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Final report

Simulation of electron bunch trajectories through a Booster Cavity

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Abstract

In the present work the Booster Cavity Simulation in Photo Injector Test Facility at DESY, Zeuthen site (PITZ) was performed. The main purpose of these simulations was to obtain a simplified model of the booster cavity which should be used later during development of the automatic procedure of optimizing the electron beam trajectory through the booster. I used the V-Code (a beam dynamic simulation program which implements the momentum approach of the beam) for obtaining the trajectories for different input parameters. A method for analysis of the simulated data was suggested. The simulated data were used to make multidimensional Legendre expansions of the output parameters (position and deflection of the beam center of mass at the end of the booster) as the function of the input parameters (beam position and deflection at the input and field phase).

1. Introduction

The goal of the Photo Injector Test Facility at DESY, Zeuthen site (PITZ) is to develop and test electron beams sources for linear colliders and free electron lasers in order to receive intense electron beams with small transverse projected emittance and reasonable longitudinal emittance [1]. The PITZ setup is shown in Fig.1.

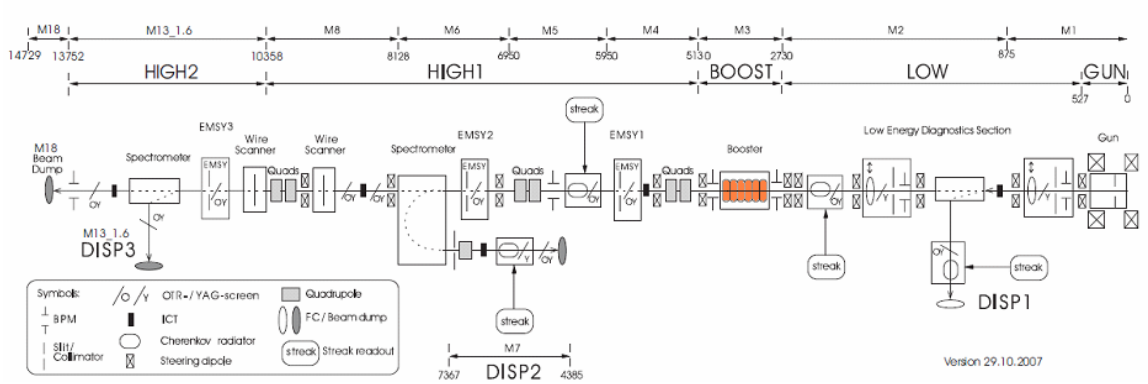
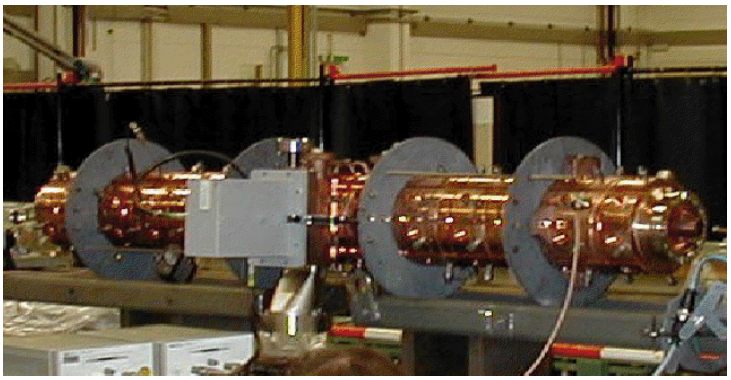


Fig. 1. PITZ setup.

One of the components of this setup is the booster cavity sector (BOOST). Inside the booster the energy of the electron bunch is increased but the electromagnetic field inside the booster can also deflect the beam causing unwanted effects on the electron bunch trains. So, in this work we want to simulate booster steering free trajectories using a beam dynamic simulation tool. The results of these simulations will be used later for testing of the in-development code for automatic optimizing of the electron bunch path through the booster.

2. Theory

2.1. The booster



The booster cavity currently used at the PITZ facility (Fig. 2) is an L-band normal-conducting fourteen-cell Cut Disc Structure (CDS) copper resonator. Expected energy gain is about 25 MeV.

Fig. 2. The booster cavity at the PITZ.

2.2. V-Code program

The real booster can be simulated using of the V-Code program. V-Code is a beam dynamics simulation tool which is based on the moment approach. It means that the particle distribution function is represented like a discrete set of characteristic moments.

