Calculating $gg \to t\bar{t} + jets$ at Tree Level

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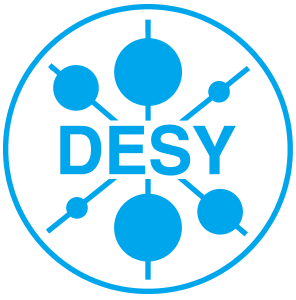


Abstract

Nowadays, experiments at LHC are in the process of testing the standard model and theoretical predictions are required. We are interested in Feynman diagrammatic approach of $gg \longrightarrow t\bar{t} + n\,gluons$ which is a partonic component of $pp \longrightarrow t\bar{t} + jets$. The results will provide background for future discoveries in LHC. The aim of this project is to provide a complete calculation at tree-level for $gg \longrightarrow t\bar{t} + n\,gluons$ by using various programs. We use Diana (Feynman Diagram Analysis) to generate all diagrams for each process. By using Form, the colour structures are extracted and calculated explicitly, and the partial amplitudes are simplified and manipulated. The final program of calculation for the amplitude is in Mathematica. The calculation of the squared matrix element of the following processes: $gg \longrightarrow t\bar{t}$, $gg \longrightarrow t\bar{t} + g$ and $gg \longrightarrow t\bar{t} + gg$ are shown in this project.

1 Introduction

The LHC experiments are leading the way to solve mystery of particle physics. Although the center-of-mass energy is now lower than the original plan, the data continues coming out. This sets a great opportunity of collaboration between experimental and theoretical physicists. The numbers of needed predictions of Standard model in LHC experiments were listed by experimentalists in Les Houches conference known as Les Houches Wish list [9]. Because proton is not an elementary particle, it consists of quarks and gluons, the particular process in proton-proton collision can be decomposed into many partonic subprocesses as shown in table: [10]

Process	Contribution(Tree-level)
$pp \to t\bar{t} + jj$	100%
$qg \to t\bar{t} + qg$	47.1%
$gg \to t\bar{t} + gg$	43.8%
$qq' \to t\bar{t} + qq'$, $q\bar{q} \to t\bar{t} + q'\bar{q}'$	6.2%
$gg \to t\bar{t} + q\bar{q}$	1.6%
$q\bar{q} \to t\bar{t} + gg$	1.2%

Generally, the complexity of calculation in QCD theory requires a lot of effort and techniques, even at tree-level. In this project, we provide the calculation steps of $gg \to t\bar{t} + n\,gluons$ by using Feynman diagrammatic approach which is the most fundamental way. This can be done by using several programs in combination such as, Diana, Form, Mathematica. This report is divided into three main parts. Firstly, the background theory of perturbative QCD will be introduced in section 2. Then, the set of programs and method of calculation are explained in section 3. Finally, the results of an example process is compared with previous calculations with some discussion and conclusions in sections 4 and 5.

2 Background Theory

In this section we will review a basic idea of analytical calculation in quantum chromodynamics by using Feynman diagrams, how to calculate an amplitude, and how to manipulate and simplify them.

2.1 Dirac equation and Dirac matrices

The equation describing the motion of a fermion is the Dirac equation (in momentum space)

$$(i\not{p} - m)\psi = 0, \tag{1}$$

where $\not{p} = \gamma_\mu p^\mu$ and γ_μ are Dirac matrices satisfying relation

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}. \tag{2}$$

In this project we use the Dirac representation.

$$\gamma^0 = \begin{pmatrix} \mathbf{I} & 0 \\ 0 & -\mathbf{I} \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

where $i = 1, 2, 3$ and σ^i are Pauli matrices.

Because of equation (2), the useful basic properties of Dirac matrices (only used in 4 dimensions) is shown below.

$$\begin{aligned} \gamma^\mu\gamma_\mu &= 4\cdot\mathbf{I} \\ \gamma^\mu\gamma^\nu\gamma_\mu &= -2\gamma^\nu \\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= 4g^{\nu\rho} \\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= -2\gamma^\nu\gamma^\rho\gamma^\sigma \end{aligned} \tag{3}$$

These sets of relations will provide the way to simplify the calculation in the next chapter.

