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AZIMUTHAL (AND SPIN) ASYMMETRIES IN SEMIINCLUSIVE LEPTOPRODUCTION

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- Inclusive and semiinclusive leptoproduction
- Parton transverse momentum
- Operator structure within QCD
- QCD corrections for azimuthal asymmetries
- Summary and conclusions

in collaboration with
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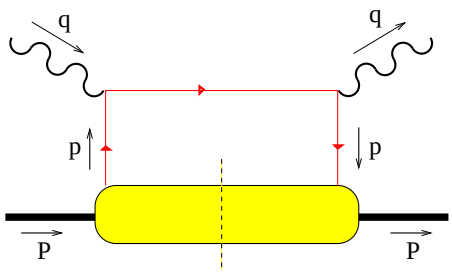
Large scale Q leads in a natural way to the use of lightlike vectors: $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ 2\,P\cdot q = \dfrac{Q^2}{x_{_B}} \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P = \dfrac{x_{_B} M^2}{Q\sqrt{2}}\, n_- + \dfrac{Q}{x_{_B}\sqrt{2}}\, n_+ \\ \\ q = \dfrac{Q}{\sqrt{2}}\, n_- - \dfrac{Q}{\sqrt{2}}\, n_+ \end{array}\right.$$



part	'components'		
	$-$	$+$	
HARD	$\sim Q$	$\sim Q$	
$H \to q$	$\sim 1/Q$	$\sim Q$	$\to \int dp^-\, d^2p_{_T} \ldots$

Note: the lightlike vectors may be rescaled into dimensionful vectors $\hat{p} = (Q/x_{_B}\sqrt{2})n_+$ and $\hat{n} = (x_{_B}\sqrt{2}/Q)n_-$

$$P = \hat{p} + \frac{M^2}{2}\,\hat{n}$$

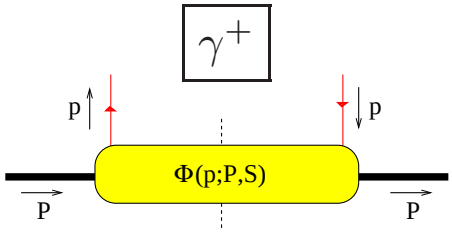
$$q = -x_{_B}\,\hat{p} + \frac{Q^2}{2x_{_B}}\,\hat{n}$$

(LEADING) QUARK DISTRIBUTION FUNCTIONS

Hadron structure enters hard processes via quark-quark correlator

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

Leading part



This part is using symmetry and hermiticity constraints parameterized with distribution functions depending on $x = p^+/P^+$, in DIS measured at $x_B = Q^2/2P\cdot q$ and in DY at $x_A = P_A\cdot q/P_A\cdot P_B$

Parameterization for a spin 1/2 hadron:

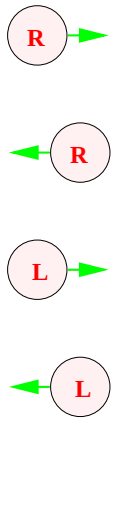
$$\Phi^{\rm twist-2}(x) = \frac{1}{2}\left\{ f_1(x)\,\not{n}_+ + S_L\, g_1(x)\gamma_5\,\not{n}_+ + h_1(x)\;\not{S}_T\,\not{n}_+\gamma_5 \right\}$$

- The functions exist for every quark flavor: $f_1^q(x) = q(x)$, $g_1^q(x) = \Delta q(x)$, $h_1^q(x) = \delta q(x)$
- Leading correlator is of the nonlocal form $\psi_+^\dagger(0)\psi_+(\xi)$ and can be expanded (OPE) into local twist-2 operators of the form $\overline{\psi}\,\gamma^+\,(D^+)^n\psi = \psi_+^\dagger\,(D^+)^n\psi_+$
- Part $\sim (M/P^+)$, with $g_T^q(x)$, is needed at order $1/Q$ in the g_2 structure function ($\psi_+^\dagger\,(D^+)^m D_T^\alpha\,(D^+)^{n-m}\psi_+$ operators)

INTERPRETING THE LEADING QUARK STRUCTURE

For a spin 1/2 hadron (e.g. nucleon) the quark distribution functions enter in a quark production matrix. Omitting the higher twist entries one obtains in quark⊗nucleon spin space

$$M^{(\mathrm{prod})} = \begin{pmatrix} f_1+g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1-g_1 & 0 & 0 \\ 0 & 0 & f_1-g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1+g_1 \end{pmatrix}$$



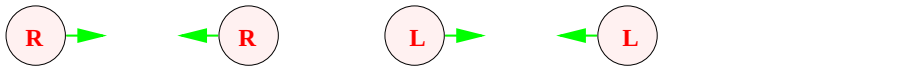
- The three distribution functions are in principle independent!
- Positivity of production matrix gives $f_1(x) \geq 0$ and $|g_1(x)| \leq f_1(x)$ but also the more stringent (Soffer) bound

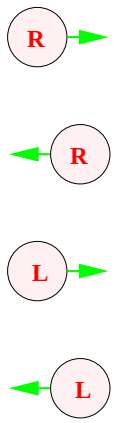
$$|h_1(x)| \leq \frac{1}{2}\left(f_1(x) + g_1(x)\right)$$

- The function h_1 is chirally odd and is not accessible in DIS

Effects of changing basis:

$$M^{(\mathrm{prod})} = \begin{pmatrix} f_1+g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1-g_1 & 0 & 0 \\ 0 & 0 & f_1-g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1+g_1 \end{pmatrix}$$





$$M^{(\mathrm{prod})} = \begin{pmatrix} f_1+h_1 & 0 & 0 & g_1+h_1 \\ 0 & f_1-h_1 & g_1-h_1 & 0 \\ 0 & g_1-h_1 & f_1-h_1 & 0 \\ g_1+h_1 & 0 & 0 & f_1+h_1 \end{pmatrix}$$

Objects in study of hadron structure (given QCD) include:

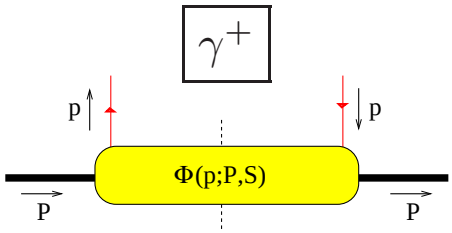
$\langle P+q\|\psi^\dagger(\xi)\Gamma\psi(\xi)\|P\rangle$	Elastic scattering $\Rightarrow$ form factor $G_\Gamma(q)$ $\Rightarrow$ spatial density/excitation spectrum
$\langle P\|\psi^\dagger(0)\Gamma\psi(\xi)\|P\rangle\big\|_{\underbrace{\xi^+=\xi_T=0}_{LC}}$	DIS cross section $\Rightarrow$ DF $f_\Gamma(x)$ $\Rightarrow$ (light-cone) parton momentum distribution
$\langle P\|\psi^\dagger(\xi)\Gamma\psi(\xi)\|P\rangle$	$\Rightarrow$ charge: $Q_\Gamma = G_\Gamma(0) = \int dx\ f_\Gamma(x)$
$\langle P\|\psi^\dagger(0)\Gamma D_T(\eta)\,\psi(\xi)\|P\rangle\big\|_{LC}$	DIS cross section at $1/Q$ $\Rightarrow$ (lightcone) higher twist correlation
$\langle P\|\psi^\dagger(0)\Gamma\psi(\xi^-,\xi_T)\|P\rangle\big\|_{\underbrace{\xi^+=0}_{LF}}$	SIDIS or DY azimuthal asymmetry $\Rightarrow$ DF $f_\Gamma(x,p_T)$ $\Rightarrow$ p_T-dependent distribution
$\langle 0\|\psi(\xi)\|P_h\rangle\,\langle P_h\|\psi^\dagger(0)\Gamma\|0\rangle\big\|_{LF}$	SIDIS or e^+e^- azimuthal asymmetry $\Rightarrow$ FF $D_\Gamma(z,k_T)$ $\Rightarrow$ parton fragmentation (or decay) function
$\langle P+\Delta\|\psi^\dagger(0)\Gamma\psi(\xi)\|P\rangle\big\|_{LC}$	DVCS cross section $\Rightarrow$ skewed parton distribution

