Evolution Equation for the Structure Function $g_2(x, Q^2)$

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based on:

V.M. Braun, G.P.K. and A.N. Manashov,
Nucl.Phys.B 603 (2001) 69-124;
Nucl.Phys.B 597 (2001) 370-398.

The scale dependence of $g_2(x,Q^2)$ still poses an outstanding theoretical problem: (Shuryak&Vainshtein'82; Bukhvostov et al'84; ...)

- ★ The main difficulty – $g_2(x,Q^2)$ presents by itself only one special projection of a more general three partons correlation function in the nucleon
- ★ A DGLAP-type evolution equation for $g_2(x,Q^2)$ in QCD does not exist or, at least, is not warranted

$$\int_0^1 dx\, x^{n-1} \underbrace{g_2^{\rm LL}(x,Q^2)}_{\rm genuine\ twist-3} = \sum_{k=0}^{3(n-3)/2} \left(\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right)^{\gamma^k_{n-3}/b} \underbrace{\langle\!\langle O^k_{n-3}(\mu)\rangle\!\rangle}$$

matrix elements of GGG and $\bar{q}Gq$ composite local operators

The theoretical challenge:

$$\boxed{\text{can the complicated pattern of quark-gluon correlations be reduced to a few effective degrees of freedom?}}$$

– If no, then one cannot relate results of the measurements of $g_2(x,Q^2)$ at different Q^2 without model assumptions.

+ If yes, then one can find a meaningful approximation to the scale dependence that introduces a "minimum" amount of nonperturbative parameters.

Example: Evolution equation for the flavor-nonsinglet contribution to g_2 in the multi-color limit, $N_c \to \infty$ (Ali, Braun, Hiller'91)

$$Q^2 \frac{d}{dQ^2} g_2^{\rm NS}(x,Q^2) = \frac{\alpha_s}{4\pi} \int_x^1 \frac{dz}{z} P^{\rm NS}(x/z)\, g_2^{\rm NS}(z,Q^2)\,,$$

$$P^{\rm NS}(z) = \left[\frac{4C_F}{1-z}\right]_+ + \delta(1-z) \left[C_F + \frac{1}{N_c}\left(2 - \frac{\pi^2}{3}\right)\right] - 2C_F\,,$$

? Is it possible to extend this analysis to the flavor-singlet sector after taking into account the twist-3 three-gluon correlations

Our main results:

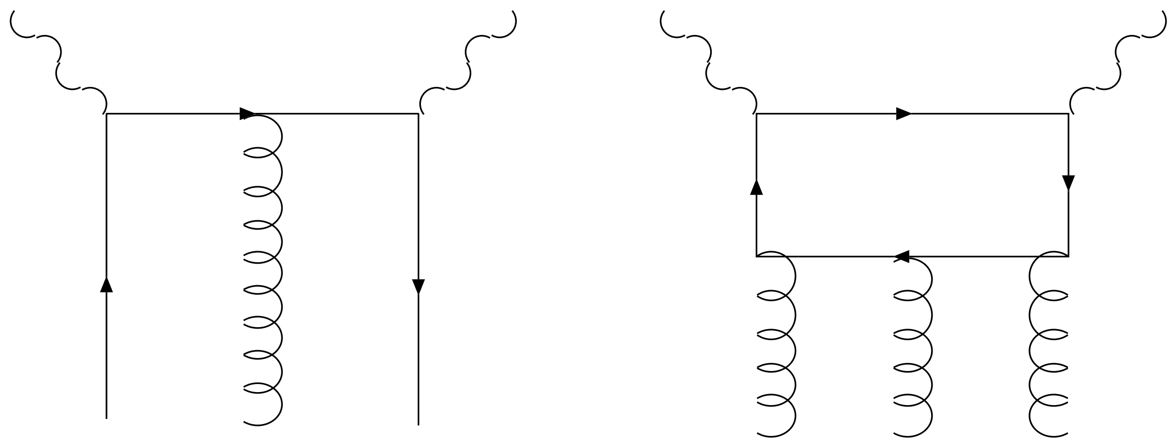
- ♡ The (infinite) set of GGG and $\bar{q}Gq$ operators can be reduced to only two degrees of freedom =transverse quark and gluon densities
- ♡ $g_2(x,Q^2)$ is dominated by contributions of these two densities
- ♡ In this approximation, genuine twist-3 part of $g_2(x,Q^2)$ satisfies a two-channel DGLAP-type evolution equation

