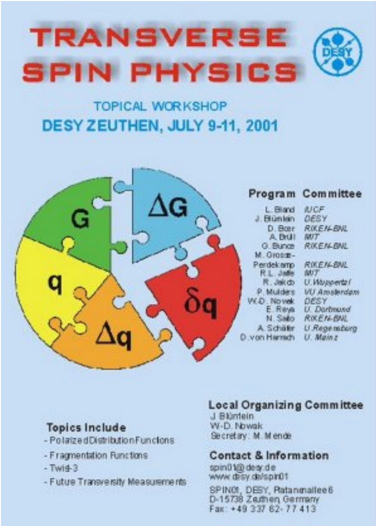


Interference Fragmentation Functions

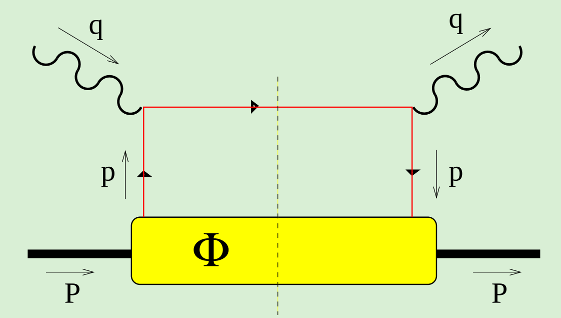
Marco Radici

INFN Pavia

based also on work of

- A. Bianconi - Univ. Brescia
- S. Boffi - Univ. Pavia
- D. Boer - VU Amsterdam
- R. Jakob - Univ. Wuppertal

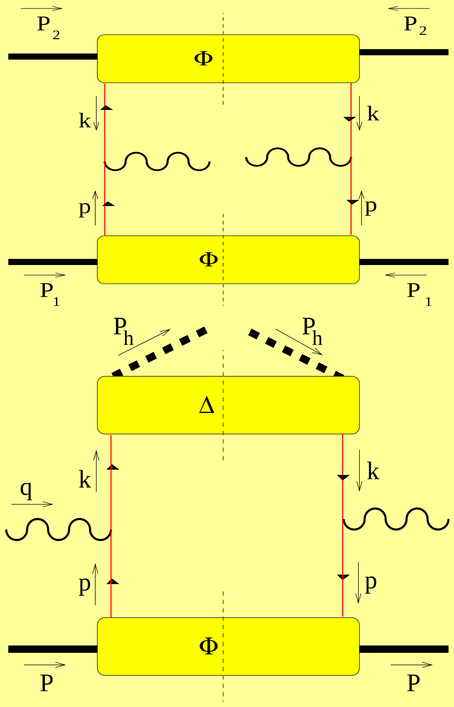
Hunting for transversity



No DIS, unless

$\delta m \sim o(1/Q)$

Double Spin Asymmetries (DSA)



$$A^{DY} = \frac{d\sigma(p^\uparrow p^\uparrow) - d\sigma(p^\uparrow p^\downarrow)}{d\sigma(p^\uparrow p^\uparrow) + d\sigma(p^\downarrow p^\downarrow)} \propto h_1 \overline{h}_1$$

low rates, small $\overline{h}_1$

Soffer ineq. → small A^{DY}

(Schäfer, Stratmann, Vogelsang...)

$ep^\uparrow \longrightarrow e'\Lambda^\uparrow X \qquad A^\Lambda \propto h_1 H_1$

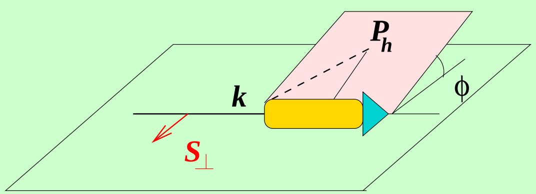
low rate, $q^\uparrow \xrightarrow{\text{pol. transf.}} \Lambda^\uparrow$?

Single Spin Asymmetries (SSA)

$ep^\uparrow \longrightarrow e'\pi^\pm X$

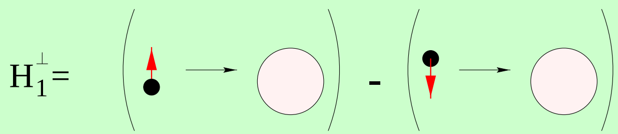
twist-3 $\longrightarrow o(1/Q)$

Collins effect



$\sin\phi \sim \vec{P}_h \times \vec{k} \cdot \vec{S}_\perp$

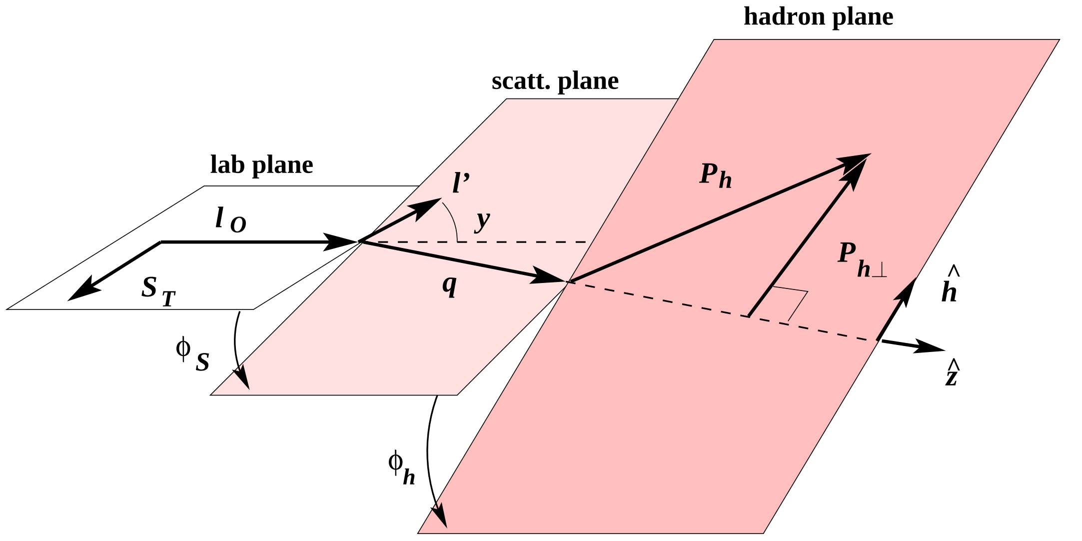
$$A^\perp = \frac{d\sigma(p^\uparrow) - d\sigma(p^\downarrow)}{d\sigma(p^\uparrow) + d\sigma(p^\downarrow)} \propto (f_1) h_1 H_1^\perp$$



$T-$rev. invar. $\Rightarrow \langle \sin\phi \rangle = 0$

FSI $\Rightarrow \langle \sin\phi \rangle \neq 0 \quad H_1^\perp$ is $T-$odd

Collins effect



SMC: $A^{\perp}(\pi^+) = 11\% \pm 6\%$

$A^{\perp}(\pi^-) = -2\% \pm 6\%$

HERMES: $e\vec{p} \to e'\pi X$

$A^{\parallel} \sim \sin\phi \times \left(|\vec{s}_\perp| \sim \frac{1}{Q}\right)$

