^2



δΛ/MeV
20
40
60
80
100
120
10^{-3}
$5\cdot10^{-2}$
10^{-2}
X_{min}


0.30
0.25
0.20
0.15
0.10
0.05
$\alpha_s(Q^2)$
Λ [MeV]
300
200
100
10^1
10^2
10^3
10^4
10^5
Q^2 [GeV^2]

CONCLUSIONS AND OUTLOOK

A) THEORY:

- EVOLUTION IN TWIST 2 UNDERSTOOD LO/NTLO
  (ONE AN. DIM → F_2 NTLO, MISSING)

  - F_2, F_L

- LO THEORY $\alpha_S \ln \frac{1}{x} \sim 1$ FADIN–KURAEV–LIPATOV

- PROBLEM : UNITARITY LIMIT $x \to 0$

  ⟶ 1. ATTEMPT : FAN DIAGRAMS

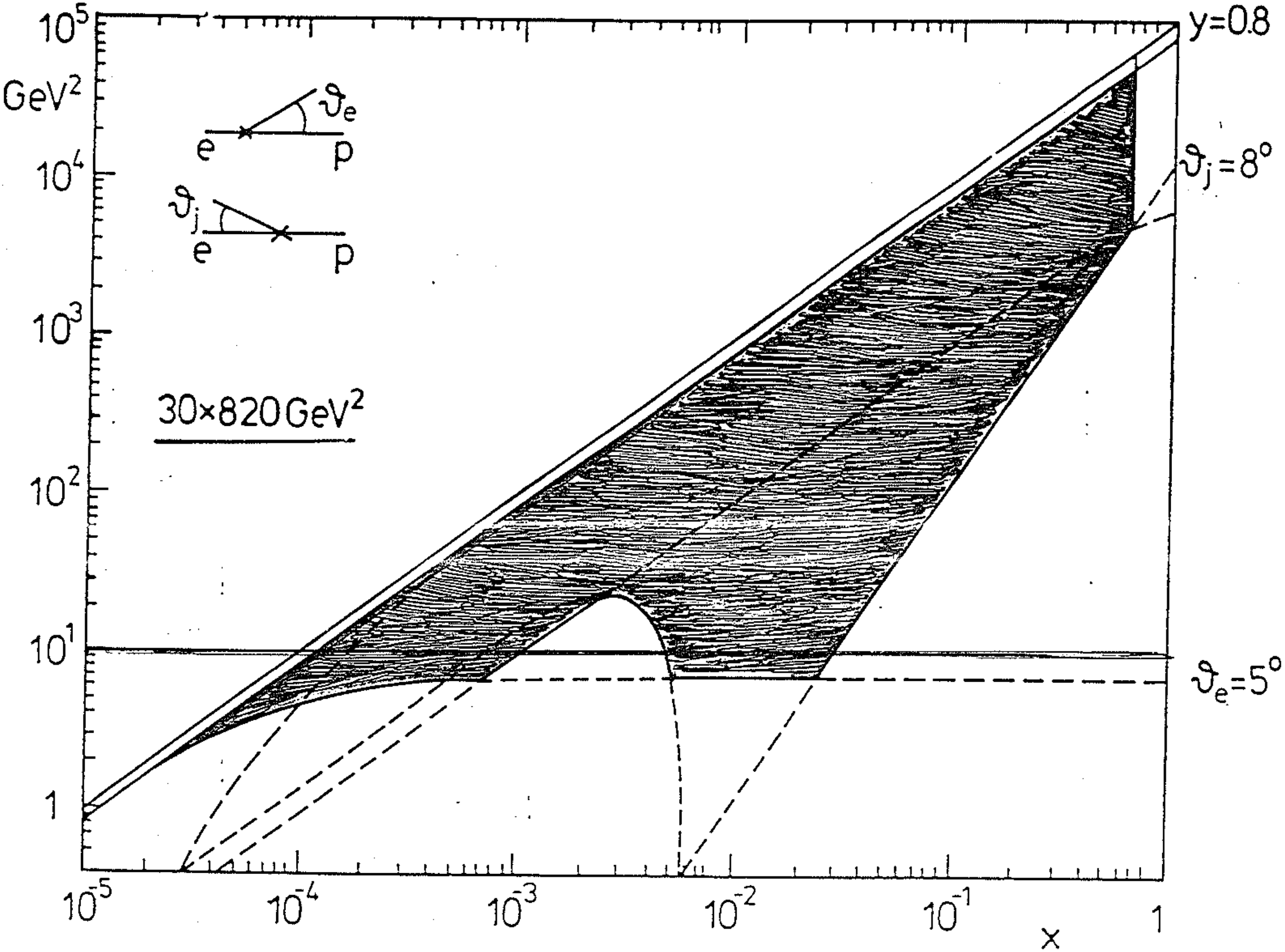
  ⟶ NEED : MORE RIGOUROS PROOF.

