Determination of the beam energy at TESLA using radiative dimuon events

Helge Todt

Institut für Physik (Universität Potsdam)
Am Neuen Palais 10, 14469 Potsdam, Germany
Email: htodt@web.de

Using events radiative to the Z pole, $e^-e^+ \to \mu^-\mu^+\gamma$, with $Z^0 \to \mu^+\mu^-$ decay, allows to determine the beam energy by measuring the angle of the muon w.r.t. the photon and requiring energy-momentum conservation. Events were simulated with PYTHIA *and the fast detector simulation program* SIMDET *was used. Beamstrahlung was accounted for by using the package $K\acute{\iota}\rho\kappa\eta$. Beam energy measurements were done for center-of-mass energies 200 to 800 GeV.*

1 Introduction

For high precision measurement of e.g. the Higgs boson mass at the linear collider TESLA a good knowledge of the beam energy is essential. The aim is to reach an energy precision of better than $\Delta E_b/E_b = 10^{-4}$. The basic device proposed for beam energy measurements is a magnet spectrometer positioned upstream of the interaction point (IP), see also [TDR01]. To check the reliability of this device (cross-check) it was suggested to perform an independent method that uses $e^+e^- \to \mu^+\mu^-\gamma$ radiative return events. The dimuon channel is the easiest in which this study should be performed: only two-prong events with well-identified muons are accounted for. $\sqrt{s} = 2E_{beam}$ measurements require precise information on the muon to photon angles ϑ_+ and ϑ_- and rely on energy-momentum conservation to coplanar events with one hard initial state radiation (ISR) photon being radiated collinear to the beams. Knowledge of M_{Z^0} (and Γ_{Z^0}) to a precision of few MeV is important to achieve E_b values with the anticipated accuracy. The impact of beamstrahlung and background from reactions with dimuon and large missing energy topology has also been investigated.

2 Method and Kinematics

The reaction with the least background is assumed to be

$$e^-e^+ \to Z^0/\gamma \to \mu^-\mu^+\gamma.$$

If one and only one ISR photon is radiated (s. fig. 1 and fig. 2), the effective c.m. energy can be calculated from the angles of the muon w.r.t. the photon using 3-body kinematics.

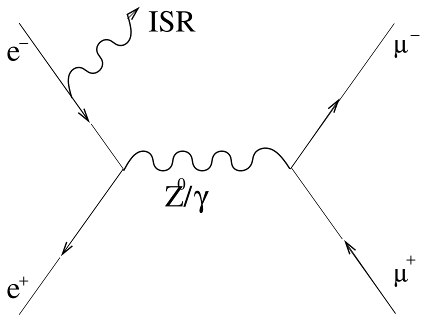

Fig. 1: *The process* $e^-e^+ \rightarrow Z^0/\gamma \rightarrow \mu^-\mu^+\gamma$

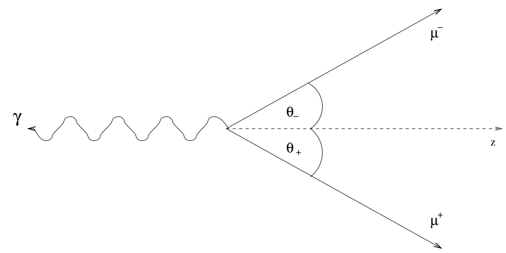

Fig. 2: *The muon pair and the ISR photon*

The muon mass is negligible ($\approx 0.1\,\mathrm{GeV}$) and for photons it equals zero, so the c.m. energy is with good approximation:

$$E_{CMS} = \sqrt{s} \approx |\vec{p_{\mu^-}}| + |\vec{p_{\mu^+}}| + |\vec{p_\gamma}|. \tag{1}$$