For each moment a time equation can be found, then these differential equations can be integrated by means of numerical methods after setting up all acting forces and proper initial conditions.

We consider the 6-dimensional (6D) phase space with space coordinates $\vec{r} = (x, y, z)$ and normalized momentum $\vec{p} = (p_x, p_y, p_z)$. The evolution of the density distribution function of particles $f(\vec{r}, \vec{p}, \tau)$ (where $\tau = c \cdot t$ is equivalent time), can be expressed by the Vlasov equation:

$$\frac{\partial f}{\partial \tau} + \frac{\partial f}{\partial \vec{r}} \cdot \frac{\vec{p}}{\gamma} + \frac{\partial f}{\partial \vec{p}} \cdot \frac{\vec{F}}{m_0 c^2} = 0, \tag{1}$$

where γ represents the relativistic factor, $\vec{F}$ - the applied forces, m_0 - the particles rest mass and c – the speed of light in free space.

Equation (1) is correct for any forces with slow variation in space. Both coulomb forces within a charged particle beam and forces from external electromagnetic fields meet this condition. So, the Vlasov equation can be solved for beam dynamics simulations of charged particle beams in accelerators. But it's rather difficult to solve a partial differential equation for a time varying 6D density distribution function using classic numerical methods. More suitable way is to consider the particle distribution function as a discrete set of characteristic moments.

The raw moments $<\mu>$ can be represented by means of the normalized distribution function $\bar{f}(\vec{r}, \vec{p}, \tau)$:

$$< \mu >= \int_{\Omega} \mu \cdot \bar{f}(\vec{r}, \vec{p}, \tau) d\Omega, \tag{2}$$

where $\Omega = \{\vec{r}, \vec{p}\}$ is the whole phase space,

$$\bar{f}(\vec{r}, \vec{p}, \tau) = \frac{f(\vec{r}, \vec{p}, \tau)}{\int f(\vec{r}, \vec{p}, \tau) d\Omega}, \int_{\Omega} \bar{f}(\vec{r}, \vec{p}) d\Omega = 1. \tag{3}$$

The first order raw moments in Cartesian coordinates allow determining the overall position and the overall momentum of a particle distribution. In present work we are interested only in first raw moments.

$$\mu \in \{x, y, z, p_x, p_y, p_z\} \tag{4}$$

The time evolution of the moment parameters can be expressed as follows:

$$\frac{\partial < \mu >}{\partial \tau} = \frac{\partial}{\partial \tau} \int \mu \cdot f d\Omega = \int (f \frac{\partial \mu}{\partial \tau} + \mu \frac{\partial f}{\partial \tau}) d\Omega \tag{5}$$

Also, the time derivative of the moment parameter can be reformulated:

$$\frac{\partial \mu}{\partial \tau} = \frac{\partial \mu}{\partial < \vec{r} >} \frac{\partial < \vec{r} >}{\partial \tau} + \frac{\partial \mu}{\partial < \vec{p} >} \frac{\partial < \vec{p} >}{\partial \tau} \qquad (6)$$

After inserting (1) and (6) in the equation (5) and applying a partial integration together with the moment definition (2) one can receive the fundamental time evolution equation:

$$\frac{\partial < \mu >}{\partial \tau} = < \frac{\partial \mu}{\partial < \vec{r} >} >< \frac{\vec{p}}{\gamma} > + < \frac{\partial \mu}{\partial < \vec{p} >} >< \frac{\vec{F}}{m_0 c^2} > + < \frac{\partial \mu}{\partial \vec{r}} \frac{\vec{p}}{\gamma} > + < \frac{\partial \mu}{\partial \vec{p}} \frac{\vec{F}}{m_0 c^2} > \qquad (7)$$

For solving this equation (7) by means of a time integration method all arguments on the right hand side should be expressed in terms of time dependent bunch parameters (moments). This can be achieved by Taylor expanding $1/\gamma$ and $\vec{F}$ in an operation point defined by the particle distribution and utilizing a truncation according to the regarded order of moments. For simulations with V-Code a paraxial approximation of the field distribution within the beam line elements is used. More details could be found in [2, 3, 4].