- HEAVY FLAVOURS : LO , NEED NTLO

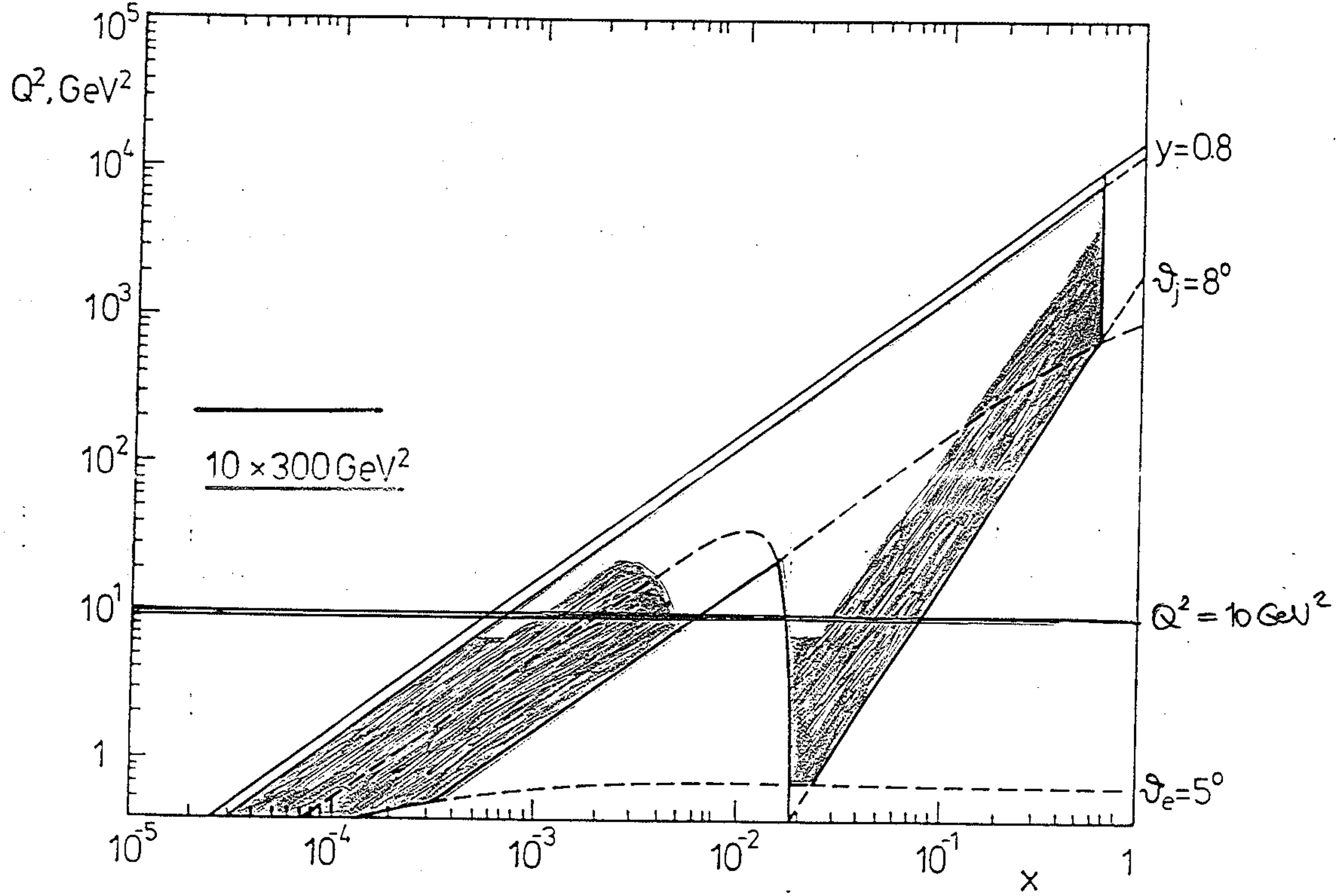
B) EXPERIMENT: HERA

NEED 10 ... 