# Some Aspects of Fragmentation in $e^+e^-$

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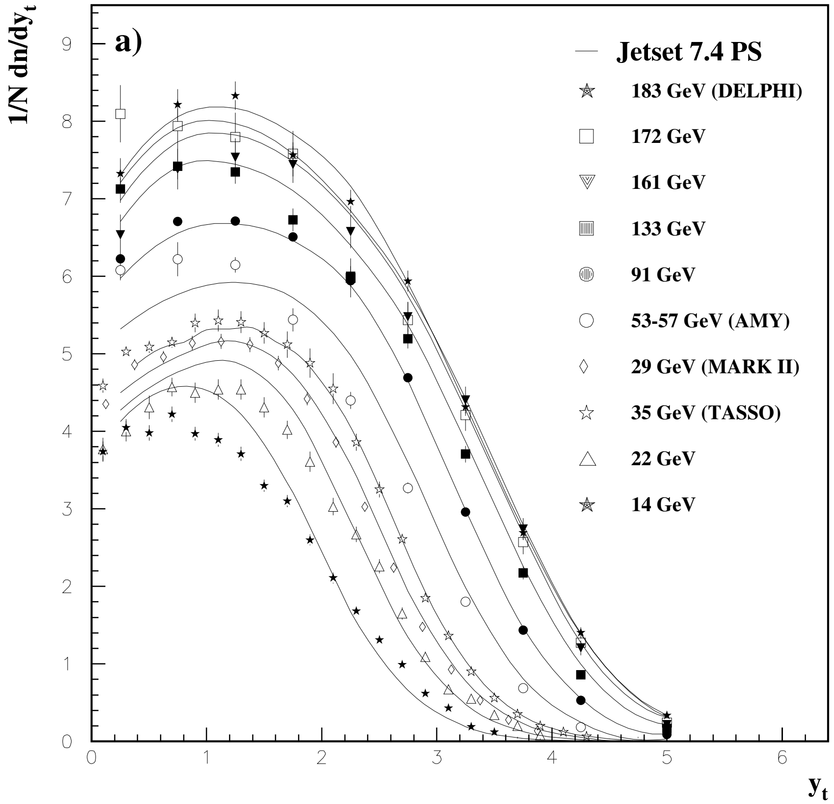
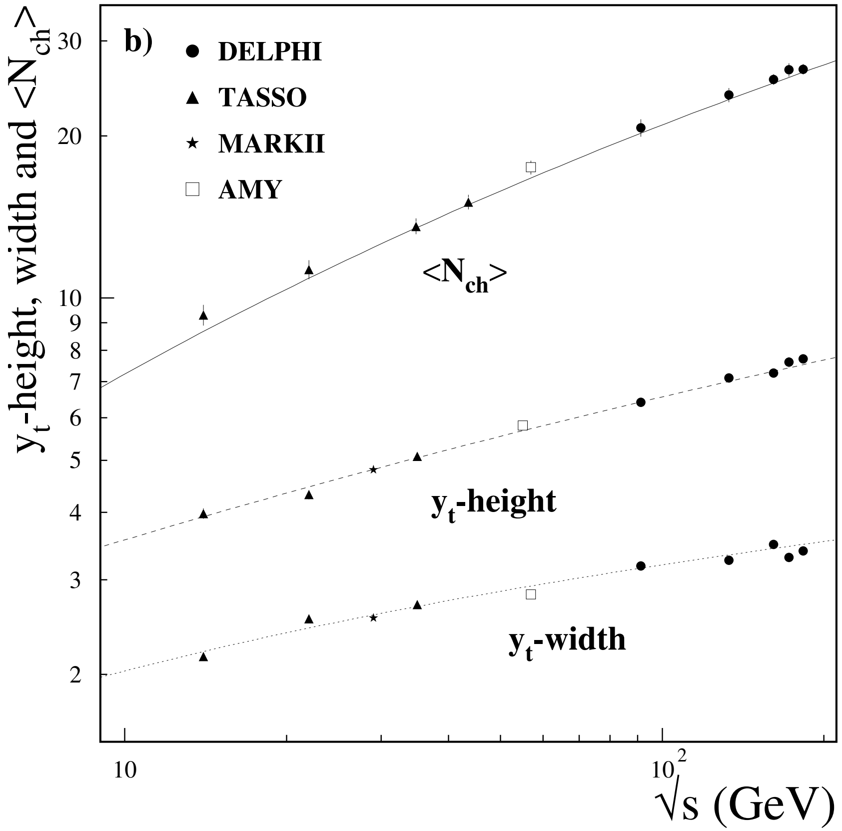
Overview:

1) E-depd. for ch. hadrons: coherence.
2) Charged & id.; flavour dependence.
3) Leading particles, b-fragmentation
4) Regularity in particle rates.
5) Gluon jet measurements, scales.
6) Scaling violations g vs. q.
7) g vs. q multiplicity.
8) Id. particles g vs. q; baryons!
9) Id. particles g vs. q vs. $e^+e^-$
10) Summary