(SUBLEADING) QUARK DISTRIBUTION FUNCTIONS

Hadron structure enters hard processes via quark-quark correlator

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

Leading part



This part is using symmetry and hermiticity constraints parameterized with distribution functions depending on $x = p^+/P^+$, in DIS measured at $x_B = Q^2/2P\cdot q$ and in DY at $x_A = P_A\cdot q/P_A\cdot P_B$

Parameterization including subleading order for a spin 1/2 hadron:

$$\begin{aligned}\Phi(x) \;=\; & \frac{1}{2}\left\{ f_1(x)\,\not{n}_+ + S_L\, g_1(x)\gamma_5\,\not{n}_+ + h_1(x)\,\frac{[\not{S}_T,\not{n}_+]\gamma_5}{2}\right\} \\ & + \frac{M}{2P^+}\left\{ e(x) + g_T(x)\,\gamma_5\,\not{S}_T + S_L\, h_L(x)\,\frac{[\not{n}_+,\not{n}_-]\gamma_5}{2}\right\} \\ & + \frac{M}{2P^+}\left\{ f_T(x)\,\epsilon_T^{\rho\sigma}\,S_{T\rho}\gamma_\sigma - i\,S_L\,e_L(x)\,\gamma_5 + h(x)\,\frac{i\,[\not{n}_+,\not{n}_-]}{2}\right\}\end{aligned}$$

T-odd

[T-odd parts will be needed for fragmentation functions]

TWIST-3 OPERATORS

Subleading quark-quark correlators are via the QCD-eom connected to twist-3 operators of the type $\psi_+^\dagger\,(D^+)^m D_T^\alpha\,(D^+)^{n-m}\psi_+$, appearing in specific (nonlocal) quark-quark-gluon correlator

$$\Phi^\alpha_{D\,ij}(x,y) \sim F.T.\,\langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\eta)\,iD_T^\alpha(\eta)\,\mathcal{U}(\eta,\xi)\,\psi_i(\xi)|P,S\rangle\big|_{LC}\,,$$

Parameterization for a spin 1/2 hadron:

$$\Phi^\alpha_D(x,y) = \frac{1}{2}\Big\{G_D(x,y)\,i\epsilon_T^{\alpha\beta}S_{T\beta}\,\not{n}_+ + \tilde{G}_D(x,y)\,S_T^\alpha\,\gamma_5\,\not{n}_+ + H_D(x,y)\,S_L\,\gamma_5\gamma_T^\alpha\,\not{n}_+ + E_D(x,y)\,\gamma_T^\alpha\,\not{n}_+\Big\},$$

Equations of motion $[(i\not{D}-m)\psi=0]$:

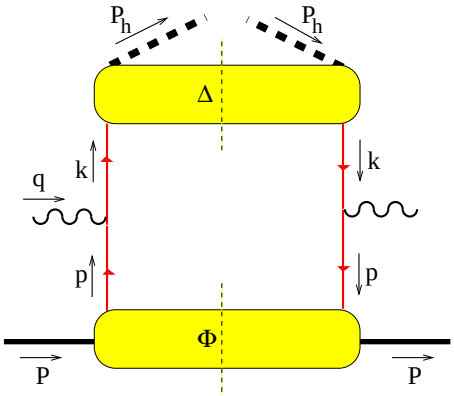
$$\int dy\,\Big[G_D(x,y)+\tilde{G}_D(x,y)\Big] = x\,g_T(x) - \frac{m}{M}\,h_1(x) + ix\,f_T(x)$$

$$2\int dy\;H_D(x,y) = x\,h_L(x) - \frac{m}{M}\,g_1(x) - ix\,e_L(x)$$

$$2\int dy\;E_D(x,y) = x\,e(x) + \frac{m}{M}\,f_1(x) + ix\,h(x)$$

Large scale Q leads in a natural way to the use of lightlike vectors: $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$

$$\left.\begin{aligned} q^2 &= -Q^2 \\ P^2 &= M^2 \\ P_h^2 &= M_h^2 \\ 2\,P\cdot q &= \frac{Q^2}{x_{_B}} \\ 2\,P_h\cdot q &= -z_h\,Q^2 \end{aligned}\right\} \longleftrightarrow \left\{\begin{aligned} P_h &= \frac{z_h\,Q}{\sqrt{2}}\,n_- + \frac{M_h^2}{z_h\,Q\sqrt{2}}\,n_+ \\ q &= \frac{Q}{\sqrt{2}}\,n_- - \frac{Q}{\sqrt{2}}\,n_+ + q_T \\ P &= \frac{x_{_B}\,M^2}{Q\sqrt{2}}\,n_- + \frac{Q}{x_{_B}\sqrt{2}}\,n_+ \end{aligned}\right.$$



	$-$	$+$	
$q \to h$	$\sim Q$	$\sim 1/Q$	$\to \int dk^+ \ldots$
HARD	$\sim Q$	$\sim Q$	
$H \to q$	$\sim 1/Q$	$\sim Q$	$\to \int dp^- \ldots$

Three external momenta (P, P_h, q) $\to$ transverse direction relevant

Non-collinearity: $q_T = q + x_{_B}\,P - \dfrac{P_h}{z_h} = -\dfrac{P_{h\perp}}{z_h}$

SEMI-INCLUSIVE LEPTOPRODUCTION

- inclusive DIS $\longleftrightarrow$ $q(x) = f_1^q(x)$ and $g(x)$ via evolution
- Polarization $\longleftrightarrow$ $\Delta q(x) = g_1^q(x) = q_R(x) - q_L(x)$, $\Delta G(x)$

- SIDIS $\longleftrightarrow$ tagging to get f_1^q and g_1^q for $q = u,\, d,\, s,\, c$
- Polarimetry $\longleftrightarrow$ measure $\delta q(x) = h_1^q(x) = q_\uparrow(x) - q_\downarrow(x)$
- Azimuthal dependence $\longleftrightarrow$ study intrinsic quark $\boldsymbol{k}_T$
- Single spin asymmetries $\longleftrightarrow$ T-odd fragmentation functions

TRANSVERSE MOMENTS

- We need SIDIS, DY or 2PI e^+e^-(two hadrons!) to probe transverse momentum balance
- In SIDIS e.g.

$$\begin{array}{ccccccc} & & & & & & \text{EXP} \\ & & & & & & \downarrow \\ q_T & = & q + x_B\, P - \dfrac{P_h}{z_h} & = & & & \dfrac{P_{h\perp}}{z_h} \\ & & \uparrow \quad \uparrow & & & & \\ & & p_{\rm quark} \quad p'_{\rm quark} & & & & \end{array}$$

