Calculating $gg \to t\bar{t} + jets$ at Tree Level

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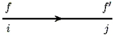
Motivation

- We are interested in $gg \to t\bar{t} + n\,gluons$ at tree level.
- This is a partonic part of $pp \longrightarrow t\bar{t} + jets$.
- This is a background for Higgs searches at the LHC.

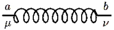
QCD Feynman Rules

- Draw the Feynman diagrams of the process
- Label each line with a momentum
- Associate particular structures as follows:

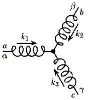
Fermion propagator : $\frac{i(\not{k}+m)\delta^i_j\delta^f_{f'}}{k^2-m^2+i\epsilon}$



Gluon propagator : $\frac{-i\delta^b_a g_{\mu\nu}}{k^2+i\epsilon}$

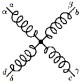


Three gluons vertex : $-gf^{abc}\left[g^{\alpha\beta}(k_1-k_2)^\gamma+g^{\beta\gamma}(k_2-k_3)^\alpha+g^{\gamma\alpha}(k_3-k_1)^\beta\right]$

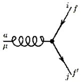


Four gluons vertex :

$$-ig^2\left[f^{abe}f^{cde}(g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma})+f^{ace}f^{bde}(g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\gamma\beta})+f^{ade}f^{bce}(g^{\alpha\beta}g^{\delta\gamma}-g^{\alpha\gamma}g^{\delta\beta})\right]$$



Fermion vertex : $ig\gamma^\mu\delta^{f'}_f(t^a)^j_i$



QCD Feynman Rules

- Associate the external particles as follows:

$$\begin{aligned}
&\textit{Initial external fermion}: \; u(k,s) \\
&\textit{Final external fermion}: \; \overline{u}(k,s) \\
&\textit{Initial external antifermion}: \; \overline{v}(k,s) \\
&\textit{Final external antifermion}: \; v(k,s) \\
&\textit{Initial external gluon}: \; \varepsilon_\mu(k) \\
&\textit{Final external gluon}: \; \varepsilon^*_\mu(k)
\end{aligned}$$

- Sum every term of diagrams and sum over colour, polarization, and spin

$gg \longrightarrow t\bar{t} + n\,gluons$

- The number of diagrams is increased rapidly when the number of outgoing gluons is increased

Process	$gg \longrightarrow t\bar{t}$	$gg \longrightarrow t\bar{t} + g$	$gg \longrightarrow t\bar{t} + gg$
Number of diagrams	3	16	123

- The complete calculation can be done by using several programs in combination.

The Method

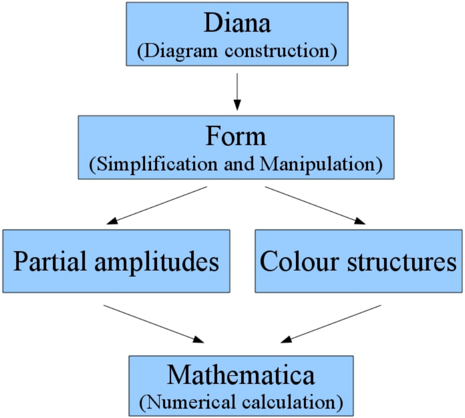
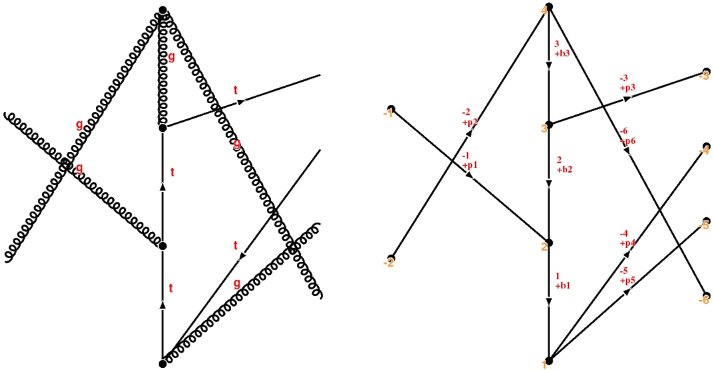

Diana
(Diagram construction)
Form
(Simplification and Manipulation)
Partial amplitudes
Colour structures
Mathematica
(Numerical calculation)

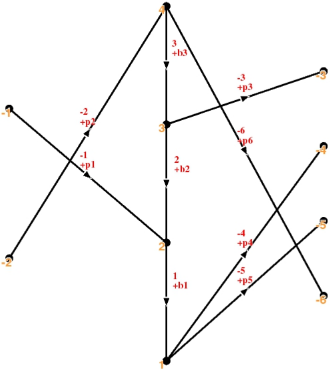
Diagram Construction

- We ask Diana for the diagram construction.