100 pb^{-1} + LOW $\sqrt{s}$ OPTION

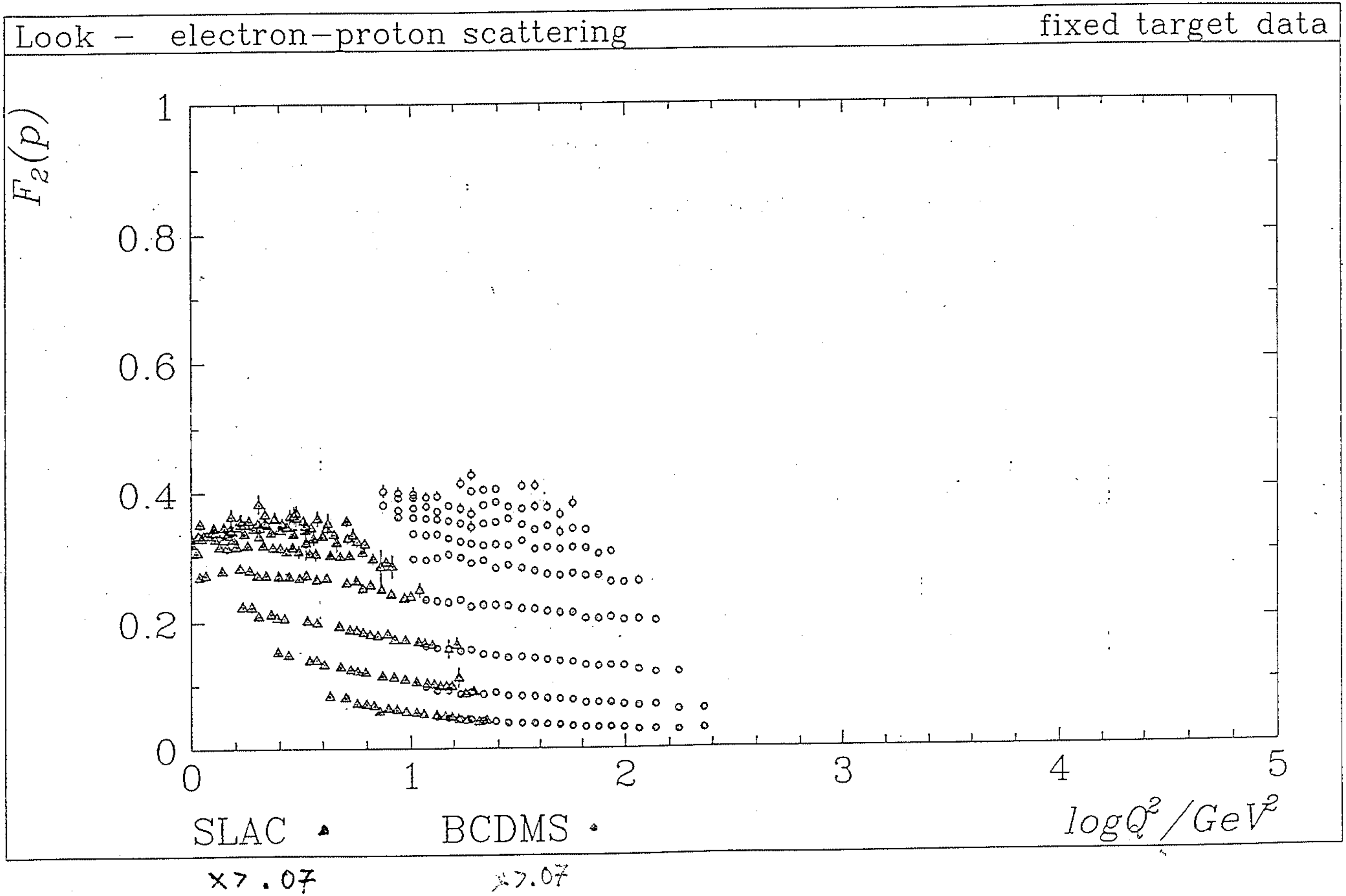
⟶ WIDE F_2, F_L RANGE ! ⟶ F_L !
⟶ VALENCE!