2.2 QCD Feynman Rules

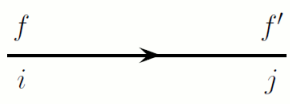
The strong interaction is governed by quantum chromodynamic theory (QCD) which has the Lagrangian:

$$\begin{aligned} L &= \overline{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + gA^a_\mu\overline{\psi}\gamma^\mu t^a\psi \\ &\quad -gf^{abc}(\partial_\mu A^a_\nu)A^{\mu b}A^{\nu c} - \frac{1}{4}g^2(f^{abc}A^a_\mu A^b_\nu)(f^{ecd}A^{\mu c}A^{\nu d}). \end{aligned} \tag{4}$$

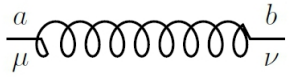
QCD Feynman rules (using Feynman gauge) following by this Lagrangian are

- Draw the Feynman diagrams of the process
- Label each line with a momentum
- Associate particular structures as follows:

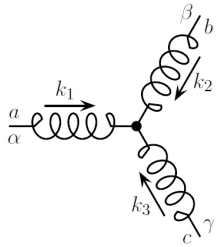
Fermion propagator : $\frac{i(k\!\!\!/+m)\delta^i_j\delta^f_{f'}}{k^2-m^2+i\epsilon}$



Gluon propagator : $\frac{-i\delta^b_a g_{\mu\nu}}{k^2+i\epsilon}$

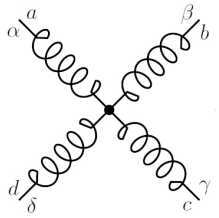


Three gluons vertex : $-gf^{abc}\left[g^{\alpha\beta}(k_1-k_2)^\gamma + g^{\beta\gamma}(k_2-k_3)^\alpha + g^{\gamma\alpha}(k_3-k_1)^\beta\right]$

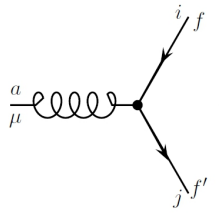


Four gluons vertex :

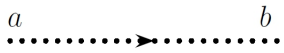
$$-ig^2\left[f^{abe}f^{cde}(g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma})+f^{ace}f^{bde}(g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\gamma\beta})+f^{ade}f^{bce}(g^{\alpha\beta}g^{\delta\gamma}-g^{\alpha\gamma}g^{\delta\beta})\right]$$



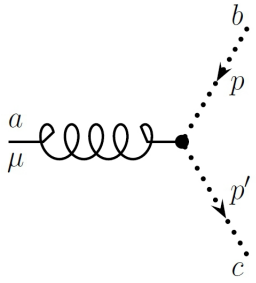
Fermion vertex : $ig\gamma^{\mu}\delta_f^{f'}(t^a)_i^j$



Ghost propagator : $\frac{i\delta_{ab}}{k^2+i\epsilon}$



Ghost vertex : $-gf^{abc}p'^{\mu}$



- Associate the external structures as follows:
  - *Initial external fermion* : $u(k,s)$
  - *Final external fermion* : $\overline{u}(k,s)$
  - *Initial external antifermion* : $\overline{v}(k,s)$
  - *Final external antifermion* : $v(k,s)$
  - *Initial external gluon* : $\epsilon_{\mu}(k)$
  - *Final external gluon* : $\epsilon_{\mu}^{*}(k)$
- Sum every term of diagrams and sum over colour, polarization, and spin

2.3 Colour Algebra

In this subsection, we will review the basic properties of Lie algebra because considering the properties of Lie group will also reduce complexity in simplification of Feynman amplitude. When one deals with QCD, the colour part of the diagram can be computed independently from the other part.

The symmetry behind non-abelian gauge field theory is the group SU(3). Following the non-abelian gauge field theory, the generator of SU(3) Lie algebra are called Gell-Mann matrices, and the commutation relation of this algebra is

$$\left[t^a, t^b\right] = if^{abc}t^c, \tag{5}$$

where the number f^{abc} is called structure constant. The structure constants also satisfy Jacobi identity,

$$f^{ade}f^{bcd} + f^{bde}f^{cad} + f^{cde}f^{abd} = 0. \tag{6}$$

The matrices t^a are traceless, and the trace of the product of two generators is chosen to be

$$tr\left(t^a t^b\right) = C\delta^{ab}, \tag{7}$$

The sum of the product of the same generator is given by

$$\sum_a t^a t^a = C_F \cdot \mathbf{I}, \tag{8}$$

where C and C_F are constants of this representation. For SU(N), these constants are given by,

$$C = \frac{1}{2},\ C_F = \frac{N^2-1}{2N}.$$

Combining equations (5) and (7) together yields the structure constants:

$$f^{abc} = -\frac{i}{C}tr\left(\left[t^a, t^b\right] t^c\right). \tag{9}$$

Moreover, the Fierz identity is also very helpful:

$$\sum_a \left(t^a\right)_{ij}\left(t^a\right)_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}\right). \tag{10}$$

2.4 Colour Decomposition

In particular, the Feynman amplitude can be described in terms of product of generators of SU(N) (colour structures) multiplied by a kinematic function called partial amplitude.