? How do the transverse quark and gluon densities come about

★ The OPE analysis of the structure function $g_2(x, Q^2)$:

$$W_{\mu\nu}^{(A)} = \frac{1}{p\cdot q}\varepsilon_{\mu\nu\alpha\beta}q^\beta\left\{s^\beta g_1(x_B,Q^2)+\left[s^\beta-\frac{s\cdot q}{p\cdot q}p^\beta\right]g_2(x_B,Q^2)\right\}$$
$$= \frac{1}{2\pi}\,\mathrm{Im}\int d^4x\, e^{iqx}\langle p,s|T\, j_{[\mu}(x/2)j_{\nu]}(-x/2)|p,s\rangle\,,$$

- Leading-order quark and gluon contributions to the OPE of the T-product of electromagnetic currents



- Is expressed in terms of the C-even nonlocal quark-antiquark-gluon operator

$$S_\beta(u,v,-u) = S_\beta^+(u,v,-u)+S_\beta^-(-u,v,u)\,,$$
$$S_\beta^\pm(a,b,c) = \frac{1}{2}\bar q(ax)[ig\widetilde G_{\beta x}(bx)\pm gG_{\beta x}(bx)\gamma_5]\,\not{x}q(cx)$$

and three-gluon operators

$$\widetilde O_\beta(u,v,w)=\frac{ig}{2}f^{abc}G^a_{x\alpha}(ux)\widetilde G^b_{x\beta}(vx)G^c_{x\alpha}(wx)\,.$$

★The odd moments of the structure function at high scale Q^2:

$$[g_2(n,Q^2)]^{\text{tw}-3} = \sum_{q=u,d,s,\ldots} \frac{e_q^2}{2} \left[\frac{1}{n} \Delta q_T(n,Q^2) + \frac{\alpha_s}{\pi} \underbrace{C_g^T(n)}_{\text{coeff.func.}} \Delta g_T(n,Q^2) \right]$$

- The twist-3 distributions $\Delta q_T(x)$ and $\Delta g_T(x)$ measure the specific projections of $\bar{q}Gq$ and GGG states

$$\int_{-u}^{u} dv \, \langle p,s| \Big[(u+v)\, S_\mu^+(u,v,-u) + (u-v) S_\mu^-(u,v,-u) \Big] |p,s\rangle$$

$$= -i \left[s_\mu - p_\mu \frac{(sx)}{(px)} \right] \int_{-1}^{1} d\xi \; \mathrm{e}^{2i\xi upx} \;\; \Delta q_T(\xi,\mu^2)$$

and

$$\int_{-u}^{u} dv \, \langle p,s| \Big[(u+v) \widetilde{O}_\mu(v,u,-u) + 2u \widetilde{O}_\mu(u,v,-u) + (u-v) \widetilde{O}_\beta(u,-u,v) \Big] |p,s\rangle$$

$$= 2\,[s_\mu(px) - p_\mu(sx)] \int_{-1}^{1} d\xi \; \mathrm{e}^{2i\xi upx} \;\; \xi \, \Delta g_T(\xi,\mu^2)\,,$$

- To the LL accuracy, g_2 is expressed through the quark distribution

$$g_2^{\text{LL}}(x,Q^2) \quad = \quad \underbrace{g_2^{WW}(x,Q^2)}_{\substack{\text{Wandzura}-\text{Wilczek} \\ \text{contribution}}} + \frac{1}{2} \sum_q e_q^2 \int_x^1 \frac{dy}{y} \Delta q_T(y,Q^2)$$

Gluon contrib. arises entirely through the evol. eqs

- To the NL accuracy (Ji&Osborne'01, BKM'01)

$$g_2^{\rm NL}(x,Q^2) = \underbrace{\cdots}_{\substack{\mathcal{O}(\alpha_{\rm S})\ \text{quark} \\ \text{corrections}}} + \frac{1}{2}\sum_q e_q^2\, \frac{\alpha_s}{\pi}\int_x^1 \frac{dy}{y}\, C_g^T(x/y)\, \Delta g_T(y,Q^2)\,,$$

$$= \text{Finite part of the box diagram in the background gluon field}$$

Twist-3 gluon coefficient function

$$C_g^T(x) = -(1-x)\left[1+\ln\frac{x}{1-x}\right] - (1-x)^3\left[\frac{2}{3}\ln^2(1-x) - \frac{19}{9}\ln(1-x) + \frac{83}{54} - \frac{\pi^2}{9}\right] + \mathcal{O}((1-x)^4)$$

- Twist-3 transverse parton distributions:

$$\left.\begin{array}{l}\Delta q_T(x,Q^2)\\ \Delta g_T(x,Q^2)\end{array}\right\} = \text{transverse spin degrees of freedom that are preserved by QCD interaction}$$

▷ Do not have probabilistic interpretation

▷ Have the same support properties as twist-2 distributions

▷ Satisfy the constraints(=Burkhardt-Cottingham sum rule)