$\sim$ twist-3

RHIC: $pp^{\uparrow} \to \pi X$

$$\frac{d\sigma_{OT}}{d\Omega_l\, dx\, dz\, d^2\vec{P}_{h\perp}} \sim \;\ldots + \sin(\phi_S+\phi_h)\mathcal{F}\left[\hat{h}\cdot\vec{k}_\perp\;;\; h_1(x,p_\perp^2)\;,\; H_1^\perp(z,k_\perp^2)\right]$$

$$\int d\phi_S\, d\phi_h\, d|\vec{P}_{h\perp}|\;\left[|\vec{P}_{h\perp}|\sin(\phi_S+\phi_h)\; d\sigma_{OT}\right] \longrightarrow\; h_1(x)\; H_1^{\perp\,(1)}(z)$$ (Boer, Mulders)

- Need $\Omega_l(=y,\phi_S), \phi_h, |\vec{P}_{h\perp}|, x, z$
- $d/d\vec{P}_{h\perp} \;\Rightarrow$ no collinear factorization $\Rightarrow$ complicated LO evolution
- $P_{h\perp}^2 \ll Q^2 \;\Rightarrow$ soft gluon radiation $\Rightarrow$ $A^\perp$ suppression by Sudakov ff (Boer)
- $H_1^\perp$ from $e^+e^- \to \pi^+\pi^- X$ but (Sudakov suppression)^2!
- Model $H_1^\perp \;\Rightarrow$ model $\pi -$ jet FSI ;   (see Kundu's talk)

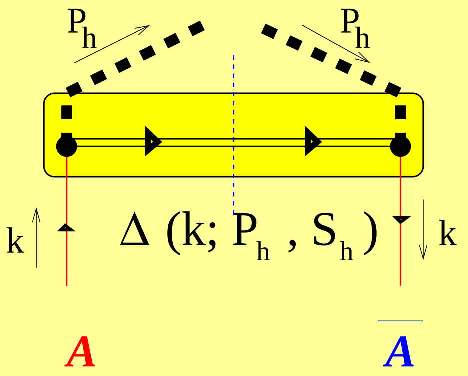
Collins function & FSI

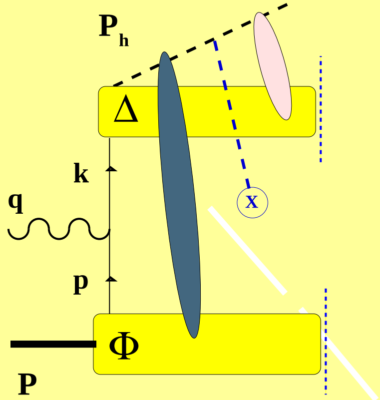
$$\Delta_{ij}(k,P_h,S_h) = \sum\!\!\!\!\!\!\int_X \frac{d^4\zeta}{(2\pi)^4}\, e^{ik\cdot\zeta} \langle 0|\psi_i(\zeta)|P_h,S_h;X\rangle \langle P_h,S_h;X|\overline{\psi}_j(0)|0\rangle$$

$$= (\text{unpol. } h) = A_1 M_h + A_2 \not{P}_h + A_3 \not{k} + \frac{A_4}{M_h}\sigma_{\mu\nu}P_h^\mu k^\nu$$

$$\Delta^{[i\sigma^{i-}\gamma_5]} = \ldots + \frac{\epsilon_\perp^{ij} k_{\perp j}}{M_h} H_1^\perp \qquad \text{with} \qquad H_1^\perp \propto \int dk^+ A_4 \Big|_{k^- = P_h^-/z}$$

Toy model (Phys. Rev. D**62** (2000) 034008)

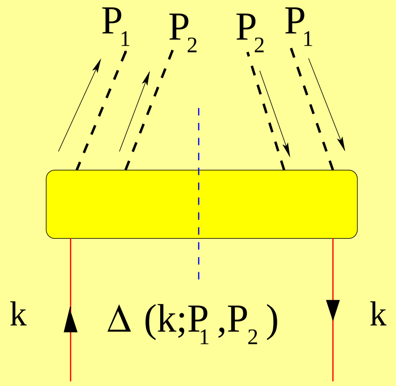




$$\Delta = \frac{-i}{\not{k}-m} u(P_h)\overline{u}(P_h) \frac{-i}{\not{k}-m}\, \delta\left((P_h-k)^2 - M^2\right)$$

- No FSI $\Rightarrow$ $A_4 = 0$
- No factorization breaking
- $A \to A + e^{i\phi} A$ (uniform damping from ext. pot.)
  $\Rightarrow \Delta = 2(1+\cos\phi) A\overline{A} \Rightarrow A_4 = 0$
- $A \to A + e^{i\phi}\not{k} A$ (dressing propagator or $\pi-$ jet FSI)
  $\Rightarrow A_4 = \frac{2M_h}{k^2-m^2}\sin\phi$ T-odd

Interference Fragmentation Functions

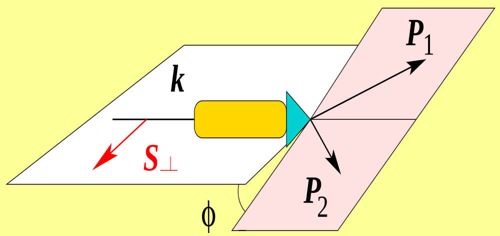


FSI: $(P_1\text{-}P_2)$; $\sum_X (P_1\text{-}X) + (P_2\text{-}X) \sim 0$

$$\sum_X |(P_1\text{-}P_2)_{l=0} + (P_1\text{-}P_2)_{l=1} + ..; X > < X; (P_1\text{-}P_2)_{l=0} + (P_1\text{-}P_2)_{l=1} + ..|$$

$$\sim \; |(P_1\text{-}P_2)_{l=0} >< (P_1\text{-}P_2)_{l=1}| + |(P_1\text{-}P_2)_{l=1} >< (P_1\text{-}P_2)_{l=0}|$$

$\Rightarrow$ SSA $A^{hh} \neq 0$

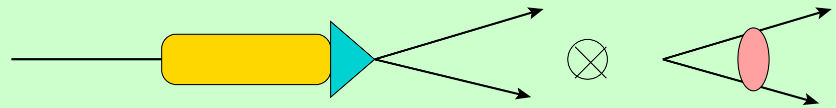


"bowling effect" ; $R = \frac{P_1 - P_2}{2}$

$\sin\phi \sim \vec{P}_1 \times \vec{P}_2 \cdot \vec{S}_\perp$ $\quad \frac{d}{d\vec{R}_\perp}$ only $\Rightarrow$ coll. fact. , no Sudakov ff , easier LO evolution

Collins, Heppelmann, Ladinski ('94): linear σ model ; [continuum] $\leftrightarrow [\sigma \to \pi^+\pi^-]$

Jaffe, Jin, Tang ('98): σ, ρ stable!