# Energy Dependence of Rapidity Distribution



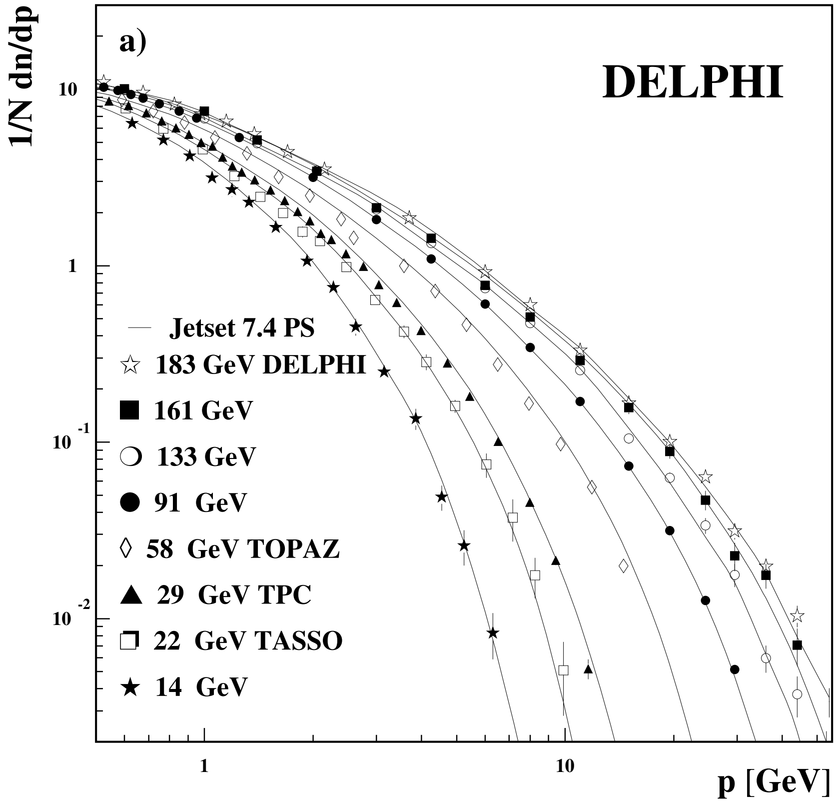



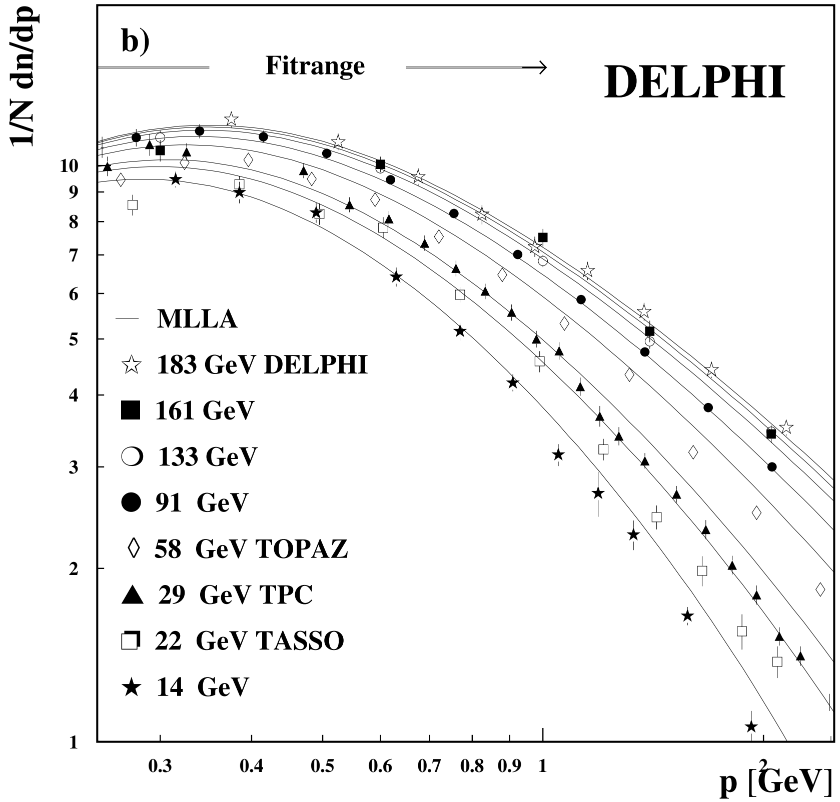

Plateau and length of rapidity $(y)$ distribution increasing with energy:

- Plateau: parton shower
- Length: longitudinal phase space

Multiplicity increase due to increase of plateau and length similar.

