Calculation of magnetic and thermodynamic characteristics of the lithium lens for the ILC positron source

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Contents

1. Short introduction to the lithium lens as a part of the ILC positron source.

The International Linear Collider (ILC) is a proposed linear particle accelerator for protons collisions. It is planned to have a collision energy of 500 GeV. The electron beam is used to get circularly polarized photons. The photons go through titanium target and they are transformed into electron-positron pairs. These pairs diverge at different angles. In order to collect and to focus electrons and positrons different devices can be used. One of them – the lithium lens (LL) – is the subject of this report.

We have an axial-symmetric volume, filled by liquid lithium at temperature $\approx 500K$, and on the top of the lens there are two wires which are supplying this device with electric current in order to provide a proper magnetic field. The flow of liquid lithium should prevent this device from overheating. The task is to calculate magnetic field to predict trajectory of positrons and to get time-dependent field of temperature.
For these tasks we use FLEXPDE [2] and COMSOL Multiphysics [3]. Some estimations and properties of LL are taken from [1]. The geometrical dimensions are given below, in fig. 1 and 2.

Nominal parameters for ILC positron source

Parameter	Value	Units
Number of positrons	$2 \cdot 10^{10}$	1/per bunch
Number of bunches	2625	
Pulse repetition rate	5	Hz
Average positron energy	5	MeV
Average photon energy	10	MeV
Bunch train length	1	ms

On fig. 1 we can see a thin Ti target ($\approx 0.15cm$), and after the target on a distance ~0.5 cm there is a short-focusing LL with thin (□ $0.05cm$) Beryllium windows astride. The transmitted beam is a very intensive, so a very important question is the possibility of overheating the LL. On fig. 2 two wires with the great current density can be seen, so one of the aims of this work is to understand what will be the dominant overheating factor: Joule heating or heating by transmitted beam.

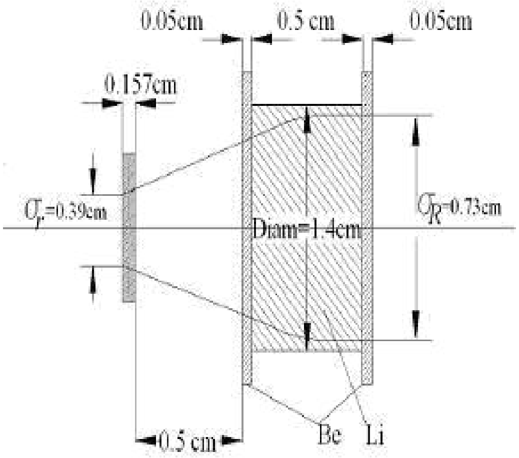


Fig.1. Geometrical dimensions of the LL

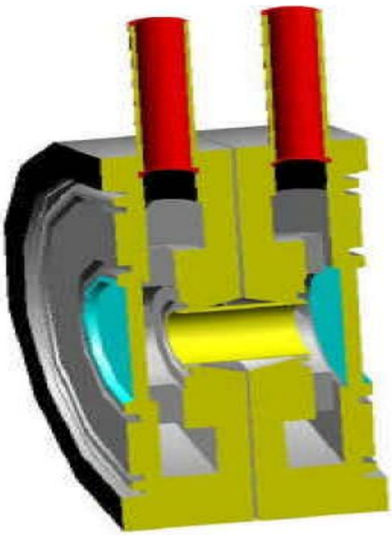


Fig. 2. Appearance of the LL

2. Maxwell equations for electric and magnetic field

2.1 Governing equations

Our first task is to calculate magnetic field inside lithium lens in order to know focus abilities and to calculate magnetic field between Ti target and LL in order to predict the effect on the trajectories of positrons. The equations for describing the electro-magnetic field for closed region are:

$$div(\vec{J}) + \frac{\partial \rho}{\partial t} = 0 \tag{2.1}$$

$$rot(\vec{H}) = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{2.2}$$

Where $\vec{J}$ - current density, ρ - charge density, $\vec{H}$ - magnetic flux density, $\vec{D} = \varepsilon \vec{E}$, where ε - dielectric conductivity, $\vec{E}$ -electric field.

In the stationary case we have $div(\vec{J}) = 0$ and $rot(\vec{H}) = \vec{J}$.

Using the fact, that $\vec{J} = -\sigma \cdot grad(V)$ and $\vec{H} = rot(\vec{A})$, where V -electric potential, $\vec{A}$ - vector potential, we can rewrite (2.1) and (2.2) as:

$$\nabla \cdot (\sigma \cdot \nabla V) = 0 \tag{2.3}$$

$$\nabla(\frac{1}{\mu_0 \mu_r} \nabla \times \vec{A}) + \sigma \nabla V = 0 \tag{2.4}$$

where ∇ - Nabla operator, μ_r - relative permeability (for Li $\mu_r \approx 1$). Two models are simulated. We start from a 2D axial symmetric model, and then we go through the 3D model to get the integral picture of magnetic field.