For the triangle formed by the three momenta (fig. 2) the following relation exists

$$\frac{|\vec{p_\gamma}|}{\sin(\pi - \vartheta_- - \vartheta_+)} = \frac{|\vec{p_{\mu^-}}| + |\vec{p_{\mu^+}}| + |\vec{p_\gamma}|}{4\cos\frac{\pi-\vartheta_--\vartheta_+}{2}\cos\frac{\vartheta_-}{2}\cos\frac{\vartheta_+}{2}}.$$

Using eq. (1) and addition theorems for sinus and cosine, one obtains

$$|\vec{p_\gamma}| = \sqrt{s}\cdot\frac{\sin(\vartheta_- + \vartheta_+)}{\sin\vartheta_- + \sin\vartheta_+ + \sin(\vartheta_- + \vartheta_+)}. \tag{2}$$

If the photon is emitted into the direction of the beampipe (z-direction), the four-momentum of the photon is

$$p_\gamma = (0, 0, \pm E_\gamma, E_\gamma) \Rightarrow |\vec{p_\gamma}| = E_\gamma,$$

So that eq. (2) becomes

$$\begin{aligned}
\frac{E_\gamma}{\frac{1}{2}\sqrt{s}} &= \frac{2\sin(\vartheta_- + \vartheta_+)}{\sin\vartheta_- + \sin\vartheta_+ + \sin(\vartheta_- + \vartheta_+)} \\
&= \frac{E_\gamma}{E_b} \equiv: \kappa.
\end{aligned}$$

On the other hand, we have $s' = (p_{\mu^-} + p_{\mu^+})^2$ and $s = (p_{\mu^-} + p_{\mu^+} + p_\gamma)^2$ and for the Z^0 decay

$$M_{Z^0}^2 = (p_{\mu^-} + p_{\mu^+})^2 = s'. \tag{3}$$

For events radiative to the Z pole, $e^-e^+ \rightarrow Z^0/\gamma \rightarrow \mu^-\mu^+\gamma$, conservation of four-momenta yields

$$\begin{aligned}
p_{e^+} + p_{e^-} &= p_{\mu^-} + p_{\mu^+} + p_\gamma \quad \text{, so} \\
\sqrt{s'} &= \sqrt{(p_{\mu^-} + p_{\mu^+})^2} \\
&= \sqrt{(p_{e^+} + p_{e^-} - p_\gamma)^2} \\
&= \sqrt{\underbrace{(p_{e^+} + p_{e^-})^2}_{=s} + \underbrace{p_\gamma^2}_{=m_\gamma=0} - 2(p_{e^+} + p_{e^-})p_\gamma} \\
&= \sqrt{s - 2(E_\gamma \underbrace{(E_{e^+} + E_{e^-})}_{=\sqrt{s}} - \vec{p_\gamma}\underbrace{(\vec{p_{e^+}} + \vec{p_{e^+}})}_{=0})} \\
&= \sqrt{s - 2E_\gamma\sqrt{s}}.
\end{aligned}$$

Hence, together with eq. (3) the center-of-mass energy $\sqrt{s}$ can be measured:

$$\begin{aligned} M_{Z^0}^2 &= s - 2E_\gamma\sqrt{s} \\ \Leftrightarrow \frac{M_{Z^0}^2}{s} &= 1 - \frac{2E_\gamma}{\sqrt{s}} = 1 - \kappa_\gamma \end{aligned}$$

$$\Rightarrow \boxed{\sqrt{s} = \frac{M_{Z^0}}{\sqrt{1-\kappa_\gamma}}} \tag{4}$$

$\sqrt{s'}$ is the reduced c.m. energy or the invariant mass of the muon system. Thus, the ratio $\sqrt{s'}/\sqrt{s} =:$ $x = \sqrt{1-\kappa}$ depends only on the angles ϑ_- and ϑ_+ and $\sqrt{s'} = x \cdot \sqrt{s}$ peaks at the Z^0 mass calculated from the four-momenta of the muons (invariant mass). For accelerators with negligible beamstrahlung the setup $\sqrt{s}$ measured by the energy spectrometers is identical with $\sqrt{s}$ obtained by eq. (4). For TESLA with its large bunch charge densities beamstrahlung decreases the c.m. energy. Therefore, from eq. (4) we expect to deduce a lower value for E_b than measured by the spectrometer.