The general strategy of colour decomposition is firstly applying equation (9) and then simplifying repeatedly by using equation (10). The final step gives the Feynman amplitude in the form:

$$M^{tree} = g^m \sum_{\sigma \epsilon S_n} \left(T^{a_{\sigma(1)}} \dots T^{a_{\sigma(n)}}\right) M^{tree}_{\sigma(1)\dots\sigma(N)}, \tag{11}$$

where m is the number of interactions, n is the number of gluons in that process, S_n is the permutation group on n elements, and we call the $M^{tree}_{\sigma(1)\dots\sigma(N)}$ as partial amplitude. Each of them is a gauge invariant property. In practice, the different colour structures at tree level can be obtained by permuting all gluons in that process.

2.5 Ward Identity

In QED (Quantum Electrodynamics) theory we know that photon has polarization. Likewise, the gauge boson in QCD which is gluon has this property. According to Feynman rules, if the process has an external gluon, the amplitude always contains $\epsilon^*_\mu(k)$. Thus, we can write the amplitude in the form:

$$M = M^\mu(k)\epsilon^*_\mu(k).$$

The external gluons are created by fermion vertex, so amplitude always contains the Dirac current term $j^\mu = \overline{\psi}\gamma^\mu\psi$. From classical equation of motion, we know that the current j^μ is conserved: $\partial_\mu j^\mu = 0$. This is still true in quantum theory, by writing differential operator in momentum space, we get

$$k_\mu M^\mu(k) = 0. \tag{12}$$

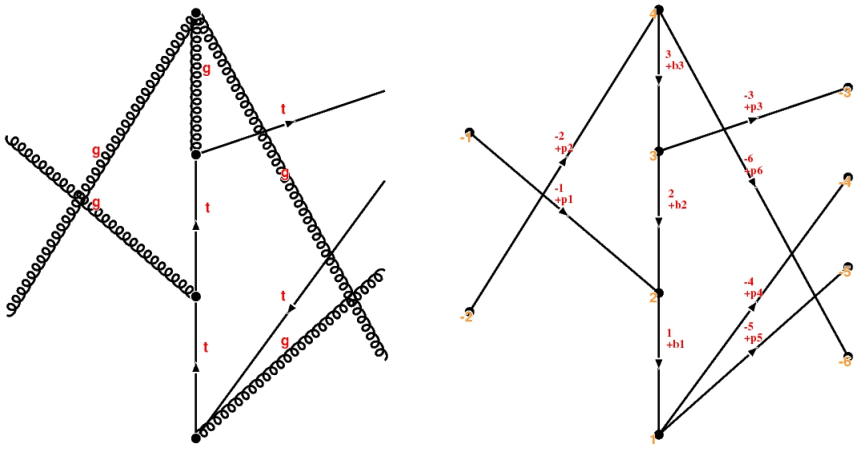
This states that if we replace the polarization vector $\epsilon^*_\mu(k)$ with momentum k_μ, the amplitude $M^\mu(k)$ always vanishes. This is a consequence of the gauge symmetry in QCD. The existence of the identity is one thing we should check for completeness.

3 The Method

In this section, the method of calculation is developed. We use many programs in combination, such as, Diana, Form, Mathematica.

3.1 Diana

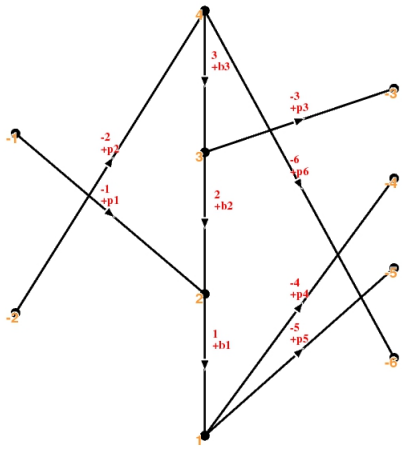
We first provide to Diana [2] the QCD Feynman rules and ask for diagram construction of the process. This is an example of a diagram produced from Diana for $gg \longrightarrow t\bar{t} + gg$ process.