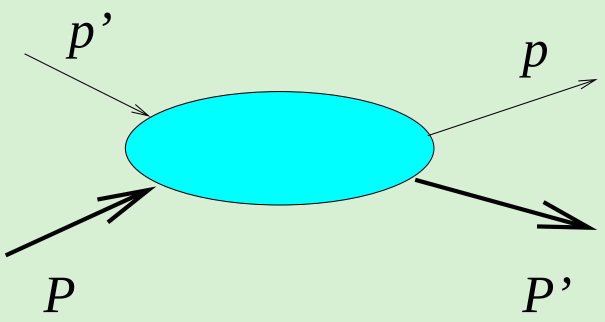


$[\sigma \to \pi^+\pi^-]_{l=0} \leftrightarrow [\rho \to \pi^+\pi^-]_{l=1}$ collinear process

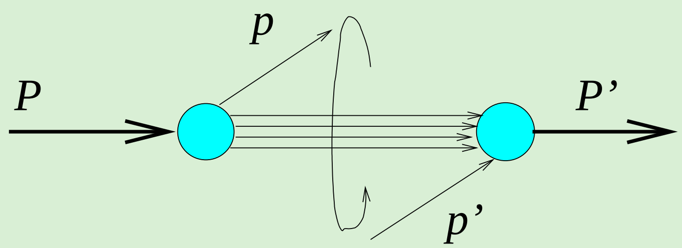
Bianconi, Boffi, Jakob, Radici

- Twist-2 general analysis: P.R. D**62** (2000) 034008
- Model calculation $q \to N\pi X$: P.R. D**62** (2000) 034009
- h_1 extraction and model calculation of $q \to \pi^+\pi^- X$: in preparation

Transversity in helicity basis (Jaffe)



quark-N forward scattering



$u-$channel disc. $\rightarrow$ DF

Constraints

twist-2

$$\Rightarrow |\psi\rangle = \begin{vmatrix} \psi_+ \\ \psi_- \\ o(1/Q) \\ o(1/Q) \end{vmatrix}$$

collinear process $+\, o(1/Q)$

$$\Rightarrow P + p' = P' + p$$

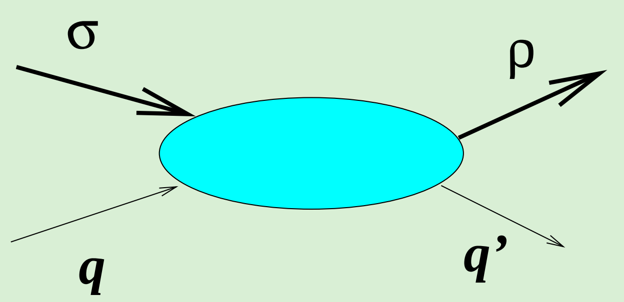
$P-$ and $T-$ invariance

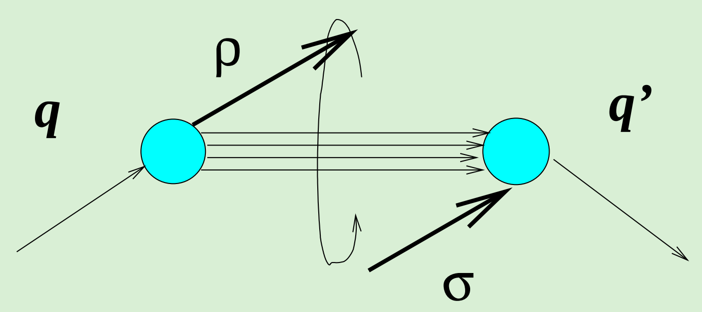
3 independent elements

	P	p'		P'	p				
a	+	+	$\Rightarrow$	+	+		f_1	$\longleftrightarrow$	a+b
b	+	−	$\Rightarrow$	+	−	$\Longrightarrow$	g_1	$\longleftrightarrow$	a−b
c	+	−	$\Rightarrow$	−	+		h_1	$\longleftrightarrow$	c

$h_1 \longleftrightarrow$ helicity flip $\overset{\text{twist-2}}{\Longleftrightarrow}$ chirality flip

Interference fragmentation functions in helicity basis (Jaffe)





Constraints

twist-2

$$\Rightarrow |\psi\rangle = \begin{vmatrix} \psi_+ \\ \psi_- \\ o(1/Q) \\ o(1/Q) \end{vmatrix}$$

collinear process $+\, o(1/Q)$

$$\Rightarrow q + \sigma = \rho + q'$$

hermiticity

$P-$ and $T-$ invariance

2 independent elements

q	σ		ρ	q'		
$\pm$	0	$\Rightarrow$	0	$\pm$	$\Longrightarrow$	$\Delta\hat{q}_I$
$\pm$	0	$\Rightarrow$	± 1	$\mp$	$\Longrightarrow$	$\delta\hat{q}_I$

$\delta\hat{q}_I \longleftrightarrow$ helicity flip $\overset{\text{twist-2}}{\Longleftrightarrow}$ chirality flip

SSA

$$A^{\pi^+\pi^-} = \frac{d\sigma(p^\uparrow) - d\sigma(p^\downarrow)}{d\sigma(p^\uparrow) + d\sigma(p^\downarrow)} \sim |\vec{S}_\perp||\vec{R}_\perp| F(M_h^2)\, h_1(x)\, \delta\hat{q}_I(z)$$

with $F(M_h^2) \propto \sin(\delta_0 - \delta_1)$; $M_h^2 = (P_1 + P_2)^2$

$z - M_h^2$ factorized dependence not general

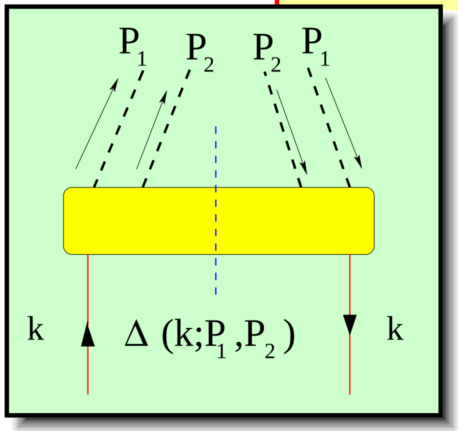
$q \to h_1 h_2 X \quad h_1, h_2$ unpol.

$$2MW^{\mu\nu} = \int dp^- dk^+ d\vec{p}_T d\vec{k}_T \, \delta(\vec{p}_T + \vec{q}_T - \vec{k}_T) \mathrm{Tr}[\Phi \gamma^\mu \Delta \gamma^\nu] \Big|_{\substack{p^+ = xP^+ \\ k^- = P_h^-/z}}$$