Example of a Diagram Contribution

- Feynman rules are defined as a set of functions in order to be used by Form.




```
#define LINE "3"
#define FERMIONLINE "1"
#define TOPOLOGY "i4_i082"
**********************
#define b1 "(+p4+p5)"
#define b2 "(-p1+p4+p5)"
#define b3 "(-p1+p3+p4+p5)"
#define NM "3"
**********************
l Rq =
1*F(3,1,li3,1,0,1)*i_*gs*GM(ai3,fi3,fi1)*
FF(2,1,-b2,mt)*i_*fdelta(fi1,fi)*
F(2,1,li1,1,0,1)*i_*gs*GM(ai1,fi,fi2)*
FF(1,1,-b1,mt)*i_*fdelta(fi2,fi5)*
F(1,1,li5,1,0,1)*i_*gs*GM(ai5,fi5,fi4)*
VV(3,li,li3,+b3,3)*i_*adelta(ai,ai3)*

V(4,li6,li2,li,-p6,+p2,-b3,3)*gs*Fabc(ai6,ai2,ai);
```

Colour Structure and Partial Amplitude

- The amplitude can be divided in two parts:

$$M = \sum_i c_i M_i^{partial}$$

- Colour structures and partial amplitudes are simplified and manipulated separately by Form.

Colour Structure

- Form reads out the different colour structures, for example,

```
#define colfactor1 "T(ai1,ai2,fi3,fi4)"

#define colfactor2 "T(ai2,ai1,fi3,fi4)"
```

where T(ai1,ai2,fi3,fi4) $= (t^{ai_1} t^{ai_2})_{fi_3 fi_4}$

- By squaring the amplitude and sum over colour, we get

$$\sum_{colour} | M |^2 = \sum_{colour} \sum_{i,j} c_i^* M_i^* c_j M_j = \sum_{i,j} (\sum_{colour} c_i^* c_j) M_i^* M_j$$

- The matrix $\sum c_i^* c_j$ can be simplified by using $\mathrm{SU}(3)$ algebra and kept in Mathematica format.

```
matrix[1,1]:={NF^-1*a^2 - 2*NF*a^2 + NF^3*a^2}
matrix[1,2]:={NF^-1*a^2 - NF*a^2}
matrix[2,1]:={NF^-1*a^2 - NF*a^2}

matrix[2,2]:={NF^-1*a^2 - 2*NF*a^2 + NF^3*a^2}
```

Partial Amplitude

- The partial amplitude is simplified by using properties of Dirac matrices and Dirac equation, and the output is in Mathematica format.

```
re1e2Sum11=
+SpinorUBar(p3,mt)*GS(p2)*SpinorV(p4,mt)*I*p1dp2^-1*e1de2
+SpinorUBar(p3,mt)*GS(e1)*SpinorV(p4,mt)*I*p1dp2^-1*p1de2
-SpinorUBar(p3,mt)*GS(e2)*SpinorV(p4,mt)*I*p1dp2^-1*p2de1;
re1e2Sum12=
-SpinorUBar(p3,mt)*GS(p2)*SpinorV(p4,mt)*I*p1dp2^-1*e1de2
-SpinorUBar(p3,mt)*GS(e1)*SpinorV(p4,mt)*I*p1dp2^-1*p1de2

+SpinorUBar(p3,mt)*GS(e2)*SpinorV(p4,mt)*I*p1dp2^-1*p2de1 ;
```

Numerical Calculation

- We use Mathematica for numerical calculation.
- The appropriate phase space point (the set of momenta, $p_1, \ldots, p_n$) is set at the beginning.
- The gluon polarization vector basis $(\varepsilon_1, \ldots, \varepsilon_m)$ are chosen.
- The representation of Dirac matrices is defined.

4 Point Process: $gg \longrightarrow t\bar{t}$

- The partial amplitudes are compared with previous calculation [6] (R. K. Ellis, W. T. Giele, Z. Kunszt, K. Melnikov. Nucl.Phys.B822:270-282, 2009)
- The phase space point is $p_1 = E(1,-\sin\theta,0,-\cos\theta)$, $p_2 = E(1,\sin\theta,0,\cos\theta)$, $p_3 = E(1,0,0,\beta)$, $p_4 = E(1,0,0,-\beta)$, where $m_t = 1.75$, $E = 10$, $\beta = \sqrt{1-m_t^2/E^2}$, and $\theta = \pi/3$.

