CANDLE Center for the Advancement of Natural Discoveries using Light Emission

Progress in Resistive Wake Fields Study

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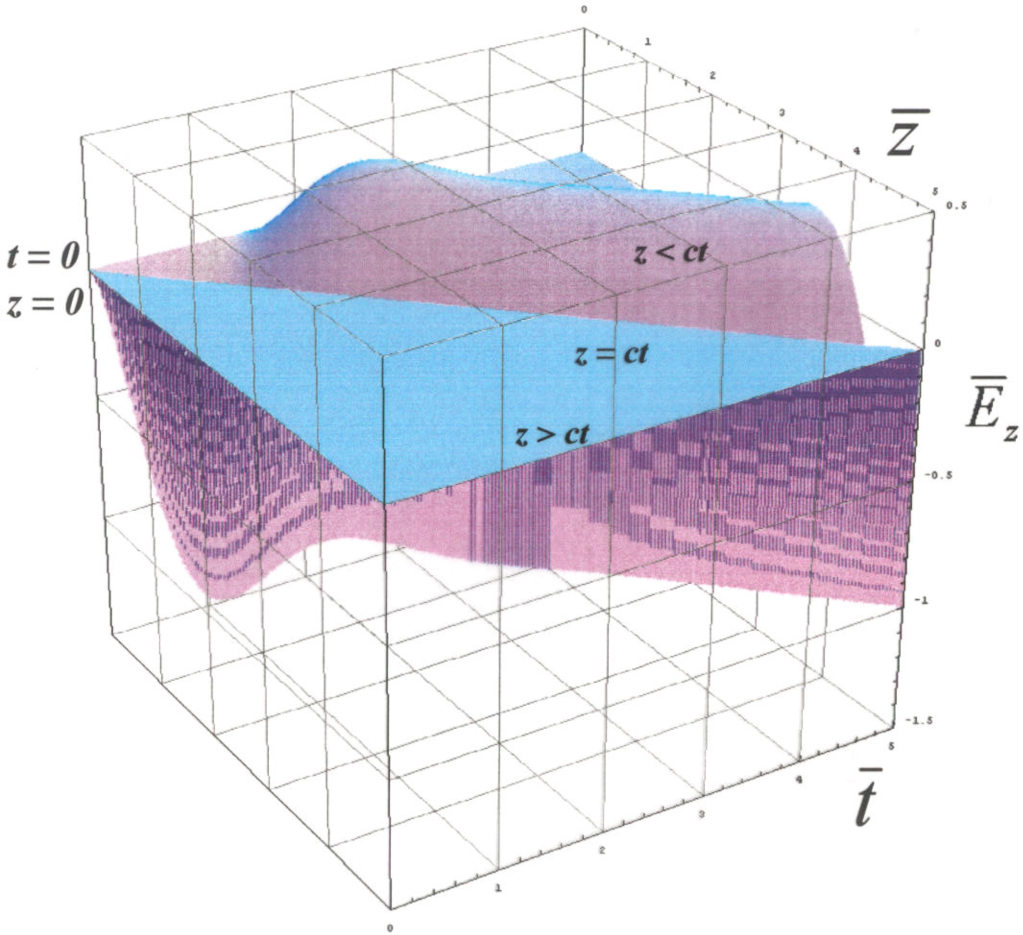
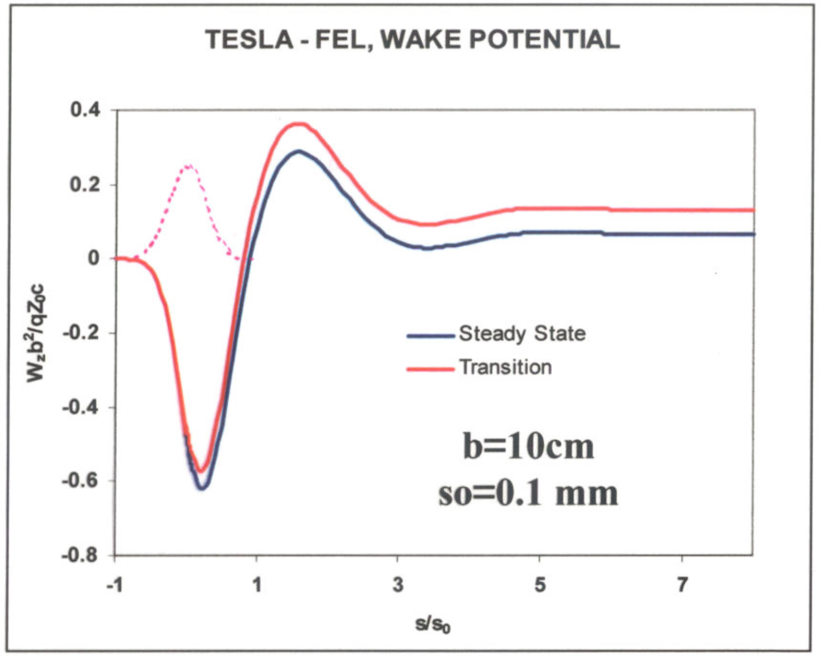
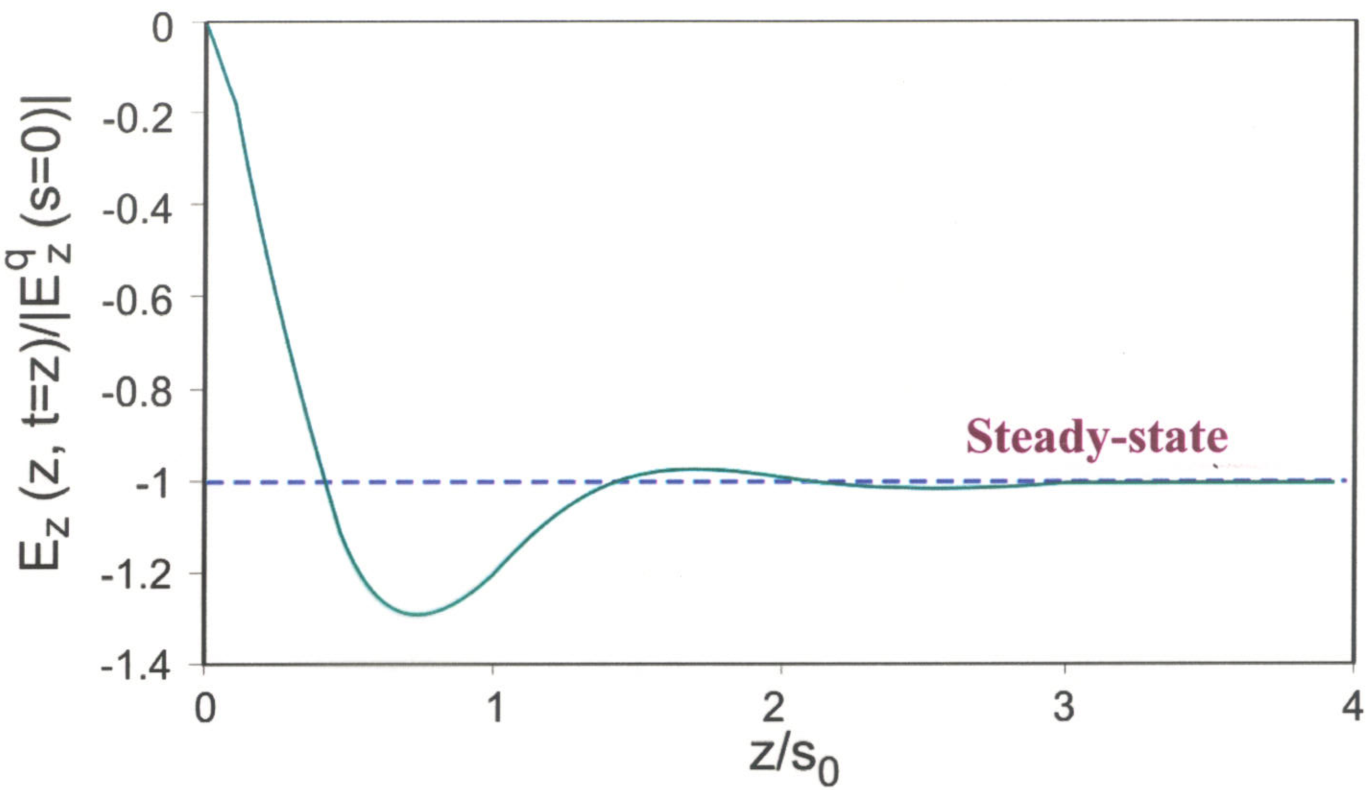
- Steady State Resistive Wakefields
- Transition Resistive Wakefields