Q^2, GeV^2
10^5
10^4
10^3
10^2
10^1
1
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x
y=0.8
$\vartheta_j = 8^\circ$
$\vartheta_e = 5^\circ$
ϑ_e
e
p
ϑ_j
e
p
30×820 GeV^2

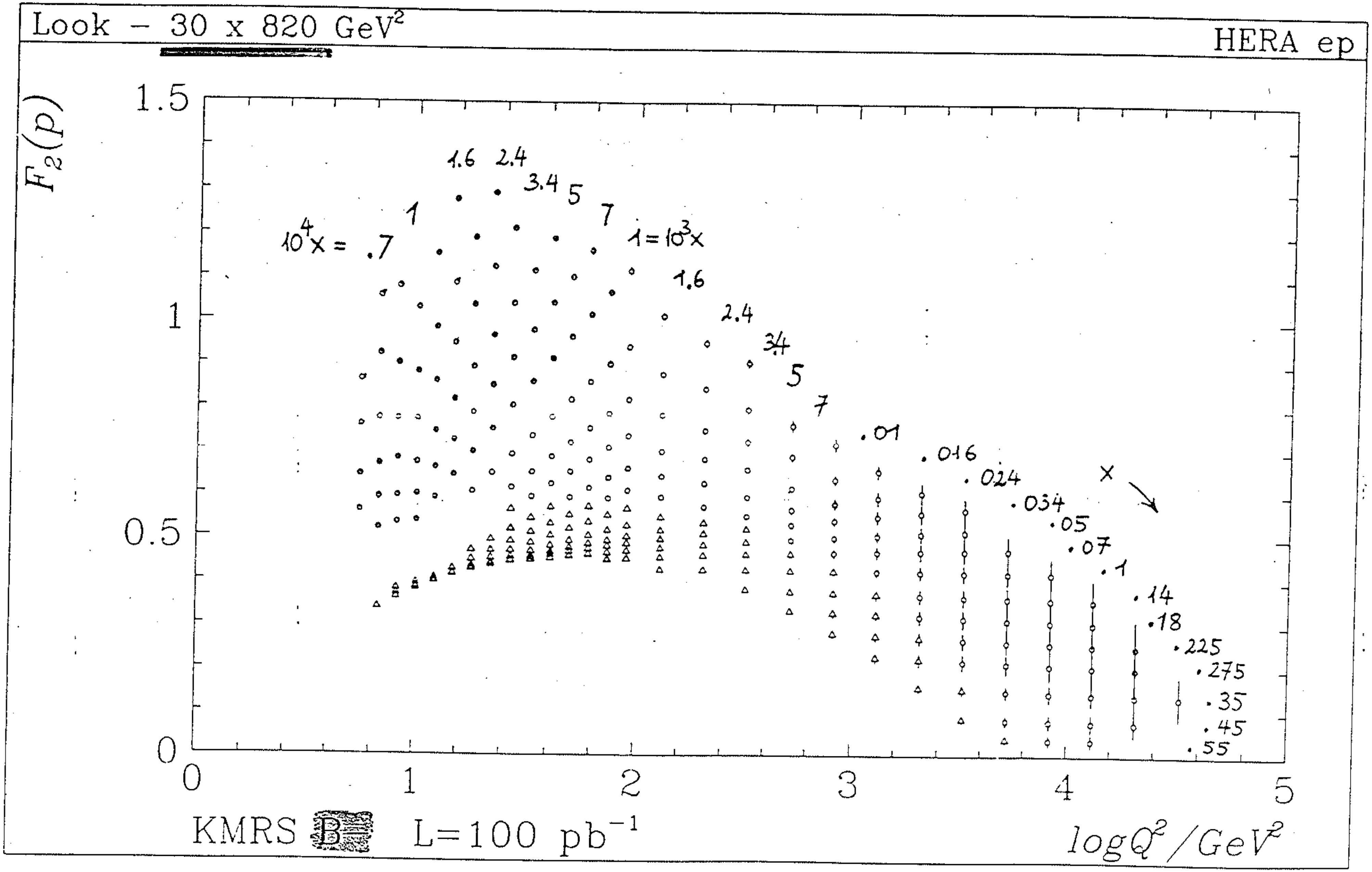


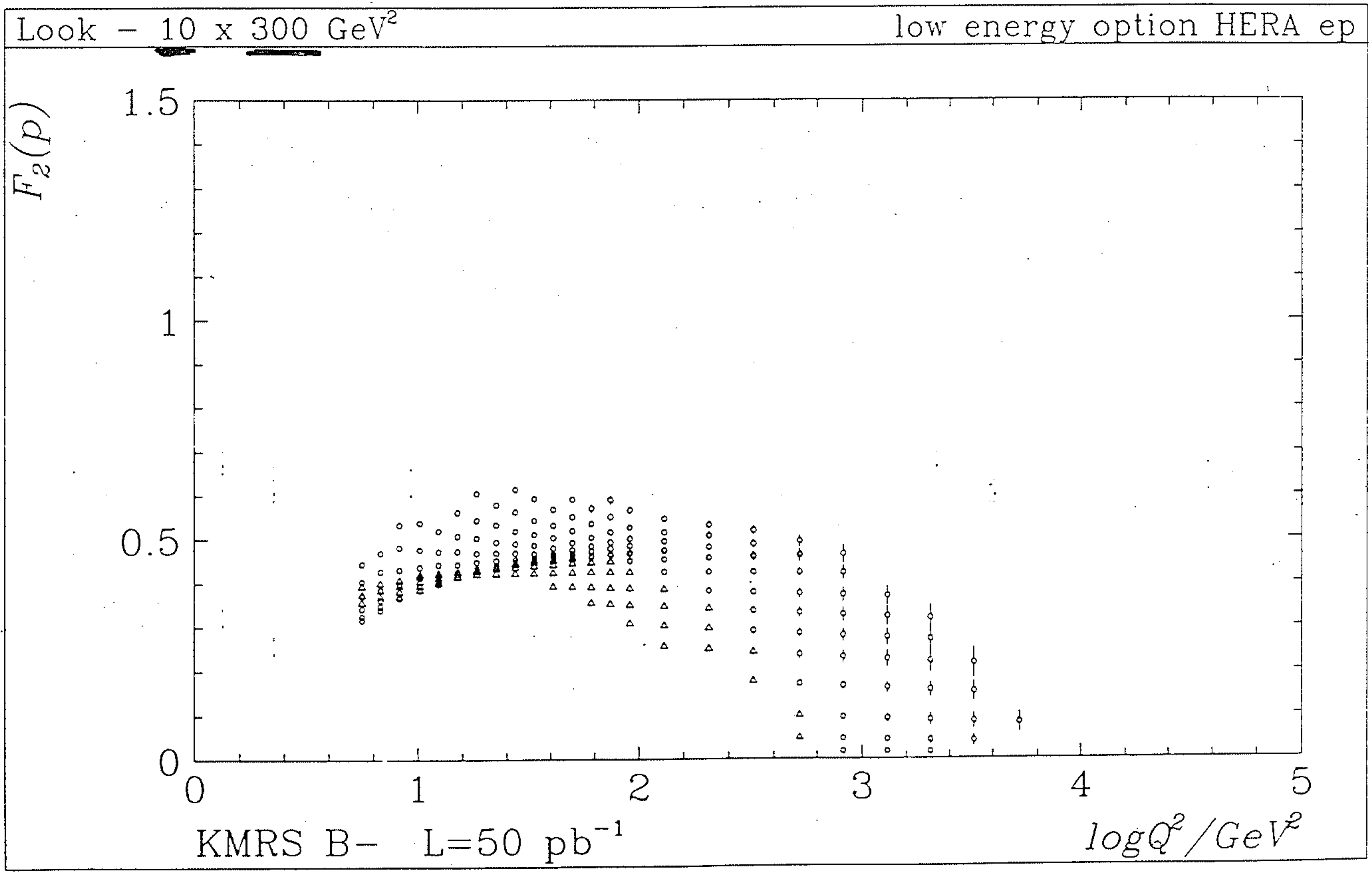
Q², GeV²
10⁵
10⁴
10³
10²
10¹
1
10 × 300 GeV²
y=0.8
ϑj=8°
Q² = 10 GeV²
ϑe=5°
10⁻⁵
10⁻⁴
10⁻³
10⁻²
10⁻¹
1
x