2.2 2D modeling

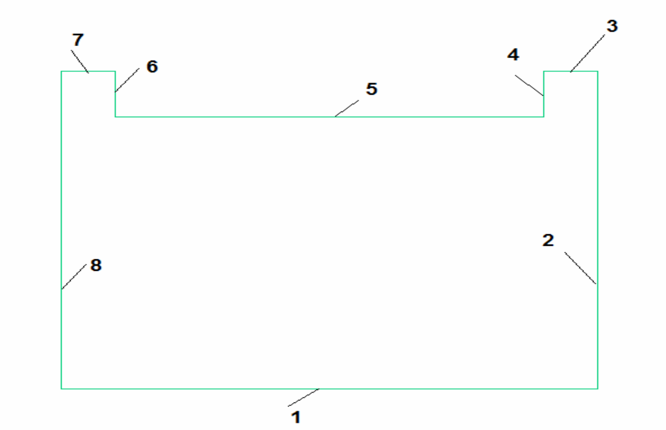


Fig. 3. Boundaries for 2D model.

For the boundary 1 , the axis of symmetry, the BC for electric and magnetic field have obvious appearance: $r = 0$. For boundaries 2,4,5,6 and 8 BC have this appearance: $\vec{n} \cdot \vec{J} = 0$.This means that current near boundary doesn't have normal component or while for the calculation of the magnetic field in the whole volume) the equation of continuity: $\vec{n} \cdot \left(\vec{J}_1 - \vec{J}_2 \right) = 0$ is used. It means that the normal components of current inside and outside the LL have the same value. All in all, this two types are similar, because in the outside region we have good vacuum: $\sigma_{ext} \to 0$. Here it must be mentioned, that during numerical calculations it is not possible to put $\sigma_{ext} = 0$, because of singularity matrix in equation (3), so, the conductivity should be slightly different from zero. For the magnetic field on this boundaries it is possible to represent continuity: $\vec{n} \times \left(\vec{B}_1 - \vec{B}_2 \right) = 0$.

The most difficult problem is to receive BC on boundaries 3 and 7. Using COMSOL it seems impossible to put boundary condition like $\int_S \vec{J} dS = I$ (as we know only a full inlet current), so, according to [1], BC for 3 and 7 will be like $V = V_{BC}$. Usually, for the 3rd boundary BC $V = ground$ will be specified, and for the 7th boundary $V = 1.5V$ or $V = 7.5V$. A few words about the fact, that current can not appear from nowhere. If we want to calculate a magnetic field along the supposed trajectory of positrons, it is better to consider boundaries 3 and 7 like external boundaries in order to avoid infinity at that region.

According to [1], a typical length of a lens is $0.5 - 1cm$ and a radius is $0.5 - 1cm$, so, two types of geometry are used. The first type is $r = 0.5cm$, $L = 1cm$, the second one is $r = 0.7cm$, $L = 0.5cm$. The applied electrical voltage is $1.5V$, Lithium electric conductivity is $\sigma = 0.108 \cdot 10^8 \dfrac{S}{m}$.

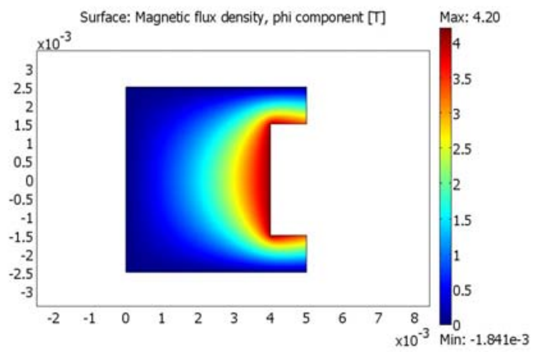


Fig 4. Magnetic flux density, norm, T

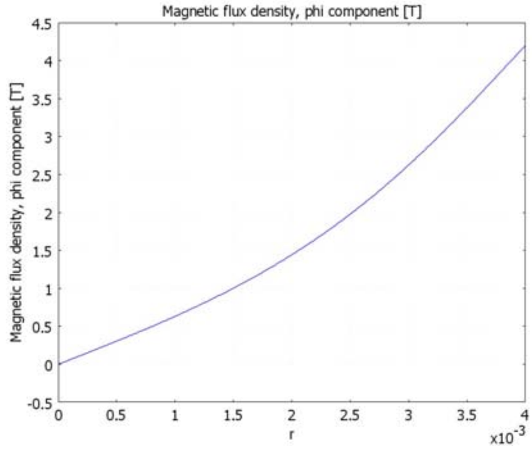


Fig 5. Magnetic flux density, cut along r direction at $z=0$, T

The type of the lens with greater radius has more non-linear dependence of the magnetic flux density on radius, which hamper to focus positrons.

2.3 3D modeling

In 3D case solving system has the same appearance like (2.3) and (2.4):

$$\nabla\cdot(\sigma\cdot\nabla(V))=0$$

$$\nabla(\frac{1}{\mu_0\mu_r}\nabla\times\vec{A})+\sigma\nabla V=0$$

where μ_r -relative permeability, σ - electric conductivity.

In order to simulate the realistic magnetic field around lithium lens, 3D model is used. The 3D lens model is simulated in two types which are different with their dimensions. First model: radius of lens $R=1cm$, length of lens $L=0.5cm$, diameter of wires $d=0.1cm$. Second model: radius of lens $R=0.7cm$, length of lens $L=1cm$, diameter of wires $d=0.1cm$.These geometrical properties is taken from [1].