In the diagram on the right, every vertex is labeled by a number (-6,-5,-4,-3,-2,-1,1,2,3,4), and the value of momentum (p1,p2,p3,p4,p5,p6,b1,b2,b3) and numbers (-6,-5,-4,-3,-2,-1,1,2,3,4) are assigned to each line. After the diagram is produced, the corresponding expression is constructed by QCD Feynman rules as shown in the box below:

```
\Begin(boson) [g,g;g; VV(num,lind:1 ,lind:2 ,vec, 3)*i_*adelta(aind:1,aind:2) ;0;spiral, 5, 2] \End(boson)
\Begin(fermion) [q,Q;q; FF(num,fnum,vec, mq )*i_*fdelta(find:1,find:2); mq;arrowLine,0,2] \End(fermion)
\Begin(ghost)[gg,GG;0; SS(num,vec,0)*i_*adelta(aind:1,aind:2);0;arrowLine,10,2] \End(ghost)
\Begin(vertex)
[g,g,g,g;4; V(num,lind:1,lind:2,lind:3,lind:4, aind:1,aind:2,aind:3,aind:4, 4)*(-i_)*gs^2]
[g,g,g;3; V(num,lind:1,lind:2,lind:3,vec:1,vec:2,vec:3, 3)*gs* Fabc(aind:1,aind:2,aind:3)]
[GG,g,gg;a; V(num,lind:2,vec:1, 1)*(-gs)*Fabc(aind:2,aind:3,aind:1)]
[Q,g,q;a; F(num,fnum,lind:2,1,0, 1)*i_*gs*GM(aind:2,find:1,find:3)]
\End(vertex)
```

These set of functions are QCD Feynman rules, but they are defined as functions in order to be used by Form. By comparing with subsection 2.2, we clearly see that FF, VV, and SS are propagators of fermions, gluons, and ghost particles respectively. Also, fermion, three-gluon, four-gluon, and ghost vertices are substituted by functions F, V and S. Actually, F, V and S stand for fermion, vector boson, and scalar particle which is general for using in any field theory. The Lorentz indices (μ, ν) are denoted as *lind*, colour indices of fermion (i, j) are denoted as *find*, and the colour indices of gluons (a, b, c) are denoted as *aind*. gs is the coupling constant, GM is the Gell-Mann matrices (t^a), and $Fabc$ is the structure constants. Following the above notations, one can easily get the expression out of the example diagram.




```
#define LINE "3"
#define FERMIONLINE "1"
#define TOPOLOGY "i4_i082"
**********************
#define b1 "(+p4+p5)"
#define b2 "(-p1+p4+p5)"
#define b3 "(-p1+p3+p4+p5)"
#define NM "3"
**********************
l Rq = 1*F(3,1,li3,1,0,1)*i_*gs*GM(ai3,fi3,fi1)*
FF(2,1,-b2,mt)*i_*fdelta(fi1,fi)*
F(2,1,li1,1,0,1)*i_*gs*GM(ai1,fi,fi2)*
FF(1,1,-b1,mt)*i_*fdelta(fi2,fi5)*
F(1,1,li5,1,0,1)*i_*gs*GM(ai5,fi5,fi4)*
VV(3,li,li3,+b3,3)*i_*adelta(ai,ai3)*

V(4,li6,li2,li,-p6,+p2,-b3,3)*gs*Fabc(ai6,ai2,ai);
```

As an example above, the vertices numbered as 1, 2, 3, and 4 are expressed by $F(1,1,li5,1,0,1) * i * s * GM(ai5,fi5,fi4)$, $F(2,1,li1,1,0,1) * i * gs * GM(ai1,fi,fi2)$, $F(3,1,li3,1,0,1) * i * gs * GM(ai3,fi3,fi1)$, $V(4,li6,li2,li,-p6,+p2,-b3,3) * gs * Fabc(ai6,ai2,ai)$ respectively. Likewise, the propagators (lines) numbered as 1, 2, and 3 are expressed by $FF(1,1,-b1,mt) * i_*fdelta(fi2,fi5)$, $FF(2,1,-b2,mt)*i*fdelta(fi1,fi)$, $VV(3,li,li3,+b3,3)*i*adelta(ai,ai3)$. The output is the diagram contribution written in Form.