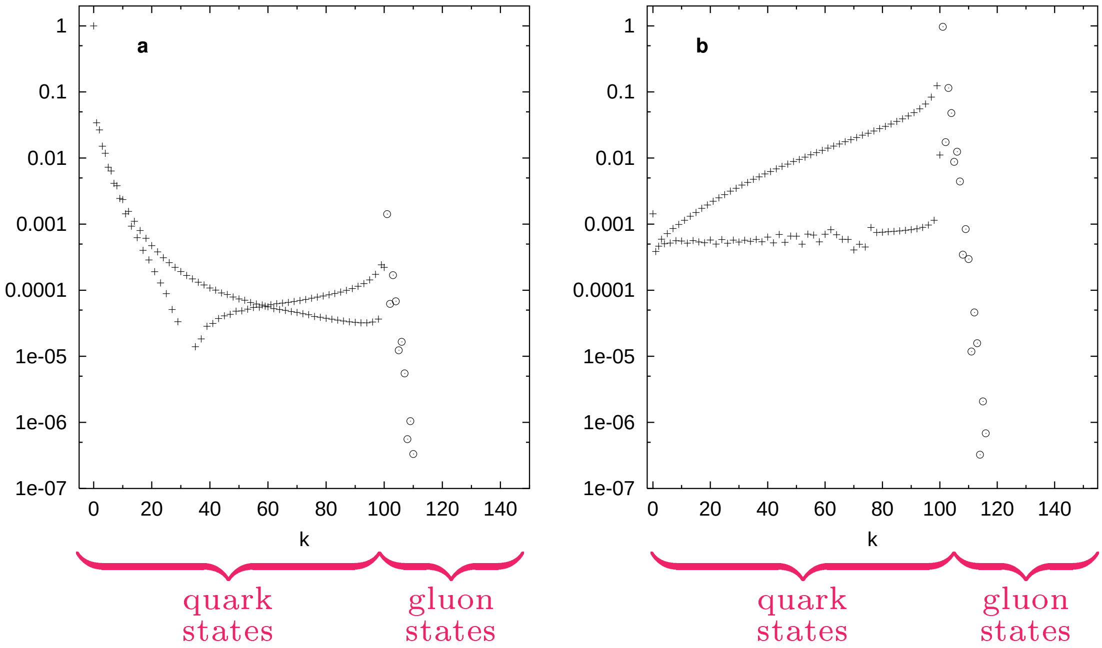
$$\int_0^1 dx\, \Delta q(x,Q^2) = \int_0^1 dx\, \Delta g_T(x,Q^2) = 0 \Longrightarrow \int_0^1 dx\, g_2(x,Q^2) = 0$$

★ Expansion of the transverse densities over multiplicatively renormalizable operators

$$\Delta q(N+3,Q^2) = \sum_{k=0}^{3N/2} w^q_{N,k} \langle\!\langle \mathcal{O}_{N,k}(\mu^2)\rangle\!\rangle\, L^{\gamma_{N,k}/b}\,,$$

$$\Delta g_T(N+3,Q^2) = \sum_{k=0}^{3N/2} w^g_{N-1,k} \langle\!\langle \mathcal{O}_{N,k}(\mu^2)\rangle\!\rangle\, L^{\gamma_{N,k}/b}\,,$$

The expansion coefficients $w^q_{N,\alpha}$ and $w^g_{N-1,\alpha}$ for $N=100$:



The index k enumerates the operators ordered according to their anomalous dimension

- $\Delta q(N+3,Q^2)$ is dominated by only two states – the lowest quark ($k=0$) and the lowest gluon ($k=N+1$) levels;
- $\Delta g(N+3,Q^2)$ is dominated by the lowest gluon ($k=N+1$) level.

- Approximate two-channel evol.eq. for the flavor-singlet quark and gluon transverse spin distributions:

$$Q^2\frac{d}{dQ^2}\Delta q_T(x;Q^2) = \frac{\alpha_s}{4\pi}\int_x^1 \frac{dy}{y}\left[P^T_{qq}(x/y)\Delta q_T(y;Q^2) + P^T_{qg}(x/y)\Delta g_T(y;Q^2)\right],$$

$$Q^2\frac{d}{dQ^2}\Delta g_T(x;Q^2) = \frac{\alpha_s}{4\pi}\int_x^1 \frac{dy}{y}P^T_{gg}(x/y)\Delta g_T(y;Q^2)\,,$$

