SUSY scale from unification

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Abstract

The evaluation of SUSY energy scale is extremely important in order to know if there is any chance to observe SUSY at LHC and at future colliders. Assuming gauge unification at high energy scale, this evaluation can be done with or without the $b-\tau$ unification predicted in GUT frameworks. The single scale approach has been used and we found that M_{SUSY} cannot be greater than about 8.0 TeV. The $b-\tau$ unification occurs only when the threshold corrections are included. We also studied the Yukawa couplings as functions of SUSY scale, $\tan\beta$ and the threshold corrections. Moreover, the prediction for the GUT energy scale and unified gauge coupling is given.

Contents

1 Introduction

The recent detection of the Higgs particle at LHC certainly represents a milestone which has once again confirmed the predictive effectiveness of the Standard Model (SM) of the electroweak interactions. However, it is hardly to believe that the SM is the last step toward the unification, in a simple principle, of all the fundamental interactions. Several phenomena or problems suggest the presence of physics beyond the SM. One can remind just few of them, like the baryonic/leptonic number violation processes necessary at scale larger than the electroweak one in order to yield baryogenesis, the problem related to the separation of very different energy scales in a field theory with scalars (*hierarchy problem*), the hint of a unification of all coupling constants for extreme large energy, the particular structure of fermion masses, etc.
Furthermore, the SM does not include the gravitational force, whose quantum effects become certainly relevant at the Planck scale $\left(\sim 2.4 \times 10^{18}\,\text{GeV}\right)$. This energy scale represents a natural cut-off of any field theory since above this level one expect a dramatic change of the space-time structure, and for this reason theoreticians look for a more comprehensive and fundamental theory of all interactions. The presence of such a natural cut-off above which QFT looses its validity exacerbates the hierarchy problem. The huge gap between the electroweak energy scale, given by the Higgs mass, and the Planck energy scale struggle to be separated once the radiative corrections are considered (quadratic divergences). Indeed, the square of Higgs boson mass suffers for ultraviolet divergent radiative corrections which are quadratic functions of the cut-off. These large quantum contributions to the square of the Higgs boson mass would inevitably make the mass huge, comparable to the scale at which new physics appears. To avoid this one should assume an incredible and unnatural fine-tuning cancellation between the quadratic radiative corrections and the Higgs bare mass. For this reason one expects that some new phenomena should be at work at large energy scale, like for example some sort of symmetry capable to solve or at least to make mild the problem.
In this scenario, Supersymmetry (SUSY) is able to solve the hierarchy problem, or at least to make milder it till to a more acceptable level. SUSY can explain how a tiny Higgs mass can be protected from quantum corrections, removing the power-law divergences of the radiative corrections to the Higgs mass. The recent measurement of the Higgs mass

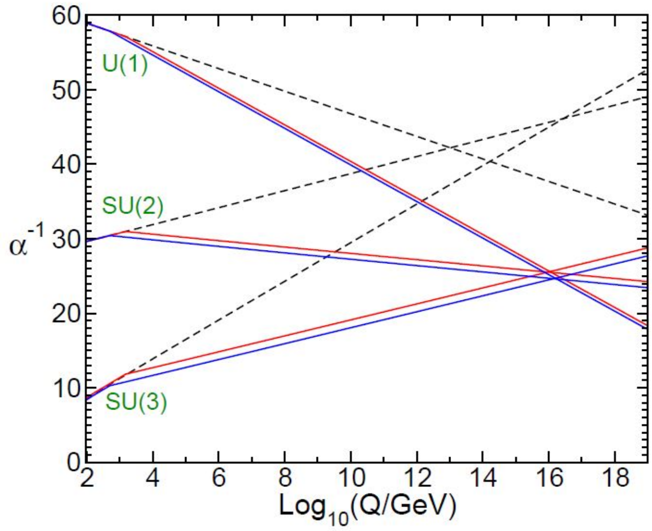

Figure 1: Two-loop running of the inverse gauge couplings in the SM (dashed lines) and the MSSM (solid lines). The two set of solid lines represent a different choice for M_{SUSY} in the single scale approach: the blue lines correspond to $M_{SUSY} = 0.5\,\text{TeV}$ while the red lines correspond to $M_{SUSY} = 1.5\,\text{TeV}$. This plot is taken by [1].