3.2 Form

The manipulation steps are done in Form [3]. Form reads out the different colour structures and manipulates them by using colour algebra. The amplitude is stripped out of colour structures as the example below:

```
#define colfactor1 "T(ai1,ai2,fi3,fi4)"
#define colfactor2 "T(ai2,ai1,fi3,fi4)"
```

```
re1e2Sum11=+SpinorUBar(p3,mt)*GS(p2)*SpinorV(p4,mt)*I*p1dp2^-1*e1de2
+SpinorUBar(p3,mt)*GS(e1)*SpinorV(p4,mt)*I*p1dp2^-1*p1de2
-SpinorUBar(p3,mt)*GS(e2)*SpinorV(p4,mt)*I*p1dp2^-1*p2de1;
re1e2Sum12=-SpinorUBar(p3,mt)*GS(p2)*SpinorV(p4,mt)*I*p1dp2^-1*e1de2
-SpinorUBar(p3,mt)*GS(e1)*SpinorV(p4,mt)*I*p1dp2^-1*p1de2
+SpinorUBar(p3,mt)*GS(e2)*SpinorV(p4,mt)*I*p1dp2^-1*p2de1 ;
```

where $T(a_1, a_2, f_3, f_4) = (t^{a_1}t^{a_2})_{f_3 f_4}$, *SpinorUbar* is $\overline{u}(k, s)$, *SpinorV* is $v(k, s)$. In this step, any two vectors which have the same Lorentz index are contracted and defined as scalar product, for example, $p_1^{\mu}(\varepsilon_2)_{\mu}$ is $p1de2$. Furthermore, $GS(v_1, ..., v_n)$ is $\not{v}_1...\not{v}_n$ After the colour decomposition step, the partial amplitudes are simplified by using equations (2), (3) moving the appropriate momenta to the corners of the spinor line, and applying the Dirac equation (1) in the end. Lastly, we express the output in Mathematica format.

In the next step, we simplify colour structure. In equation (11), we can rewrite as

$$M = \sum_i c_i M_i^{partial},$$

where c_i is a i^{th} colour structure. By squaring the amplitude and sum over colour, we get

$$\sum_{colour} \mid M \mid^2 = \sum_{colour} \sum_{i,j} c_i^* M_i^* c_j M_j = \sum_{i,j} (\sum_{colour} c_i^* c_j) M_i^* M_j \tag{13}$$

The matrix $\sum c_i^* c_j$ can be simplified into the polynomial of SU(N) constants by using SUn.prc (subroutine written by J. Vermaseren [3]) which contains equations (7), (9), (10), and also kept in Mathematica format as the example below:

```
matrix[1,1]:={NF^-1*a^2 - 2*NF*a^2 + NF^3*a^2}
matrix[1,2]:={NF^-1*a^2 - NF*a^2}
matrix[2,1]:={NF^-1*a^2 - NF*a^2}
matrix[2,2]:={NF^-1*a^2 - 2*NF*a^2 + NF^3*a^2}
```

3.3 Mathematica

The last step is to calculate numerically two pieces of output in Mathematica. In this step, everything is basically converted to Mathematica. The appropriate phase space points (energy and momentum of every incoming and outgoing particle) are set as the input numbers at the beginning. The gluon polarization vector basis is chosen, and the representation of Dirac matrices is defined. Consequently, colour structure matrix and partial amplitude are combined together by equation (13). In order to obtain the squared matrix element, we further sum over spin and helicity, or average in the case that it is a property of initial particle.

4 Results

In this section, the results are compared to previous calculations in papers [6], [7], and [8] by using the same phase space points and definitions. In all cases the Ward identity has been verified.

4.1 4 point amplitude ($gg \longrightarrow t\bar{t}$)

We are interested in comparison with results from recent papers both in partial amplitudes and squared matrix elements.