t = 0
z = 0
z < ct
z = ct
z > ct
$\overline{Z}$
$\overline{E}_z$
$\overline{t}$


TESLA - FEL, WAKE POTENTIAL
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
$W_z b^2/qZ_0c$
Steady State
Transition
b=10cm
so=0.1 mm
-1
1
3
5
7
s/s_0

zo=5so

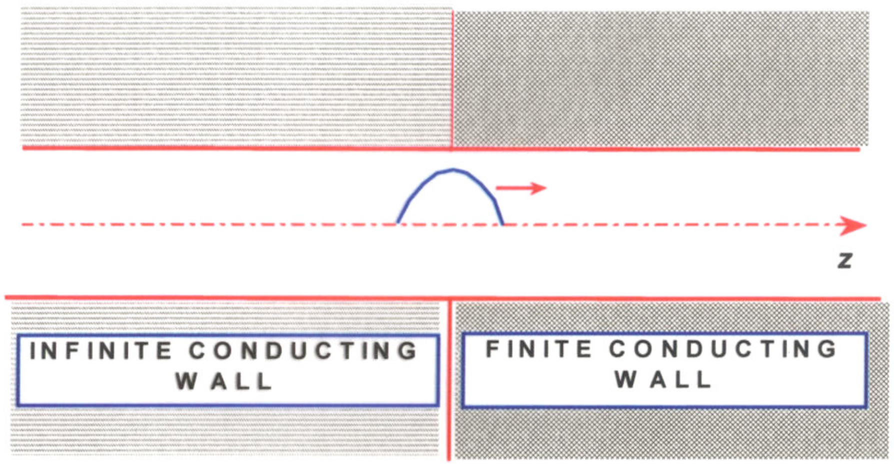


Retarding electric field experienced by driving charge

$$E_z^+(z,t) = E_z^{st}(ct - z) - E_z^{st}(ct + z)$$

Transition length ~ $s_0 = \left(2cb^2\varepsilon_0 / \sigma_c\right)^{1/3}$

The charge transition wakefields in resistive pipe



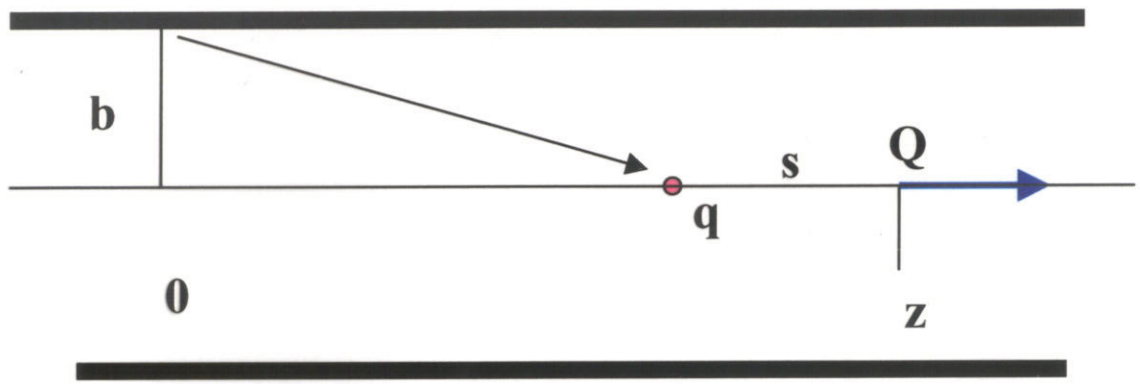
$$\vec{E} = \vec{E}^{st} + \vec{E}^{tr}$$

$\vec{E}^{st}$ - the steady-state solution

Transition fields $$E_z = \sum_{n=1}^{\infty} A_n^{\pm} e^{\pm j p_n^{\pm} z} J_0\left(u_n^{\pm}\frac{r}{b}\right)$$

Fields Matching at z=0 → $$A_n^{+} = \frac{qZ(\omega)}{J_0(u_n^{+})}\prod_{m=1}^{\infty} S_{m,n}$$

Transition fields.