- SIDIS (q_T-integrated): Sensitive to p_T-integrated correlations.

$$\int d^2q_T \int d^2p_T\, d^2p'_T\ \delta^2(p_T + q_T - p'_T)\ldots = \left[\int d^2p_T \ldots\right]\left[\int d^2p'_T \ldots\right]$$

- SIDIS (q_T-weighted): Sensitive to p_T-weighted correlations

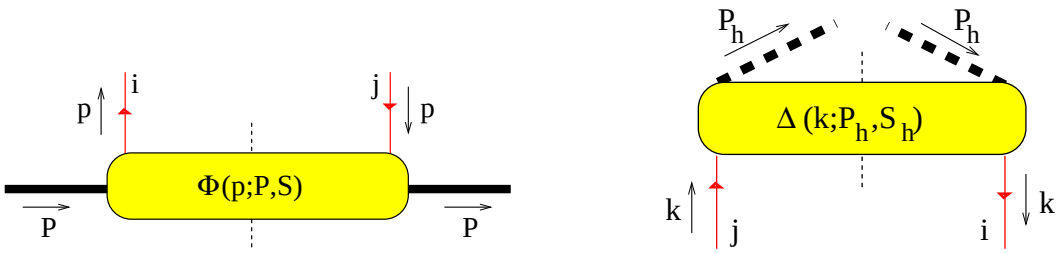
$$\int d^2q_T\ q_T^\alpha \int d^2p_T\, d^2p'_T\ \delta^2(p_T + q_T - p'_T)\ldots = \left[\int d^2p_T \ldots\right]\left[\int d^2p'_T\ p_T'^\alpha \ldots\right] - \left[\int d^2p_T\ p_T^\alpha \ldots\right]\left[\int d^2p'_T \ldots\right]$$

$$\downarrow \qquad\qquad \downarrow$$

transverse moments

- Experimentally: Weighting with $q_T^\ell \Rightarrow \int d^2P_{h\perp}\ |P_{h\perp}|\cos(\phi_h^\ell)\ldots$

SOFT PART IN SIDIS



Hard inclusive (polarized) deep inelastic leptoproduction involves the soft part Φ integrated over all momenta except $p^+ \equiv x\,P^+$

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

Hard semi-inclusive (polarized) deep inelastic lepton-hadron scattering involves soft part Φ integrated over p^- with $p^+ \equiv x\,P^+$ and p_T

$$\Phi_{ij}(x,p_T) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=0}$$

Fragmentation into a hadron involves soft part integrated over k^- leaving $P_h^- = zk^-$ and $P_{h\perp} = -zk_T$

$$\Delta_{ij}(z,k_T) = \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3}\; e^{ik\cdot\xi}\,\langle 0|\psi_i(\xi)|P_h,X\rangle\langle P_h,X|\overline{\psi}_j(0)|0\rangle\Big|_{\xi^-=0}$$

QUARK SPIN STRUCTURE

For azimuthal asymmetries in semi-inclusive leptoproduction or Drell-Yan scattering one needs transverse momentum dependence

Dirac structure for a polarized (spin 1/2) hadron:

$$\Phi_0(x,p_T) = \left\{ f_1(x,p_T^2) + i\, h_1^\perp(x,p_T^2)\, \frac{\not{p}_T}{M} \right\} \not{n}_+$$

$$\Phi_L(x,p_T) = \left\{ S_L\, g_{1L}(x,p_T^2)\, \gamma_5 + S_L\, h_{1L}^\perp(x,p_T^2)\gamma_5\, \frac{\not{p}_T}{M} \right\} \not{n}_+$$

$$\Phi_T(x,p_T) = \left\{ f_{1T}^\perp(x,p_T^2)\, \frac{\epsilon_{T\,\rho\sigma} p_T^\rho S_T^\sigma}{M} + g_{1T}(x,p_T^2)\, \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M}\, \gamma_5 \right.$$
$$\left. + h_{1T}(x,p_T^2)\, \gamma_5\, \not{S}_T + h_{1T}^\perp(x,p_T^2)\, \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M}\, \frac{\gamma_5\, \not{p}_T}{M} \right\} \not{n}_+$$

- For fragmentation functions time reversal symmetry does not give constraints
- The functions $h_1^\perp$ and $f_{1T}^\perp$ are T-odd
- Operator structure $\psi_+^\dagger(0)\psi_+(\xi)$ allows a density interpretation.
- For fragmentation: $f \to D$, $g \to G$, $h \to H$ or hat-notation

Quark p_T-dependent distributions

$$\begin{pmatrix} f_1 + g_{1L} & \frac{|p_T|}{M} e^{i\phi} g_{1T} & \frac{|p_T|}{M} e^{-i\phi} h_{1L}^{\perp} & 2\, h_1 \\ \frac{|p_T|}{M} e^{-i\phi} g_{1T} & f_1 - g_{1L} & \frac{|p_T|^2}{M^2} e^{-2i\phi} h_{1T}^{\perp} & -\frac{|p_T|}{M} e^{-i\phi} h_{1L}^{\perp} \\ \frac{|p_T|}{M} e^{i\phi} h_{1L}^{\perp} & \frac{|p_T|^2}{M^2} e^{2i\phi} h_{1T}^{\perp} & f_1 - g_{1L} & -\frac{|p_T|}{M} e^{i\phi} g_{1T} \\ 2\, h_1 & -\frac{|p_T|}{M} e^{i\phi} h_{1L}^{\perp} & -\frac{|p_T|}{M} e^{-i\phi} g_{1T} & f_1 + g_{1L} \end{pmatrix}$$

- Three new functions with straightforward interpretation: g_{1T} corresponds to probability of finding longitudinally polarized quarks in a transversely polarized target; $h_{1L}^{\perp}$ to that of transversely polarized quarks in a longitudinally polarized target
- Naming convention: quark polarization/chirality: f, g, or h; target polarization in subscript L or T
- Matrix representation in Dirac space allows derivation of bounds

  Bacchetta, Boglione, Henneman and M, PRL 58 (2000) 712

- h-functions are chiral-odd!

BOUNDS ON k_T DEPENDENT FUNCTIONS

$$\begin{pmatrix} f_1 + g_{1L} & \frac{|p_T|}{M} e^{i\phi} g_{1T} & \frac{|p_T|}{M} e^{-i\phi} h_{1L}^{\perp} & 2\, h_1 \\ \frac{|p_T|}{M} e^{-i\phi} g_{1T} & f_1 - g_{1L} & \frac{|p_T|^2}{M^2} e^{-2i\phi} h_{1T}^{\perp} & -\frac{|p_T|}{M} e^{-i\phi} h_{1L}^{\perp} \\ \frac{|p_T|}{M} e^{i\phi} h_{1L}^{\perp} & \frac{|p_T|^2}{M^2} e^{2i\phi} h_{1T}^{\perp} & f_1 - g_{1L} & -\frac{|p_T|}{M} e^{i\phi} g_{1T} \\ 2\, h_1 & -\frac{|p_T|}{M} e^{i\phi} h_{1L}^{\perp} & -\frac{|p_T|}{M} e^{-i\phi} g_{1T} & f_1 + g_{1L} \end{pmatrix}$$