that results to be of the order of the top quark mass has worsen the so called small hierarchy problem which still represents an unnatural feature of the Minimal Supersymmetric extension of the Standard Model (MSSM). In this SUSY framework the gap between the low energy scale and the Planck energy scale is filled by the introduction of new matter required by SUSY, giving the hope to discover new physics at LHC.
The introduction of new particles can give also an answer to the problem of dark matter. In fact, as consequence of forbidding proton decay, the neutralino must be absolutely stable, being the Lightest Supersymmetric Particle (LSP), and it is a candidate for dark matter. Moreover, assuming SUSY as a broken symmetry of the Nature, the unification of the three gauge coupling constants is predicted at high energy scale $\left(\sim 10^{16}\,\text{GeV}\right)$. On the other hand, in the Standard Model the gauge unification is not reached, even if the three gauge couplings tend to get closer at high energy scale, as shown in Figure 1. For these reasons, the idea of gauge unification at high energy scale arise naturally. The aim of this project is to study the level of viability of SUSY models after the LHC measurements. Particularly, the evaluation of the SUSY energy scale (M_{SUSY}) at which the new particles predicted can be detected is really important because it gives an indication of the chance to test these new theories. This evaluation is done assuming the unification of the three gauge coupling constants at high energy scale. The goal is to give an update of previous estimate of M_{SUSY} given for instance in [2, 3].
In the second section, it is explained how the running can be performed, giving the definition the SUSY scale in the single scale approach, introducing the RGE and specifying the experimental values taken into account. The third and fourth sections give an explanation of the threshold effects and of how the Yukawa couplings can be included in the running. In the fifth there is reported in detail the iterative method used. The sixth section is dedicated to results related to both cases without and with the $b-\tau$ unification. Finally, the last section is the conclusions.

2 RGE, Unification and SUSY scale

The SM is extended by SUSY and the effective scale where we have the transition from SM to SUSY is denoted as SUSY scale $M_{\rm SUSY}$. In general we can have more than one scales because the spectrum of the new SUSY particles can be not degenerate: for instance, squark and lepton masses can differ for a factor of about 3. However, for the sake of simplicity, it is common to use a single effective scale: in the single scale approach or the so called "common scale approach" [4], all the new particles predicted by SUSY have the same mass. It would be more suitable to adopt a multi-scale approach in order to describe the more natural situation in which there is a severely split mass spectrum. However this approach is more complicated and it is currently not available. Therefore, in the single energy scale approach the energy scale $M_{\rm SUSY}$ can be properly viewed as the effective energy of SUSY breaking. This means that sparticles have different masses, which are below and above $M_{\rm SUSY}$. In this way, the energy $M_{\rm SUSY}$ represents the energy at which the intersection between the SM asymptotic solution and the MSSM asymptotic solution occurs.

Our aim is to evaluate the range of $M_{\rm SUSY}$ in agreement with the recent experimental data and the unification gauge coupling constants[1]. For this reason, we developed a program in *Mathematica* which resolves all the RGEs (other programs able for such a purpose are RunDec [5], SPheno [6] and SoftSusy [7]). The RGEs are differential equations through which we can translate physical quantities at an input scale into a set of predictions at an another energy scale. We consider in our program RGE at two-loop order

$$\frac{d}{dt}X = \frac{1}{16\pi^2}\beta_X^{(1)} + \frac{1}{\left(16\pi^2\right)^2}\beta_X^{(2)}\,, \tag{1}$$

where $t = \ln\left(M/M_0\right)$ and X is a given coupling. Here, M_0 represents the energy scale at which we impose the initial conditions of the RGEs or, in other words, the energy scale at which we impose the renormalization conditions. The β functions in the SM and in the MSSM are well known and are given by [8, 9, 10, 11] and [12], respectively. We rightly assume that only the Yukawa couplings for the heaviest particles of each family (i.e. top and bottom quarks and tau lepton) give a considerable contributions. At two loops the gauge couplings and Yukawa couplings are all coupled in the RGEs.