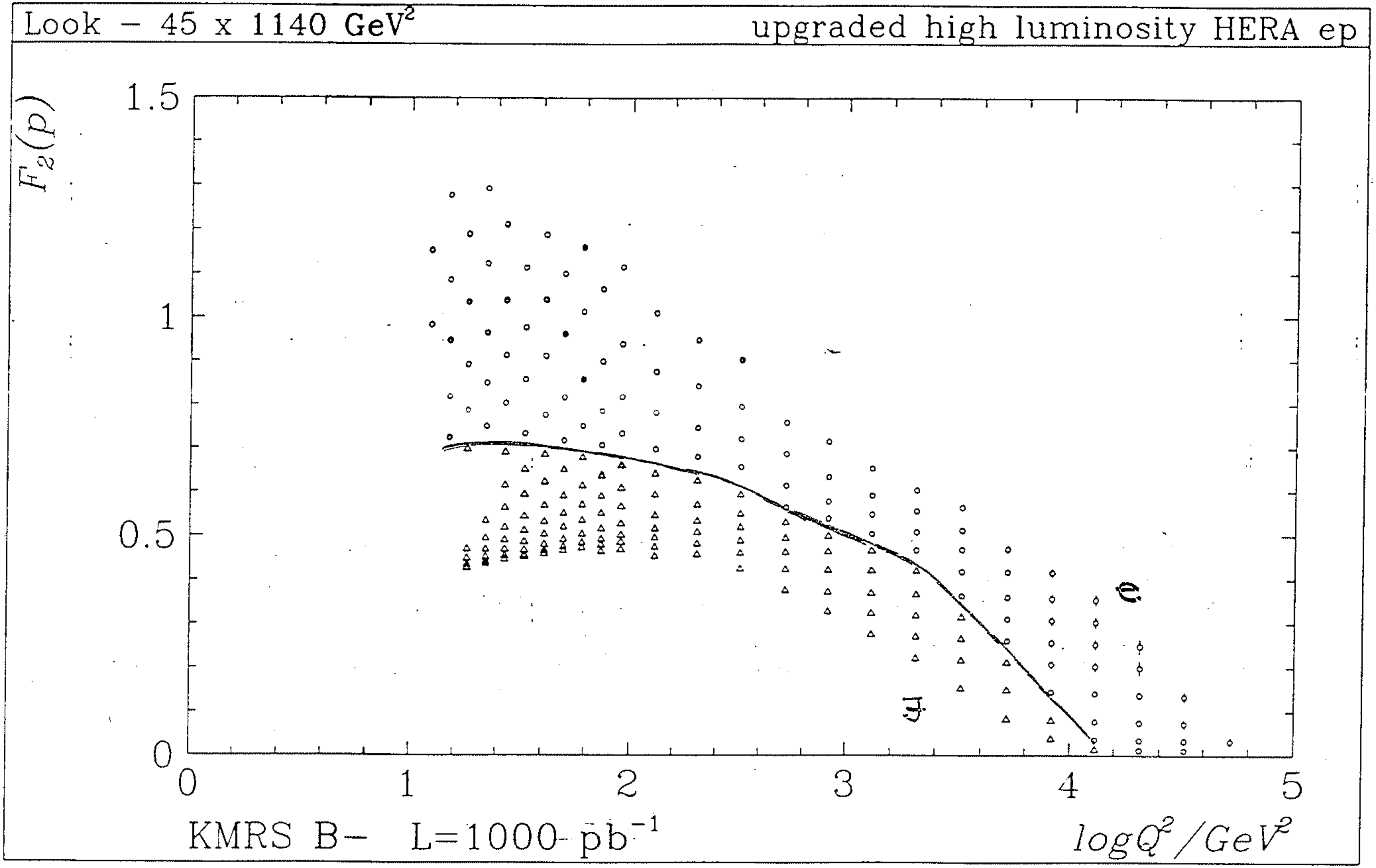


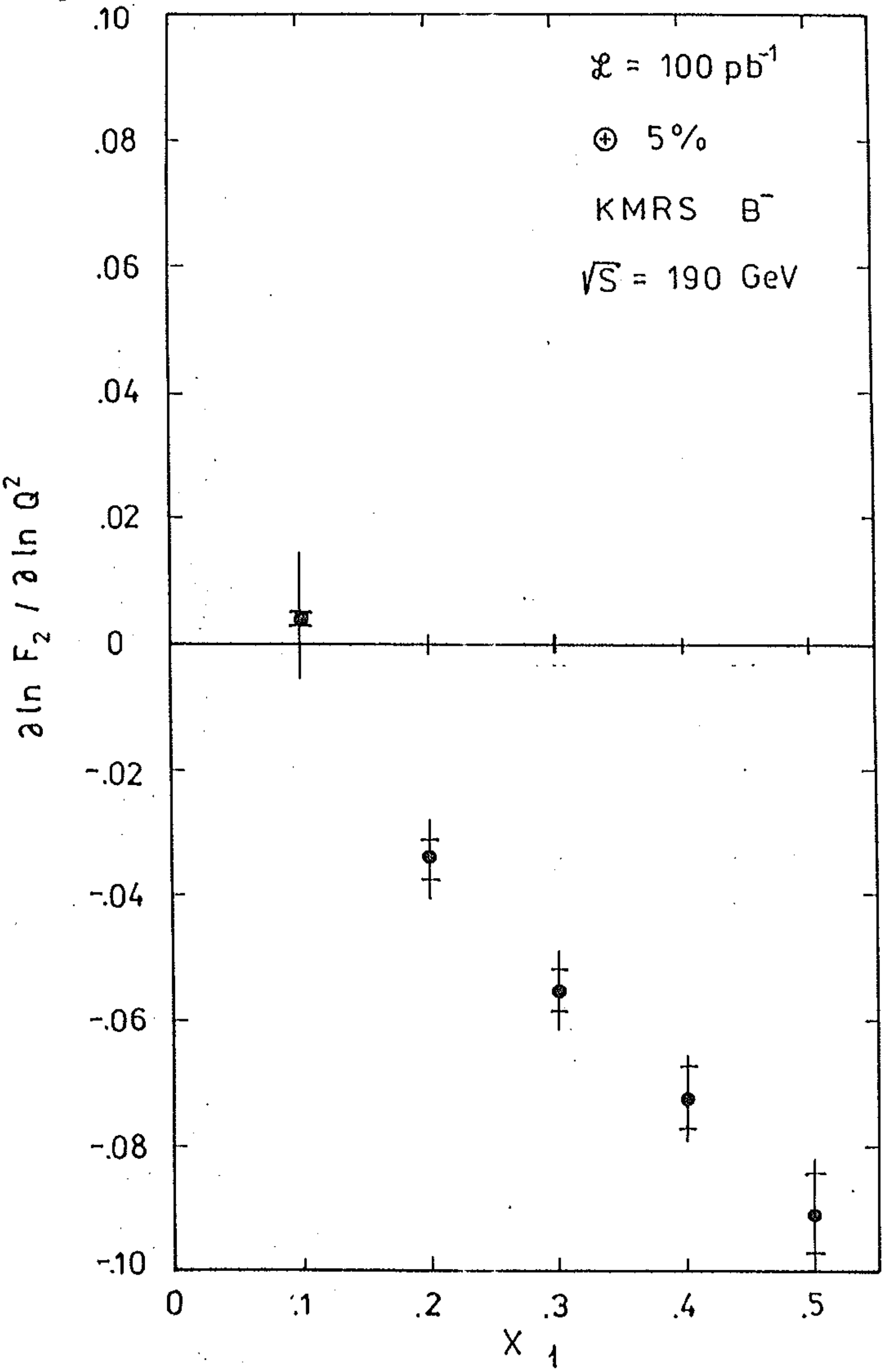
Look - electron-proton scattering
fixed target data
$F_2(p)$
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
$log Q^2 / GeV^2$
SLAC
BCDMS
x7.07
x7.07

