TOMOGRAPHY RECONSTRUCTION OF THE TRANSVERSE PHASE SPACE USING TWO QUADRUPOLES

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September 12, 2007

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Abstract

Photo-Injector Test Facility at DESY in Zeuthen (PITZ) is being built to develop and optimize electron sources to be used in Free Electron Lasers such as FLASH and the future XFEL in DESY Hamburg. Current upgrade of the machine includes several new beam diagnostic modules like a multiscreen module for transverse phase space tomography reconstruction. This module should operate for beam momenta between 15 and 40 MeV/c. At such energies it is difficult to deliver a beam matched to the optics of the beam line which is necessary for correct reconstruction. For that purpose, a second hardware solution has to considered. The purpose of this report is to describe the work done in this direction.

A setup of two quadrupoles and an observation screen located downstream has been investigated. Brief theoretical overview will be presented. It will be shown that the chosen setup is capable of delivering full π rotation of the beam in the phase space. This will be confirmed with data from numerical simulations and results from tomography reconstruction will be shown.

1 INTRODUCTION

The Photo-Injector Test Facility at DESY in Zeuthen (PITZ) is a test stand that has been working on research and development of laser driven electron sources for free electron lasers (FEL) and linear colliders. PITZ consists of a laser driven RF gun, a booster cavity for further acceleration and modules for beam diagnostics. Currently the machine is being upgraded to PITZ2 with an extended diagnostic section and a new booster cavity. A simplified layout can be seen on Fig. 1.

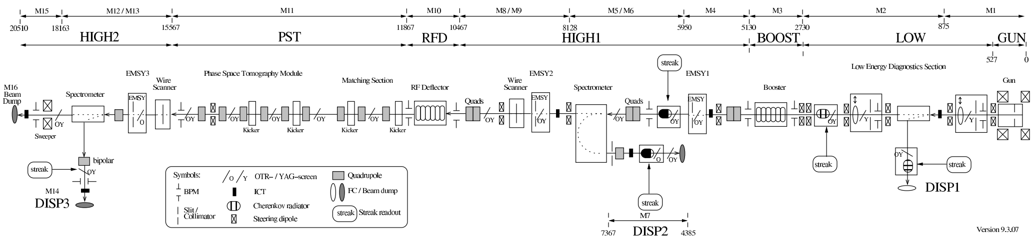

Figure 1: Simplified PITZ2 layout.

One of the new diagnostics modules is a multiscreen setup for phase-space tomography reconstruction. It will have four screens and three pairs of FODO cells in-between them. The phase advance between screens is required for 45 °. However, in general, the beam Twiss parameters are not matched to the setup and therefore an upstream matching section is required.

In reference [1] it has been shown that a good matching along the tomography section is possible for one of the transverse planes. For that reason, an alternative approach has been undertaken and preliminary studies were performed during this International Summer Student Program.

This project report is structured as follows: the next section presents the basic theoretical beam dynamics and related matrix algebra which is the background knowledge for this work, followed by modelling of the used physical setup and numerical simulations. The last section describes some results of tomography reconstruction.

2 BASIC BEAM DYNAMICS

A beam can be understood as a group of particles each of which can be described by its three positions and three canonical momentum coordinates (x, p_x, y, p_y, z, p_z). Such a six-dimensional vector is known as phase space. It can be simplified into two transverse - (x, p_x) and (y, p_y), and a longitudinal - (z, p_z) components. Beam physics practice and theory is more interested in the angle enclosed between each transverse component of the momentum and the longitudinal one, i.e. $x' = \frac{p_x}{p_z}$, known as the divergence of the beam.

Since a real beam consists of over 10^{10} particles, it is more efficient to use statistical means when describing the behaviour of the beam. Such are the second moments of the distribution - rms beam size $\sigma_{x,y}$, divergence $\sigma_{x',y'}$ and the correlation between them i.e. covariance - $\sigma_{xx',yy'}$.