In the running from a low to a high energy scale, we need to add separately every particle at its individual threshold, including matching conditions in agreement with the decoupling theorem [13]. In the transition between SM and MSSM, the matching conditions due to the introduction of all the superpartners must be take into account. Furthermore, since in the SM the β functions are evaluated in the $\overline{MS}$ renormalization scheme while in the MSSM they are evaluated using the $\overline{DR}$ renormalization scheme, we need also to take into account the relations related to transition between these two different schemes of renormalization. Then, we need to impose a matching condition in order to decouple the heavy particles not present in the SM and to ensure the change of regularization from DREG to DRED. For these reasons, we use the data[2] reported in the second table of [14]. These data are evaluated performing a Monte Carlo analysis with 1σ uncertainties. Furthermore, these values have already been transformed from $\overline{MS}$ to $\overline{DR}$ scheme. Since there are reported

[1] We will consider the cases with and without $b-\tau$ Yukawa couplings unification at the GUT scale.

[2] By convention, we call these data as experimental data in the sense that they are obtained by evolving the experimental data taken at low energy scale to $\mu = M_{SUSY}$.

three different set of data related to three different values of M_{SUSY}, we have performed a parabolic fit in order to cover all the range between 0.5 and 10.0 TeV.
In order to impose the unification of gauge coupling constants, for a given M_{SUSY} we could:

- evolve firstly α_1 and α_2 from M_{SUSY};
- find their intersection point which defines the values of α_{GUT} and the energy M_{GUT};
- evolve backward α_3 using (α_{GUT}, M_{GUT}) as initial point.

In this way, for each value of M_{SUSY} we obtain a value of α_3 at M_{SUSY} which must be compare with the experimental data. Hence, we obtain an estimate for the range of M_{SUSY} in agreement with $\alpha_3\,(M_{SUSY})$, obtained by the running.
This is precisely the purpose of such a work. It is important to point out that, if we follows the same procedure in the framework of the SM, we will not find compatibility between the unification and the experimental values within the experimental errors.

3 Threshold corrections

The threshold corrections arise from including new particles in the running of couplings, as the decoupling theorem states. In general, there are two kind of threshold corrections, known as low and high thresholds. The former is related to the introduction of superpartners in the transition between the SM and the MSSM, whereas the latter arises from the introduction of the new particles predicted by the GUTs[3]. We have taken into account only the low threshold corrections in the only case of single scale approach as in [14]. However, in the more natural and true multi-scale approach, we should include threshold corrections for each superparticle.
On the other hand, we are also interested in the running of Yukawa couplings, giving particular attention to the $b-\tau$ unification (see [15] for recent study in similar direction). Although the running masses are well known at low energy scale, this is not true for the Yukawa couplings. Indeed, the threshold corrections (see [14, 16])

$$y_t^{SM} = y_t^{MSSM}\sin\beta\,, \tag{2}$$
$$y_b^{SM} = (1+\eta)\,y_b^{MSSM}\cos\beta\,, \tag{3}$$
$$y_\tau^{SM} = y_\tau^{MSSM}\cos\beta\,, \tag{4}$$

must be take into account. Here η is the threshold parameter on bottom Yukawa coupling and the angle β is defined by the ratio of VEVs taken by two Higgs doublet in the MSSM, i.e. $\tan\beta \equiv \frac{v_u}{v_d}$. These threshold corrections are related to $\tan\beta$-enhanced corrections, which modify the tree-level relations ($\eta = 0$), to weak effects due to electroweak symmetry breaking and to the possibility of having a split sparticle spectrum (multi-scale approach). In this framework, η and β are free parameters. Consequently, there is some arbitrariness in the initial conditions at M_{SUSY} for the differential equations of Yukawa couplings. The infrared fixed point of the renormalization group equation for the top Yukawa coupling yields that the high energy situation is even more arbitrary. Indeed, a small difference in initial conditions of top Yukawa coupling at low energy scale makes a huge difference at

[3] For instance, in $SU\,(5)$ we need to include the so called leptoquark.