BLÜMLEIN, KLEIN



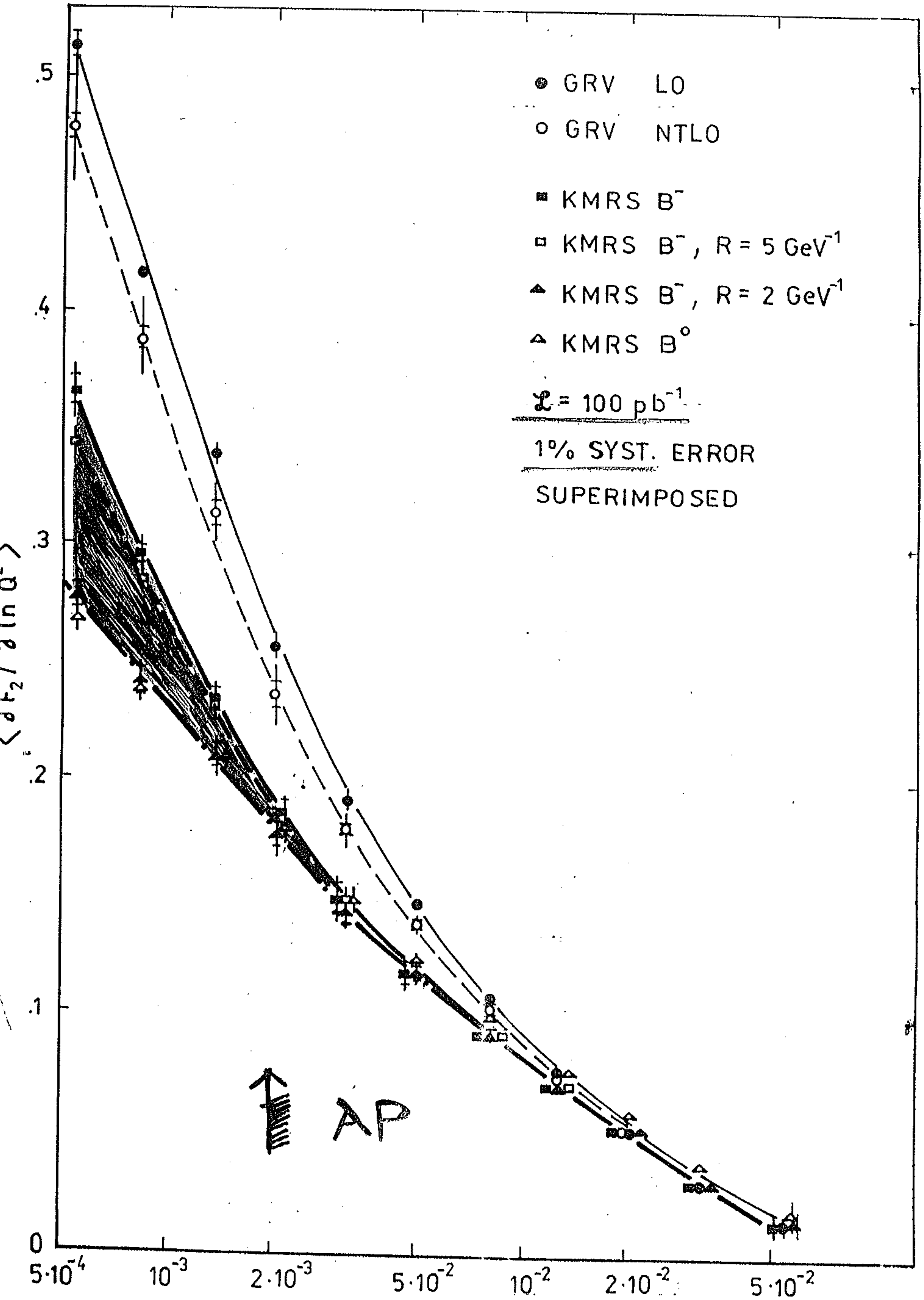

Look - 10 x 300 GeV^2
low energy option HERA ep
$F_2(p)$
1.5
1
0.5
0
0
1
2
3
4
5
KMRS B- L=50 pb^{-1}
$log Q^2/GeV^2$


Look - 45 x 1140 GeV²
upgraded high luminosity HERA ep
F₂(p)
1.5
1
0.5
0
0
1
2
3
4
5
μ
e
KMRS B- L=1000 pb⁻¹
logQ²/GeV²



$$\frac{1}{F_2}\frac{\partial F_2}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\,\frac{\int_x^1 dz\,\frac{1}{z}P_{NS}(z)\,F_2\left(\frac{x}{z},Q^2\right)}{F_2(x,Q^2)} \simeq \alpha_s(Q^2)\,C(x)$$

$x \gtrsim 0.25$ ↑



GRV LO
GRV NTLO
KMRS B⁻
KMRS B⁻, R = 5 GeV⁻¹
KMRS B⁻, R = 2 GeV⁻¹
KMRS B°
ℒ = 100 pb⁻¹
1% SYST. ERROR
SUPERIMPOSED
⟨∂F₂ / ∂ ln Q²⟩
.5
.4
.3
.2
.1
0
5·10⁻⁴
10⁻³
2·10⁻³
5·10⁻²
10⁻²
2·10⁻²
5·10⁻²
X
AP

ALL WE ARE WAITING FOR
THE JULY '92 - RESULTS
ON

$$\partial F_2 / \partial \ln Q^2$$

$$x \sim 10^{-4} \ldots 10^{-3}$$

$$\mathcal{L} > 1\ \mathrm{pb}^{-1}$$