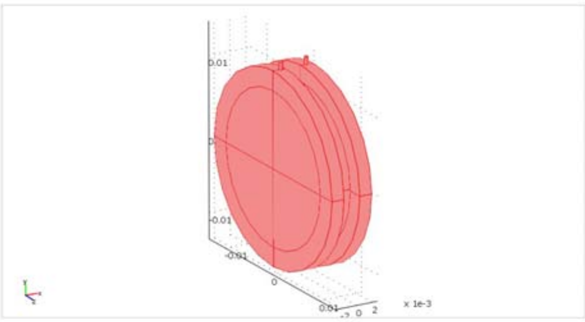


Fig. 6. First model

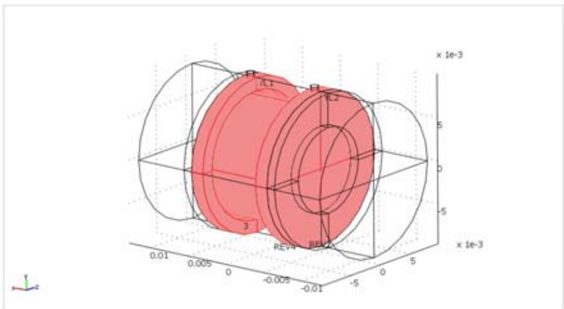
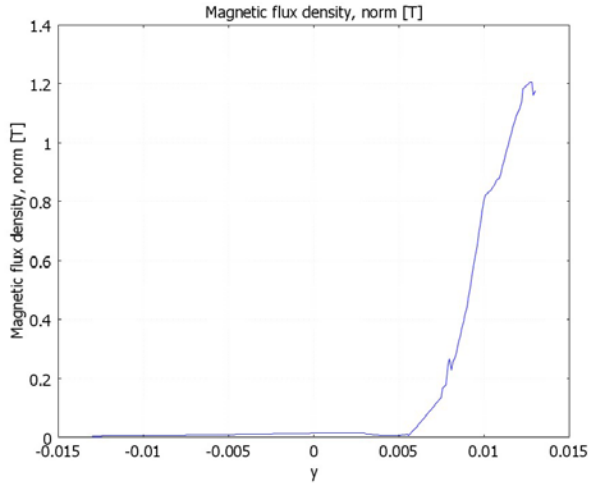


Fig. 7. Second model

For outside boundary BCs are: for magnetic field: $\vec{n}\times\vec{A}=0$, for electric field: $\vec{n}\cdot\vec{J}=0$, where $\vec{n}$ is outer unit normal vector. For the internal boundary the BCs of continuity are: $\vec{n}\times(\vec{H}_1-\vec{H}_2)=0$ and $\vec{n}\cdot(\vec{J}_1-\vec{J}_2)=0$. For the boundaries which have definite potential, BCs are: $V=V_1$ or $V=V_2$, where V_1,V_2 – given values, and $\vec{n}\times\vec{A}=0$. Result of numerical simulation for first model are presented in fig. 8 -11.



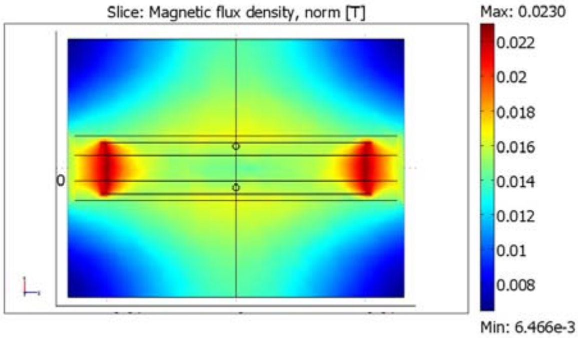
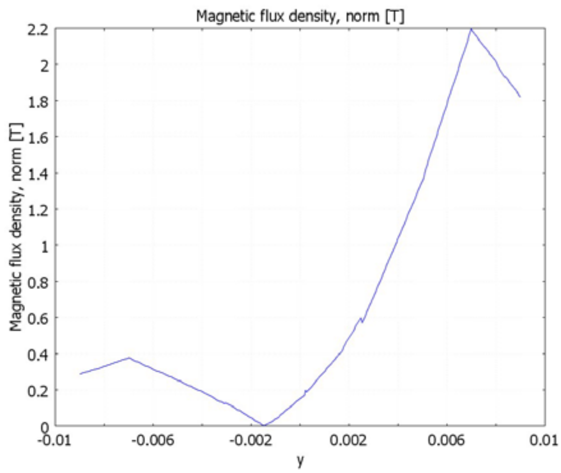


Fig 8. Cut of magnetic flux density along Y axes, norm. Fig. 9. Magnetic flux density, ZX plane.

It may be noticed, that in the ideal case i.e. without wires on the top of the LL and rather big length of LL, (at least $L \geq R$) magnetic flux distribution along Y axes must be linear, but we have strongly non-linear dependence of magnetic flux because of two reasons: there are two wires with great current density, which makes our task non-axial symmetric, and because of the shortness of the LL. That is why the second model is simulated.