A quantity of extreme importance for the FELs is the so called beam emittance defined as the phase space area. In the horizontal transverse plane the emittance is

$$\epsilon_{x,rms} = \sqrt{\sigma_x \sigma_{x'} - \sigma_{xx'}^2}. \tag{1}$$

The above, in turn, can be written using the determinant of the beam-sigma matrix

$$\Sigma_x = \begin{pmatrix} \sigma_x & \sigma_{xx'} \\ \sigma_{xx'} & \sigma_{x'} \end{pmatrix} \tag{2}$$

or $\epsilon_{x,rms} = \sqrt{|\Sigma_x|}$. Similarly the 6-dimensional emittance is defined as $\epsilon_{6D} = \sqrt{|\Sigma_{6D}|}$.

The beam emittance is not constant although the normalized emittance is conserved in six-dimensional phase-space as long as there are no interactions between particles and/or the environment. The normalised emittance is defined as

$$\epsilon_{u,N} = \beta\gamma\epsilon_{u,rms}, \tag{3}$$

where β and γ are the relativistic Lorentz factors and $\epsilon_{u,rms}$ is the rms emittance for plane (u, u').

Another definition of the beam uses the optics of the machine or the so-called Twiss prameters - β_u, α_u and γ_u. The relationship between the rms values and Twiss parameters can be seen from Fig. 2.

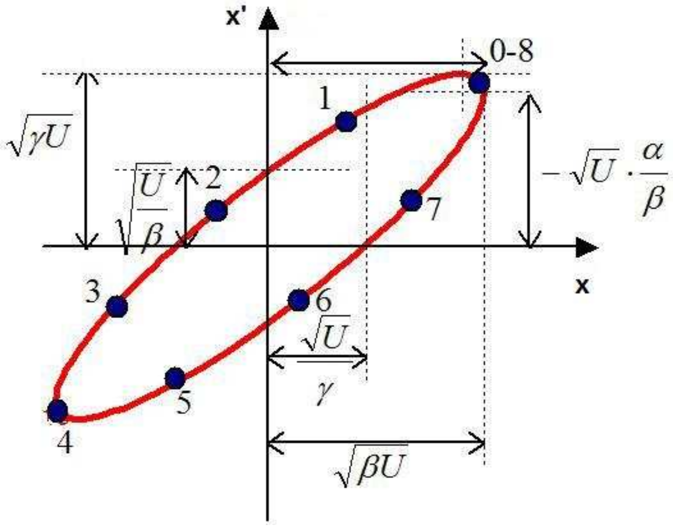

Figure 2: Phase-space ellipse described by the Twiss parameters.

In linear approximation, the propagation of a single particle along the machine axis is described using the matrices of the beamline elements such as quadrupoles, drift spaces, etc. When there is a focusing quadrupole for one of the transverse planes, i.e. $K > 0$ for negatively charged particles, the transfer matrix is given by

$$\begin{pmatrix} cos(\sqrt{K}L) & 1/\sqrt{K}sin(\sqrt{K}L) \\ -\sqrt{K}sin(\sqrt{K}L) & cos(\sqrt{K}L) \end{pmatrix}, \tag{4}$$

where L is the effective length.

For the case in which there presents a defocusing quadrupole, i.e. $K < 0$, the matrix will look like

$$\begin{pmatrix} cosh(\sqrt{|K|}L) & 1/\sqrt{|K|}sinh(\sqrt{|K|}L) \\ -\sqrt{|K|}sinh(\sqrt{K}L) & cosh(\sqrt{|K|}L) \end{pmatrix}. \tag{5}$$

In a drift space the linear motion is defined by

$$\begin{pmatrix} 1 & L_{drift} \\ 0 & 1 \end{pmatrix}. \tag{6}$$

Thus, the transverse coordinates of a particle travelling between points z_0 and z_n can be found using

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{(z_n)} = M_n M_{n-1} \cdots M_0 \begin{pmatrix} x \\ x' \end{pmatrix}_{(z_0)} \tag{7}$$

The particles always lie on ellipses as they propagate along the machine axis. Each particle moves on its own ellipse and in terms of beam rms values this is described by phase advance which can be explicityly expressed as:

$$\mu_x(z/z_0) = \phi(z) - \phi(z_0) = \int \frac{dz}{\beta_x(z)} \tag{8}$$

where the limits of the integral runs from the z_0 to z.

3 PHASE SPACE TOMOGRAPHY

In general, tomography is understood as imaging by sectioning. Tomography reconstruction for a wide variety of purposes can be done using various tomography techniques. The resulting image can be 2D or 3D, depending on the need. The techniques employ one of many available algorithms to reconstruct the original distribution from the projections. Some of the well-known algorithms are Algebraic Reconstruction Technique (ART), Filtered Backprojection, Fourier Transformation, Maximum Entropy Method (MENT) and so on.