Scattered field reach test particle q at a distance

$$z \approx \frac{b^2 + s^2}{2s}$$

For TESLA-FEL

b=10cm
S= 25 mkm
Z= 400 m

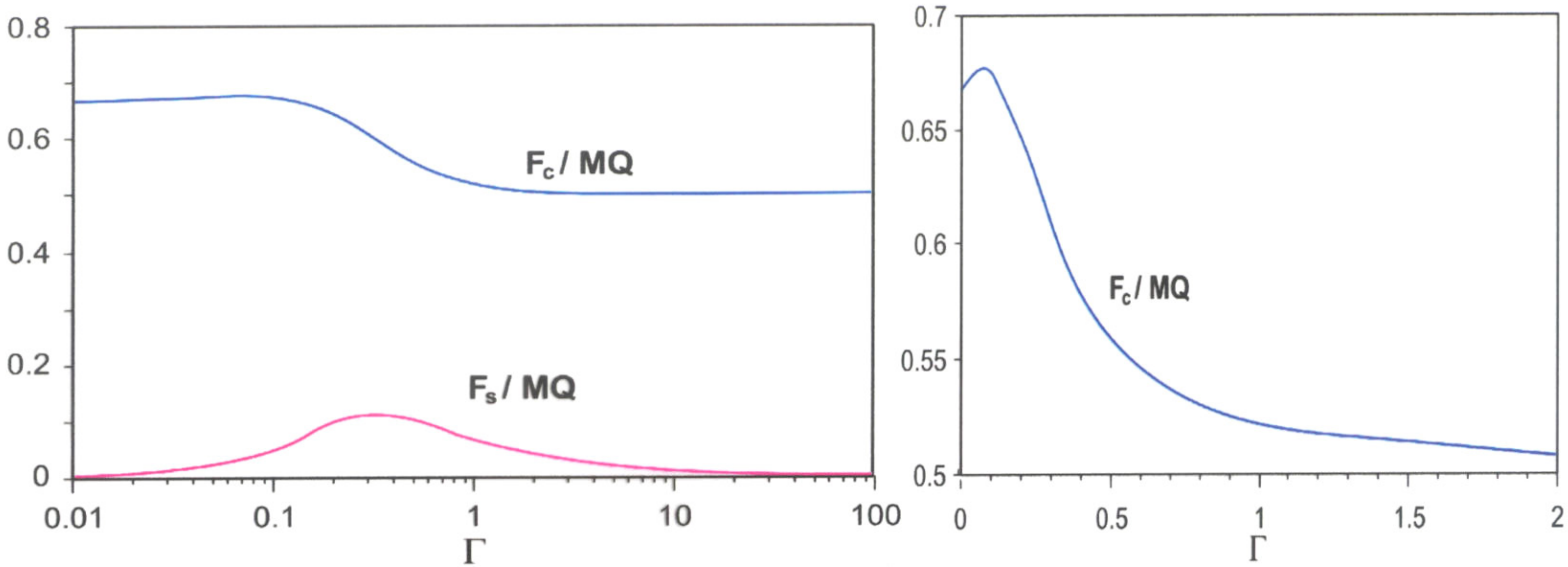
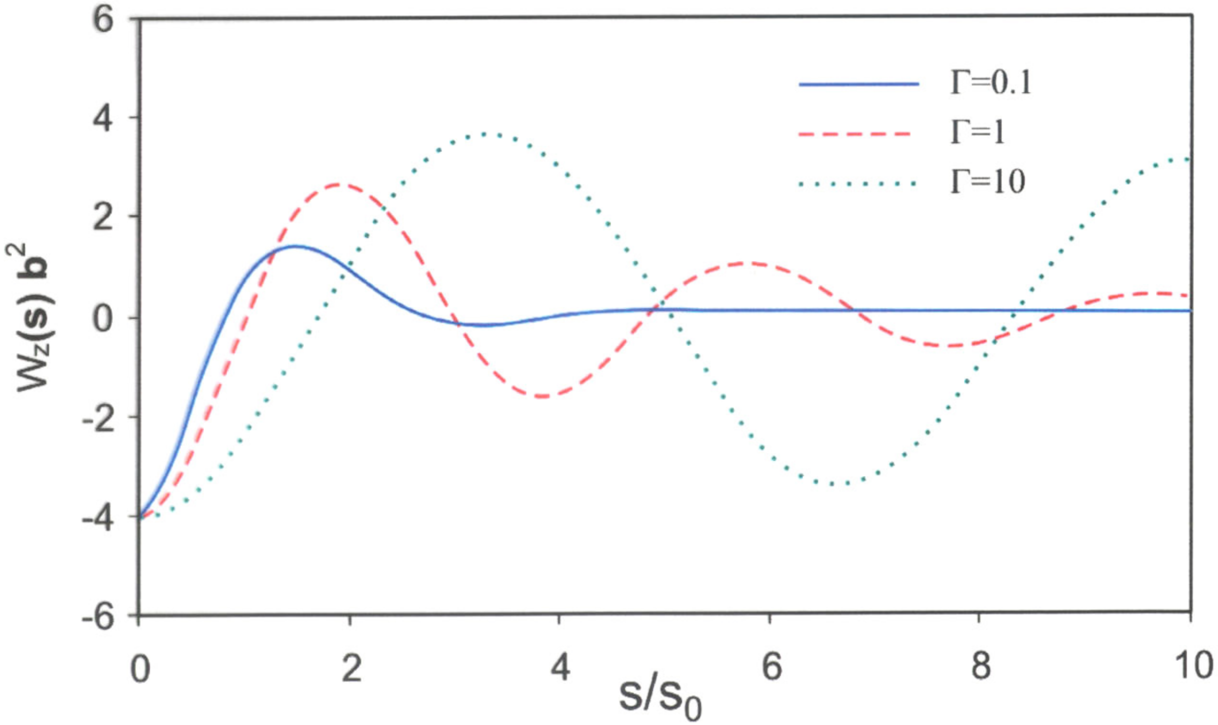