Bounds from 1-dimensional subspaces:

$$f_1 \geq 0$$
$$|g_{1L}| \leq f_1$$

Bounds from 2-dimensional subspaces:

$$|h_1| \leq \frac{1}{2}\,(f_1 + g_{1L}) \leq f_1$$
$$|h_{1T}^{\perp(1)}| \leq \frac{1}{2}\,(f_1 - g_{1L}) \leq f_1$$
$$|g_{1T}^{(1)}| \leq \frac{|\boldsymbol{p}_T|}{2M}\sqrt{(f_1 + g_{1L})\,(f_1 - g_{1L})} \leq \frac{|\boldsymbol{p}_T|}{2M}\, f_1$$
$$|h_{1L}^{\perp(1)}| \leq \frac{|\boldsymbol{p}_T|}{2M}\sqrt{(f_1 + g_{1L})\,(f_1 - g_{1L})} \leq \frac{|\boldsymbol{p}_T|}{2M}\, f_1$$

Note: including T-odd functions: $g_{1T} \to g_{1T} + i\, f_{1T}^{\perp}$ and $h_{1L}^{\perp} \to h_{1L}^{\perp} + i\, h_1^{\perp}$

FULL FRAGMENTATION SPIN STRUCTURE

Quark fragmentation functions accessible via azimuthal asymmetries

$$\begin{pmatrix} D_1 + G_{1L} & \frac{|k_T|}{M}\, e^{i\phi}\, G_{1T} & \frac{|k_T|}{M}\, e^{-i\phi}\, H_{1L}^{\perp} & 2\, H_1 \\ \frac{|k_T|}{M}\, e^{-i\phi}\, G_{1T}^{*} & D_1 - G_{1L} & \frac{|k_T|^2}{M^2}\, e^{-2i\phi}\, H_{1T}^{\perp} & -\frac{|k_T|}{M}\, e^{-i\phi}\, H_{1L}^{\perp} \\ \frac{|k_T|}{M}\, e^{i\phi}\, H_{1L}^{\perp *} & \frac{|k_T|^2}{M^2}\, e^{2i\phi}\, H_{1T}^{\perp} & D_1 - G_{1L} & -\frac{|k_T|}{M}\, e^{i\phi}\, G_{1T} \\ 2\, H_1 & -\frac{|k_T|}{M}\, e^{i\phi}\, H_{1L}^{\perp *} & -\frac{|k_T|}{M}\, e^{-i\phi}\, G_{1T}^{*} & D_1 + G_{1L} \end{pmatrix}$$

- The fragmentation functions depend on the ratio of (light-cone) momenta $z = P_h^- / k^-$ and on the transverse momentum of the quark wrt hadron (k_T)
- T-odd functions appear as imaginary parts of off-diagonal entries for fragmentation functions: $G_{1T} + i\, D_{1T}^{\perp}$ and $H_{1L}^{\perp} + i\, H_1^{\perp}$
- T-odd functions appear in single (or triple) spin asymmetries!
- For ρ-production one needs extension to spin 1

A. Bacchetta and P.M., hep-ph/0104176

FRAGMENTATION FUNCTIONS FOR SPIN 0

Quark fragmentation functions into unpolarized or spin 0 hadrons as entries in the quark decay matrix in Dirac space

$$M^{(\mathrm{dec})} = \begin{pmatrix} D_1 & i\frac{|k_T|\,e^{-i\phi}}{M_h}\,H_1^\perp \\ -i\frac{|k_T|\,e^{+i\phi}}{M_h}\,H_1^\perp & D_1 \end{pmatrix} \begin{matrix} \mathrm{R} \\ \mathrm{L} \end{matrix}$$

$$\begin{matrix} \mathrm{R} & \mathrm{L} \end{matrix}$$

- Note for fragmentation into spin 0 hadrons (e.g pions) there are two (!) fragmentation functions, D_1 and $H_1^\perp$

  J. Collins, NP B396 (1993) 161

- In quark decay time reversal invariance cannot be used $\longrightarrow$ T-odd fragmentation functions, in this case $H_1^\perp$

- The function $H_1^\perp$ is also chiral-odd!
  It can be used to probe chiral-odd distribution functions

INTEGRATED AND WEIGHTED DISTRIBUTIONS

From the p_T-dependent distributions we recover the p_T-integrated results and obtain p_T-weighted distributions

$$\begin{aligned}
\Phi(x) &= \int dp^-\, d^2\boldsymbol{p}_T\; \Phi(p,P,S) \\
&= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\,\mathcal{U}\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0} \\
\Rightarrow\quad \Phi^\alpha_\partial(x) &= \int dp^-\, d^2\boldsymbol{p}_T\; p_T^\alpha\, \Phi(p,P,S) \\
&= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\, \langle P,S|\, i\partial^\alpha \left(\overline{\psi}(0)\,\mathcal{U}\psi(\xi)\right)|P,S\rangle\Big|_{\xi^+=\xi_T=0}
\end{aligned}$$

$$\begin{aligned}
\Phi(x) &= \frac{1}{2}\left\{ f_1\, \not{n}_+ + S_L\, g_1\gamma_5\, \not{n}_+ + h_1\, \frac{[\not{S}_T,\, \not{n}_+]\gamma_5}{2} \right\} \\
\frac{1}{M}\,\Phi^\alpha_\partial(x) &= \frac{1}{2}\left\{ g_{1T}^{(1)}\, S_T^\alpha\, \gamma_5\, \not{n}_+ + S_L\, h_{1L}^{\perp(1)}\, \frac{[\not{n}_+,\gamma^\alpha]\gamma_5}{2} \right. \\
&\qquad \left. + f_{1T}^{\perp(1)}\, \epsilon_T^{\alpha\beta} S_{T\beta}\, \not{n}_+ + h_1^{\perp(1)}\, \frac{i\,[\not{n}_+,\gamma^\alpha]}{2} \right\}
\end{aligned}$$

T-odd

Transverse moments: $g_{1T}^{(1)} = g_{1T}^{(1)}(x) \equiv \int d^2\boldsymbol{p}_T\, \frac{\boldsymbol{p}_T^2}{2M^2}\, g_{1T}(x,\boldsymbol{p}_T)$

WHERE DO THE FUNCTIONS SHOW UP

- twist t of correlation functions
  $\Rightarrow$ behavior $(1/Q)^{t-2}$
- k_T-dependent functions
  $\Rightarrow$ azimuthal dependence (ϕ_ℓ, ϕ_h, ϕ_S)
- cross sections are chirally even

$$\sigma_{000} \Longrightarrow f_1 \otimes D_1$$

$$\langle \frac{Q_T}{M_h} \sin\phi_h^\ell \; \sigma_{L00} \rangle \Longrightarrow \lambda_e \frac{M}{Q} \left(e - \frac{m}{Mx} f_1 \right) \otimes H_1^\perp$$

- # of spin vectors is even in case of a T-even combination

$$\sigma_{LL0} \Longrightarrow \lambda_e S_L \, g_1 \otimes D_1$$

$$\langle \cos(\phi_S^\ell + \phi_{S_h}^\ell) \; \sigma_{0TT} \rangle \Longrightarrow |S_T| \, |S_{hT}| \, h_1 \otimes H_1$$