In this project, the existing code for tomography reconstruction that was investigated was the last one, i.e. Maximum Entropy Method (MENT). This method, as it is so-called, is said to maximise the entropy of the distribution [4]. This algorithm was developed by Gerald Minerbo [5].

In general in tomography reconstruction methods, the observed data is given by a number of projections [6],

$$P_n(s) = \int f(x_n(s,t)y_n(s,t)dt \tag{9}$$

where the n= 1,2,3....N projections. Each projection represents a particular view of the object - these can be seen in Fig. 3.

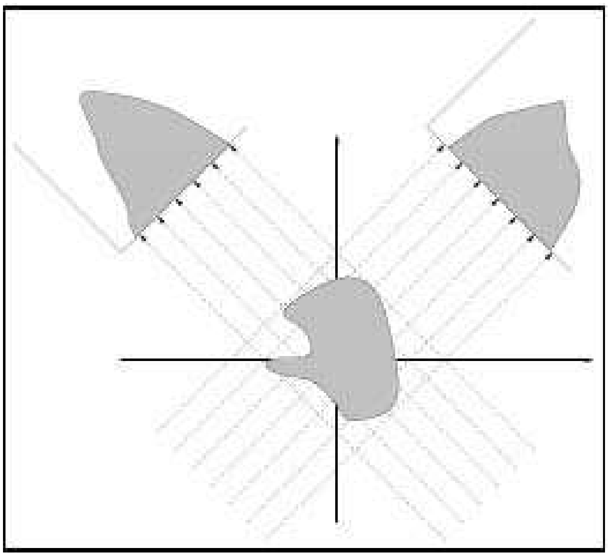

Figure 3: Two projections at different viewing angles of an object as used in tomography reconstruction.

Precise reconstruction requires small angular step and consequently big number of projections.

4 TOMOGRAPHY RECONSTRUCTION WORK AND RESULTS

Precise measurement of the transverse phase space distribution of the beam is essential if bright particle (electron) beams are to be used to drive free electron lasers. At PITZ, measurement needs to be done for its full energy range in operation. In this regard tomographic image reconstruction techniques may be a solution in this particular context since it seems to not limit the number of views we may take. Further more, there are several advantages, one of which is that it requires no a priori assumptions on the phase space distribution [3]. Thus, MENT was selected for the purpose of this project.

The setup used for these studies consists of two quadrupole magnets and an observation screen further downstream. The position of reconstruction is required to be EMSY1 where the emittance is measured using single slit scan technique and one is interested also in the distribution of the particles in the phase space. The beam is rotated in the phase space with the help of quadrupole lenses. The observation screens have to be defined so that the beam performs full π rotation in the phase space in small angular steps. This screen may differ according to operating energies. For these studies beam momentum of 32 MeV/c was used. A simple layout of the set up is shown in Fig. 4.

Initially, space charge was ignored in the design and hence linear beam transport was assumed. This allows rough estimation of the position of the screen, range of quadrupole gradients and covered phase advances. For that purpose, a few short

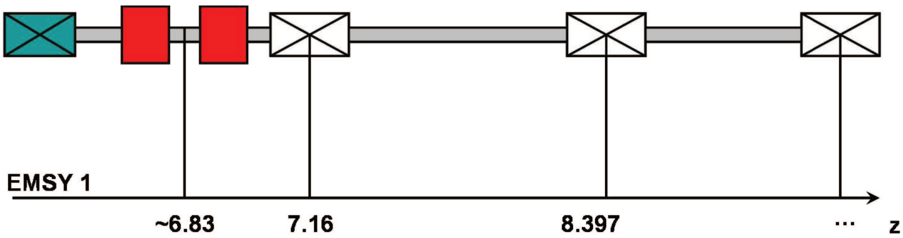

Figure 4: Simple layout of the set up.

programs in MATLAB and C++ were written and later the obtained results were used for beam tracking with ASTRA. Later, the influence of the space charge forces was included for both of the screens taken into account. If the reconstruction is required to be done simultaneously for both planes then the further downstream screen is to be used. Some positive results have been obtained - numerically, using MAD [7], angular step of 5° was obtained, as seen in Fig. 5 where phase advances at each projection for cases with and without space charge were compared against each other.