2.3. Algorithm for finding the booster transformation function

The electron beam in front of the booster can be characterized with input parameters. In our work we are interested only in the first order moments, such as position of the electron bunch center of mass and its deflection ($x_0, y_0, x_0' \propto p_{x0}, y_0' \propto p_{y0}$). Also, we can modify the phase of the electromagnetic field inside the booster (φ) and see how this parameter influences on our results. The moments dependence from the field amplitude is expected to be linear, thus we exclude this parameter from the analysis

The purpose of simulations was to receive the first order moments $x, y, x' \propto p_x, y' \propto p_y$ but after the booster. So, the moments of interest (μ) after the booster can be performed as functions of the initial parameters (8), in the paper we called this function a “transformation function”.

$$\mu = f^{\mu}(x_0, x_0', y_0', \varphi), \qquad (8)$$

In this expression the dependence from y_0 is excluded because we always can rotate the initial coordinate system in a proper angle for receiving y_0 is equal to 0. Using V-Code the beam trajectory simulations were performed for selected input parameters.

As a result, the set of output parameters were received. These simulated data can be expanded as a sum of functions:

$$f^{\mu}(x_0, x_0', y_0', \varphi) = \sum_{i,j,k,l} c^{\mu}_{ijkl} \psi_{ijkl}, \tag{9}$$

where ψ_{ijkl} - the basis functions, c^{μ}_{ijkl} - constants.

At the same time, each basis function can be written as follows:

$$\psi_{ijkl} = \overline{P}_i(x_0)\overline{P}_j(x_0')\overline{P}_k(y_0')\overline{P}_l(\varphi), \tag{10}$$

where $\overline{P}_n$ are normalized *n*-th order Legendre polynomials

After multiplying the equation (9) in sequence by all of the basis functions ψ_{pqrs}, we can receive all expanding constants (11) because of the property of orthogonality of basis functions (13).

$$c_{pqrs} = \left\langle f^{\mu}_{simulated}(x_0, x_0', y_0', \varphi) \cdot \psi_{pqrs} \right\rangle, \tag{11}$$

where the averaging brackets $\langle\ \rangle$ mean:

$$\langle g \rangle = \int_{\Omega} g dx dx' dy' d\varphi \tag{12}$$

$$\left\langle \psi_{ijkl} \psi_{pqrs} \right\rangle = \delta_{ijkl,pqrs} = \delta_{ip}\delta_{jq}\delta_{kr}\delta_{ls} \tag{13}$$

3. Simulation setup and conditions

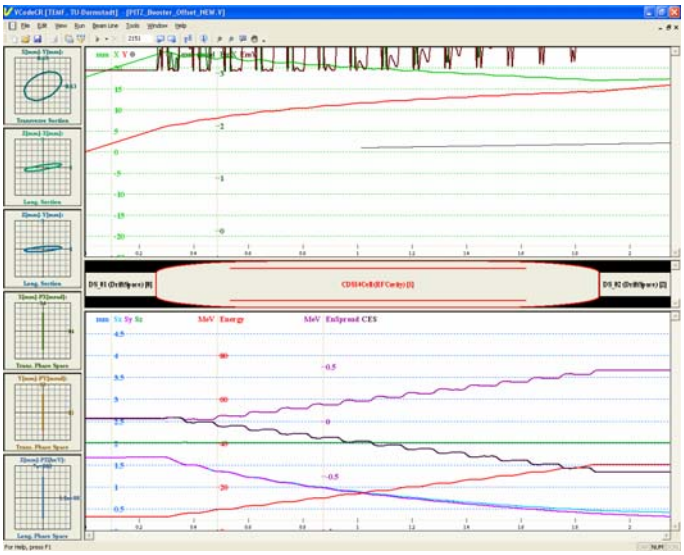


Fig. 3. Screenshot of the V-Code program, the booster.

V-Code program was used for the real booster simulation [4]. The tube radius at the drift space element is 0.019 m. The length of the booster is 1.627 m, the length of the first and the second drift spaces (between the first beam position monitor (BPM) and the beginning of the booster and between the end of the booster and second BPM) are both 0.2615 m. The field frequency inside the booster is 1.3 GHz and the Field Amplitude = 28.85 MV/m. In Fig. 3 the simulation setup of the V-Code is shown.