4.1.1 Partial amplitude

First, in terms of partial amplitude, the primitive amplitude (The definition of primitive amplitude is described by [4]) and the partial amplitude are the same. So, we can compare with primitive amplitude by considering carefully the ordering of arguments in primitive amplitude.
According to [6], the definition of gluon polarization vectors are stated as follows:

$$\begin{aligned} p_\mu &= E(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \\ \varepsilon_\mu^{\pm}(p) &= \frac{1}{\sqrt{2}}(0, \cos\theta\cos\phi \mp i\sin\phi, \cos\theta\sin\phi \pm i\cos\phi, -\sin\theta). \end{aligned} \tag{14}$$

For the top quark spinors we use

$$u_+(p) = \begin{pmatrix} \sqrt{E+m} \\ 0 \\ p_z\sqrt{E+m} \\ (p_x+ip_y)\sqrt{E+m} \end{pmatrix},\ u_-(p) = \begin{pmatrix} 0 \\ \sqrt{E+m} \\ (p_x-ip_y)\sqrt{E+m} \\ -p_z\sqrt{E+m} \end{pmatrix}$$

$$v_+(p) = \begin{pmatrix} p_z\sqrt{E+m} \\ (p_x+ip_y)\sqrt{E+m} \\ \sqrt{E+m} \\ 0 \end{pmatrix},\ u_-(p) = \begin{pmatrix} (p_x-ip_y)\sqrt{E+m} \\ -p_z\sqrt{E+m} \\ 0 \\ \sqrt{E+m} \end{pmatrix} \tag{15}$$

The phase point, gluon momenta (p_1 and p_2) and top-antitop quark momenta (p_3 and p_4) are set as

$$p_1 = E(1, -\sin\theta, 0, -\cos\theta),\ p_2 = E(1, \sin\theta, 0, \cos\theta),\ p_3 = E(1, 0, 0, \beta),\ p_4 = E(1, 0, 0, -\beta).$$

where $m_t = 1.75$, $E = 10$, $\beta = \sqrt{1 - m_t^2/E^2}$, and $\theta = \pi/3$. Then, the results are:

Helicities	Partial amplitude (colour structure 1)	Primitive amplitude
$+_{\bar{t}}, +_1, +_2, +_t$	0.0009048290295650407i	0.000905i
$+_{\bar{t}}, -_1, +_2, +_t$	-0.0432975854852175i	-0.043298i
$+_{\bar{t}}, +_1, -_2, -_t$	0.4285349594597339i	0.4285350i
$+_{\bar{t}}, -_1, +_2, -_t$	-0.14284498648657798i	-0.142845i

Helicities	Partial amplitude (colour structure 2)	Primitive amplitude
$+_{\bar{t}}, +_t, +_1, +_2$	0.0026594841523586 43i	0.026595i
$+_{\bar{t}}, +_t, -_1, +_2$	-0.12726077377146208i	-0.127261i
$+_{\bar{t}}, -_t, +_1, -_2$	1.2595550056154345i	1.259555i
$+_{\bar{t}}, -_t, -_1, +_2$	-0.41985166853847816i	-0.4198517

4.1.2 Squared matrix element

In accordance with [7], the formula for squared matrix element in tree-level diagram is given by

$$P = \sum_{all\ helicities} \mid M \mid^2 = \frac{2(N^2-1)}{N(1-\beta^2y^2)^2}[N^2(1+\beta^2y^2) - 2]\{1 + 2\beta^2(1-y^2) - \beta^4[1 + (1-y^2)^2]\} \tag{16}$$

where $\beta = \sqrt{1 - \frac{4m_t^2}{s}}$, s is center of mass energy, and $y = \cos\theta$. Notice that the coupling constant is equal to unity. To compare with the squared matrix element the result of the random phase space point is:

$p_1 = \frac{s}{2}(1,0,0,1),\ p_2 = \frac{s}{2}(1,0,0,-1),\ p_3 = \frac{s}{2}(1,\beta\sin\theta\cos\phi,\beta\sin\theta\sin\phi,\beta\cos\theta),\ p_4 = p_1 + p_2 - p_3$

The angles θ, ϕ are random variables in Mathematica. The convention of gluon polarization is the same as equation (14), and $m_t = 1.72$. Thus, after summing over all helicities (2 spins of 2 fermions and 2 polarizations of 2 gluons), the results are

s	$\cos\theta$	Numerical result of the method	Analytical result of the equation (16)
2	0.549637	61.148204544340906	61.148204544340935‘
2	-0.421996	57.178176605675446	57.17817660567546‘
20	-0.0446397	38.127257714904346	38.12725771490433‘
20	0.842497	413.9748159148358	413.97481591483586‘
200	-0.601096	116.54246966893957	116.54246966893972‘
200	0.90523	772.2986631763597	772.2986631763583‘

4.2 5 point amplitude ($gg \longrightarrow t\bar{t} + g$)