For the second model, in order to make current distribution more homogeneous inside the lens, a copper ring is implied on the side face of the lens. In this case, the potential along the flanks decreases with the coordinate not as fast as without copper, since copper is a good conductor.



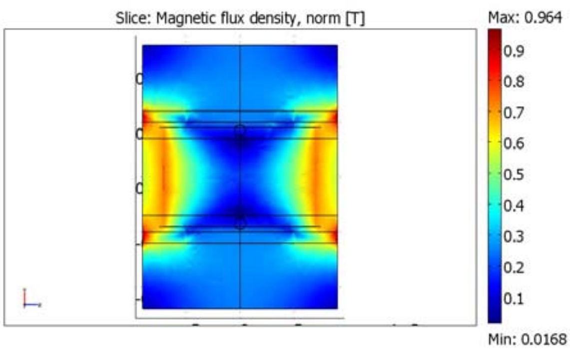


Fig. 10. Cut of magnetic flux density distribution along y, norm.

Fig. 11. Magnetic flux density distribution, cut along X axes.

Now, it is possible to see on fig. 10 there is enough linear distribution of magnetic flux comparing with fig. 8.

3. Problem of cooling lithium lens

According to [1], a full current 166 kA is needed in order to supply a proper magnetic field in lithium lens (~ 1 T). After each pulse inside the lens a energy Q = 1000 Joules is released, so the approximate gain of temperature is 170 K. The melting point of Beryllium is near 1560 K, so, starting from initial temperature 500 K (453 K is the point of melting Li), we need only six pulses to melt the lens. Using fact, that frequency of pulses is 5 Hz, the working time is $\approx$1-2 sec. It is a very rough estimation, because here the cooling period is neglected, for a more precise estimation we need to built a two dimensional axial-symmetric model.

Short describing of input data: there is a Li lens with dimensions, which are taken from [1]. On entrance boundary there is a Li flux at the speed approximately $10 m/s$, [1], Li lens is restricted by Beryllium windows with thickness $\approx 0.5mm$ (melting temperature $T = 1500K$) from both sides. Also there is a pulsed beam of positrons with known function of dissipated energy. The pulse duration is $1ms$, the repetition rate is $5Hz$. Our task is to calculate the temperature field and to prevent melting of Be windows.

Lithium lens has ~ 0.5 cm of radius and ~ 1 cm of the length, so if the liquid lithium has the flowing speed ~10 m/s, during each pulse (~ 1ms) the lithium inside the lens can be renewed approximately 4 times, and the gain temperature is 4 times smaller then in the case without flow of lithium.

In order to simulate this problem we need to solve heat transfer equation, Navier – Stokes equation and continuity equation in order to define Joule heating for heat transfer equation. For the heat transfer equation we need to find the temperature field. This transfer equation has the form:

$$\rho C_p \frac{\partial T}{\partial t} + \nabla(-k \cdot \nabla T) = Q - \rho C_p \nabla T \cdot U \,, \tag{3.1}$$

where:

T – temperature, $[K]$

k – heat conduction coefficient, $[J / kg \cdot K]$

Q – heat source, $[W / m^3]$

ρ – density, $[kg / m^3]$

U – velocity vector, $[m / s]$

C_p – heat capacity at constant pressure, $[J / kg \cdot K]$

In order to find the heat source function, we need to know the current distribution inside the lens. Thus we need to solve the continuity equation

$$\nabla(\sigma \nabla V) = 0 \,, \tag{3.2}$$

where

σ – is conductivity, $[S / m]$

V – electric potential, $[V]$

And, at lastly, we need the Navier-Stokes equation in order to solve heat transfer equation in the assumption of a flow of lithium. It is better to start from the case of laminar flow of Navier-Stokes equation. On low mach's numbers steady-state Navier-Stokes equation reads

$$\nu \cdot div(grad(U)) - grad(P) = \rho \cdot U \cdot grad(U) \tag{3.3}$$

Where

P – pressure, $[Pa]$

ν – dynamic viscosity, $[Pa \cdot s]$

and the continuity equation

$$\nabla U = 0 \tag{3.4}$$

if there is no external sources of liquid lithium except inlet boundary

Or, in the assumption of low-compressible media, the following equation is correct

$$\nu \cdot div(grad(U)) - grad(P) = \rho \cdot U \cdot grad(U) + div((\frac{2}{3}\eta - k_{dv}) grad(U)) \,,$$

where k_{dv} -dilatational viscosity, but then only equation 3.3 will be used. Also we need an equation for pressure. Here we will use a "slightly-compressible" model of lithium:

$$P(\rho) = P_0 + L \cdot (\rho - \rho_0) \,, \tag{3.5}$$

where ρ_0 - is the density at room temperature, P_0 is the pressure at density ρ_0, L characterize the coefficient of compressibility. L is chosen large enough to enforce the near-incompressibility of the fluid, but not large enough to erase the other terms of the equation in the finite precision of the computer arithmetic.