Generalization of hadronic matrix elements of nonlocal quark operator

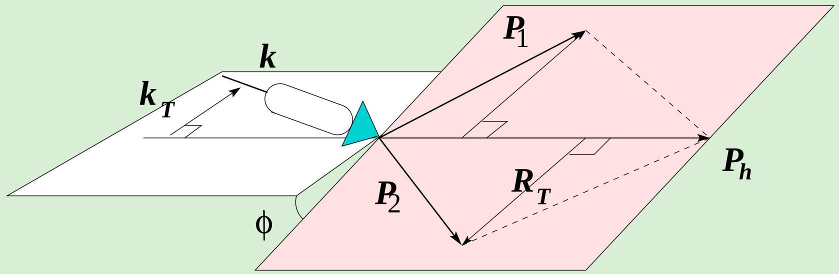
$$\Delta(k; P_1, P_2) = \sum\!\!\!\!\!\!\int_X \frac{d^4\xi}{(2\pi)^4} \frac{d^4 P_X}{(2\pi)^4} \, e^{ik\cdot\xi} \, \langle 0|\psi(\xi)|P_1, P_2, X\rangle\langle X, P_2, P_1|\overline{\psi}(0)|0\rangle$$

$$= C_1(M_1 + M_2) + C_2 \not{P}_1 + C_3 \not{P}_2 + C_4 \not{k} + \frac{C_5}{M_1}\sigma_{\mu\nu}P_1^\mu k^\nu + \frac{C_6}{M_2}\sigma_{\mu\nu}P_2^\mu k^\nu + \frac{C_7}{M_1+M_2}\sigma_{\mu\nu}P_1^\mu P_2^\nu + \frac{C_8}{M_1 M_2}\gamma_5 \epsilon_{\mu\nu\rho\sigma}\gamma^\mu P_1^\nu P_2^\rho k^\sigma$$



Hermiticity	$\Longrightarrow$	$C_i^* = C_i \; ; \; i = 1-4, 5-8$
T-reversal inv.	$\Longrightarrow$	$C_i^* = C_i \quad i = 1-4$; $C_i^* = -C_i \quad i = 5-8$
no FSI	$\Longrightarrow$	$C_5 = C_6 = C_7 = C8 = 0$!

Twist-2 Interference Fragmentation Functions

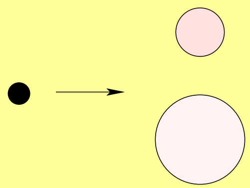


$$\left.\begin{array}{l} \vec{P}_{hT} = 0 \quad P_1^2 = M_1^2 \quad P_2^2 = M_2^2 \\ R_T^2 = \frac{1}{z}\left(\frac{z_1 z_2}{z} M_h^2 - z_2 M_1^2 - z_1 M_2^2\right) \\ \Delta^{[\Gamma]} \propto \int dk^+ dk^- \ldots \end{array}\right\} -7$$

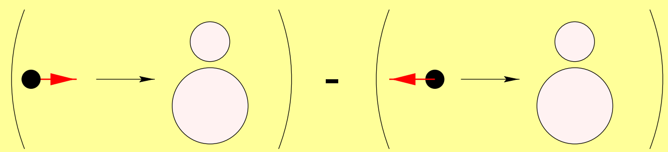
$k, P_1, P_2 \to$ 12 d.o.f.

$\Delta^{[\Gamma]}(z, \xi, k_T^2, R_T^2, \vec{k}_T \cdot \vec{R}_T)$ with $z = P_h^- / k^-$; $\xi = z_1 / z$

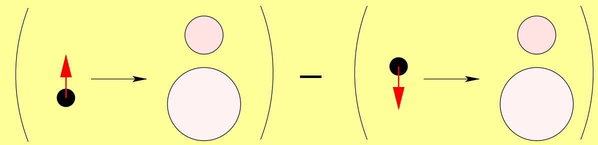
$$\Delta^{[\gamma^-]} = D_1$$

$D_1(x)=$ 

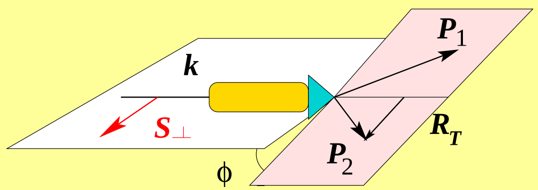
$$\Delta^{[\gamma^- \gamma_5]} = \frac{\epsilon_T^{ij} R_{Ti} k_{Tj}}{M_1 M_2} G_1^\perp$$

$G_1^\perp =$ 

$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} H_1^\perp + \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} H_1^{\sphericalangle}$$

$H_1^\perp, H_1^{\sphericalangle} =$ 

"bowling effect"



(mislead. notat. $\widetilde{H}_1^\perp$)

$eH^{\uparrow} \to e'(h_1 h_2)X$ cross section at twist 2

$$
\frac{d\sigma_{OT}}{d\Omega\, dx\, dz\, d\xi\, d^2\vec{P}_{h\perp}\, dM_h^2\, d\phi_R} = \frac{d\sigma_{OO}}{d\Omega\, dx\, dz\, d\xi\, d^2\vec{P}_{h\perp}\, dM_h^2\, d\phi_R} + |\vec{S}_\perp| \frac{\alpha_{em}^2\, s\, x}{2(2\pi)^3 Q^4} \Bigg\{
$$

$$
A(y)\ \sin(\phi_h - \phi_S)\ \mathcal{F}\left[\hat{\vec{h}}\cdot\vec{p}_T\ \frac{f_{1T}^{\perp}\, D_1}{M}\right] + A(y)\ \Big(\sin \leftrightarrow \cos\Big)\mathcal{F}\left[\hat{\vec{h}} \leftrightarrow \hat{\vec{g}}\right]
$$

$$
+B(y)\ \sin(\phi_h + \phi_S)\ \mathcal{F}\left[\hat{\vec{h}}\cdot\vec{k}_T\ \frac{h_1\, H_1^{\perp}}{M_1 + M_2}\right] + B(y)\ \Big(\sin \leftrightarrow \cos\Big)\mathcal{F}\left[\hat{\vec{h}} \leftrightarrow \hat{\vec{g}}\right]
$$

$$
+\ |\vec{R}_\perp|\ B(y)\ \sin(\phi_R + \phi_S)\ \mathcal{F}\left[\frac{h_1\, H_1^{\sphericalangle}}{M_1 + M_2}\right]
$$

$$
-|\vec{R}_\perp|\ A(y)\ \cos(\phi_h - \phi_S)\ \left(\sin(\phi_h - \phi_R)\ \mathcal{F}\left[\hat{\vec{h}}\cdot\vec{k}_T\ \hat{\vec{h}}\cdot\vec{p}_T\ \frac{g_{1T}\, G_1^{\perp}}{M M_1 M_2}\right] + \cos(\phi_h - \phi_R)\ \mathcal{F}\left[\hat{\vec{h}}\cdot\vec{k}_T \leftrightarrow \hat{\vec{g}}\cdot\vec{k}_T\right]\right)
$$

$$
+|\vec{R}_\perp|\ A(y)\ \sin(\phi_h - \phi_S)\ \left(\sin(\phi_h - \phi_R)\ \mathcal{F}\left[\hat{\vec{h}}\cdot\vec{k}_T\ \ \hat{\vec{g}}\cdot\vec{p}_T\ \frac{g_{1T}\, G_1^{\perp}}{M M_1 M_2}\right] + \cos(\phi_h - \phi_R)\ \mathcal{F}\left[\hat{\vec{h}}\cdot\vec{k}_T \leftrightarrow \hat{\vec{g}}\cdot\vec{k}_T\right]\right)
$$