3.1. Acceptable initial parameters for the 1st order moments of electron bunch

In the present work we are interested only in the first order moments. So, at the beginning we found out the maximal proper initial parameters for ensemble ($x_{0\_max} = y_{0\_max} = 17.76 \cdot 10^{-3} m, x_0' = y_0' = 0.2738 mrad$). Criteria for acceptable parameters were the smoothness and the completeness of line which represented the electron bunch track (Fig. 3).

Also, the output beam energy as a function of the field phase was investigated for different field amplitudes (Fig. 4). Using the calculated data as the base, we can consider the Field Phase = 66 degree as a central phase with a maximum average beam output energy (at that phase electron bunch in phase with a field) and add +45 or -45 degrees for receiving the range of interest ($\varphi_{max} = 111°, \varphi_{min} = 11°$). It is obvious, the more the field amplitude the more is the bunch energy gain at the same field phase (the linear dependence is presented in Fig. 5).

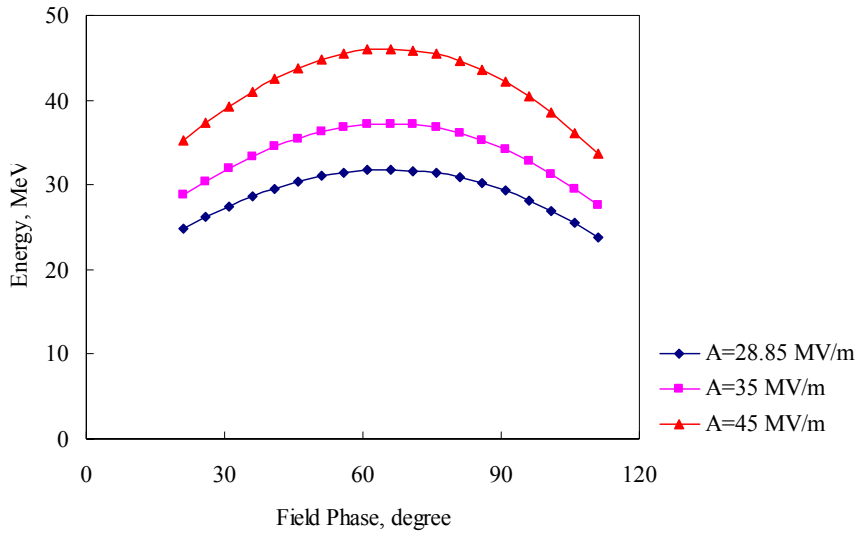


Fig. 4. The output beam energy as a function of the field phase for different field amplitudes.

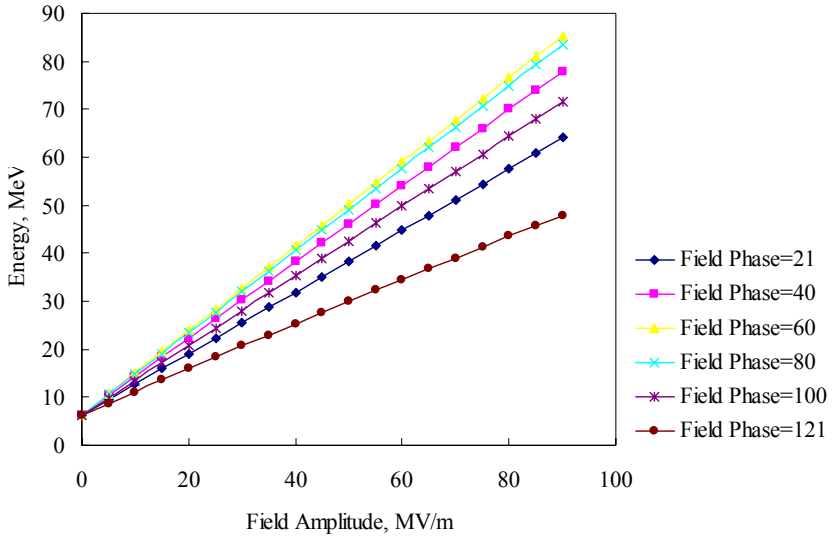


Fig. 5. The beam energy as a function of the field amplitude.