Helicities	Partial amplitude (colour structure 1)	Primitive amplitude [6]
$+_{\bar{t}}, +_1, +_2, +_t$	0.0009048290295650407i	0.000905i

- The squared matrix element is compared with previous calculation[7] (W. Bernreuther, A. Brandenburg, Z. G. Si, P. Uwer. Nucl.Phys. B690 (2004) 81-137)
- The phase space point is $p_1 = \frac{s}{2}(1,0,0,1)$, $p_2 = \frac{s}{2}(1,0,0,-1)$, $p_3 = \frac{s}{2}(1,\beta\sin\theta\cos\phi,\beta\sin\theta\sin\phi,\beta\cos\theta)$, $p_4 = p_1+p_2-p_3$, where $\beta = \sqrt{1-4m_t^2/s}$, $\cos\theta$ is the angle between incoming particle and outgoing particle, s is center of mass energy.

s	$\cos\theta$	Numerical result of the method	Analytical result of [7]
20	0.842497	413.9748159148358	413.9748159148358
200	0.90523	772.2986631763597	772.2986631763583

5 Point Process: $gg \longrightarrow t\bar{t} + g$

- The partial amplitudes are compared with previous calculation [6] (R. K. Ellis, W. T. Giele, Z. Kunszt, K. Melnikov. Nucl.Phys.B822:270-282, 2009)
- The phase space point is
  $p_1 = E\xi(-1,1,0,0)$, $p_2 = E\xi(-\sqrt{2},0,1,1)$, $p_3 = E(1,0,0,\beta)$, $p_4 = E(1,0,0,-\beta)$, $p_5 = p_1 + p_2 - p_3 - p_4$,
  where $m_t = 1.75$, $E = 10$, $\beta = \sqrt{1 - m_t^2/E}$, and $\xi = 2/(1+\sqrt{2}+\sqrt{3})$

Helicities	Partial amplitude (colour structure 1)	Primitive amplitude [6]
$+_{\bar{t}}, +_t +_1, +_2, +_5$	-0.0005332686176129279 - 0.00013689856022906747i	-0.000533-0.000137i

- The squared matrix element is compared with previous calculation [8] (S. Dittmaier, P. Uwer, S. Weinzierl. arXiv:0810.0452, hep-ph)
- The phase space point is $p_1 = (500,0,0,500)$, $p_2 = (500,0,0,-500)$,
  $p_3 = (458.53317553852783, 207.0255169909440, 0, 370.2932732896167)$,
  $p_4 = (206.6000026080000, -10.65693677252589, 42.52372780926147, -102.39982104210421085)$,
  $p_5 = (334.8668220067217, -196.3685802184181, -42.52372780926147, -267.8934522475083)$.

	Squared matrix element ($10^{-3} GeV^{-2}$)
Result of [8] Version 1	0.6566843362709776
Numerical result of the method	0.6566843357688175

6 Point Process: $gg \longrightarrow t\bar{t}+gg$

- $gg \longrightarrow t\bar{t}+gg$ compared with Madgraph.
- We use the benchmark phase space point [8] (S. Dittmaier, P. Uwer, S. Weinzierl. arXiv:0810.0452, hep-ph):
- $p_1 = (2100, 0, 0, 2100)$, $p_2 = (2800, 0, 0, -2800)$,
  $p_3 = (1581.118367308447, 1254.462316247655, -766.9360998604944, -554.7905976902205)$,
  $p_4 = (1460.449317799282, -975.9731477430979, -466.5314749495881, 965.6402060944737)$,
  $p_5 = (545.4084744819, 218.7220720302516, 472.0439121434804, -163.7241712507502)$,
  $p_6 = (1313.023840410371, -497.2112405348086, 761.423662666602, -947.1254371535031)$,
  where $m_t = 174$.

	Squared matrix element [8] ($10^{-10}\,GeV^{-4}$)
Numerical result of the method	2.34651551922455
MadGraph	2.34651551922455

Summary

- By using several programs in combination, the complete method of calculation is given.
- The example results of $gg \longrightarrow t\bar{t} + n\, gluons$ at tree level agree well with previous calculations.
- The advantage of the method is that we can compute the different processes by the same method with minimal changes.

References I

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Thank you