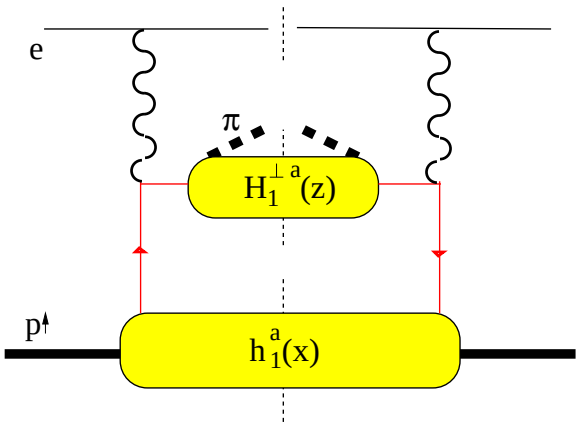
$$\langle \frac{Q_T}{M} \cos(\phi_S^\ell - \phi_h^\ell) \; \sigma_{LT0} \rangle \Longrightarrow \lambda_e \, |\boldsymbol{S}_T| \, g_{1T}^{(1)} \otimes D_1$$

- # of spin vectors is odd in case of a T-odd combination (single spin asymmetries)

$$\langle \frac{Q_T}{M_h} \sin(\phi_h^\ell) \; \sigma_{0L0} \rangle \Longrightarrow S_L \frac{M}{Q} \, h_L \otimes H_1^{\perp(1)}$$

$$\langle \frac{Q_T}{M_h} \sin(\phi_h^\ell + \sin\phi_S^\ell) \; \sigma_{0T0} \rangle \Longrightarrow |S_T| \, h_1 \otimes H_1^{\perp(1)}$$

MEASURING h_1 VIA A SINGLE SPIN ASYMMETRY



J. Collins, NPB 396 (1993) 161

$$\left\langle \frac{Q_T}{M_\pi} \sin(\phi_h^\ell + \phi_S^\ell) \right\rangle_{OTO} = \frac{2\pi\alpha^2\, s}{Q^4} \, |\boldsymbol{S}_T| \, 2(1-y) \sum_{a,\bar{a}} e_a^2 \, x_B \, h_1^a(x_B) H_1^{\perp(1)a}(z_h)$$

How large can $H_1^{\perp(1)}(z)$ become?

$$|H_1^{\perp(1)}(z, -z\boldsymbol{k}_T)| \quad = \quad |\frac{\boldsymbol{k}_T^2}{2M_\pi^2} \, H_1^\perp(z, -z\boldsymbol{k}_T)| \leq \frac{|\boldsymbol{k}_T|}{2M_\pi} \, D_1(z, -z\boldsymbol{k}_T)$$

With assumption

$$D_1(z, -z\boldsymbol{k}_T) \quad = \quad D_1(z) \, \frac{R_\pi^2(z)}{\pi \, z^2} \, e^{-\boldsymbol{k}_T^2 \, R_\pi^2}$$

one finds

$$|H_1^{\perp(1)}(z)| \leq \underbrace{\frac{\sqrt{\pi}}{4M_\pi R_\pi(z)}}_{\mathcal{O}(1)} \, D_1(z)$$

RELATIONS IMPOSED BY LORENTZ INVARIANCE

Recall the various $\langle\overline{\psi}(0)\psi(\xi)\rangle$ correlators

$$\Phi(x) = \frac{1}{2}\left\{ f_1 \not{n}_+ + S_L\, g_1 \gamma_5 \not{n}_+ + h_1 \frac{[\not{S}_T, \not{n}_+]\gamma_5}{2} \right\}$$
$$+ \frac{M}{2P^+}\left\{ e + g_T\, \gamma_5 \not{S}_T + S_L\, h_L \frac{[\not{n}_+, \not{n}_-]\gamma_5}{2} \right\}$$
$$\text{T-odd} \quad + \frac{M}{2P^+}\left\{ f_T\, \epsilon_T^{\rho\sigma} S_{T\rho}\gamma_\sigma - i\, S_L\, e_L\, \gamma_5 + h \frac{i\,[\not{n}_+, \not{n}_-]}{2} \right\}$$

$$\frac{1}{M}\Phi^\alpha_\partial(x) = \frac{1}{2}\left\{ g_{1T}^{(1)}\, S_T^\alpha\, \gamma_5 \not{n}_+ + S_L\, h_{1L}^{\perp(1)} \frac{[\not{n}_+, \gamma^\alpha]\gamma_5}{2} \right.$$
$$\text{T-odd} \quad \left. + f_{1T}^{\perp(1)}\, \epsilon_T^{\alpha\beta} S_{T\beta} \not{n}_+ + h_1^{\perp(1)} \frac{i\,[\not{n}_+, \gamma^\alpha]}{2} \right\}$$

Covariant form for $\langle\overline{\psi}(0)\,\psi(\xi)\rangle \Rightarrow$ Lorentz invariance relations

$$\underbrace{g_T - g_1}_{g_2} = \frac{d}{dx} g_{1T}^{(1)} \qquad \underbrace{h_L - h_1}_{\frac{1}{2} h_2} = -\frac{d}{dx} h_{1L}^{\perp(1)}$$

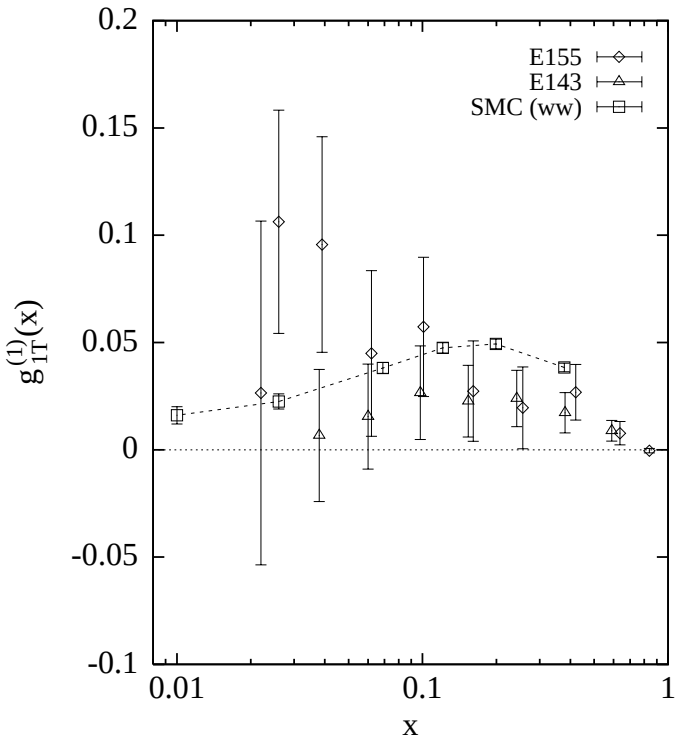
$$\text{T-odd} \quad f_T = -\frac{d}{dx} f_{1T}^{\perp(1)} \qquad h = -\frac{d}{dx} h_1^{\perp(1)}$$

A.P. Bukhvostov, E.A. Kuraev, L.N. Lipatov; Tangerman & M

Estimate of $g_{1T}^{(1)}$

A.M. Kotzinian & PJM
M. Boglione & PJM

- datapoints: SLAC g_2-data: $g_{1T}^{(1)}(x) = -\int_x^1 dy\, g_2(y)$ (including E155, preliminary)
- line: above relation with $g_2(x) = g_2^{WW}(x)$