GUT energy scale. It would be really recommended to integrate these differential equations backwards. In this way, we could study better the high energy region and estimate the value of β which is a fundamental parameter in the MSSM phenomenology. Furthermore, the threshold corrections related to η must be taken into account as shown in (3). For these reasons, we impose the initial value of the top Yukawa at M_{GUT}, checking later his agreement with the experimental data at M_{SUSY}.

4 Including Yukawa contributions in the RGE

As regard the $b-\tau$ unification, imposing the initial condition of y_τ at GUT energy scale requires a very high precision since the error of y_τ at M_{SUSY} is too small. Therefore, we use the mean value of y_τ at M_{SUSY} as initial condition of RGE and resolve its differential equation up to M_{GUT}. Here, we impose the $b-\tau$ unification using $y_\tau(M_{GUT})$ as initial condition of the bottom Yukawa RGE which is consequently resolved backward. In this way, the agreement of $b-\tau$ unification with the experimental data at M_{SUSY} is given by checking the obtained value of $y_b(M_{SUSY})$ with its experimental value at M_{SUSY}. Hence, in order to characterize the theory at GUT energy scale and to impose the $b-\tau$ unification, knowing M_{GUT} by studying the running of gauge coupling constants, for a given M_{SUSY}, β and η we could:

- evolve backward y_t from M_{GUT} to M_{SUSY}, using some arbitrary initial condition;
- evolve y_τ using its mean experimental value, from bottom to top;
- evolve backward y_b imposing $y_b(M_{GUT}) = y_\tau(M_{GUT})$ as initial condition.

In this way, for each value of M_{SUSY}, β and η we obtain the values of $y_t(M_{SUSY})$ and $y_b(M_{SUSY})$ which must be checked with the experimental data at M_{SUSY}.
However, the two-loop corrections to β functions complicate considerably the problem. Unlike the one-loop case, each differential equation of gauge couplings depends on the running of all gauge coupling constants and all Yukawa couplings, implying that they must be resolve simultaneously. Therefore, we cannot resolve firstly the differential equations for g_1 and g_2, resolve backward the differential equation for g_3, imposing the unification, and then studying the running of Yukawa couplings.
All previous arguments suggest anyway that the differential equations for g_1, g_2 and y_τ should be resolved from low to hight energy scales, whereas those for g_3 and the other Yukawa couplings should be integrated imposing initial conditions at GUT energy scale, in order to impose the gauge unification and the $b-\tau$ unification and to give a prediction on the value of β as function of threshold parameter η.

5 Method

We need to employ an iterative method to resolve all the RGEs as we want. For a given value of M_{SUSY}, β and η, the first step of the iterative method is to find the one-loop solutions of all the couplings. In Figure 2 there is shown the steps necessary to find the $i+1$ solution knowing the i solution. Furthermore, we need to require that the theory remains perturbative during the running.
Hence, the inputs of the program are

$$\{M_{SUSY},\, g_1(M_{SUSY}),\, g_2(M_{SUSY}),\, y_\tau(M_{SUSY}),\, y_t(M_{GUT}),\, \tan\beta,\, \eta\}\;. \tag{5}$$