The continuity equation:

$$\frac{\partial \rho}{\partial t} + div(\rho U) = 0,$$

(3.6)

Using 3.5 and 3.6 we can write

$$\frac{\partial P}{\partial t} = -L\rho_0 \cdot div(U).$$

In steady state $\frac{\partial P}{\partial t} = -div(grad(P))$, according to smoothing operators in PDE's, so

$$div(grad(P)) = M \cdot div(U), \quad (3.7)$$

where M has dimension $\frac{Pa \cdot s}{r^2}$. In 2D axial-symmetric case equation 3.7 looks like $div(grad(P)) = M \cdot (\frac{\partial U_z}{\partial z} + \frac{1}{r}\frac{\partial (r \cdot U_r)}{\partial r})$, where U_z, U_r - are the velocity components in cylindrical system of coordinates, M is chosen as $5 \cdot 10^6 \cdot \frac{viscosity}{x_1^2}$, where x_1-characteristic dimension of LL. Thus, equations 3.1-3.3 and 3.7 completely describe current task.

3.2 Boundary conditions

3.2.1 Navier – Stokes equation

As usual, for boundary 1 BC is performed as $r = 0$. For boundaries 2, 4, 5, 6, 8 condition $\vec{U} = 0$ is used. It is worth to mention that in the case of the Navier-Stokes equation the boundary condition "wall, no slip" is only correct for laminar flow. In turbulent flow there is no liquid sticking to the wall. For the inlet boundary 7 and for the outlet boundary 3 the BCs are: $\vec{U}_z = 0$ and $\frac{\partial \vec{U}_r}{\partial z} = 0$. Other words it's possible to consider no viscose stress on the boundary: $grad(U) \cdot \vec{n} = 0$. Generally speaking, it is not necessary to put exact value of speed on boundaries 3 and 7, we can select such difference in pressure on that boundaries which supply the speed in the middle of the lens as we need. It is possible to calculate Reynolds number like $\mathrm{Re} = \frac{\rho V L}{\nu}$, where ν – is the dynamic viscosity. For lithium the properties are (at temperature 520K): $\rho = 510 kg/m^3$, $\nu = 5 \cdot 10^{-4} Pa \cdot s$, $V \approx 10 m/s$, $L \approx 10 mm$ and resulting $\mathrm{Re} \approx 10^5$. From table we can find $\mathrm{Re}_{cr} \approx 1000$, where Re_{cr} is crucial number for transition "laminarity - turbulence", so $\mathrm{Re} \approx 10^5$ is much bigger than R_{cr} and it's a turbulence model in our case. In order to make laminar flow, it is better to decrease the inflowing speed to $0.05 - 0.1 m/s$, but, probably, cooling process would not be efficient at this speed.

Because of high turbulence model and relatively low kinematic viscosity of lithium ($0.98 \cdot 10^{-6} \frac{m^2}{s}$, comparing with water viscosity $1.01 \cdot 10^{-6} m^2/s$), the numerical calculation at real viscosity does not converge. The minimal viscosity, at which calculation has stable convergence, was $1 \cdot 10^{-2} \frac{m^2}{s}$. This viscosity is used for numerical calculations.

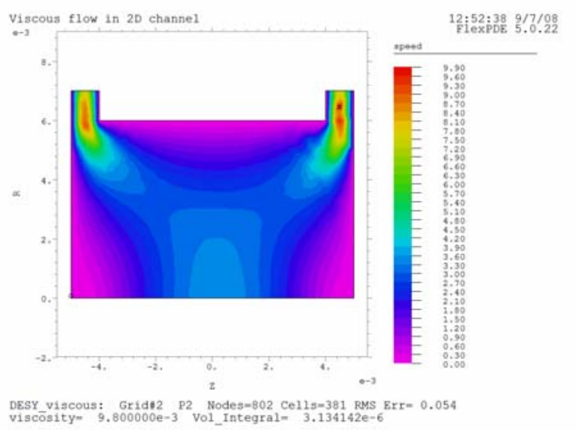


Fig. 12. Field of velocity, viscosity $1 \cdot 10^{-2} \frac{m^2}{s}$

On boundary 3 we put pressure equal to $10^5 Pa$, on boundary 7 -$1.45 \cdot 10^6 Pa$. For this difference of pressure we have speed $10 \frac{m}{s}$ at the entrance to the LL. Field of velocity depicted on fig. 12 is used for cooling the LL. Average speed in the middle cross-section of the LL is $2m/s$.

3.2.2 Heat transfer equation

For the boundaries 2, 4, 5, 6, 8 it is possible to put the condition of thermal isolation $-\vec{n} \cdot (-k\nabla T) = 0$ because the adjacent media is vacuum. For boundary 7 the best suitable condition is constant temperature $T = T_0$, because inflowing lithium is coming from big capacity with constant temperature. For boundary 3 it is possible to put convective flux $\vec{n} \cdot (-k\nabla T) = 0$ but also (as boundary 3 adjoin to big capacity of Li) it's possible to put $\vec{n} \cdot (-k\nabla T) = -k \cdot (T_{\inf} - T)$, where k - is thermal-conductivity coefficient, $T_{\inf} = T_0$.