LIGHTCONE CORRELATORS

$$
\begin{aligned}
\Phi(x) &= \int dp^-\, d^2\boldsymbol{p}_T\; \Phi(p,P,S) \\
&= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\,\mathcal{U}\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0} \\
\Rightarrow\quad \Phi^\alpha_\partial(x) &= \int dp^-\, d^2\boldsymbol{p}_T\; p_T^\alpha\, \Phi(p,P,S) \\
&= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\, \langle P,S|i\partial^\alpha\left(\overline{\psi}(0)\,\mathcal{U}\psi(\xi)\right)|P,S\rangle\Big|_{\xi^+=\xi_T=0} \\
\Phi^\alpha_D(x,y) &= \int dp^-\, d^2\boldsymbol{p}_T\, dp_1^-\, d^2p_{1T}\, \Phi_D(p,p_1,P,S) \\
&= \int \frac{d\xi^-}{2\pi}\,\frac{d\eta^-}{2\pi}\; e^{i\,p_1\cdot(\xi-\eta)+ip\cdot\eta} \\
&\qquad \times \langle P,S|\overline{\psi}(0)\,\mathcal{U}\,iD^\alpha(\eta)\,\mathcal{U}\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=\eta^+=\eta_T=0} \\
\Phi^\alpha_D(x) &= \int dp^-\, d^2\boldsymbol{p}_T\, d^4p_1\, \Phi_D(p,p_1,P,S) \\
&= \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\,\mathcal{U}\,iD^\alpha\,\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}
\end{aligned}
$$

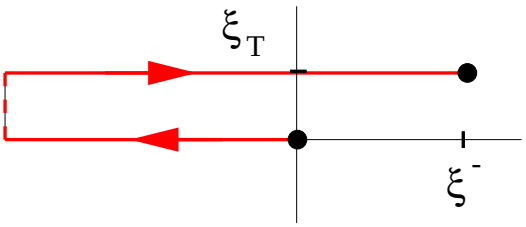
Similarly $\Phi^\alpha_A(x,y)$, $\Phi_F^{+\alpha}(x,y)$, etc.

Ellis, Furmanski, Petronzio
Jaffe, Soldate
Belitsky, Braun, ...

Color gauge invariance and twist analysis

Including longitudinal gluons and transverse momenta we get:

$$\Phi(x,\boldsymbol{p}_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{2\,(2\pi)^3}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\,\mathcal{U}(0,-\infty) \\ \mathcal{U}(-\infty,\xi^-)\psi(\xi)|P,S\rangle\Big|_{\xi^+=0}$$



$$\mathcal{U}(x,y) = \mathcal{P}\, e^{-ig\int_x^y d\eta^-\, A^+(\eta)}\Big|_{x_T=y_T=\eta_T}$$

Integrated results:

$$\Phi(x) = \int \frac{d\xi^-}{4\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\,\mathcal{U}\psi(\xi)|P,S\rangle\Big|_{LC}$$

$$\Phi^\alpha_\partial(x) = \int \frac{d\xi^-}{4\pi}\, e^{ip\cdot\xi}\Big\{ \langle P,S|\overline{\psi}(0)\,\mathcal{U}\, iD^\alpha_T\psi(\xi)|P,S\rangle\Big|_{LC} \\ -\langle P,S|\overline{\psi}(0)\,\mathcal{U}\int_{-\infty}^{\xi^-} d\eta^-\,\mathcal{U}\, g\, F^{+\alpha}(\eta)\, U\,\psi(\xi)|P,S\rangle\Big|_{LC}\Big\} \\ = \Phi^\alpha_D(x) - \Phi^\alpha_A(x)$$

In $A^+=0$ gauge: $F^{+\alpha} = \partial^+ A^\alpha_T \Longrightarrow A^\alpha_T(\xi) = \int_{-\infty}^{\xi^-} d\eta^-\, F^{+\alpha}(\eta)$

EQUATIONS OF MOTION

$(i \not{D} - m)\psi = 0 \qquad \Longrightarrow$

$$\frac{1}{M}\,\Phi_D^\alpha(x) = \frac{1}{2}\left\{\left(x g_T - \frac{m}{M}\,h_1\right) S_T^\alpha\,\gamma_5\,\not{n}_+ + S_L\left(x h_L - \frac{m}{M}\,g_1\right)\frac{[\not{n}_+,\gamma^\alpha]\gamma_5}{2} - x f_T\,\epsilon_T^{\alpha\beta} S_{T\beta}\,\not{n}_+ - x h\,\frac{i\,[\not{n}_+,\gamma^\alpha]}{4}\right\}$$

T-odd

T-odd

Interaction dependent pieces: $\Phi_A \equiv \Phi_D - \Phi_\partial$

$$\frac{1}{M}\,\Phi_A^\alpha(x) = \frac{1}{2}\left\{\underbrace{\left(x g_T - g_{1T}^{(1)} - \frac{m}{M}\,h_1\right)}_{x\tilde{g}_T} S_T^\alpha\,\gamma_5\,\not{n}_+ + S_L\left(x h_L - h_{1L}^{\perp(1)} - \frac{m}{M}\,g_1\right)\frac{[\not{n}_+,\gamma^\alpha]\gamma_5}{2} - \left(x f_T + f_{1T}^{\perp(1)}\right)\epsilon_T^{\alpha\beta} S_{T\beta}\,\not{n}_+ - \left(x h + 2h_1^{\perp(1)}\right)\frac{i\,[\not{n}_+,\gamma^\alpha]}{4}\right\}$$

T-odd

T-odd

$$g_T(x) = \int_x^1 dy\ \frac{g_1(y)}{y} + \left[\tilde{g}_T(x) - \int_x^1 dy\ \frac{\tilde{g}_T(y)}{y}\right]$$
$$\frac{g_{1T}^{(1)}(x)}{x} = \int_x^1 dy\ \frac{g_1(y)}{y} - \int_x^1 dy\ \frac{\tilde{g}_T(y)}{y}$$

$$h_L(x) = 2x\int_x^1 dy\ \frac{h_1(y)}{y^2} + \left[\tilde{h}_L(x) - 2x\int_x^1 dy\ \frac{\tilde{h}_L(y)}{y^2}\right]$$
$$\frac{h_{1L}^{\perp(1)}(x)}{x^2} = -\int_x^1 dy\ \frac{h_1(y)}{y^2} + \int_x^1 dy\ \frac{\tilde{h}_L(y)}{y^2}$$

$$f_T(x) = \left[\tilde{f}_T(x) - \int_x^1 dy\ \frac{\tilde{f}_T(y)}{y}\right]$$
$$\frac{f_{1T}^{\perp(1)}(x)}{x} = \int_x^1 dy\ \frac{\tilde{f}_T(y)}{y}$$

$$h(x) = \left[\tilde{h}(x) - 2x\int_x^1 dy\ \frac{\tilde{h}(y)}{y^2}\right]$$
$$\frac{h_1^{\perp(1)}(x)}{x^2} = \int_x^1 dy\ \frac{\tilde{h}(y)}{y^2}$$