Fig.1. The waves amplitude

Fig.2 Longitudinal wake potential



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Wake potential :

$$W_z(s) = W_z^R(s) + W_z^D(s)$$

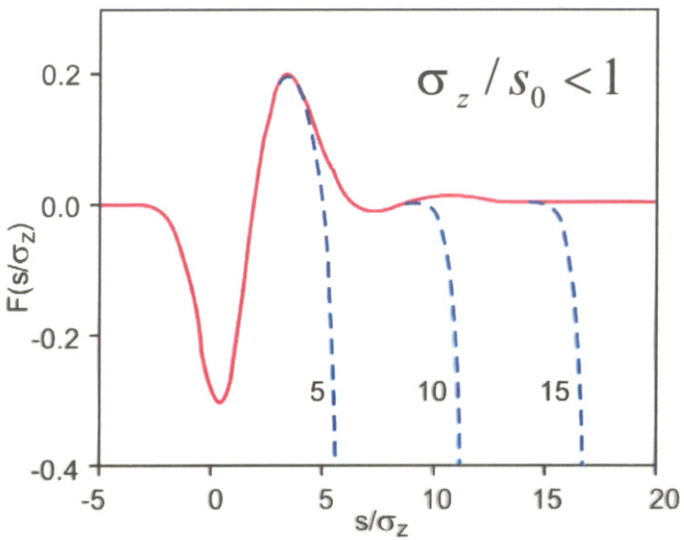
Resonator term : ***(K. Bane)*** ***(correction)***

$$W_z^R(s) = -\frac{8}{b^2 MQ} e^{-\frac{\beta}{M}\frac{s}{s_0}} \left\{ F_c \cos\frac{\alpha}{M}\frac{s}{s_0} + F_s \sin\frac{\alpha}{M}\frac{s}{s_0} \right\}$$

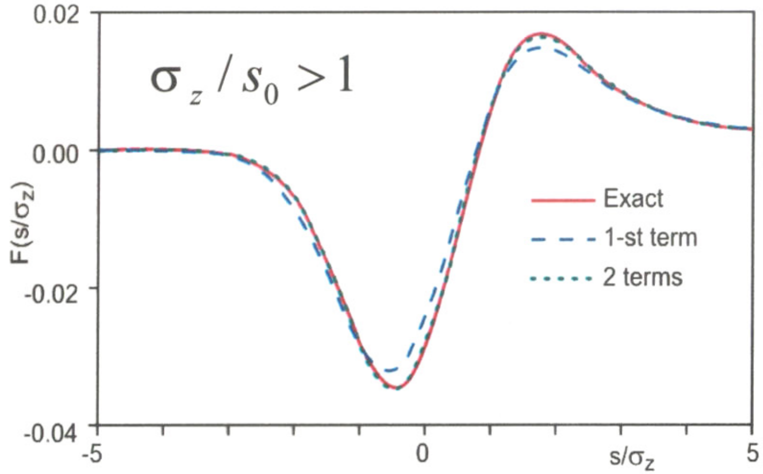
Diffusion term:- analytical presentation

$$W_z^D(s) = \frac{16\sqrt{2}}{b^2\pi} \int_0^\infty \frac{x^2(1+x^2\Gamma)}{8\cdot(1+x^2\Gamma)^4 + x^6} e^{-\frac{s}{s_0}\frac{x^2}{1+x^2\Gamma}} dx$$