3.2. The function of automatical loading the initial data and receiving the data in V-Code

For receiving the view of booster transformation function a large number of calculations should be done. The task was to receive the set of output first order moments after the booster for different initial parameters (from $-x_{0\_\max}$ to $x_{0\_\max}$, from $-y_{0\_\max}$ to $y_{0\_\max}$, from $-p_{x0\_\max}$ to $p_{x0\_\max}$, from $-p_{y0\_\max}$ to $p_{y0\_\max}$ and from $\varphi_{\max}$ to $\varphi_{\min}$ with defined steps), like a 4D matrix. For solving this problem a C++ program for automatical loading data in V-Code was written. Using this program one can set up the set of input parameter and after the calculations receives the file with array of output data.

4. Results and discussion

4.1. Obtaining the booster transformation function

In this paper only on-crest beams were considered ($\varphi = 66°$), so, the following transformation functions $f^{\mu}(x_0, x_0', y_0')$ were obtained. Since the V-Code assumes only linear external forces, only linear basis functions were considered ($\psi_{000}, \psi_{100}, \psi_{010}, \psi_{001}$, where $\psi_{ijk} = \overline{P}_i(x_0)\overline{P}_j(x_0')\overline{P}_k(y_0')$). In Fig. 6 some 2D projections of the transformation function are presented. In all of these plots one of the input parameter is fixed and the other two are varied. As expected, the dependences are all linear.