Moments: $[f]_n \equiv \int dx\; x^{n-1}\, f(x)$

$$[g_T]_n = \frac{1}{n}\,[g_1]_n + \frac{n-1}{n}\,[\tilde{g}_T]_n + \frac{m}{M}\,\frac{n-1}{n}\,[h_1]_{n-1}$$
$$[g_{1T}^{(1)}]_n = \frac{1}{n+1}\left([g_1]_{n+1} - [\tilde{g}_T]_{n+1} - \frac{m}{M}\,[h_1]_n\right)$$

$$[h_L]_n = \frac{2}{n+1}\,[h_1]_n + \frac{n-1}{n+1}\,[\tilde{h}_L]_n + \frac{m}{M}\,\frac{n-1}{n+1}\,[g_1]_{n-1}$$
$$[h_{1L}^{\perp(1)}]_n = -\frac{1}{n+2}\left([h_1]_{n+1} - [\tilde{h}_L]_{n+1} - \frac{m}{M}\,[g_1]_n\right)$$

$$[f_T]_n = \frac{n-1}{n}\,[\tilde{f}_T]_n$$
$$[f_{1T}^{\perp(1)}]_n = \frac{1}{n+1}[\tilde{f}_T]_{n+1}$$

$$[h]_n = \frac{n-1}{n+1}\,[\tilde{h}]_n$$
$$[h_1^{\perp(1)}]_n = \frac{1}{n+2}\,[\tilde{h}]_{n+1}$$

$$\frac{G_T(z)}{z} = -\int_z^1 dy\ \frac{G_1(y)}{y^2} + \left[\frac{\tilde{G}_T(z)}{z} + \int_z^1 dy\ \frac{\tilde{G}_T(y)}{y^2}\right]$$

$$G_{1T}^{(1)}(z) = -\int_z^1 dy\ \frac{G_1(y)}{y^2} + \int_z^1 dy\ \frac{\tilde{G}_T(y)}{y^2}$$

$$H_L(z) = -2\int_z^1 dy\ \frac{H_1(y)}{y} + \left[\tilde{H}_L(z) + 2\int_z^1 dy\ \frac{\tilde{H}_L(y)}{y}\right]$$

$$z\, H_{1L}^{\perp(1)}(z) = \int_z^1 dy\ \frac{H_1(y)}{y} - \int_z^1 dy\ \frac{\tilde{H}_L(y)}{y}$$

$$\frac{D_T(z)}{z} = \left[\frac{\tilde{D}_T(z)}{z} + \int_z^1 dy\ \frac{\tilde{D}_T(y)}{y^2}\right]$$

$$D_{1T}^{\perp(1)}(z) = -\int_z^1 dy\ \frac{\tilde{D}_T(y)}{y^2}$$

$$H(z) = \left[\tilde{H}(z) + 2\int_z^1 dy\ \frac{\tilde{H}(y)}{y}\right]$$

$$z\, H_1^{\perp(1)}(z) = -\int_z^1 dy\ \frac{\tilde{H}(y)}{y}$$

Moments: $[D]_n \equiv \int dz\ z^{n-1}\, f(z)$

$$[G_T]_n = -\frac{1}{n+1}[G_1]_n + \frac{n+2}{n+1}[\tilde{G}_T]_n + \frac{m}{M_h}\frac{n+2}{n+1}[H_1]_{n+1}$$
$$[G_{1T}^{(1)}]_{n+1} = -\frac{1}{n+1}\left([G_1]_n - [\tilde{G}_T]_n - \frac{m}{M_h}[H_1]_{n+1}\right)$$

$$[H_L]_n = -\frac{2}{n}[H_1]_n + \frac{n+2}{n}[\tilde{H}_L]_n + \frac{m}{M_h}\frac{n+2}{n}[G_1]_{n+1}$$
$$[H_{1L}^{\perp(1)}]_{n+1} = \frac{1}{n}\left([H_1]_n - [\tilde{H}_L]_n - \frac{m}{M_h}[G_1]_{n+1}\right)$$

$$[D_T]_n = \frac{n+2}{n+1}[\tilde{D}_T]_n$$
$$[D_{1T}^{\perp(1)}]_{n+1} = -\frac{1}{n+1}[\tilde{D}_T]_n$$

$$[H]_n = \frac{n+2}{n}[\tilde{H}]_n$$
$$[H_1^{\perp(1)}]_{n+1} = -\frac{1}{n}[\tilde{H}]_n$$

EVOLUTION OF DISTRIBUTION FUNCTIONS

$$\frac{d}{d\tau} f(x,\tau) = \frac{\alpha_s(\tau)}{2\pi} \int_x^1 \frac{dy}{y} P^{[f]}\left(\frac{x}{y}\right) f(y,\tau),$$

$$\frac{d}{d\tau} [f]_n(\tau) = \frac{\alpha_s(\tau)}{2\pi} A_n^{[f]} [f]_n(\tau)$$

Twist-2 functions

$$P^{[f_1]}(\beta) = C_F \left[\frac{3}{2}\delta(1-\beta) + \frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[g_1]}(\beta) = C_F \left[\frac{3}{2}\delta(1-\beta) + \frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[h_1]}(\beta) = C_F \left[\frac{3}{2}\delta(1-\beta) + \frac{2\beta}{(1-\beta)_+}\right]$$

Interaction-dependent functions (for large N_c)

$$P^{[\tilde{g}_T]}(\beta) = P^{[\tilde{f}_T]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\delta(1-\beta) + \frac{1+\beta}{(1-\beta)_+}\right]$$

$$P^{[\tilde{h}_L]}(\beta) = P^{[\tilde{h}]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\delta(1-\beta) + \frac{3\beta-1}{(1-\beta)_+}\right]$$

$$P^{[\tilde{e}]}(\beta) = P^{[\tilde{e}_L]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\delta(1-\beta) + \frac{3-\beta}{(1-\beta)_+}\right]$$

Braun, Balitsky, Koike, Tanaka, Ji, Osborne, ...

EVOLUTION OF AZIMUTHAL ASYMMETRIES

Large N_c

From the twist-2 and interaction-dependent functions one obtains the evolution of the transverse moments

Azimuthal asymmetries (large N_c, omitting quark mass-terms)

$$\frac{d}{d\tau}\, g_{1T}^{(1)}(x,\tau) = \frac{\alpha_s(\tau)}{4\pi}\, N_c \int_x^1 dy \left\{ \left[\frac{1}{2}\,\delta(y-x) + \frac{x^2+xy}{y^2(y-x)_+} \right] g_{1T}^{(1)}(y,\tau) + \frac{x^2}{y^2}\, g_1(y,\tau) \right\}$$

$$\frac{d}{d\tau}\, h_{1L}^{\perp(1)}(x,\tau) = \frac{\alpha_s(\tau)}{4\pi}\, N_c \int_x^1 dy \left\{ \left[\frac{1}{2}\,\delta(y-x) + \frac{3x^2-xy}{(y-x)_+} \right] h_{1L}^{\perp(1)}(y,\tau) - \frac{x}{y}\, h_1(y,\tau) \right\}$$

Evolution of transverse moments is also driven by the twist-2 polarized quark distribution functions g_1 and h_1.