3 Analysis

The invariant mass of the $\mu^-\mu^+$ pair has two distinct peaks, one is at $\sqrt{s'} \simeq \sqrt{s}$ and the other at $\sqrt{s'} \simeq M_{Z^0}$. The second peak is formed by radiation of one (or more) photons from the incoming electron or positron, reducing the effective $\mu^-\mu^+$ mass to the Z^0 pole where the cross section is large.
Two methods are applied to select events which fulfil the conditions of the formula and allow to measure $\sqrt{s}$:

1. no photon is detected, it is assumed that the photon vanishes through the beampipe,
2. a photon is detected and a cut on the radiative photon energy is applied.

3.0 No Photon detected

3.0.1 Event selection method A

Method A selects events with $M_{\mu^-\mu^+} \simeq M_{Z^0}$ and no photon in the detector. The cuts are made as follows:

A1	Photon	no photon detected for $E_\gamma > 0.2\,\mathrm{GeV}$, $\pi - 0.005 > \vartheta_\gamma > 0.005\,\mathrm{rad}$ (detector acceptance)
A2	Z^0 pole	$\lvert M_{\mu^-\mu^+} - m_{Z^0}\rvert < 3.0\,\mathrm{GeV}$
A3	$\mathrm{sign}(z_\mu)$	both muons emitted in same hemisphere

Tab. 1: *Cuts of method A*

Cut A1 accumulates events with photon(s) produced along the z-direction. Cut A2 selects events, that have the muon system on the well known Z^0 mass. To avoid unreasonable $\sqrt{s}$ from eq. (4) (at low beam energies), A3 rejects wrong topology events of opposite-hemisphere muons.It turned out that these cuts are not sufficient two exclude of two-photon background events to a great extend (s. fig. 3), which are included in the $\sqrt{s}$ determination, see fig. (4).

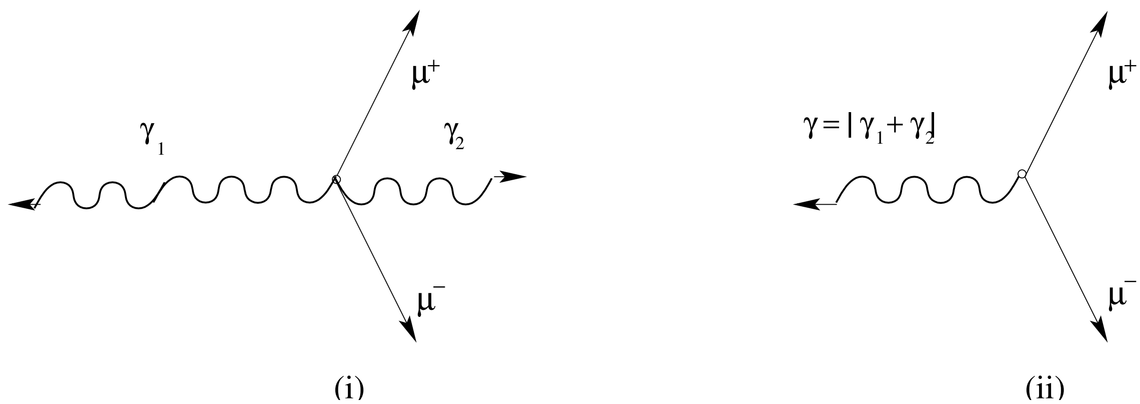

Fig. 3: *Two photon events* (i) *and their reconstruction* (ii) *using eq. (4).*

3.0.2 Event selection method B

To reject dimuon events with background topologies, i.e. those that are not consistent with single ISR down the beampipe, the following cuts are applied (selection method B) [Del98].