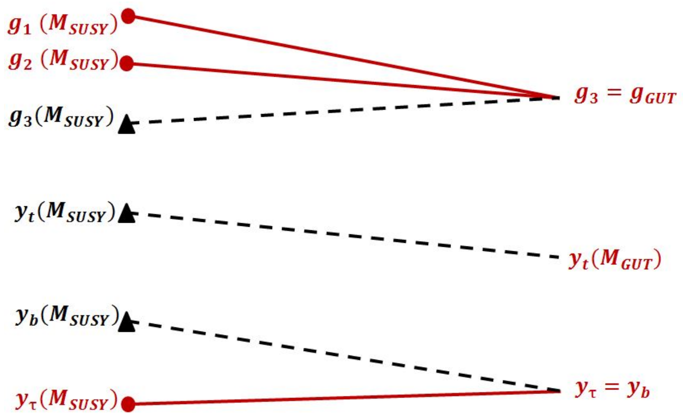

Figure 2: Steps of the iterative method implemented in the program which resolve the RGEs of the gauge and Yukawa couplings, imposing the gauge unification and the $b-\tau$ unification. All the red solid lines mean that the differential equations are resolved from low to high energy scales, while all the black dashed lines mean that the RGEs are resolved backwards. Furthermore the red circles represent the initial conditions of RGEs, whereas the black triangles are the results which must be checked with the experimental data at M_{SUSY}.

The first is the energy scale at which the constants must be run using the MSSM RGEs in the common scale approach. The other three values are taken by the experimental data. In particular $y_\tau(M_{SUSY})$ is always equal to the mean value, whereas $g_1(M_{SUSY})$ and $g_2(M_{SUSY})$ are taken randomly following their normal distribution. For a given value of all the other inputs, we generate 200 different pairs of $g_1(M_{SUSY})$ and $g_2(M_{SUSY})$. The last three input values are arbitrary. We have that

- M_{SUSY} comes from 0.5 TeV to 10.0 TeV with a step equal to 0.25 TeV, having overall 39 points;
- $y_t(M_{GUT})$ is defined from 0.3 to 1.0 with a step equal to 0.025, having overall 28 points[4];
- $\tan\beta$ is defined from 5 to 60 with a step of 5.0, having overall 12 points;
- η comes from -0.6 to 0.6 with a step equal to 0.05, having 24 points.

On the other hand, the outputs of the program are

$$\{g_3(M_{SUSY}),\, y_t(M_{SUSY}),\, y_b(M_{SUSY}),\, \alpha_3(M_{GUT}),\, M_{GUT}\}\,. \tag{6}$$

The first three values are used to check the agreement with the experimental data. The check is performed asking if the obtained values are or not are within 3 sigma from their mean values. In this way, we can discern the good choices of $(M_{SUSY},\, y_t(M_{GUT}),\, \tan\beta,\, \eta)$ from the bad ones, in the agreement with data and unifications. Furthermore, the last two values $(\alpha_3(M_{GUT}),\, M_{GUT})$ describe instead the high energy region at which GUTs can be implemented.

[4]The previous analysis has shown that there are not allowed values for $y_t(M_{GUT})$ above 1.0.

6 Results

We now report and discuss all of our results. First of all, we will consider what happens by resolving all the RGEs without the $b-\tau$ unification, i.e. imposing only the gauge unification. Then we will focus on the case of $b-\tau$ unification.

Case without $b-\tau$

We find that the requirements of gauge unification and of agreement with experimental data yield M_{SUSY} to be less than 8.0 TeV, as shown in Figure 3a. Here and in all the next histograms, the probability represents the relative number of points within bins. In the histogram there is reported the marginalized distribution of M_{SUSY}. We are in agreement with [2], in which the estimate of allowed M_{SUSY} is $10^{3.0\pm1.0}$ GeV. This is an important result since it shows that the gauge unification can be reached in the framework of MSSM. Indeed, the last measurements taken at ATLAS state the mass of the SUSY partners should be greater than about 1.7 TeV. The Figure 3a shows that there are situations in which M_{SUSY} is larger that 2 TeV. However, we must remember that in the single energy scale approach M_{SUSY} represents an *effective* energy, meaning that sparticles can also have mass below M_{SUSY} The principle of naturalness yields that SUSY energy scale can be split by a factor of 2 or 3. A greater split would leads to problem of so called little hierarchy.
The allowed values of threshold parameter η are shown in Figure 3c. Since we do not impose the $b-\tau$ unification, the only constraint on η is the requirement of perturbative couplings. For large and negative values of η the bottom and tau Yukawa couplings become nonperturbative. This is indeed shown in Figure 3c: the large and negative values of η are disadvantaged.