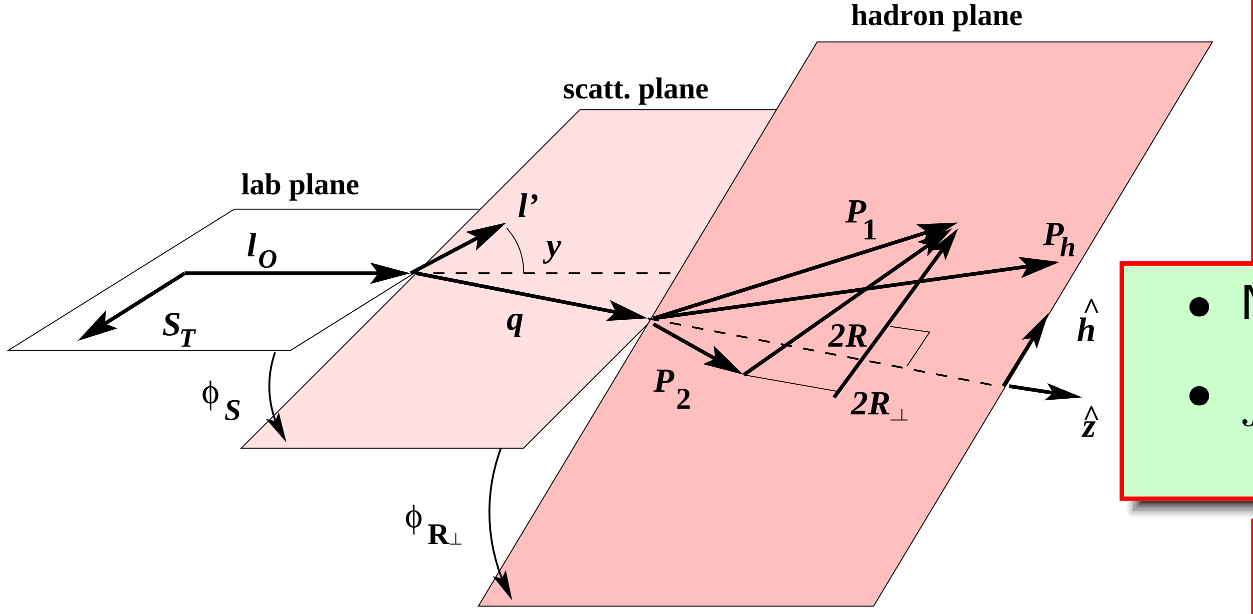
$$
+B(y)\ \left(\cos(3\phi_h - \phi_S)\ \mathcal{F}\left[\hat{\vec{h}}\cdot\vec{k}_T\ \ \hat{\vec{h}}\cdot\vec{p}_T\ \ \hat{\vec{g}}\cdot\vec{p}_T\ \frac{h_{1T}^{\perp}\, H_1^{\perp}}{M^2 (M_1 + M_2)}\right] - \sin(3\phi_h - \phi_S)\ \mathcal{F}\left[\hat{\vec{h}}\cdot\vec{k}_T \leftrightarrow \hat{\vec{g}}\cdot\vec{k}_T\right]\right)
$$

$$
+B(y)\ \sin(2\phi_h)\ \left(\cos(\phi_h - \phi_S)\ \mathcal{F}\left[\hat{\vec{h}}\cdot\vec{k}_T\ (\ \hat{\vec{h}}\cdot\vec{p}_T\ )^2\ \frac{h_{1T}^{\perp}\, H_1^{\perp}}{M^2 (M_1 + M_2)}\right] + \sin(\phi_h - \phi_S)\ \mathcal{F}\left[\hat{\vec{h}} \leftrightarrow \hat{\vec{g}}\right]\right)
$$

$$
+B(y)\ \cos(2\phi_h)\ \left(\cos(\phi_h - \phi_S)\ \mathcal{F}\left[\hat{\vec{g}}\cdot\vec{k}_T\ (\ \hat{\vec{h}}\cdot\vec{p}_T\ )^2\ \frac{h_{1T}^{\perp}\, H_1^{\perp}}{M^2 (M_1 + M_2)}\right] - \sin(\phi_h - \phi_S)\ \mathcal{F}\left[\hat{\vec{h}} \leftrightarrow \hat{\vec{g}}\right]\right)
$$

$$
+|\vec{R}_\perp|\ B(y)\ \sin(2\phi_h + \phi_R - \phi_S)\mathcal{F}\left[\left((\hat{\vec{h}}\cdot\vec{p}_T)^2 - (\hat{\vec{g}}\cdot\vec{p}_T)^2 + 2\ \hat{\vec{h}}\cdot\vec{p}_T\ \ \hat{\vec{g}}\cdot\vec{p}_T\ \right)\frac{h_{1T}^{\perp}\, H_1^{\sphericalangle}}{2M^2 (M_1 + M_2)}\right]\Bigg\}
$$

Extraction of transversity



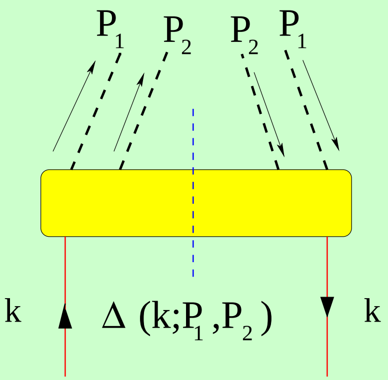
- Need $\Omega_l(=y,\phi_S)$, $\phi_{R_\perp}$, x, z
- $\int d^2\vec{P}_{h\perp} \rightarrow$ no suppression from Sudakov ff

$$\frac{d\sigma_{OT}}{dy\, d\phi_S\, dx\, dz\, d\phi_{R_\perp}\, dM_h^2\, d^2\vec{P}_{h\perp}} = \ldots\ldots + \frac{B(y)|\vec{S}_\perp|}{M_1+M_2}\sin(\phi_S+\phi_{R_\perp})\,\mathcal{F}\Big[h_1(x,p_\perp^2)\,,\ \int d\xi\,|\vec{R}_\perp|\,H_1^{\sphericalangle}(z,\xi,M_h^2,k_\perp^2,\vec{k}_\perp\cdot\vec{R}_\perp)\Big]$$

$$\frac{<d\sigma_{OT}>}{dy\,dx\,dz\,dM_h^2} = \int d\phi_S\, d\phi_{R_\perp}\, d^2\vec{P}_{h\perp}\ \sin(\phi_S+\phi_{R_\perp})\ d\sigma_{OT}$$

$$= \frac{\pi\alpha_{em}^2 sx}{(2\pi)^3 Q^4}\,\frac{B(y)|\vec{S}_\perp|}{2(M_1+M_2)}\,h_1(x)\ H_1^{\sphericalangle}(z,M_h^2)$$