B1	Photon	no photon detected for
		$E_\gamma > 0.2\,\mathrm{GeV}$, $\pi - 0.005 > \vartheta_\gamma > 0.005\,\mathrm{rad}$
B2	one ISR photon	$\lvert\sqrt{s'} - M_{\mu^-\mu^+}\rvert < 0.025\sqrt{s_{set.}}$
		$\lvert p_{x\mu^-} - p_{x\mu^+}\rvert < 0.5\,\mathrm{GeV}$
		$\lvert p_{y\mu^-} - p_{y\mu^+}\rvert < 0.5\,\mathrm{GeV}$
		$\lvert\lvert p_{z\mu^-} + p_{z\mu^+}\rvert - p_\gamma\rvert < 0.5\,\mathrm{GeV}$
		$\lvert E_{z\mu^-} + E_{z\mu^+} + \lvert p_\gamma\rvert - \sqrt{s}\rvert < 0.5\ \ \mathrm{GeV}$
B3	range considered	$\lvert\sqrt{s_{set.}} - \sqrt{s_{ang.}}\rvert < 15\,\mathrm{GeV}$

Tab. 2: *Cuts of method B*

B2 eliminates most of the multiple ISR photon events calculated from the muon angles. In addition, these selection criteria are also very efficient in discarding background events from

$$e^+e^- \rightarrow \begin{array}{l} e^+e^-\mu^-\mu^+ \\ W^+W^- \rightarrow \mu^+\nu\mu^-\nu \\ \tau^+\tau^- \rightarrow \mu^+\nu\nu\mu^-\nu\nu \end{array} \tag{5}$$

Cut B3 defines the range for $\sqrt{s}$ calculation close to the Z^0 boson.

3.1 One Photon detected

If a photon is detected, it must have an angle larger than 5 mrad - in fact the angle must be even greater, because the Low Angle Tagger (up to 30 mrad) is almost blind to low energetic photons. In this case $\vartheta_{+/-}$ are the angles between the μ^+ resp. μ^- and the photon and not the angle w.r.t. the beampipe (z-direction). Since the photon is detected, its energy is known. To ensure radiative returns to the Z^0 resonance we require that the photon energy E_γ is in the range $\lvert E_\gamma - 0.53\sqrt{s_{set.}} - 23\rvert < 10\,\mathrm{GeV}$ (cut C3). Thus, $\sqrt{s}$ is given by $\sqrt{s} = E_\gamma + E_{\mu^+} + E_{\mu^-}$. Unfortunately, the resolution on $\sqrt{s}$ was found to be large and a relative large amount of $e^+e^- \rightarrow e^+e^-\mu^-\mu^+$ background events were accepted. The last surviving event sample could however substantially reduced by the cut $\lvert\sqrt{s'}(= \sqrt{s_{set.}} \cdot \sqrt{1-\kappa}) - M_{\mu^-\mu^+}\rvert < 0.025\sqrt{s_{set.}}$ (cut C2).

C1	Photon	only one photon detected for $E_\gamma > 0.2\,\text{GeV}$, $\pi - 0.005 > \vartheta_\gamma > 0.005\,\text{rad}$
C2	one ISR photon	$\lvert\sqrt{s'} - M_{\mu^-\mu^+}\rvert < 0.025\sqrt{s_{set.}}$
C3	radiative return peak	$\lvert E_\gamma - (0.53 \cdot \sqrt{s_{set.}} - 23.01)\rvert < 10\,\text{GeV}$
C4	range considered	$\lvert\sqrt{s_{set.}} - \sqrt{s_{ang.}}\rvert < 15\,\text{GeV}$

Tab. 3: *Cuts of method C*

3.2 Results

The results on $\sqrt{s}$ measurements from 200 to 800 GeV are summarized in Table 4.
Here, non-photon and one-photon events are combined which fulfil selection criteria B. $\sqrt{s}$ was estimated by measuring the muon angles only and is denoted as $\sqrt{s_{ang.}}$. For each energy, 100000 $\mu^-\mu^+\gamma$ events were generated which corresponds to an integrated luminosity as given in Table 4. The value of $\langle\sqrt{s_{ang.}}\rangle$ is simply the average of the individual $\sqrt{s_{ang.}}$ values from the events surviving the selection procedure ($N_{acc.}$ in Table 4). The statistical errors given, $\Delta\langle\sqrt{s_{ang.}}\rangle$, were scaled from fit procedure errors at LEP II energies[Del02].