- The twist-3 splitting functions:

$$\begin{aligned}
P^T_{qq}(x) &= \left[\frac{4C_F}{1-x}\right]_+ + \delta(1-x)\left[C_F + \frac{1}{N_c}\left(2-\frac{\pi^2}{3}\right)\right] - 2C_F\,,\\
P^T_{gg}(x) &= \left[\frac{4N_c}{1-x}\right]_+ + \delta(1-x)\left[N_c\left(\frac{\pi^2}{3}-\frac{1}{3}\right) - \frac{2}{3}n_f\right]\\
&\quad + N_c\left(\frac{\pi^2}{3}-2\right) + N_c\ln\frac{1-x}{x}\left(\frac{2\pi^2}{3}-6\right),\\
P^T_{qg}(x) &= -4n_f\left[x-2(1-x)^2\ln(1-x)\right].
\end{aligned}$$

▶ Very similar to well-known expressions for the twist-2 DGLAP kernels

▶ The quark splitting function $P^T_{qq}(x)$ is the same in the flavor singlet and flavor-nonsinglet channels

▶ The approximation has a high accuracy for all integer moments

$$
\begin{aligned}
\frac{3}{2}\langle e_q^2\rangle^{-1} g_2^{\mathrm{LL}}(3,Q^2) &= \underbrace{L}_{\alpha_s(Q^2)/\alpha_s(\mu^2)}{}^{10.56/b}\langle\!\langle S_0^0\rangle\!\rangle\,, \\
\frac{5}{2}\langle e_q^2\rangle^{-1} g_2^{\mathrm{LL}}(5,Q^2) &= \\
& L^{10.99/b}\Big[1\,\langle\!\langle S_2^2\rangle\!\rangle - 2\,\langle\!\langle S_2^1\rangle\!\rangle + 3\,\langle\!\langle S_2^0\rangle\!\rangle + 0.97\,\langle\!\langle G_2^0\rangle\!\rangle\Big] \\
& +L^{17.35/b}\Big[-0.974\,\langle\!\langle G_2^0\rangle\!\rangle\Big]
\end{aligned}
$$

▶ These results have to be compared with the exact numerical expressions (Bukhvostov et al '84)

$$
\begin{aligned}
&\frac{3}{2}\langle e_q^2\rangle^{-1} g_2^{\mathrm{LL}}(3,Q^2) = L^{10.56/b}\langle\!\langle S_0^0\rangle\!\rangle\,, \\
&\frac{5}{2}\langle e_q^2\rangle^{-1} g_2^{\mathrm{LL}}(5,Q^2) = \\
&\qquad L^{10.74/b}\Big[0.42\langle\!\langle S_2^2\rangle\!\rangle - 2.56\langle\!\langle S_2^1\rangle\!\rangle + 2.78\langle\!\langle S_2^0\rangle\!\rangle + 0.97\langle\!\langle G_2^0\rangle\!\rangle\Big] \\
&\quad + \quad L^{13.52/b}\Big[0.34\langle\!\langle S_2^2\rangle\!\rangle - 0.15\langle\!\langle S_2^1\rangle\!\rangle - 0.26\langle\!\langle S_2^0\rangle\!\rangle - 0.11\langle\!\langle G_2^0\rangle\!\rangle\Big] \\
&\quad + \quad L^{17.68/b}\Big[0.13\langle\!\langle S_2^2\rangle\!\rangle + 0.49\langle\!\langle S_2^1\rangle\!\rangle + 0.40\langle\!\langle S_2^0\rangle\!\rangle - 0.91\langle\!\langle G_2^0\rangle\!\rangle\Big] \\
&\quad + \quad L^{18.17/b}\Big[0.11\langle\!\langle S_2^2\rangle\!\rangle + 0.21\langle\!\langle S_2^1\rangle\!\rangle + 0.08\langle\!\langle S_2^0\rangle\!\rangle + 0.06\langle\!\langle G_2^0\rangle\!\rangle\Big]
\end{aligned}
$$

! The leading gluon contribution and the corresponding anomalous dimensions are correctly reproduced

Conclusions & Further questions

- The structure function $g_2(x, Q^2)$ is dominated by contribution of the twist-3 quark and gluon transverse distributions, $\Delta q_T(x, Q^2)$ and $\Delta g_T(x, Q^2)$, at large scales

- $\Delta q_T(x, Q^2)$ and $\Delta g_T(x, Q^2)$ are the particular components of quark-antiquark-gluon and three-gluon parton correlation functions that possess the lowest anomalous dimension

- The "leakage" of transverse spin to genuine three-particle degrees of freedom at lower scales is small due to the specific pattern of the QCD evolution

- In this approximation, $g_2(x, Q^2)$ satisfies the two-channel DGLAP evolution equations

? Phenomenological models for $\Delta q_T(x, Q^2)$ and $\Delta g_T(x, Q^2)$

? Small–$x_{\rm Bj}$ behavior of $\Delta q_T(x, Q^2)$ and $\Delta g_T(x, Q^2)$