3.2.3 Results of simulation

Heat transfer equation in cylindrical system of coordinates has the form:

$$\mathrm{div}(\mathrm{k} \cdot \mathrm{grad(T)}) + \mathrm{Q} = \rho \cdot \mathrm{C_p} \frac{\partial T}{\partial t} + \rho \cdot \mathrm{C_p} (\mathrm{U_z} \cdot \frac{\partial T}{\partial z} + \mathrm{U_r} \cdot \frac{\partial T}{\partial r}),$$

where the summand $\rho \cdot \mathrm{C_p} (\mathrm{U_z} \cdot \frac{\partial T}{\partial z} + \mathrm{U_r} \cdot \frac{\partial T}{\partial r})$ presents a convective flux. The function of sources Q is the sum of two components: $Q = Q_{joule} + Q_{beam}$, $Q_{joule} = \frac{J^2}{\sigma}$, where J – current density, σ – electrical conductivity. $Q_{beam} = Q_0 + \frac{A}{w\sqrt{\pi/2}} \exp(\frac{r}{w}^2)$ - Gaussian function, this is a fixed data. Firstly, heat transfer equation is solved without convective flux summand in order to compare lens cooling by flowing lithium.

All numerical simulations are made for two values of voltage: $1.5V$ and $8V$ just to compare the influence of joule heating on general heating. The initial temperature is $520K$ (liquid state lithium). Numerical calculation under the assumption that the applied voltage is $8V$ are shown in fig.13 -16.

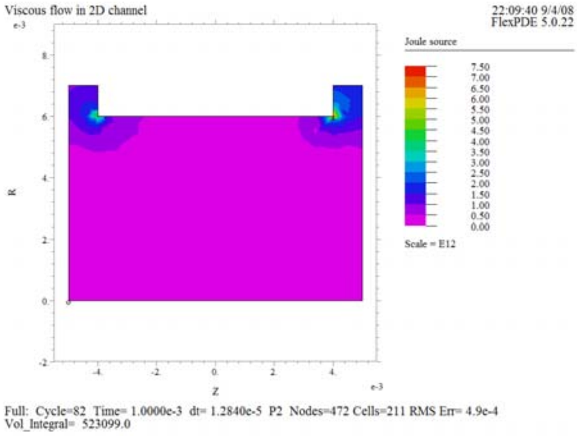


Fig. 13. Heating by current, voltage $8V$,W/m^3

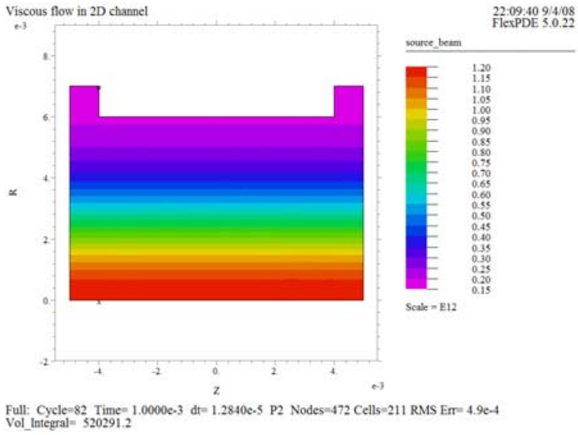


Fig. 14 Heating by transmitted beam, W/m^3

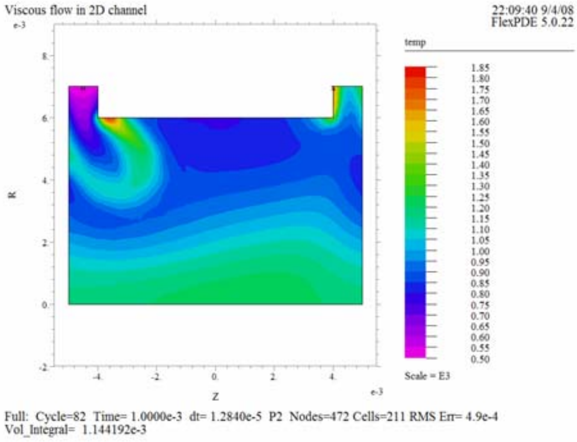


Fig. 15. Temperature after first pulse, $1ms$,
heating by transmitted beam and current

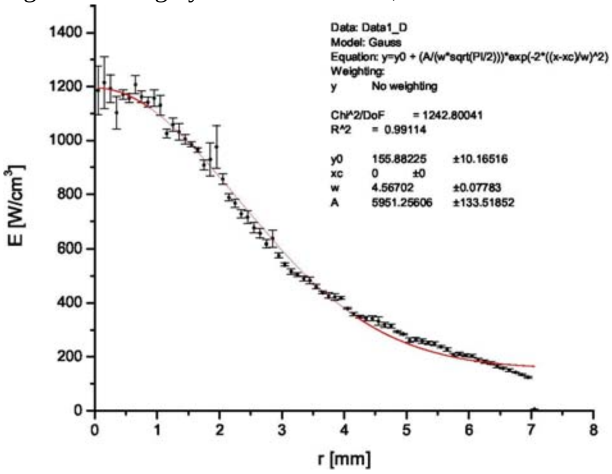


Fig. 16. Gaussian heating function of transmitted beam.