Spectator model

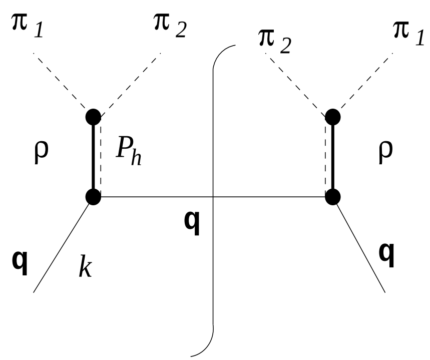


$$\sum_X |X> \sim |q\,;\,(k-P_1-P_2)^2 \equiv k_h^2 = m_q^2>$$

$$\Delta = \delta(k_h^2 - m_q^2)\frac{\theta(k_h^+)}{(2\pi)^3} < 0|\psi(0)|\pi^+,\pi^-,q>< q,\pi^+,\pi^-|\overline{\psi}(0)|0>$$

$$\equiv \delta(k_h^2 - m_q^2) \quad \widetilde{\Delta}$$

$$\Delta^{[\Gamma]} = \frac{\mathrm{Tr}[\widetilde{\Delta}\Gamma]}{8(1-z)(P_1+P_2)^-}\bigg|_{k^2=\frac{z}{1-z}k_T^2+\frac{m_q^2}{1-z}+\frac{M_h^2}{z}}$$



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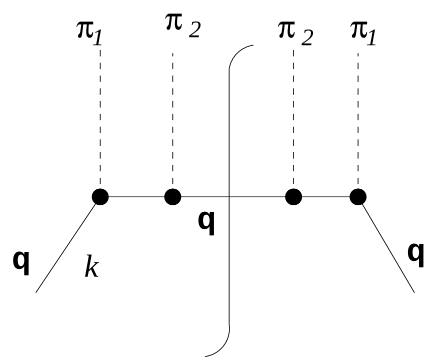

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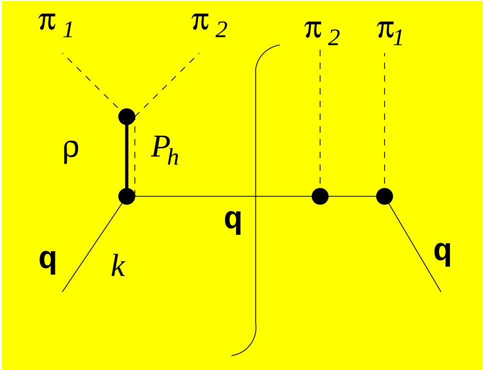
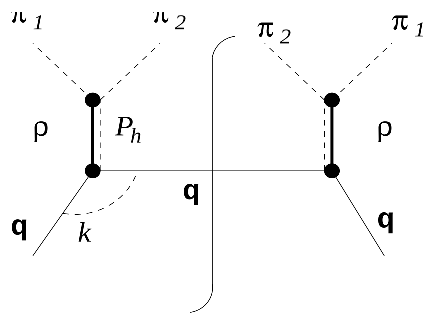


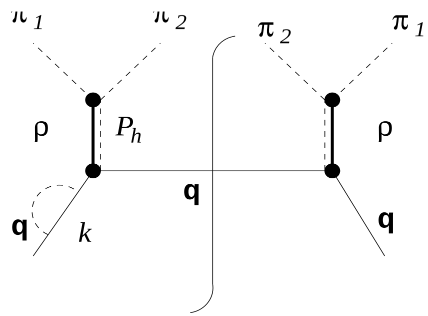
$+$ h.c.

$$\sim \; T_{\pi\pi}^{L=0}(s \leq 1\mathrm{GeV})$$

$+$



$+$



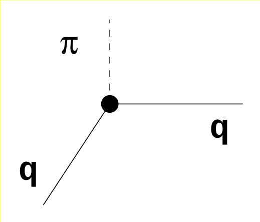
$+$ h.c. $+ \ldots$

"Feynman" rules

asymptotic $\sim (1-z)^{2\alpha-1} = (1-z)^{-3+2q+2|\Delta\lambda|}$ (Joffe, Khoze, Lipatov)

cut off large virtualities $\rightarrow \Lambda$ fragmenting quark $k^2 = \frac{z}{1-z}k_T^2 + \frac{1}{1-z}m_q^2 + \frac{1}{z}M_h^2 \neq m_q^2$

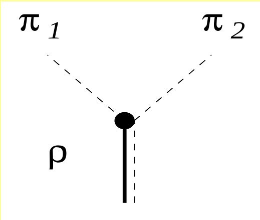
units from $\left[\int d^2\vec{k}_\perp d^2\vec{R}_\perp \, D_1(z,\xi,k_\perp^2,\vec{k}_\perp\cdot\vec{R}_\perp)\right] = \#$



$\Upsilon^{q\pi q} = N_{q\pi}\frac{1}{|\kappa^2-\Lambda_\pi^2|^{\frac{3}{2}}}\gamma_5$; $\kappa = k,\ k-P_\pi$; $N_{q\pi} = 2.564$ GeV^2

$\Lambda_\pi = 0.4$ GeV $\sim \frac{1}{3}g_{\pi NN}$

(Jakob, Mulders, Rodrigues)

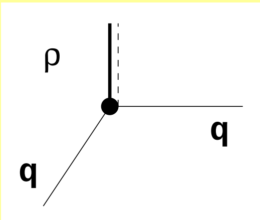


$(\Upsilon^{\rho\pi\pi})^\nu = f_{\rho\pi\pi}\ R_h^\nu$

(Joffe, Khoze, Lipatov)

resonance $(S_\rho)^{\mu\nu} = \frac{1}{P_h^2-m_\rho^2+\mathrm{i}m_\rho\Gamma_\rho}\left(-g^{\mu\nu}+\frac{P_h^\mu P_h^\nu}{P_h^2}\right)$; $\Gamma_\rho = \frac{f_{\rho\pi\pi}^2}{4\pi}\frac{m_\rho}{12}\left(1-\frac{4m_\pi^2}{m_\rho^2}\right)^{\frac{3}{2}}$

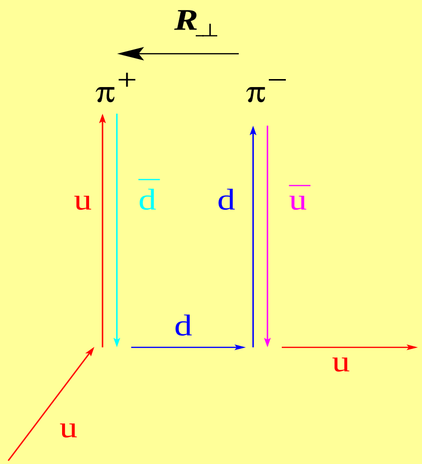
$\frac{f_{\rho\pi\pi}^2}{4\pi} = 2.84 \pm 0.50$



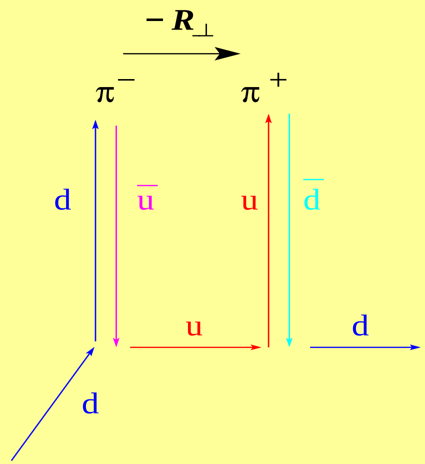
$(\Upsilon^{q\rho q})^\mu = N_{q\rho}\frac{1}{|k^2-\Lambda_\rho^2|^{\frac{3}{2}}}\gamma^\mu$; $\Lambda_\rho = 0.5$ GeV