# Energy Dependence of Momentum Spectrum

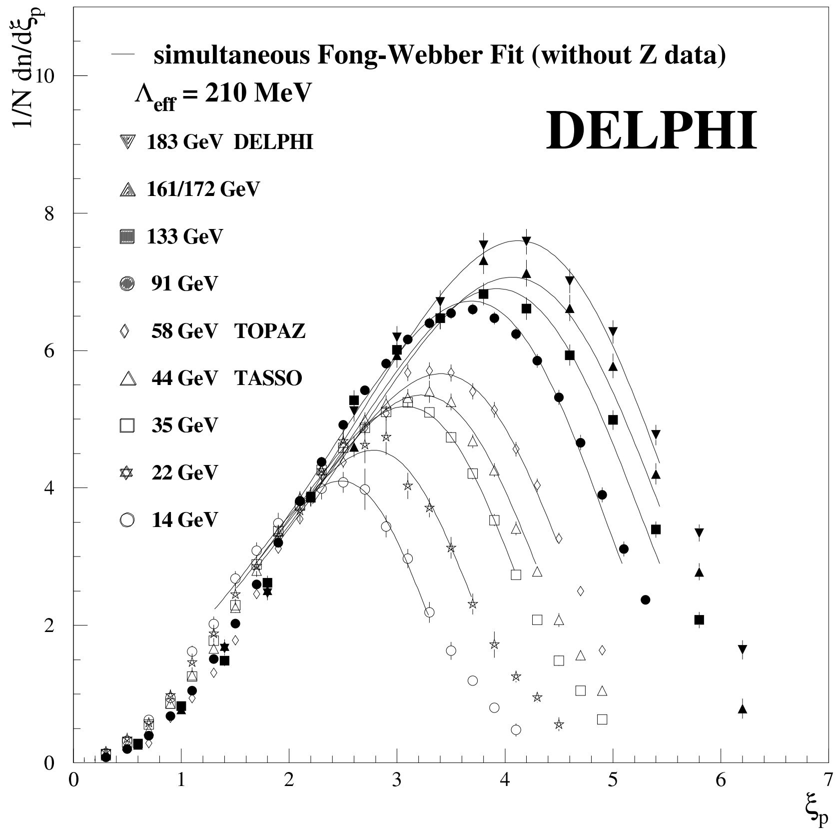


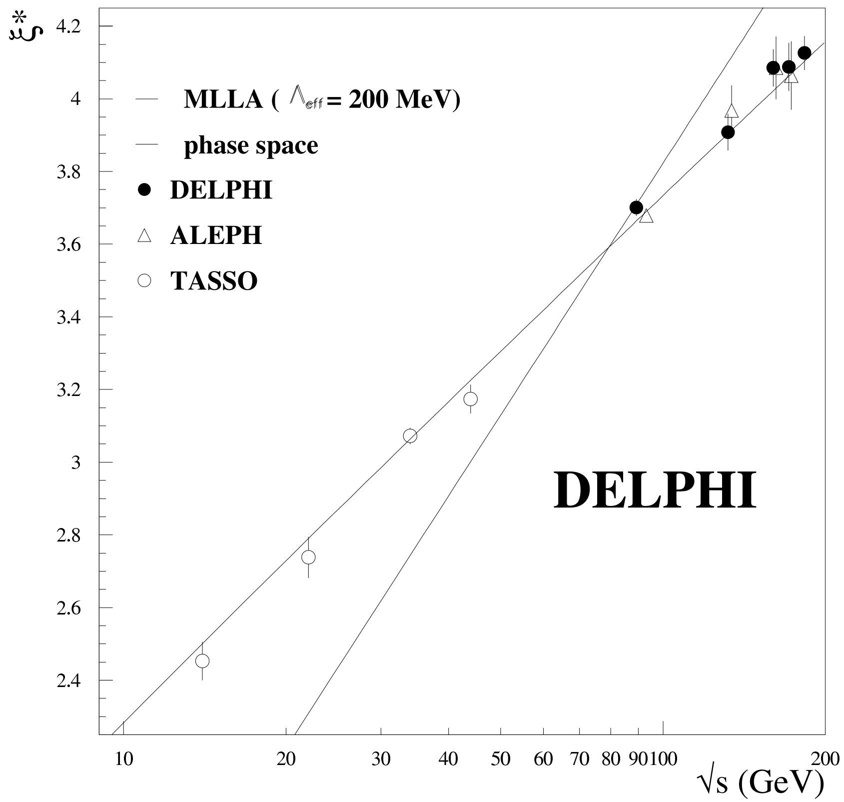




- Large $p$: "scaling" region – cross-section increases due to phase space opening.
- Small $p \lesssim 1$: $\sim$ universal x-section.

- Soft gluon coherence: soft gluons cannot "see" hard gluons emitted at high energy; only conserved total colour current resolved.

# Energy Dependence of $\log 1/x$ Spectrum



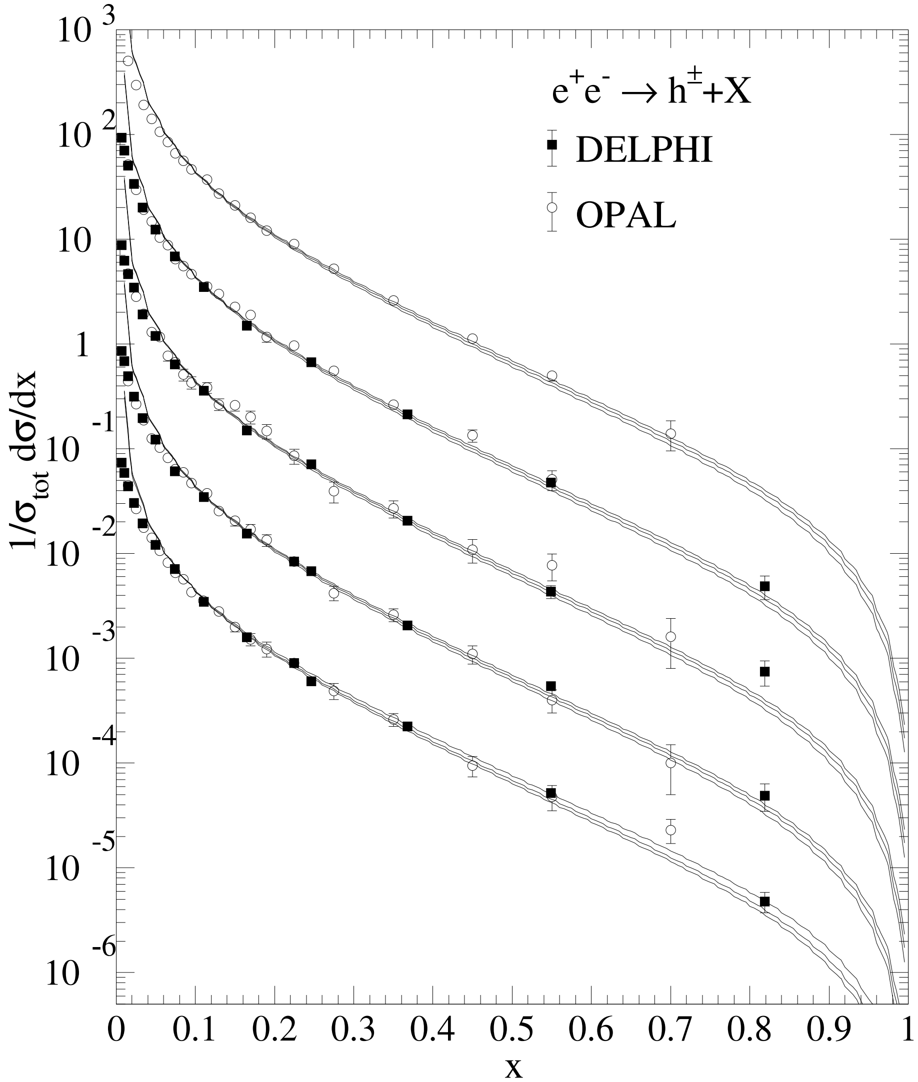

- Dominant part of x-section "hump backed plateau" described by perturbative calculation.

- $\Lambda$ parameter from distribution and peak position $\xi^*$ consistent.
- Energy increase of peak excludes incoherent model.

# Large $x$; Scaling Violations

Data shown: 133GeV to 189GeV
Fit also includes low energy data.

- Large scaled momenta $x$: ~scaling.
- Scaling violations determine $\alpha_s$.
- Analyses of LEP experiments: use PETRA/PEP/TRISTAN data.
- Recent analysis (Kniehl et al.) checked LEP high E data. Find scaling violations as expected.
- $\alpha_s(M_Z) = 0.117 \pm 0.073$ (NLO) using PEP and LEPI.