It can be seen, that maximal temperature is $1850K$ and lies in the region, where the current density is maximal. It is worth to notice that almost in the whole area $Q_{joule} \geq Q_{beam}$. Comparing figures 13 and 14, it is safe to say that maximal value of Joule heating ($7.5 \cdot 10^{12} \dfrac{W}{m^3}$) is three times more than source beam ($1.2 \cdot 10^{12} \dfrac{W}{m^3}$). Results of calculation under the assumption that applied voltage is $1.5V$ are presented on fig. 17 -18.

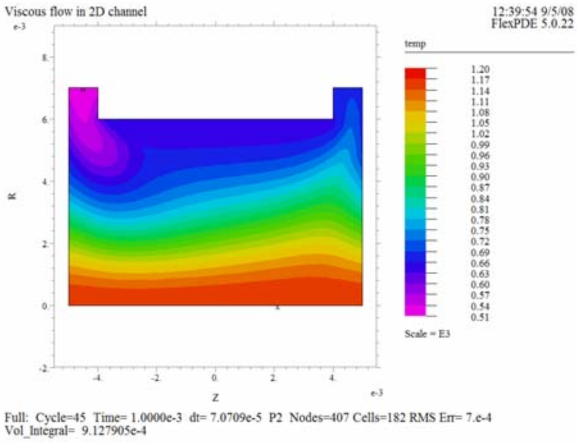


Fig. 17. Temperature after first pulse, $1ms$.

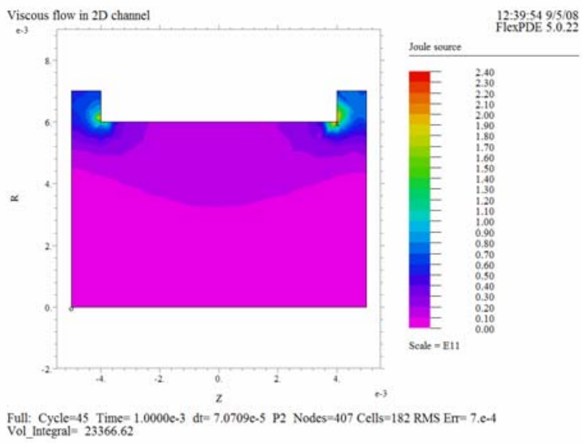


Fig. 18. Heating by current, voltage, W/m^3 $1.5V$.

Heating by current, shown on fig. 18 corresponds to full current $25kA$, heating by current, shown on figure 13 corresponds to full current $105kA$.

Now, we can see that at voltage $1.5V$ the maximal temperate is $\approx 1200K$ in the area of maximal beam intensity. Meanwhile, in this case the heating by transmitted beam is five times more than heating by current.
But before the second bunch train, after $0.2s$ after first bunch train, the temperature field inside LL becomes the same, as it was before the first bunch.
To present a more realistic model, we put beryllium windows on the left and right side of the LL.

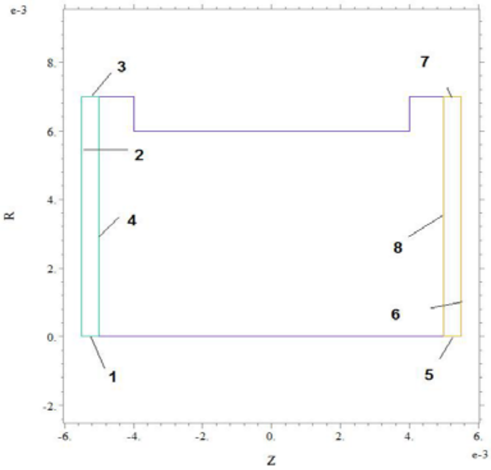


Fig. 19. Boundaries for Be windows

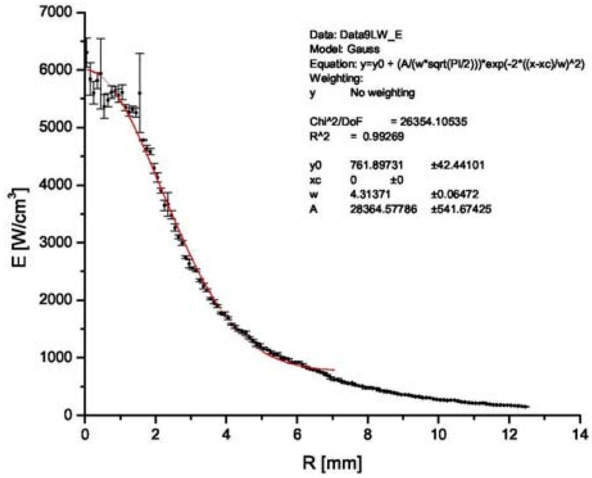


Fig. 20. Dissipated energy in left Be window.