4.2.1 Partial amplitude

For 5 points case, to have an agreement with [6], we use the definitions of equations (14) and (15), and the phase space point is:
$p_1 = E\xi(-1,1,0,0)$, $p_2 = E\xi(-\sqrt{2},0,1,1)$, $p_3 = E(1,0,0,\beta)$, $p_4 = E(1,0,0,-\beta)$, $p_5 = p_1+p_2-p_3-p_4$ where $E = 10$, $\beta = \sqrt{1-\frac{m_t^2}{E}}$, and $\xi = 2/(1+\sqrt{2}+\sqrt{3})$. The result is

Helicities	Partial amplitude (colour structure 1)	Primitive amplitude
$+_{\bar{t}}, +_t +_1, +_2, +_5$	-0.0005332686176129279 - 0.00013689856022906747i	-0.000533-0.000137i
$+_{\bar{t}}, -_t +_1, +_2, +_5$	-0.004540211480727542 + 0.018665255112854117i	-0.004540+0.018665i
$+_{\bar{t}}, +_t -_1, +_2, -_5$	-0.004725600544555214 + 0.014201357628478772i	-0.004726+0.014201i
$+_{\bar{t}}, -_t -_1, +_2, +_5$	0.04578618146359473 + 0.010661037914172063i	0.045786+0.010661i

Helicities	Partial amplitude (colour structure 6)	Primitive amplitude
$+_{\bar{t}}, +_1, +_2, +_5 +_t$	-0.00025189184876626524 + 0.00014447820861754 24i	-0.000252+0.000144i
$+_{\bar{t}}, +_1, -_2, +_5 -_t$	0.005002271870438688 + 0.008870703539059638i	0.0050023+0.008871i
$+_{\bar{t}}, -_1, +_2, -_5 +_t$	0.0005614642550840552 - 0.004105486458822788i	0.000561-0.004105i
$+_{\bar{t}}, -_1, +_2, +_5 -_t$	0.02121648883576497 - 0.011994006474942 05i	0.021216-0.011994i

4.2.2 Squared matrix element

In order to compare with [8], we use the phase space point:
$p_1 = (500,0,0,500)$, $p_2 = (500,0,0,-500)$,
$p_3 = (458.53317553852783, 207.0255169909440, 0, 370.2932732896167)$,
$p_4 = (206.6000026080000, -10.65693677252589, 42.52372780926147, -102.39982104210421085)$,
$p_5 = (334.8668220067217, -196.3685802184181, -42.52372780926147, -267.8934522475083)$.
We also use the definitions of equations (14) and (15), and the mass is set to $m_t = 174$. After summing over all spins of fermions, averaging over polarizations and colours of gluons, the results are:

	Squared matrix element ($10^{-3}GeV^{-2}$)
Result of [8] Version 1	0.6566843362709776
Result of [8] Version 2	0.6566843362709785
Numerical result of the method	0.6566843357688175
MadGraph	0.6566843362709775

4.3 6 point amplitude ($gg \longrightarrow t\bar{t} + gg$)

4.3.1 Squared matrix element

The set of momenta is chosen by [8]:
$p_1 = (2100, 0, 0, 2100)$, $p_2 = (2800, 0, 0, -2800)$,
$p_3 = (1581.118367308447, 1254.462316247655, -766.9360998604944, -554.7905976902205)$,
$p_4 = (1460.449317799282, -975.9731477430979, -466.5314749495881, 965.6402060944737)$,
$p_5 = (545.4084744819, 218.7220720302516, 472.0439121434804, -163.7241712507502)$,
$p_6 = (1313.023840410371, -497.2112405348086, 761.423662666602, -947.1254371535031)$.
The top quark mass is set to $m_t = 174$. After summation and averaging over all helicities, the result is compared to the MadGraph [5] result shown as table below:

	Squared matrix element ($10^{-10}GeV^{-4}$)
Numerical result of the method	2.34651551922455
MadGraph	2.34651551922455

5 Conclusion

The complete method of calculation was given in this report. We use several programs, Diana for generating all diagrams, Form for simplification and manipulation, Mathematica for numerical calculation. The example results of $gg \longrightarrow t\bar{t} + n\,gluons$ in tree level agree well with previous calculations. The advantage of the method is that the program is flexible, i.e., we can compute the different processes by the same method with minimal changes. However, the time used for numerical calculation is rapidly increased when the number of gluons is increased. Our future task is to reduce the time of calculation for higher number of jets.

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