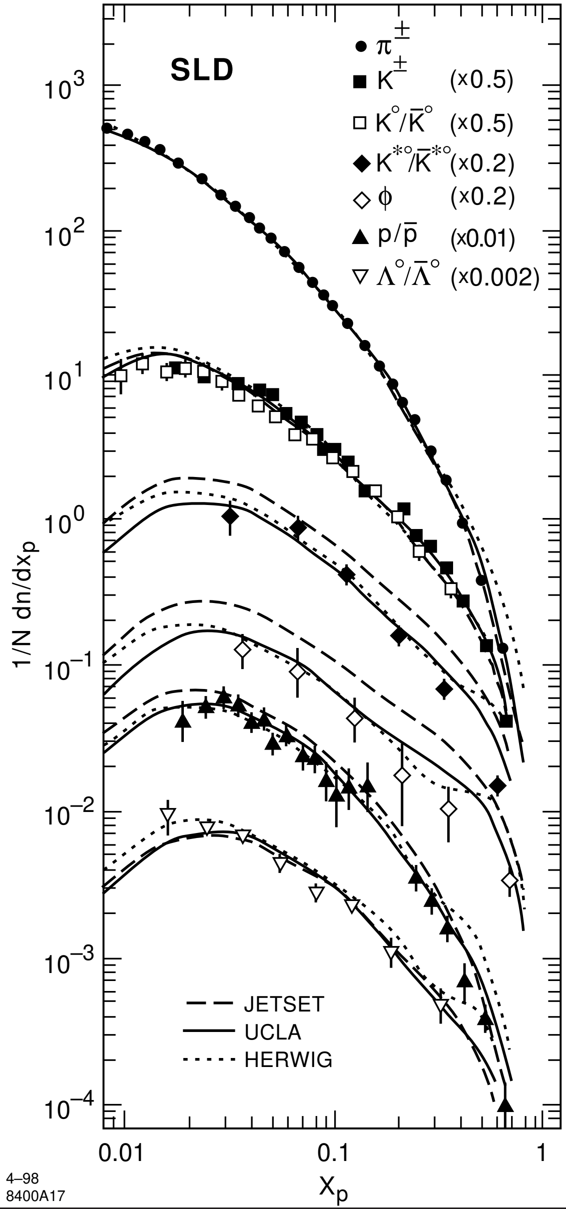


# Identified Particles : Description by Models (1)

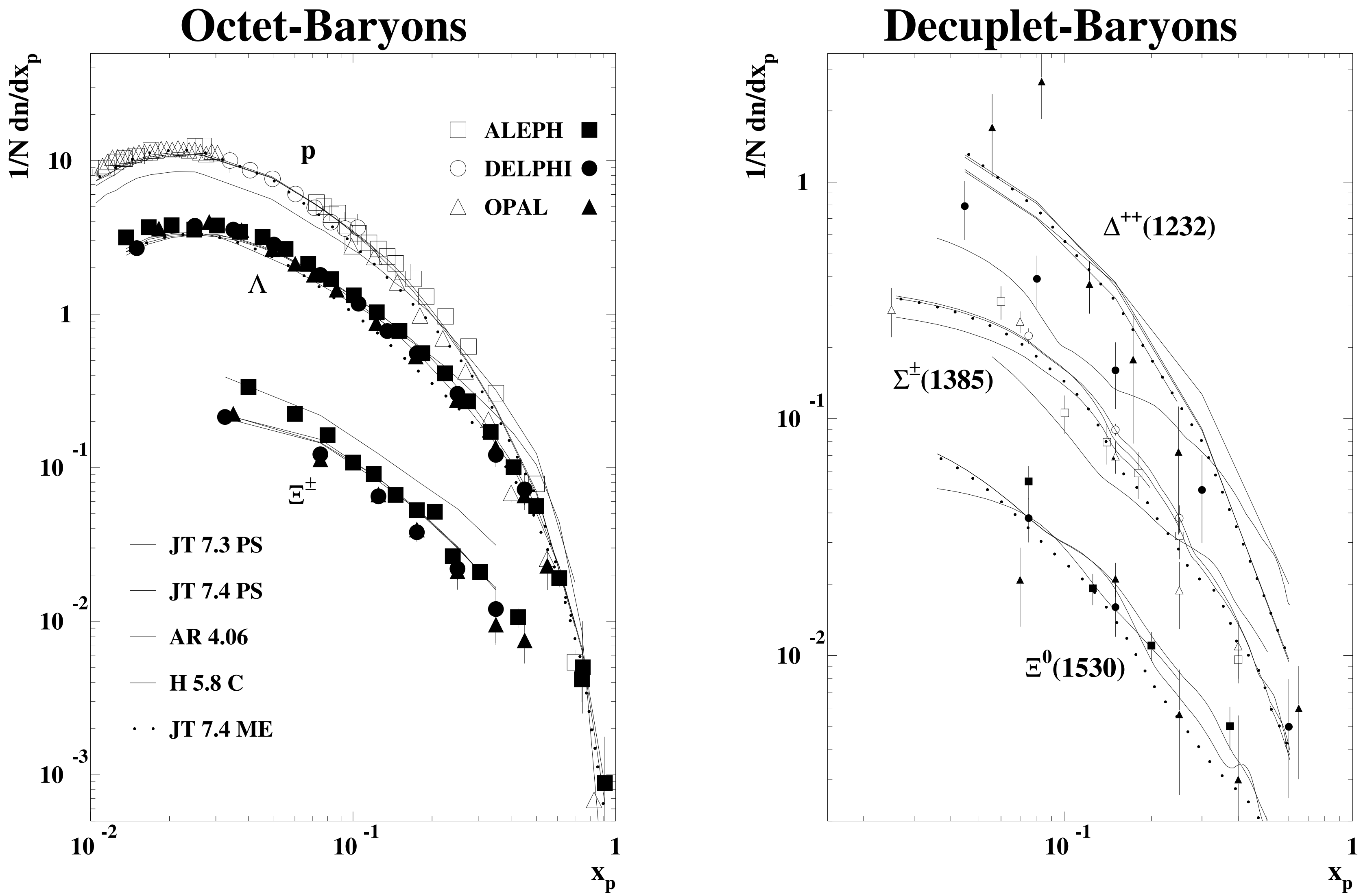
Fragmentation models give a reasonable description of the shape of the fragmentation functions for all mesons and baryons.