$N_{q\rho} = \left(\frac{1}{26}\frac{N_{q\pi}^4}{f_{\rho\pi\pi}^2}\frac{|m_q^2-\Lambda_\rho^2|^3}{|m_q^2-\Lambda_\pi^2|^6}\right)^{\frac{1}{2}}$

Approximated symmetries



$H_1^{\sphericalangle}(u \to \pi^+\pi^-)$ $\equiv$ $H_1^{\sphericalangle}(d \to \pi^-\pi^+)$



$$2MW^{\mu\nu} = \int \dots \int dk^+ d^2\vec{k}_\perp \dots \mathrm{Tr}[\Phi\gamma^\mu \Delta\gamma^\nu] \dots \sim w^{\mu\nu}\, f(x)\, \hat{R}_\perp H_1^{\sphericalangle}(z, M_h^2)$$

$$\Rightarrow H_1^{\sphericalangle}(d \to \pi^-\pi^+) = -H_1^{\sphericalangle}(d \to \pi^+\pi^-)$$

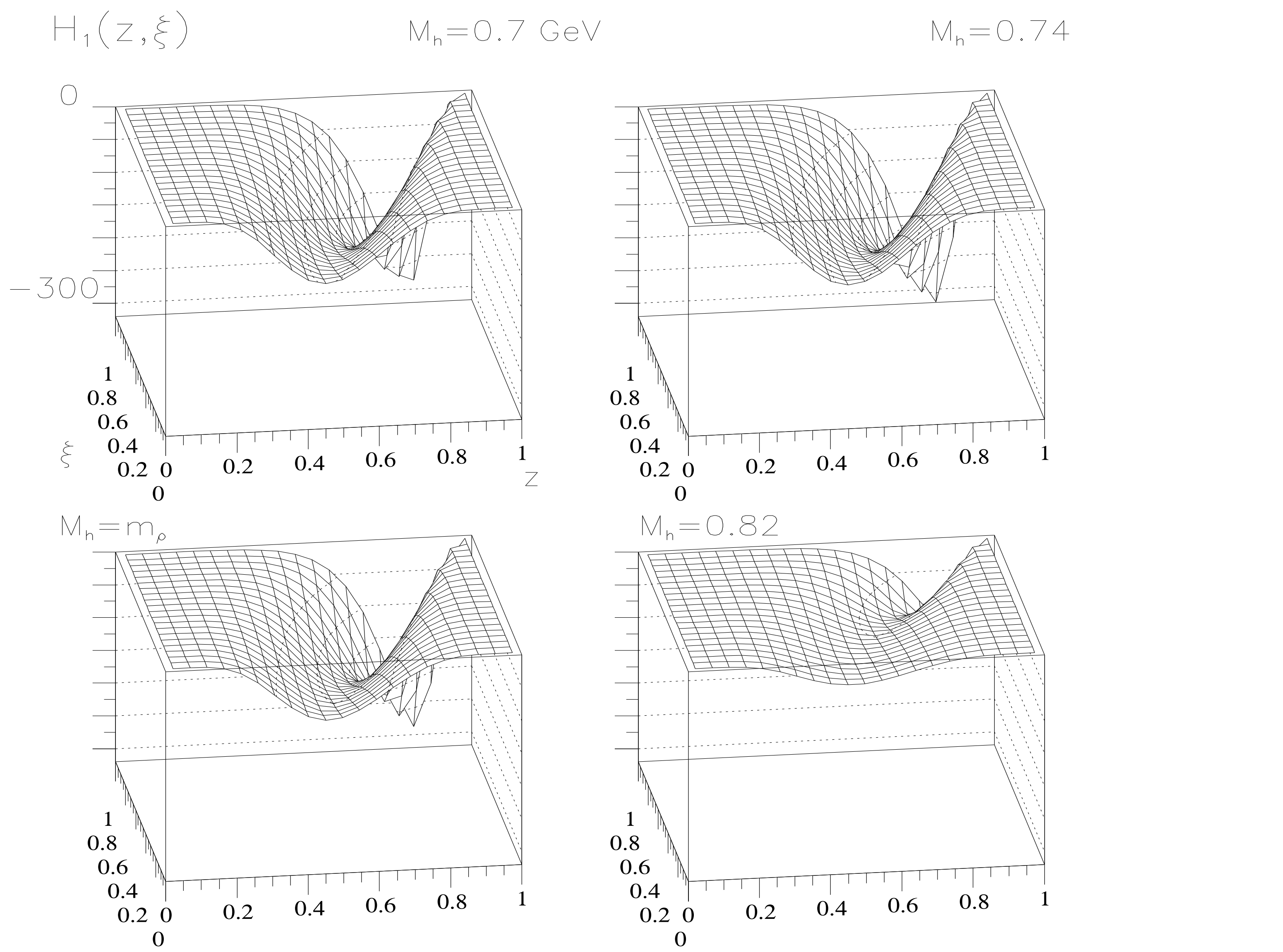
$$|p\rangle = |u,u,d\rangle \Rightarrow A(p \to \pi^+\pi^-) \propto \left(\tfrac{8}{9}h_1^u - \tfrac{1}{9}h_1^d\right) H_1^{\sphericalangle\, u}$$

$$|n\rangle = |d,d,u\rangle \Rightarrow A(n \to \pi^+\pi^-) \propto \left(\tfrac{4}{9}h_1^u - \tfrac{2}{9}h_1^d\right) H_1^{\sphericalangle\, u}$$