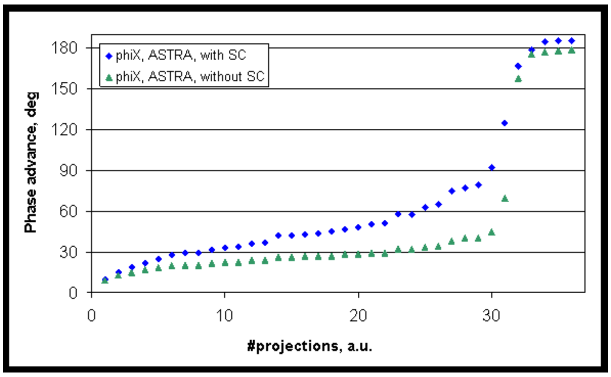

Figure 5: Phase advances at each step

The beam tracking with ASTRA afterwards was performed for a large number of particles to resemble real experiment. We managed to work on reconstruction with various number of projections and compared the values of the normalised beam emittance against the original ones. In doing the reconstruction work, it has been discovered, as expected, that the more projections there are the higher the precision is. Errors increase as the number of projections is decreased. This can be seen in Fig. 6 where the relative error in emittance is shown as a function of the number of projections. Fig. 7, 8 and 9 shows the relative error in the covariance, beam size and divergence as a function of the number of projections. From Fig. 9 it can be concluded that the performance of the reconstrucion algorithm cannot be judged using only the mentioned quantities.

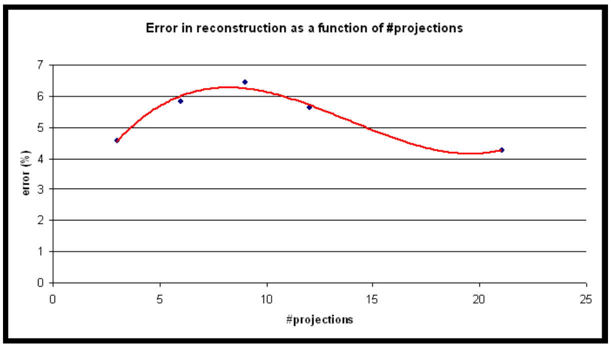

Figure 6: Relative error in percent in the normalised emittance as a function of the number of projections used.

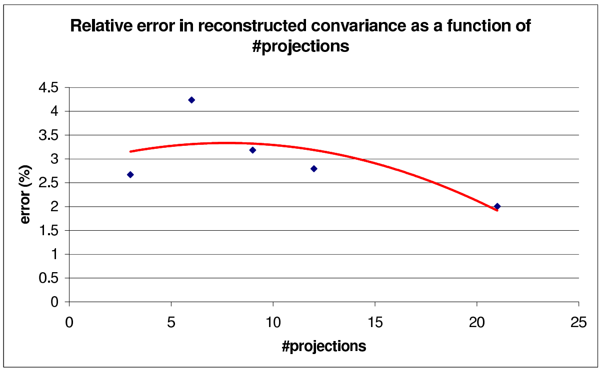

Figure 7: Relative error in percent in the covariance as a function of the number of projections used.

The following images are the results obtained from reconstruction work carried out with different number of projections.

From these images, it can be seen that the quality of the reconstructed images decreases as the number of projections are decreased, and hence the bigger error. For

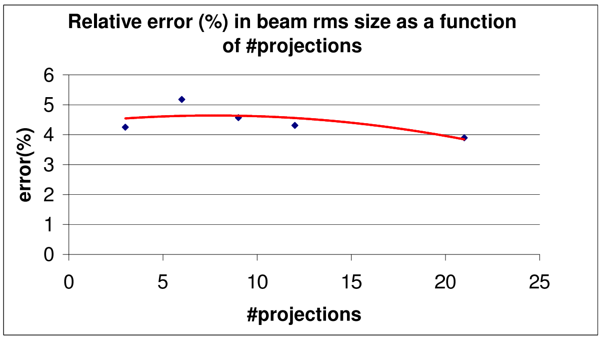

Figure 8: Relative error in percent in rms beam size as a function of the number of projections used.