But:

- Effectively one FF parameterization for mesons,
- another one for baryons
  (e.g. PYTHIA: POPCORN, extra baryon suppress.).
- Cluster fragmentation: too stiff baryons.
  Maybe missing baryon resonances?
- Rates for individual multiplets fixed by many parameters.



# Identified Particles : Description by Models (2)



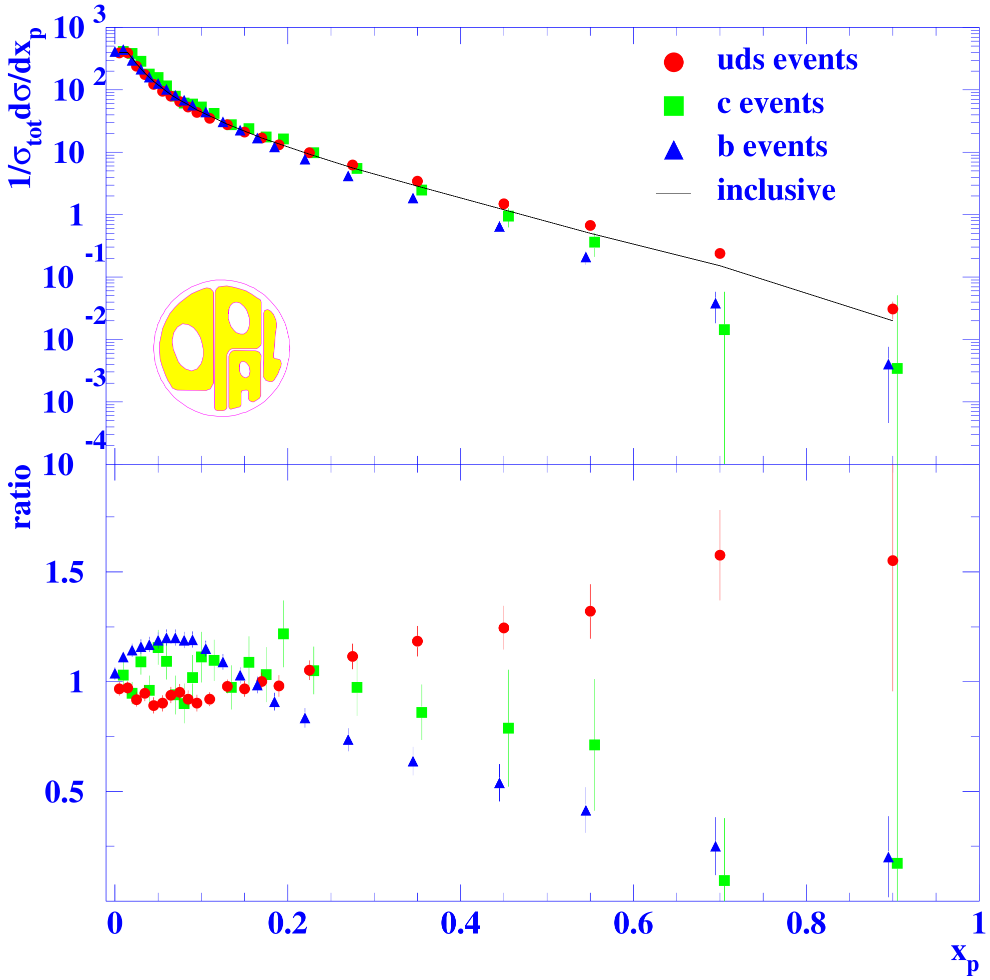
# Flavour Dependence of $x_p$-Distribution

Secondary vertex/D* tag:
allow to access flavour dependent fragmentation functions.

- Softer frag. fct. (higher multiplicity) for c and b quarks.
- Experiments agree well.
- Some importance for scaling violation analysis.
- Heavy decays shift $\xi^*$ peak:

inc:$3.656 \pm 0.003$ uds:$3.74 \pm 0.06$

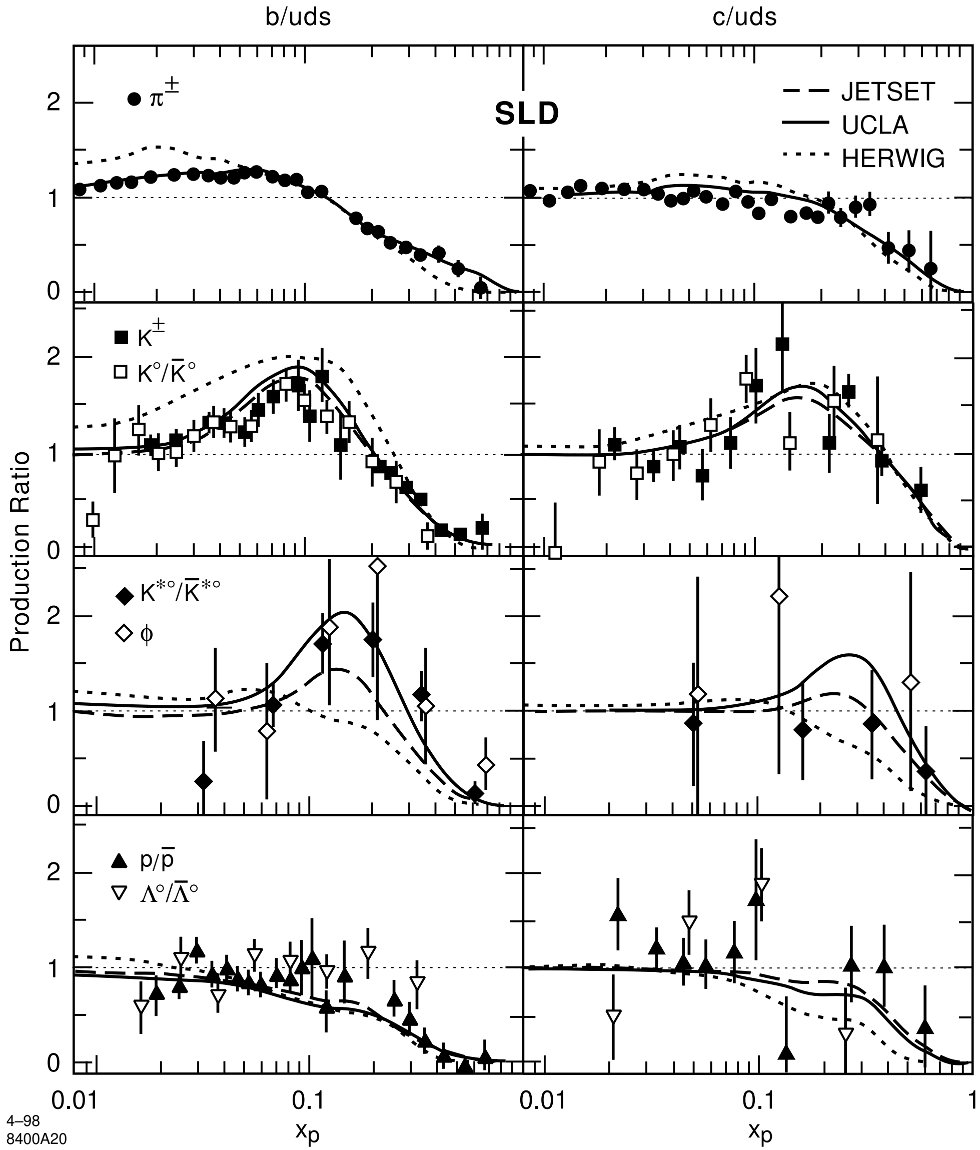
c :$3.63 \pm 0.16$ b :$3.55 \pm 0.01$



# Flavour Dependence for Identified Particles

Check ratio b/uds and c/uds:

- Moderate $x_p$ dependence for $\pi$s. General behaviour as for charged particles.
- Kaons and strange resonances with strangeness: "peak" at moderate $x_p$ due to weak decays; better visible in rapidity
- Baryons ∼decrease; less baryons from B/D decays.
- Well described by fragmentation models.

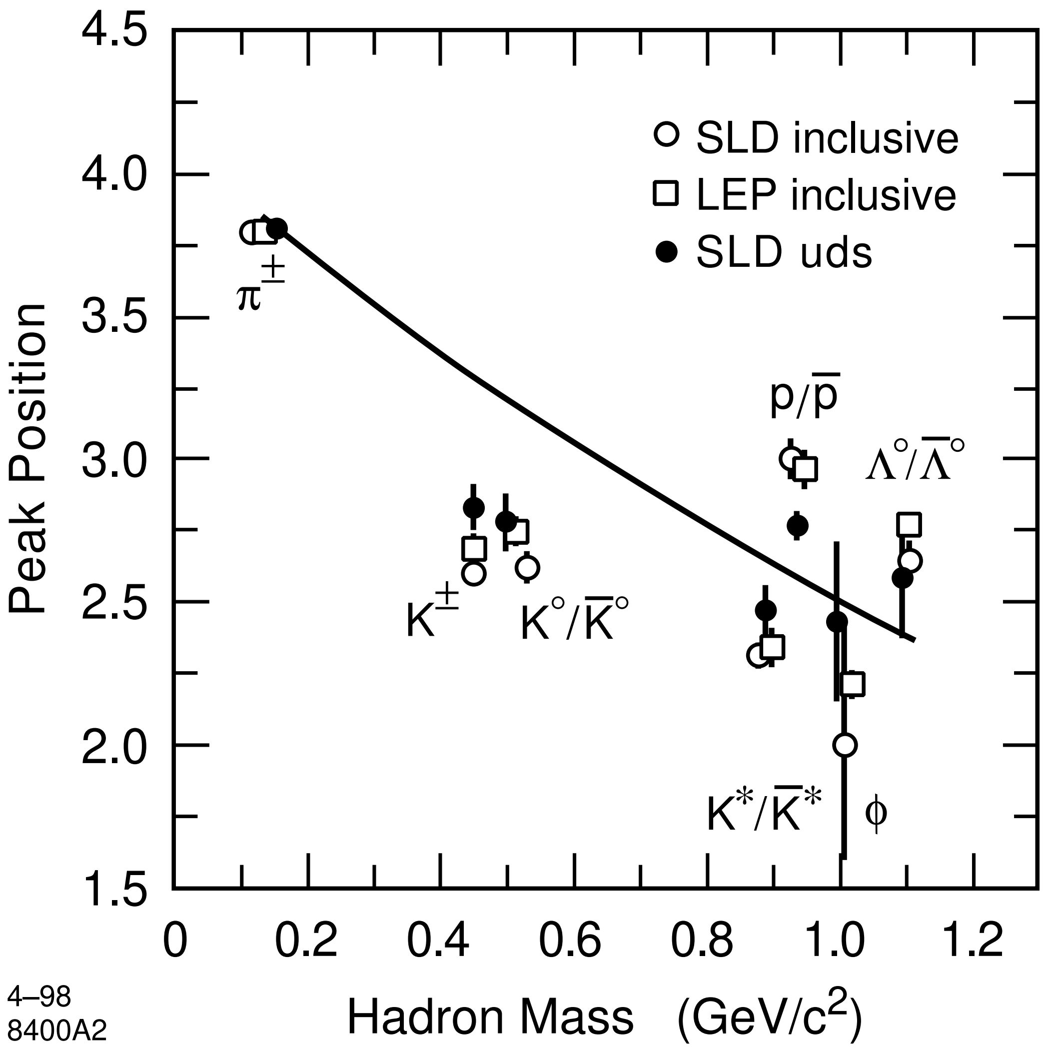


# Peak Position of $\xi$ Distribution Inclusive vs. uds

MLLA expects: $\xi^* \propto \log Q_0/m_{hadron}$ due to the "truncation" of the parton cascade

- Inclusives: jump of peak position between mesons and baryons.
- uds: jump partly compensated!
- Even light quarks do not follow simple MLLA expectation well.
- Behaviour can be ∼understood in frag. models considering decays.

No "proof" of MLLA.