test of: { twist-2, spectator model, valence quark } approximations

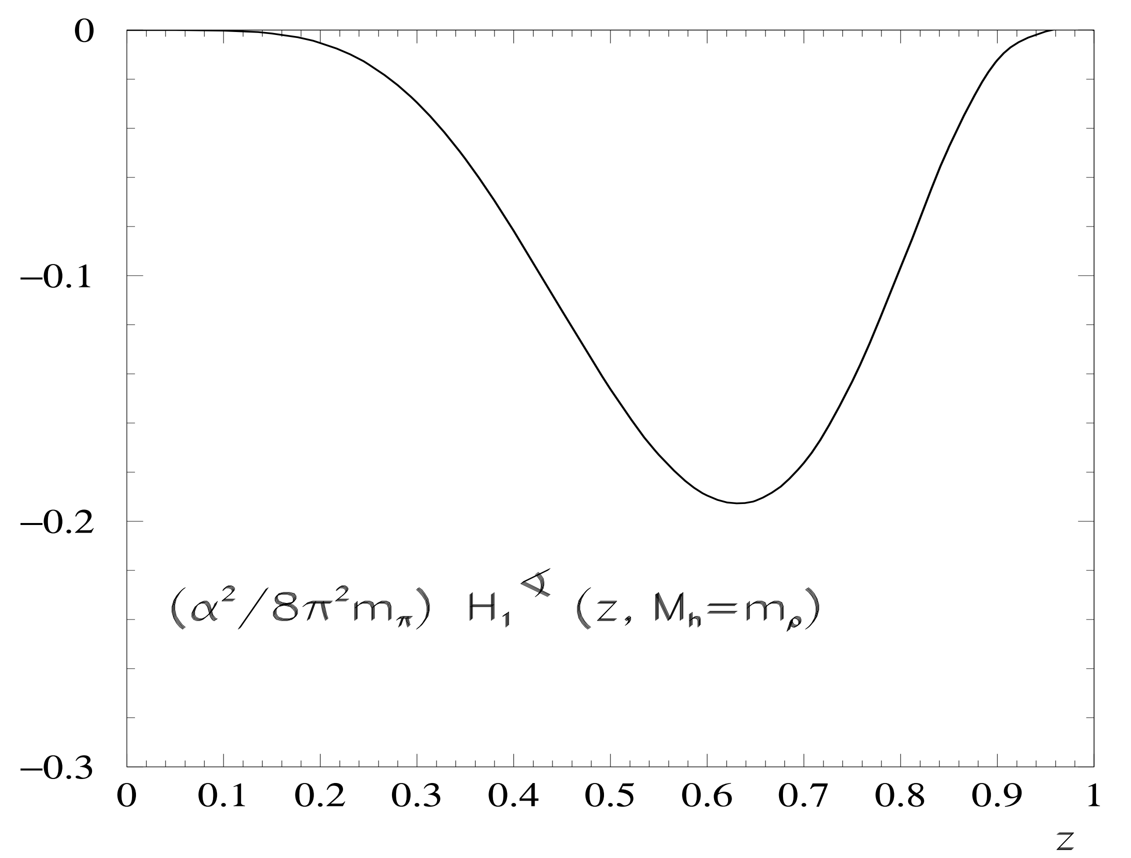
$H_1^\perp \equiv 0 \Rightarrow$ analogous of "Collins" effect higher twist than "bowling" effect

$$\int d^2\vec{k}_\perp \, H_1^{\sphericalangle\,(u\to\pi^+\pi^-)}(z,\xi,M_h^2,\vec{k}_\perp,\phi_{R_\perp}=0)$$



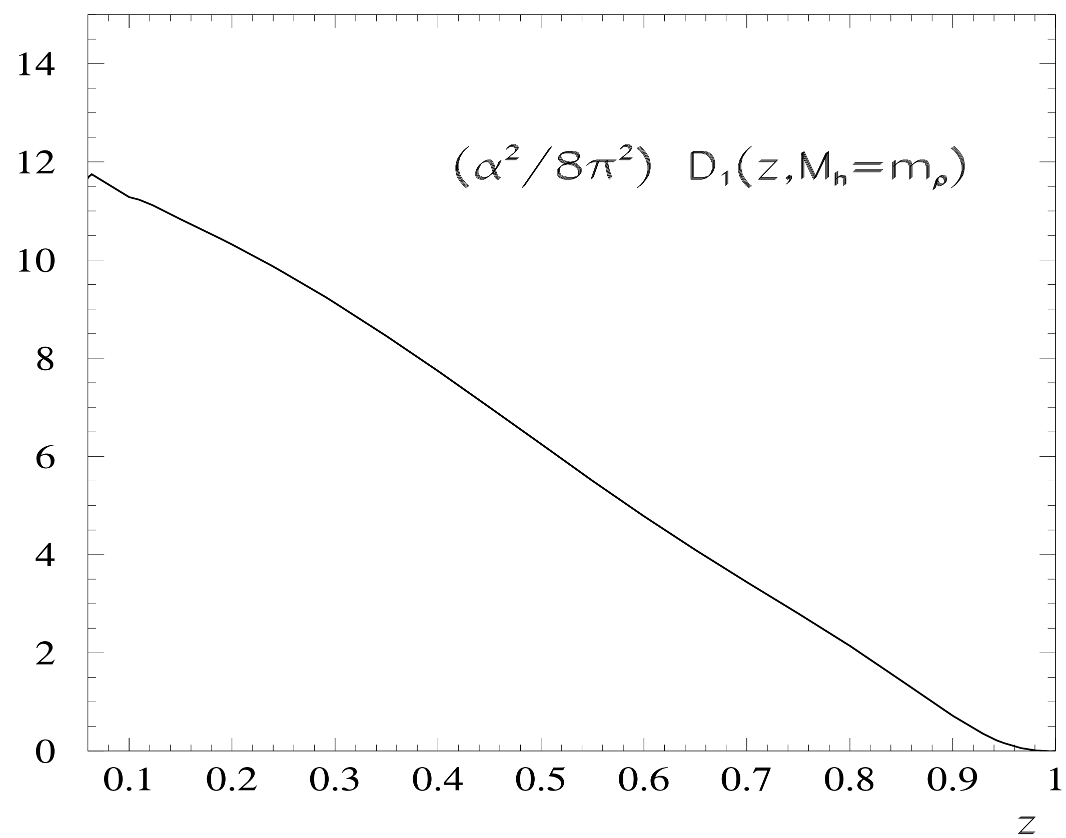
$$H_1^{\sphericalangle\,(u\to\pi^+\pi^-)}(z, M_h = m_\rho) = \int d\phi_{R_\perp}\, d^2\vec{k}_\perp\, d\xi\, |\vec{R}_\perp|\, H_1^{\sphericalangle}\left(z, \xi, M_h = m_\rho, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{R}_\perp\right)$$

$$\frac{\alpha_{em}^2}{8\pi^2 m_\pi} H_1^{\sphericalangle\,(u\to\pi^+\pi^-)}(z)$$



$$D_1^{(u\to\pi^+\pi^-)}(z, M_h = m_\rho) = \int d\phi_{R_\perp}\, d^2\vec{k}_\perp\, d\xi\, D_1\left(z, \xi, M_h = m_\rho, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{R}_\perp\right)$$

$$\frac{\alpha_{em}^2}{8\pi^2} D_1^{(u\to\pi^+\pi^-)}(z)$$



Outlooks

Experiments

- DSA:
  low rates
- 1-hadron SSA:
  require $\vec{P}_{h\perp}$;
  require $\vec{S}_\perp$;
  but are doable
  (HERMES, RHIC)
- 2-hadrons SSA:
  at RHIC $(pp^\uparrow \to \pi\pi X)$
  at EIC?
  at HERMES?
  at COMPASS?

Theory

- DSA:
  small (DY) and uncertain $(\Lambda^\uparrow)$
- 1-hadron SSA:
  not collinear factorization;
  suppression from Sudakov form factors;
  LO evolution complicated;
  asymmetry diluted;
  model hadron – jet FSI
- 2-hadrons SSA:
  Interference Fragmentation Functions
  filter h_1 at twist-2;
  model calculations;
  from $e^+e^- \to (\pi^+\pi^-)(\pi^+\pi^-)X$?
  from spin=1? (see Bacchetta's talk)