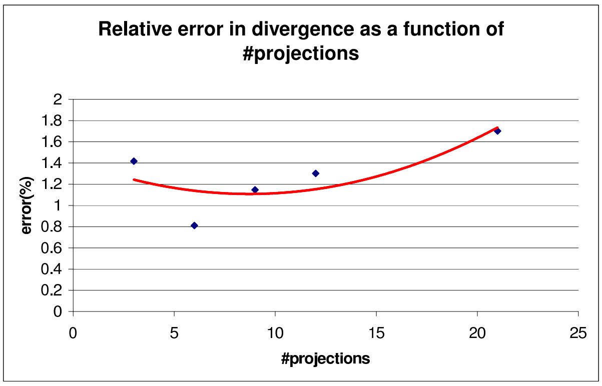

Figure 9: Relative error in percent in divergence as a function of the number of projections used.

example, in the case of 3 projections, it could not even capture tails. Thus, it is once again confirmed from these results that the smaller the steps, the higher the number of projections and the better the precision is. Some work was also done for the case when the space charge is considered.

Table 1 shows the relative errors for different projections.

Due to limited availability of time, only the work on small number of particles including space charge were completed. In reality, the larger the number, the better the results. Tomography reconstruction for this case was also carried out as the same time as this report is being written up. An example for reconstruction using

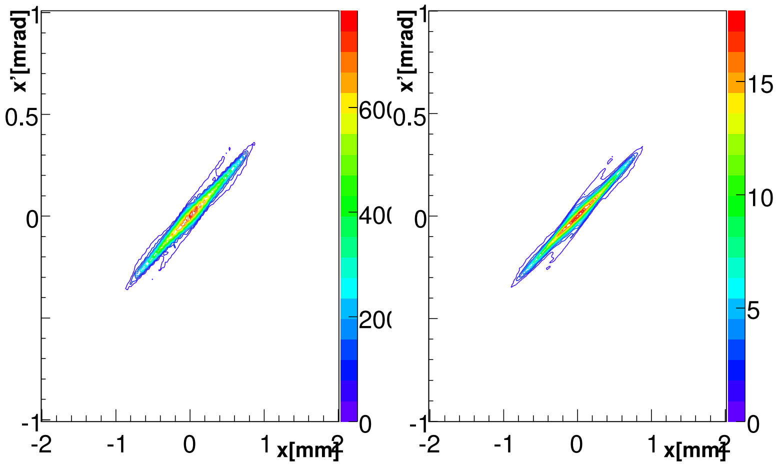

Figure 10: Original (left) versus reconstructed horizontal trace space for 21 projections.

Table 1: Relative errors in percent in terms of emittance, rms beam size, divergence and covariance as a function of number of projections used for reconstruction.

#projections	$\triangle\epsilon$	$\triangle\sigma_x$	$\triangle\sigma_{x'}$	$\triangle\sigma_{xx'}$
21	4.27	3.9	0.017	2.
12	5.63	4.31	0.013	2.79
9	6.45	4.57	0.011	3.19
6	5.84	5.18	0.008	4.24
3	4.58	4.25	0.014	2.67

12 projections is shown in Fig. 13. Here it can be seen that the linear transport used for mapping in tomography reconstruction negatively influences the results.

5 CONCLUSION AND SUMMARY

The work it has been done in this summer student project is relavant to a part of the work needed to be done for PITZ2 due to start its upgrade in the near future. From the results obtained, it indicates that the design with 2 quadrupoles is possible

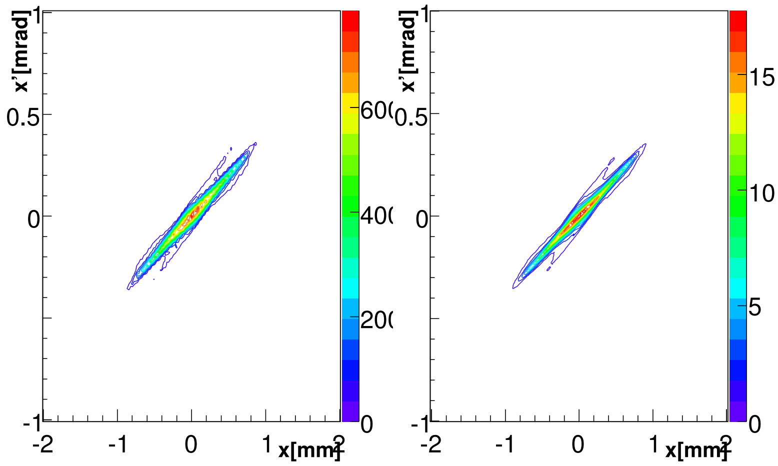

Figure 11: Original (left) versus reconstructed horizontal trace space for 12 projections.