EVOLUTION OF FRAGMENTATION FUNCTIONS

We note a generalized reciprocity valid for twist-2 *and* interaction-dependent functions:

$$P^{[f]}(\beta) = \frac{\mathcal{N}(\beta)}{(1-\beta)_+}$$

$$P^{[zD]}(\beta) = \frac{\beta^2\,\mathcal{N}(1/\beta)}{(1-\beta)_+}$$

$$A_n^{[f]} = \mathcal{A}(n) - 2\sum_{j=1}^{n}\frac{1}{j}$$

$$A_{n+1}^{[D]} = \mathcal{A}\left(-(n+1)\right) - 2\sum_{j=1}^{n}\frac{1}{j}.$$

$$P^{[f_1]}(\beta) = C_F\left[\frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[g_1]}(\beta) = C_F\left[\frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[h_1]}(\beta) = C_F\left[\frac{2\beta}{(1-\beta)_+}\right]$$

$$P^{[zD_1]}(\beta) = C_F\left[\frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[zG_1]}(\beta) = C_F\left[\frac{1+\beta^2}{(1-\beta)_+}\right]$$

$$P^{[zH_1]}(\beta) = C_F\left[\frac{2\beta}{(1-\beta)_+}\right]$$

$$P^{[\tilde{g}_T]}(\beta) = \frac{N_c}{2}\left[\frac{1+\beta}{(1-\beta)_+}\right]$$

$$P^{[\tilde{h}_L]}(\beta) = \frac{N_c}{2}\left[\frac{3\beta-1}{(1-\beta)_+}\right]$$

$$P^{[\tilde{e}]}(\beta) = \frac{N_c}{2}\left[\frac{3-\beta}{(1-\beta)_+}\right]$$

$$P^{[z\tilde{G}_T]}(\beta) = \frac{N_c}{2}\left[\frac{1+\beta}{(1-\beta)_+}\right]$$

$$P^{[z\tilde{H}_L]}(\beta) = \frac{N_c}{2}\left[\frac{3\beta-1}{(1-\beta)_+}\right]$$

$$P^{[z\tilde{E}]}(\beta) = \frac{N_c}{2}\left[\frac{3-\beta}{(1-\beta)_+}\right]$$

$$\frac{d}{d\tau} D(z,\tau) = \frac{\alpha_s(\tau)}{2\pi} \int_z^1 \frac{dy}{y} P^{[f]}\left(\frac{z}{y}\right) f(y,\tau),$$
$$\frac{d}{d\tau} [D]_n(\tau) = \frac{\alpha_s(\tau)}{2\pi} A_n^{[D]} [D]_n(\tau)$$

Twist-2 functions

$$P^{[zD_1]}(\beta) = C_F \left[\frac{3}{2}\delta(1-\beta) + \frac{1+\beta^2}{(1-\beta)_+}\right]$$
$$P^{[zG_1]}(\beta) = C_F \left[\frac{3}{2}\delta(1-\beta) + \frac{1+\beta^2}{(1-\beta)_+}\right]$$
$$P^{[zH_1]}(\beta) = C_F \left[\frac{3}{2}\delta(1-\beta) + \frac{2\beta}{(1-\beta)_+}\right]$$

Interaction-dependent functions (for large N_c)

$$P^{[z\tilde{G}_T]}(\beta) = P^{[z\tilde{D}_T]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\delta(1-\beta) + \frac{1+\beta}{(1-\beta)_+}\right]$$
$$P^{[z\tilde{H}_L]}(\beta) = P^{[z\tilde{H}]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\delta(1-\beta) + \frac{3\beta-1}{(1-\beta)_+}\right]$$
$$P^{[z\tilde{E}]}(\beta) = P^{[z\tilde{E}_L]}(\beta) = \frac{N_c}{2}\left[\frac{1}{2}\delta(1-\beta) + \frac{3-\beta}{(1-\beta)_+}\right]$$

EVOLUTION OF THE COLLINS FUNCTION

Large N_c

Like for the distribution functions one has evolution equations for the transverse moments with driving from G_1 and H_1:

$$\frac{d}{d\tau}\, z G_{1T}^{(1)}(z,\tau) = \frac{\alpha_s(\tau)}{4\pi}\, N_c \int_z^1 dy \left\{ \left[\frac{1}{2}\,\delta(y-z) + \frac{y+z}{y(y-z)_+} \right] y G_{1T}^{(1)}(y,\tau) - \frac{y^2+z^2}{2y^2 z}\, G_1(y,\tau) \right\}$$

$$\frac{d}{d\tau}\, z H_{1L}^{\perp(1)}(z,\tau) = \frac{\alpha_s(\tau)}{4\pi}\, N_c \int_z^1 dy \left\{ \left[\frac{1}{2}\,\delta(y-z) + \frac{3y-z}{y(y-z)_+} \right] y H_{1L}^{\perp(1)}(y,\tau) + \frac{1+\ln(z/y)}{y}\, H_1(y,\tau) \right\}$$

Evolution equation for Collins function obtained as imaginary part of the equation for $H_{1L}^{\perp}$

$$\frac{d}{d\tau}\, z H_1^{\perp(1)}(z,\tau) = \frac{\alpha_s}{4\pi}\, N_c \int_z^1 dy \left[\frac{1}{2}\,\delta(y-z) + \frac{3y-z}{y(y-z)_+} \right] y H_1^{\perp(1)}(y,\tau)$$

- No driving term in the large N_c evolution
- No mixing with gluons

ANOMALOUS DIMENSIONS

Large Q^2 behavior of the moments $\propto [\alpha_s(Q^2)]^{d_n}$

Twist-2: $d_n = -6\, a_n\, C_F/(11\, N_c - 2\, N_f)$
For large N_c: $d_n = -6\, a_n/11$

function	C	a_0	a_1	a_2	a_3	validity
f_1	$+$		0	-7/6	-25/12	
g_1	$-$		0	-7/6	-25/12	
h_1	$+$	$+3/2$	-1/2	-3/2	-13/6	
$\tilde{g}_T$	$-$		-1/2	-2	-17/6	large N_c
$\tilde{h}_L$	$+$		-5/2	-3	-7/2	large N_c
$\tilde{e}$	$-$		$+3/2$	-1	-13/6	large N_c
$g_{1T}^{(1)}$	$+$	-1/2	-2	-17/6	-41/12	large N_c
$h_{1L}^{\perp(1)}$	$-$	-5/2	-3	-7/2	-47/12	large N_c
zD_1	$+$		0	-7/6	-25/12	
zG_1	$-$		0	-7/6	-25/12	
zH_1	$+$	$+3/2$	-1/2	-3/2	-13/6	
$z\tilde{G}_T$ and $z\tilde{D}_T$	$-$	-1/2	-2	-17/6	-41/12	large N_c
$z\tilde{H}_L$ and $z\tilde{H}$	$+$	$+3/2$	-1	-13/6	-35/12	large N_c
$z\tilde{E}$	$-$	-5/2	-3	-7/2	-47/12	large N_c
$zG_{1T}^{(1)}$ and $zD_{1T}^{\perp(1)}$	$+$		-1/2	-2	-17/6	large N_c
$zH_{1L}^{\perp(1)}$ and $zH_1^{\perp(1)}$	$-$		$+3/2$	-1	-13/6	large N_c

Note: $\int dz\; zH_1^{\perp(1)}(z) = 0$ (Schaefer & Teryaev)

$\int dz\; e(z) = \infty$ (Jaffe & Ji)

SUMMARY

- p_T-dependent distribution and fragmentation functions $\Leftrightarrow$ azimuthal asymmetries
- Complementary to study of twist-3

$$\frac{M}{Q} \text{ contribution } \Rightarrow \text{ measurement of } Q_T \ll Q$$

- Access to $h_1^q(x)$ (transversity) in the nucleon

For results including weak interactions, see
D. Boer, R. Jakob and PM, NP B564 (2000) 471;
hep-ph/9907504

- Operator structure of lowest transverse moments of the p_T-dependent functions involves (known) twist-2 and twist-3 operators
- This also enables us to find their evolution, which in the limit $N_c \to \infty$ can be simply written down

A. Henneman, D. Boer and PM, hep-ph/0104271