Gaussian Bunch



Short Bunch



Long Bunch

$$F(\tilde{z}) = A_1\zeta^{-3/2} + A_2\zeta^{-3} + A_3\zeta^{-9/2} + A_4\zeta^{-6} + \ldots$$

$$A_1 = -\frac{(\pm\tilde{z})^{3/2}}{2^4} e^{-u}\left\{-I_{1/4}(u) + I_{-3/4}(u) \mp I_{-1/4}(u) \pm I_{3/4}(u)\right\}$$ **(A. Piwinski)**

$$A_2 = \frac{1}{2^3\sqrt{2\pi}}(\tilde{z}^2 - 1)e^{-\tilde{z}^2/2}$$ **First correction**

Steady-State Resistive Wakefields

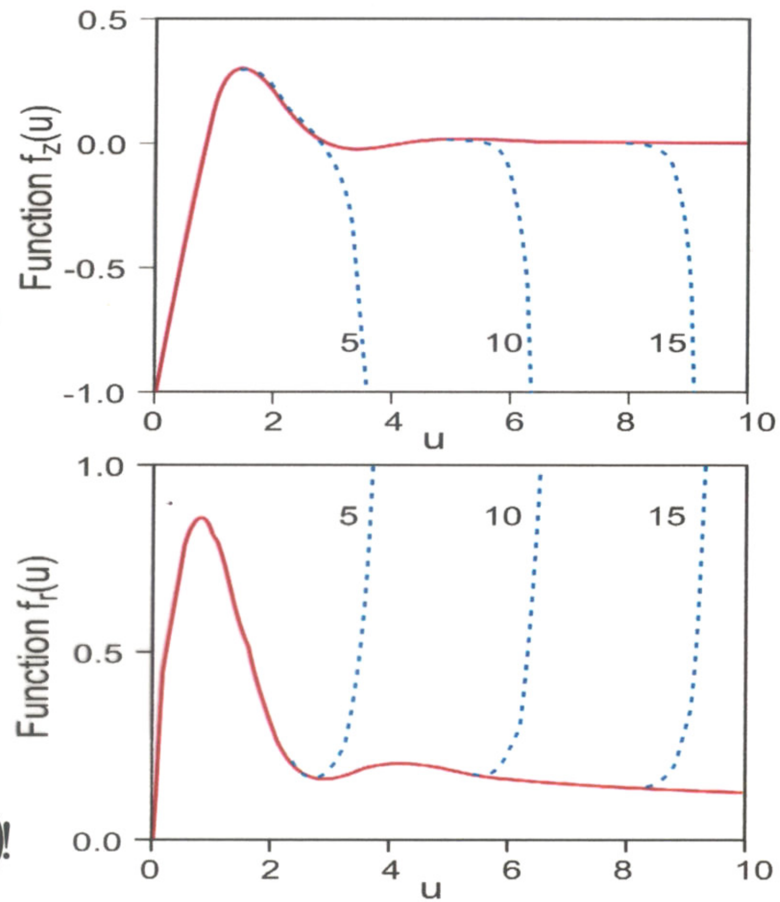
Impedance (A.Chao)

$$Z(\kappa) = \frac{2s_0}{cb^2}\left[\frac{\lambda}{\kappa} - \frac{i\kappa}{2}\right]^{-1}$$

DC conductivity (K. Bane)

$$f_z(u) = -\frac{4}{3}e^{-u}\cos\left(\sqrt{3}u\right) + \frac{4\sqrt{2}}{\pi}I_z(u)$$

$$f_r(u) = \frac{2}{3}e^{-u}\left(\sqrt{3}\sin(u) - \cos\left(\sqrt{3}u\right)\right) - \frac{8\sqrt{2}}{\pi}I_r(u)$$



Series expansion

$$f_z(s) = \sum_{k=0}^{\infty} c_k s^{3k} + \sum_{k=1}^{\infty} d_k s^{3k-3/2}$$

$$c_k = -2^{3k}/(3k!),\quad d_k = \sqrt{2/\pi}\,2^{6k-3}/(6k-3)!$$