for practical purposes. Although the results from the simulations without taking into account the space charge seems more encouraging, in real life situations space charge cannot be ignored. Including it in the simulation results in error being bigger in the reconstruction work. Thus, it is concluded that improvements will need to be done for the algorithm that will handle the situations involving the space charge. Further work on that in the future is anticipated.

6 ACKNOWLEDGEMENT

I hereby would like to take this opportunity to express my heart-felt thanks to Galina Asova, my supervisor, for all her help and patience through out my time at DESY. Because of her, I have been able to improve my programming skills in C++ and MATLAB as well as LATEX and learn the use of ROOT. It has further enhanced my motivation to follow a career in an area of Computational Physics and to become

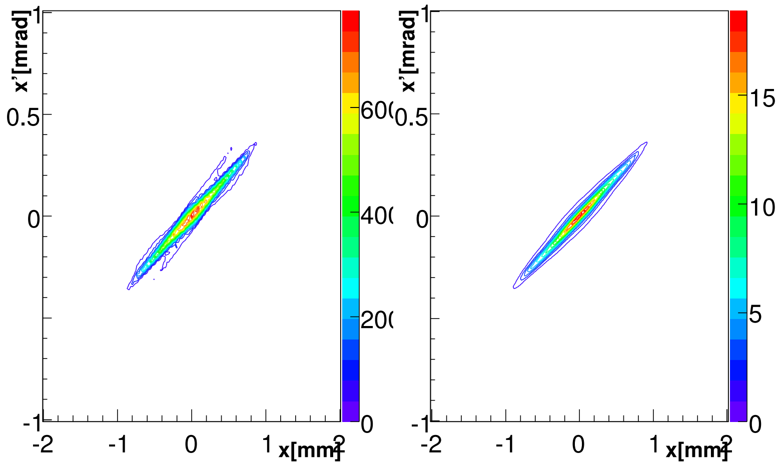

Figure 12: Original (left) versus reconstructed horizontal trace space for 3 projections.

a proficient computer programmer and a great physicist like her. Galina has been a good example for me to look up. I would also like to thank Andrea Schrader (DESY Hamburg) for her kind help before my arrival at DESY, Karl-Heinz Hiller and other staff involved in the organisation of summer student program for their help during our stay here. I also would like to thank all my fellow summer students, especially Szymon Kulis and Petty Bernitt, for their help in many ways and friendship, and I wish them good luck.

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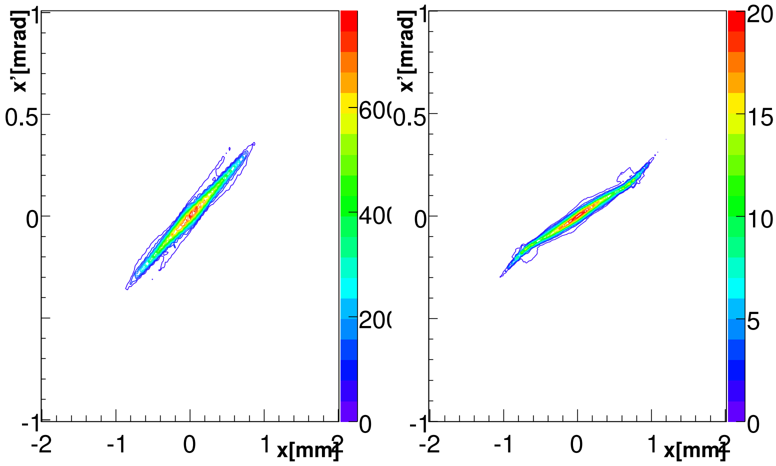

Figure 13: Original (left) versus reconstructed horizontal trace space for 12 projections including space charge in the numerical tracking.

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