Case with $b-\tau$

In this case, the range of allowed values for M_{SUSY}, shown in Figure 3b, is in agreement with the previous case. Indeed, the range of M_{SUSY} is again between 0.5 TeV and 8.5 TeV. However, this time the threshold parameter η is highly constrained by the requirement of agreement with experimental values of $y_b\,(M_{SUSY})$. Indeed, η takes on only four values, from -0.25 to -0.10 (see Figure 3d).
Now, since we have to check also the obtained values of $y_b\,(M_{SUSY})$ with the experimental data at M_{SUSY}, we expect that the range of allowed M_{SUSY} depends deeply on the value of threshold parameter η. Then, it would be very interesting to see how the range of M_{SUSY} depends on this threshold parameter. In Figure 4a there is reported the marginalized distribution of M_{SUSY} as function of the four different allowed values of η. It is important to point out that we need a large and negative value of the threshold parameter η in order to reach high values for M_{SUSY}. If we sum on all the values of η, we will find the same histogram shown in Figure 3b. However, the $b-\tau$ unification can be reached only with large threshold corrections from 10 % to 25 %.
In the Figure 4b the unified gauge coupling is given as function of the GUT energy scale. The graphs is obtained by taking all the points which are in agreement with the experimental data at low energy. There is a considerable correlation between these values since they are organized in a band. We point out that all the values of M_{GUT} are above the experimental limit coming from the proton lifetime.
As regards the Yukawa couplings, we are interested in their allowed values at GUT energy