The thickness of Be windows is $0.5mm$. The properties of Be at that conditions we have are: thermal conductivity $k = 201\frac{W}{m \cdot K}$, density $\rho = 1848\frac{kg}{m^3}$, thermal capacity $Cp = 1820\frac{J}{kg \cdot K}$, electric conductivity $\sigma = 0.313 \cdot 10^8 \frac{S}{m}$.
On boundaries 1-4 and 5-8 for Navier – Stokes equation no slip condition was supposed ($U_r = 0, U_z = 0$), this provides zero speed in solid state Be. According to equation 3.2, one should mention that Be has even more electric conductivity then Li, Boundary conditions were performed like continuity of current: $\vec{n} \cdot \left(\vec{J}_1 - \vec{J}_2\right) = 0$.
At last, for the heat-transfer equation the BCs was supposed like convective flux $\vec{n} \cdot (-k\nabla T) = 0$.
Results for applied potential $1.5V$ are presented on figures 21 - 22:

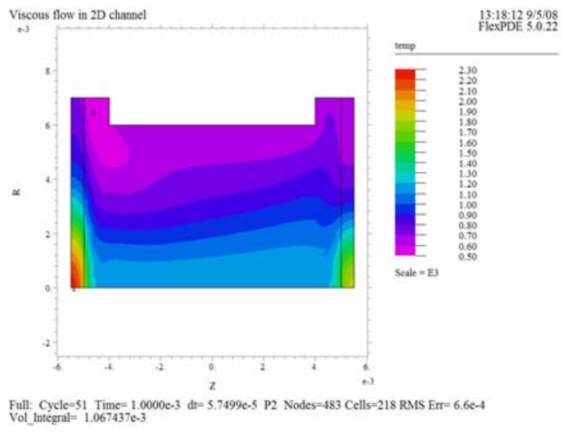


Fig. 21. Temperature after first pulse, $1ms$.

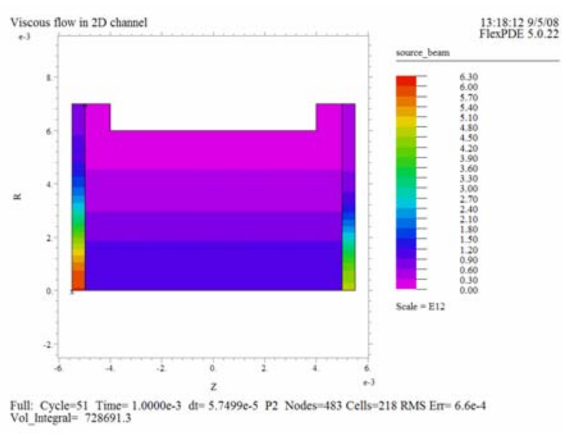


Fig. 22 Heating by transmitted beam, W/m^3.

Now the left beryllium window is the hottest part. It happens because at $r = 0$ dissipated energy in beryllium material (left window) is equal to $6\frac{kW}{cm^3}$ (see fig. 20), comparing with dissipated energy in Li material (only $1.2\frac{kW}{cm^3}$) it becomes a significant value. Even inside right window this value

($4.5\frac{kW}{cm^3}$) is higher, and, of course, there is no way for cooling mechanism inside the window ($\vec{U} = 0$) except radiation one.

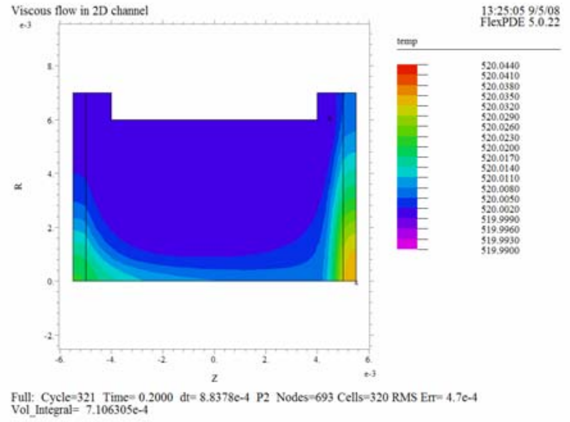


Fig. 23. Temperature field before second pulse, $0.2s$

But after $0.2s$ of cooling (remember, that between first and second bunches there is no nor heating by transmitted beam neither by current, because we do not need to supply proper magnetic field during the time between bunches) temperature field becomes similar to initial one.

4. Summary

In this work the magnetic field inside a LL was calculated. It has been found that a LL with the length $1cm$ has a more linear magnetic field dependence than the lens with the length $0.5cm$, and as the result, better positron focusing abilities. The magnetic field outside the LL is not very big comparing with the field inside the LL. It is the positive feature of the LL. To get a magnetic field of $\approx 1T$ we need $105kA$ of current. It is similar to applied potential $8V$.

It has been found that at the current of $105kA$ overheating factor is Joule heating but not the transmitted beam: the temperature gain after one pulse in the regions with great current density is $\approx 1300K$, while in the region with great intensity of dissipated energy of transmitted beam it is only $\approx 700K$. This temperature gain is more than the melting temperature of Be, although between first and second pulse liquid lithium inside the lens returns to initial temperature ($500K$).

The maximal temperature gain is inside Be windows, not inside the lens. It happens because of a greater value of dissipated energy, comparing with the same parameter inside the LL. There is no efficient cooling of Be windows, except radiation flux from the surface of Be window. A feasible solution is to increase cross-section of transmitted beam.

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