$\sqrt{s_{set.}}$ [GeV]	σ [fb]	$\int \mathcal{L}dt$ [fb^{-1}]	N_{acc}	$\langle\sqrt{s_{ang.}}\rangle - \sqrt{s_{set.}}$ [GeV]	$\Delta\langle\sqrt{s_{ang.}}\rangle$ [MeV]
200	$7.146 \cdot 10^3$	14	13465	-0.324	31
350	$2.234 \cdot 10^3$	45	10923	-0.356	35
500	$1.117 \cdot 10^3$	80	7344	-0.277	43
800	$4.544 \cdot 10^2$	220	3207	-0.272	64

Tab. 4: *Results*

The error on the beam energy $\Delta E_{beam} = \frac{1}{2}(\langle\sqrt{s_{ang.}}\rangle - \sqrt{s_{set.}})$ (s. fig. 5) is shown as a function of E_b in fig. 5. It is not zero but shifted by

$$\Delta E_{beam} = (-154 \pm 10)\,\text{MeV}$$

in the average (bias).

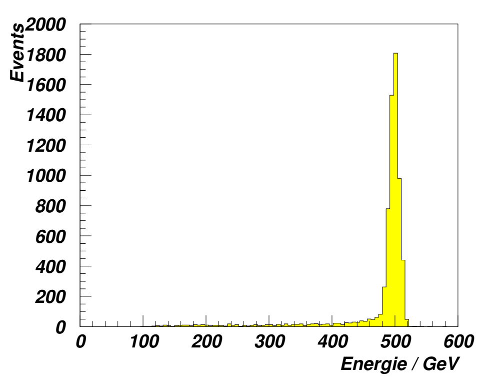

Fig. 4: *Cuts of method A: reconstructed c.m.s. energy $\sqrt{s}$ using eq. (4).*

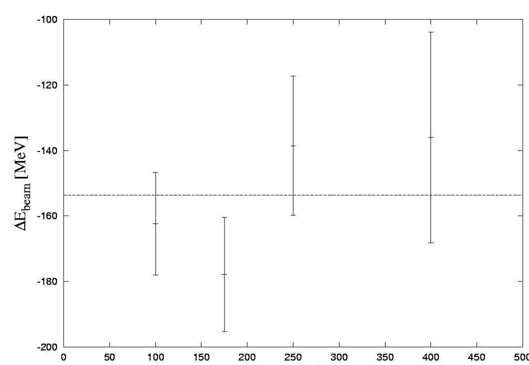

Fig. 5: *The bias of the calculated E_{beam} over the setup beam energy is linear over $\sqrt{s}$*

4 Systematics

Here we discuss qualtitatively some sources of systematic errors expected for E_b. Some values given constitute simple estimates while others are deduced from recent LEP studies. In any case, detailed investigations are mandatory for proper error estimates at a linear collider.

4.1 Multiple ISR photon events

$(13 \pm 2)\%$ of two or more ISR photon events with $E_\gamma > 1\,\mathrm{GeV}$ survive the selection cuts B. This is probably one of the reasons for the observed bias of the $\sqrt{s_{ang.}}$ refering to $\sqrt{s_{set.}}$.

4.2 Background events

Backgrounds from $e^-e^+ \rightarrow e^-e^+ \quad \mu^-\mu^+$, $e^-e^+ \rightarrow W^+W^- \rightarrow \mu^-\nu\mu^+\nu$ and from $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \mu^-\nu\nu\mu^+\nu\nu$ events normalized to the appropriate integrated luminosity were completely rejected by the cuts B. Thus, their effect to the error of the beam energy can be neglected.