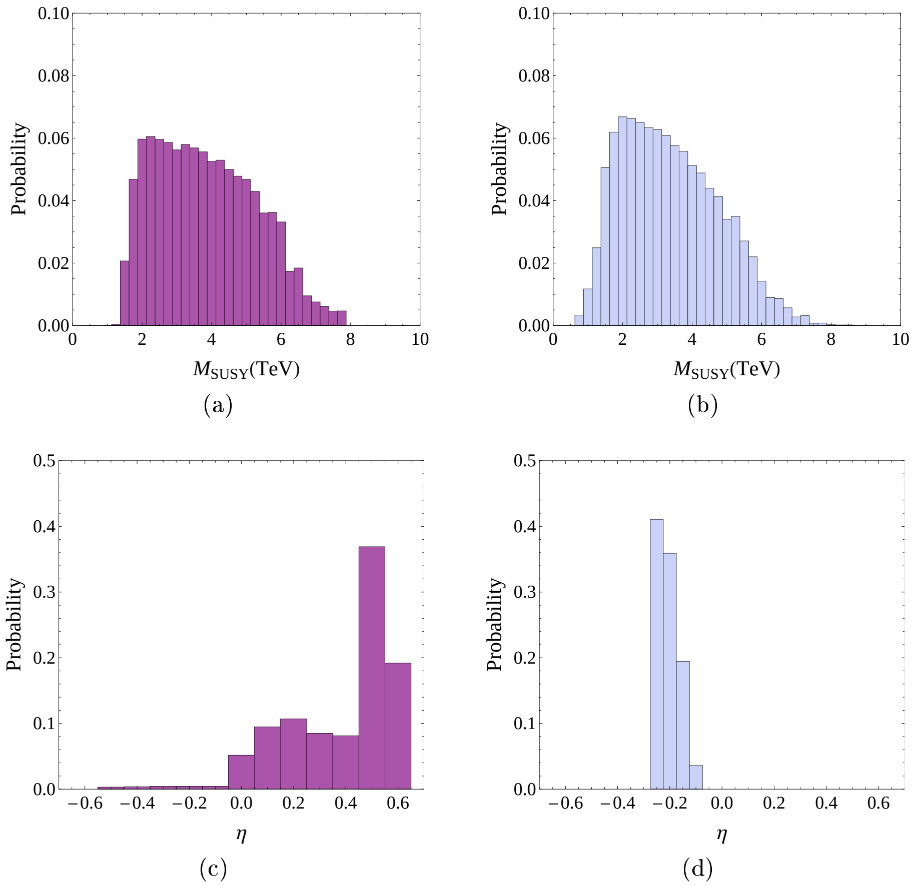

Figure 3: The histograms show the allowed values of M_{SUSY} and threshold η without and with the $b-\tau$ unification: (a) and (c) are without Yukawa unification while (b) and (c) are obtained assuming it.

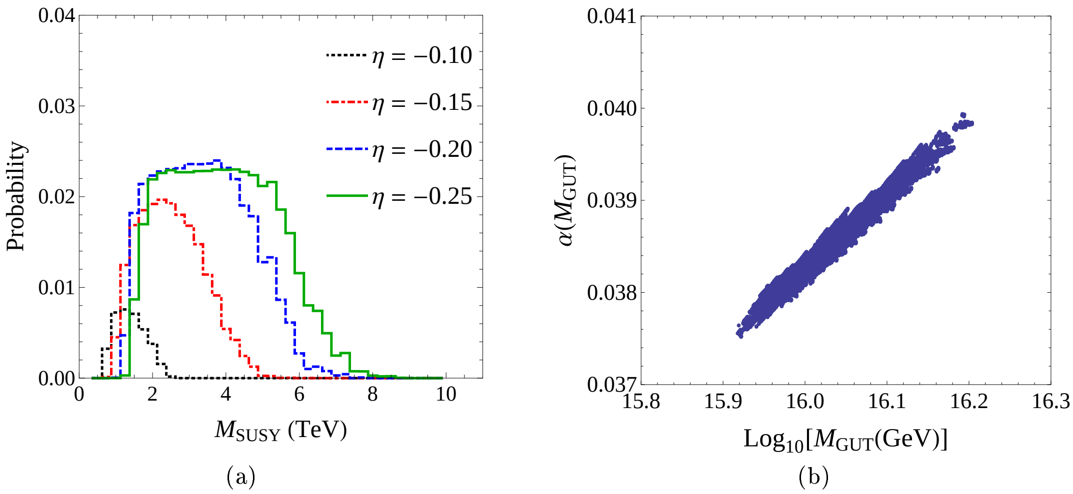

Figure 4: The plots are: (a) the range of SUSY energy scale as function of threshold corrections; (b) the unified gauge coupling as function of GUT energy scale.

scale and their correlations with the other input parameters. The Figure 5a represents the 2-dimensional histogram of the top Yukawa coupling as function of the SUSY energy scale. In this kind of histograms, the darkness of the colour is related to the number of points: darker is the colour, more probable is the bin. Most of the points are at low M_{SUSY} and low $y_t(M_{GUT})$, which is principally in the range from 0.50 to 0.60. The top Yukawa coupling at M_{GUT} and $\tan\beta$, are strongly correlated as shown in the 2-dimensional histogram 5b. This correlation is given by the relation (2). The histogram is quite uniform. However, it is important to point out that in order to have large values of top Yukawa coupling at M_{GUT} we need a very large value of $\tan\beta$.

The tau lepton Yukawa is reported in Figure 5c as function of $\tan\beta$. Again it is clear that the Yukawa couplings are strongly correlated with $\tan\beta$, since we have to impose the matching conditions (2), (3) and (4) in the transition between the SM and the MSSM. It is important to point out that $y_\tau(M_{GUT})$ is an output of the program. In Figure 5c, the hole means that the region has not been explored with the choice of input parameters. Finally, in the last 2-dimensional histogram (Figure 5d) the ratio between the two Yukawa couplings is reported as function of $\tan\beta$.