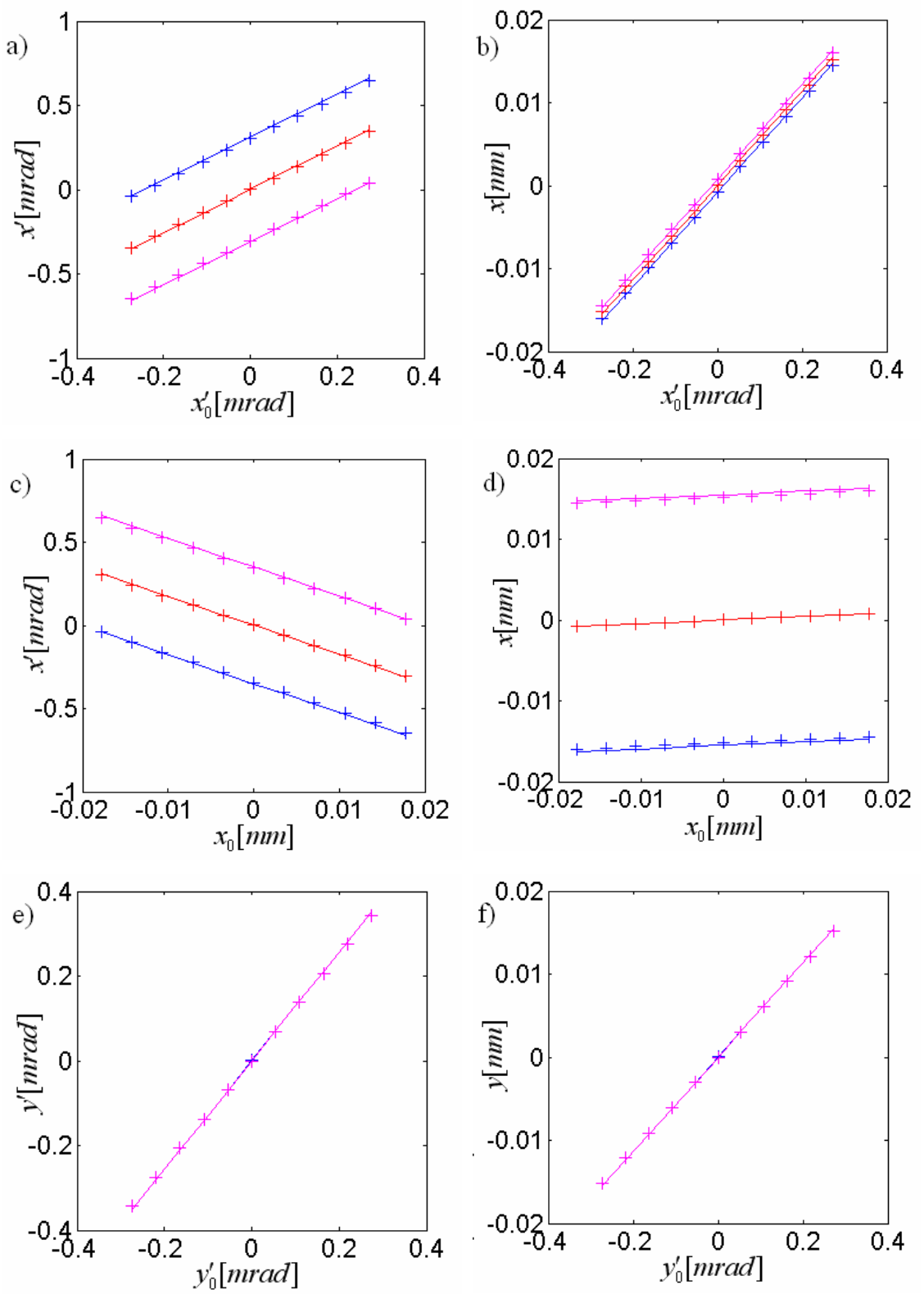


Fig. 6. 2D projections of the transformation function.

"+" – simulated data $f^{\mu}_{simulated}(x_0, x'_0, y'_0)$, "continuous line" – calculated transformation functions $f^{\mu}(x_0, x'_0, y'_0, \varphi) = \sum_{i,j,k,l} c^{\mu}_{ijkl} \psi_{ijkl}$ (9).

a, b) $y'_0 = 0$, blue line – corresponds $x_{0\_\min} = -17.76mm$, red – $x_0 = 0mm$, purple – $x_{0\_\max} = 17.76mm$;

c, d) $y'_0 = 0$, blue – $x'_{0\_\min} = -0.2738mrad$, red – $x'_0 = 0mrad$, purple – $x'_{0\_\max} = 0.2738mrad$;

e, f) $x_0 = x_{0\_\max} = 17.76mm$, blue – $x'_{0\_\min} = -0.2738mrad$, red – $x'_0 = 0mrad$, purple – $x'_{0\_\max} = 0.2738mrad$.

4.2. Electromagnetic field phase influence

Next, the phase dependence (in a range of $\varphi(11°,111°)$) on the beam position was examined. In Fig. 7 the phase scan of coordinates of the beam center of mass at the booster exit for different input parameters are presented. When the beam is perfectly aligned with the booster symmetry axis (blue cross), the position of the beam center of mass would obviously be independent on the booster field phase. But if the beam were not aligned, then we would observe more or less complex moving of the beam while changing field phase.

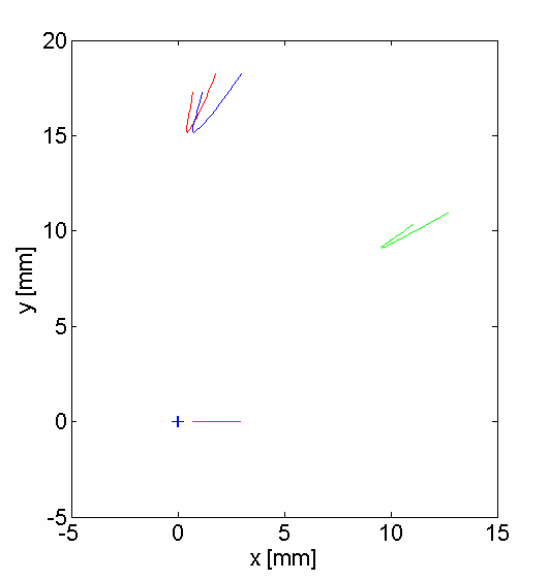


Fig. 7. Phase scan of the beam position after the booster for different input parameters $[\frac{x_0}{x_{0\_\max}}, \frac{x'_0}{x'_{0\_\max}}, \frac{y'_0}{y'_{0\_\max}}]$.

Blue cross – [0, 0, 0];
red line – [0.6, 0, 1];
green line – [0.6, 0.6, 0.6];
purple line – [1, 0, 0];
blue line – [1, 1, 1].

5. Summary

In this paper Booster Section of PITZ was considered. Simulations of Booster were performed in V-Code program, the area of acceptable parameters was found: $x_{0\_\max} = y_{0\_\max} = 17.76 \cdot 10^{-3} m, x_0' = y_0' = 0.2738, \varphi_{\max} = 111°, \varphi_{\min} = 11°$. Using C++ language the program of automatical loading and receiving data was added to the V-Code. 2D projections of the transformation function which represent the real booster were received in linear approximation. Also, the investigation of electromagnetic field phase influence on the output position of the beam was performed.

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7. References

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