4.3 Muon polar angles

Events with muons not emitted in the same hemisphere must be discarded (A3). Nevertheless these are correct events so one would expect to get a bias from cut A3, in particular at low energies.

The error expected from muon polar angle measurements to the beam energy was derived from comparis with the generated muon momenta. It is nearly negligible less than 1 MeV.

4.4 Z^0 mass

The relative error of the mass of the Z^0 of

$$\frac{\Delta M_{Z^0}}{M_Z^0} = \frac{2.2\,\mathrm{MeV}}{91.1882\,\mathrm{GeV}} \approx 0.002\%.$$

limits the precision of E_b to about 4 MeV at $E_b = 400\,\mathrm{GeV}$ and being less at smaller beam energies.

4.5 Detector aspect ratio

The measurement precision of the aspect ratio defined as the ratio of the length to the girth of the detector limits the precision of the polar angle measurement of the muons. For LEP data it was found to be the dominant systematic error of about 22 MeV. It is necessary that for TESLA this error must be substantially reduced.

4.6 QED modelling

It is also possible that there is a systematic error in modelling the ISR (and FSR plus their interferences) on generator level due to approximation order limits. Studies at LEP/,II indicate an error of few MeV to the beam energy.

5 Summary

In order to achieve a precision of $\Delta E_b/E_b = 10^{-4}$ at e.g. $\sqrt{s_{set.}} = 500\,\mathrm{GeV}$, ΔE_b should be 25 MeV in total. The statistical error of 43 MeV (Table 4) based on $80\,\mathrm{fb}^{-1}$ accumulated luminosity requires to take into account very large data samples.

Most of the systematic errors have to be carefully evaluated. From LEP studies it was found that muon polar angle mismeasurements due to imperfect knowledge of the detector aspect ratio effect dominantly the beam energy measurement.

6 Acknowledgements

I thank the people of DESY and all the summer students who made this time quite comfortable to me.
Especially I would like to thank H. Jürgen Schreiber who gave me guidance not only through my studies in particle and accelerator physics but also through the world of DESY and its tools.

References

[TDR01] TECHNICAL DESIGN REPORT,*DESY 2001-011, ECFA 2001-209, Tesla Report 2001-23*

[Del98] The DELPHI Collab., J. Libby and G. Morton, THE MEASUREMENT OF THE 1997 BEAM ENERGY USING $e^+e^- \to \mu^+\mu^-\gamma$, *DELPHI 98-152 PHYS 796*, 31 August (1998)

[Del00] The DELPHI Collab., G.W. Morton, PRELIMINARY DETERMINATION OF $\sqrt{s}$ AT LEP2 USING RADIATIVE DIMUON EVENTS IN DELPHI, *DELPHI 2000-134 CONF 433*, 30 June (2000)

[Del02] The DELPHI Collab.,K. Hamilton, P.J. Holt, G. Morton,G. Myatt, M. Nikolenko, PRELIMINARY DETERMINATION OF $\mathrm{E_{BEAM}}$ AT LEP2 USING RADIATIVE 2-FERMION EVENTS IN DELPHI, *DELPHI 2002-084 CONF 618*, 1 July (2002)

[Ros01] Christian Rosenbleck, RÜCKKEHR ZUM Z DURCH PHOTONENABSTRAHLUNG BEI LEP ENERGIEN UM 200 GEV, Rheinisch-Westfälische Technische Hochschule Aachen, December (2001)

[Rot97] Stefan Roth, MESSUNG DER MUONPAARPRODUKTION UND IHRER STRAHLUNGSKORREKTUREN MIT DEM L3-DETEKTOR BEI LEP, Rheinisch-Westfälische Technische Hochschule Aachen, 17 January (1997)