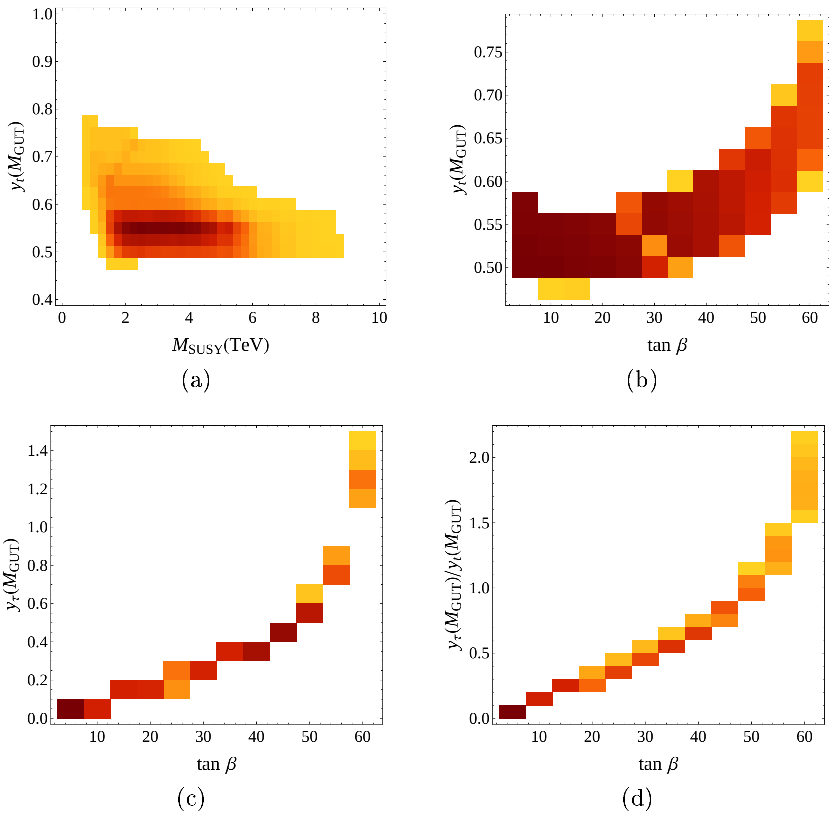

Figure 5: These 2-dimensional histograms show the correlations of Yukawa couplings with M_{SUSY} and $\tan\beta$. Here, darker is the colour, more probable is the bin.

7 Conclusions

We have theoretical and experimental hints that the SM needs to be overcome. SUSY is a possible extension of the SM that has been proposed to solve the hierarchy problem. In particular, SUSY predicts the unification of the three gauge couplings constants, suggested by the SM itself, and gives a candidate for the dark matter. The MSSM needs to introduce new particles as superpartners of the already observed SM particles. Therefore, we expect new physics that could be discovered at LHC and at future accelerators.
The aim of the project is to study the compatibility of the unification of gauge coupling constants in agreement with the recent experimental data, giving an evaluation of range of SUSY energy scale. We considered two cases: without and with $b-\tau$ unification (in the framework SUSY GUTs, as for instance $SU(5)$).
We have found that the allowed range of M_{SUSY} is between about 0.5 and 8.0 TeV in both cases considered. This range depends strongly by the threshold parameter η, which must be quite large and negative $(-0.25 \div -0.10)$ in order to achieve the $b-\tau$ unification. Without taking into account other threshold effects, related for instance to M_{GUT}, this result states that if SUSY is a broken symmetry of the Nature, then new physics could be observed at LHC.
We also found that the top quark Yukawa coupling cannot be greater than 0.9 at M_{GUT}. This variable is strongly correlated with $\tan\beta$ and with the threshold parameter η, as expected because of the matching conditions at M_{SUSY}.
Finally, the GUT region is also characterized: M_{GUT} is around 10^{16} GeV in agreement with the previous studies, and a strong correlation between M_{GUT} and α_{GUT} is shown.

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