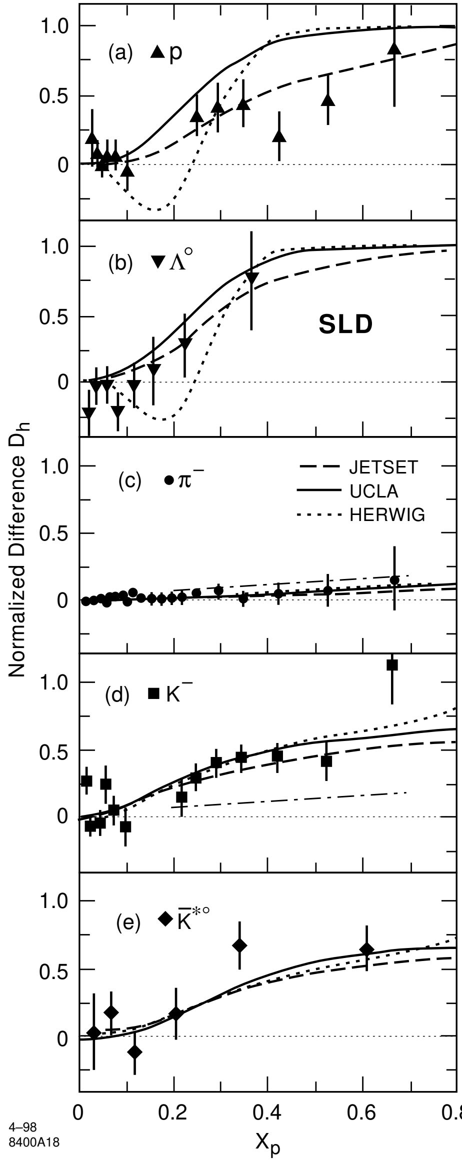


# Leading Particle Effect

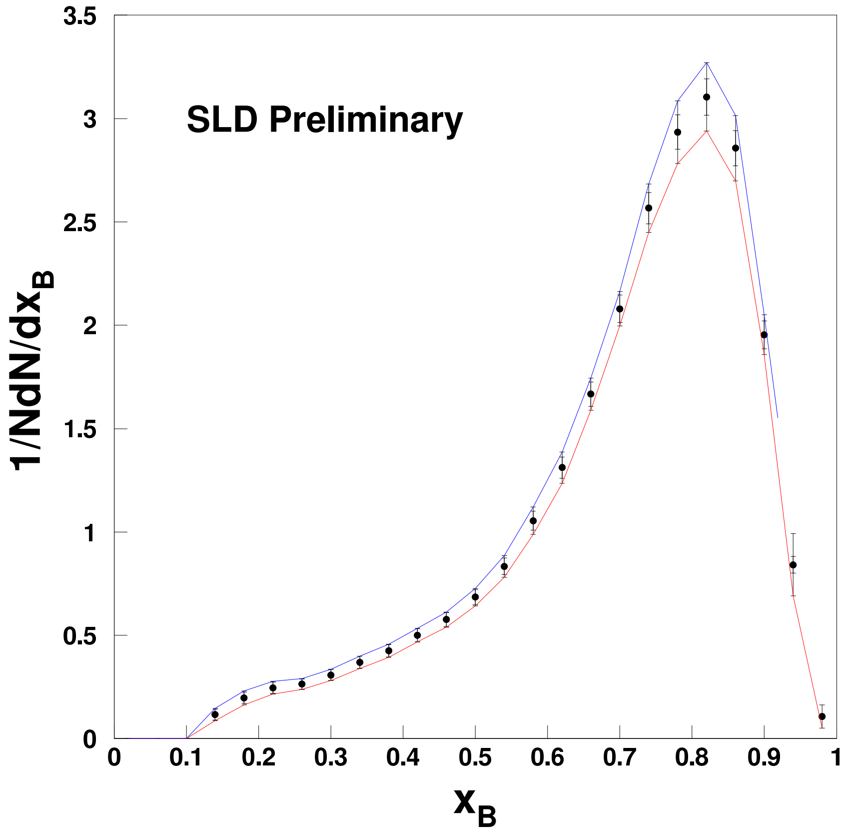
Beam polarisation/EW asymmetry allows to prepare enriched quark/anti-quark samples ($\sim 75\%$ pure). Use also uds tag. SLD shows normalised difference of production rates:

$$D_h = \frac{(N(q \to h) + N(\bar{q} \to \bar{h})) - (N(q \to \bar{h}) + N(\bar{q} \to h))}{(N(q \to h) + N(\bar{q} \to \bar{h})) + (N(q \to \bar{h}) + N(\bar{q} \to h))}$$

- Strong leading particle effect for kaons and baryons
- No leading particle effect for $\pi$s seen. Cancelation d vs. u.
- JETSET describes leading particles well, HERWIG fails. Reason: strong baryon/anti-baryon correlation from neutral cluster decay. (Need more baryon resoances?)
- Important experimental validation for EW asymmetry measurements.



# B Hadron Fragmentation




- Primary b quark fragm. pert. calculable. Interleaved with parton shower.
- Excellent tracking/vertexing allows reconstruction of B hadron energy.
- Bowler model consistent with data; others less favoured or excluded.

SLD:

$$\langle x_B \rangle = 0.710 \pm 0.003 \pm 0.006(syst.)$$

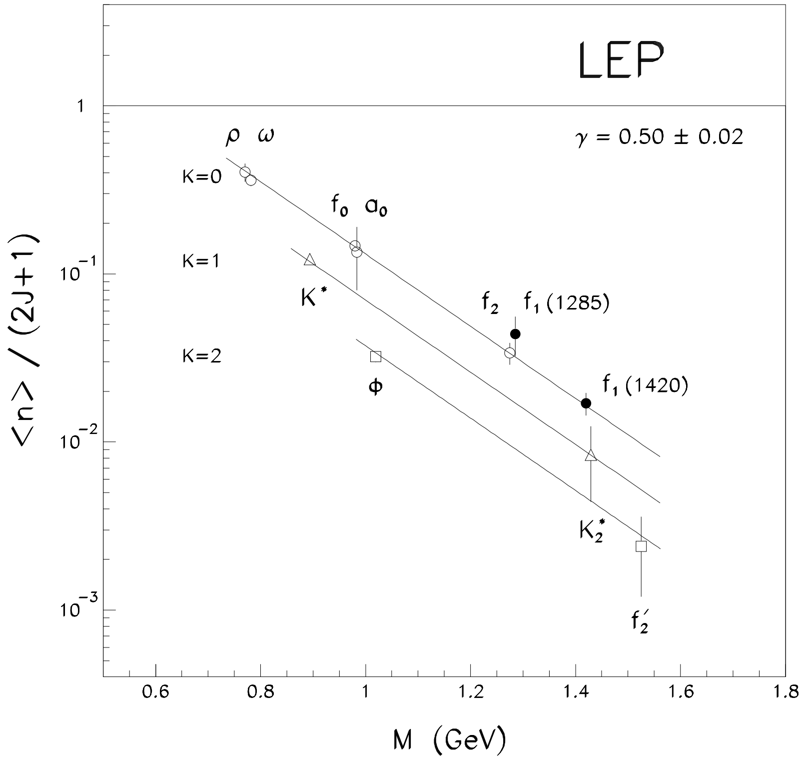
ALEPH:

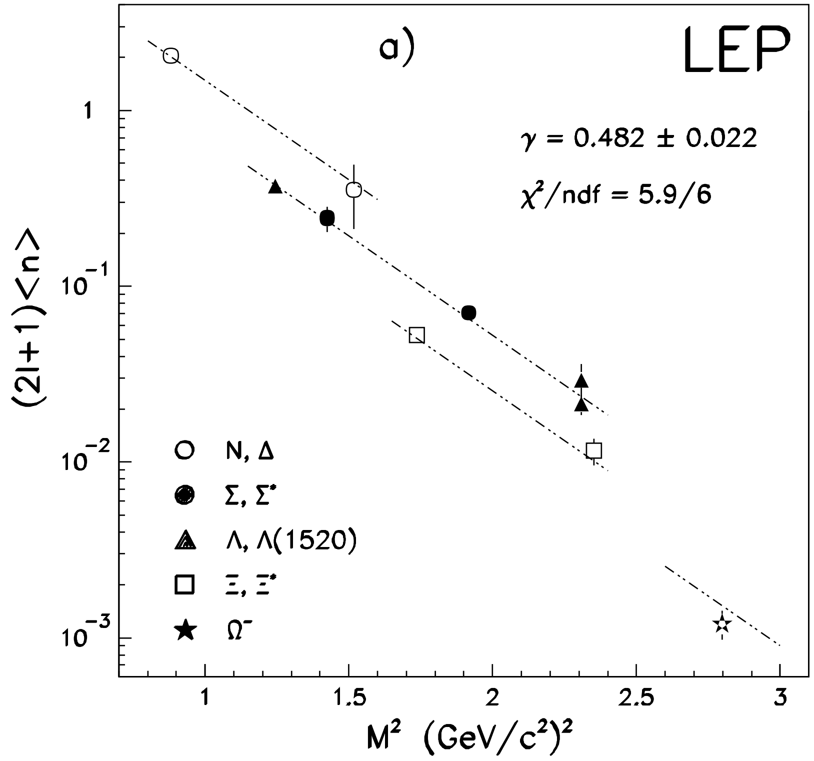
$$\langle x_B \rangle = 0.7304 \pm 0.003 \pm 0.006(syst.)$$

DELPHI:

$$\langle x_B \rangle = 0.707 \pm 0.001 \pm 0.006(syst.)$$

# Regularity of Hadron Production Rates



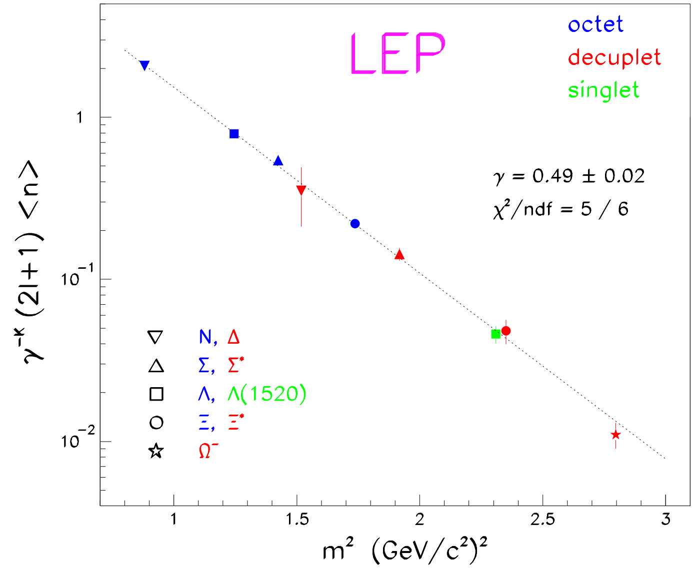



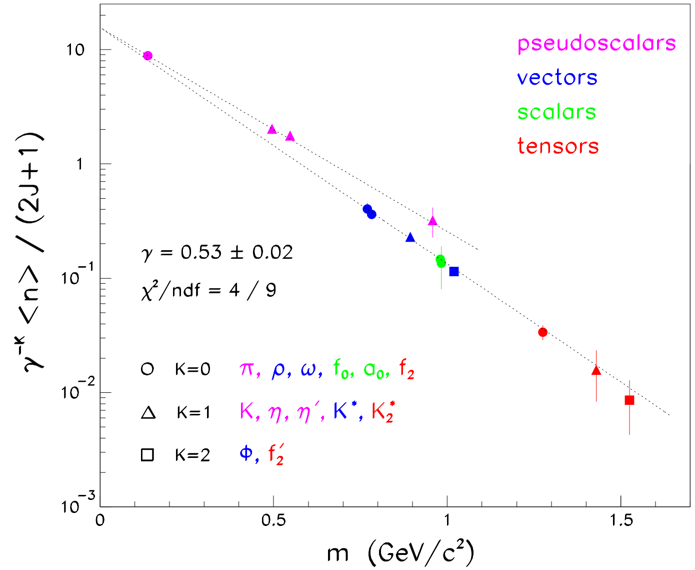

- Baryons: Sum of rates of isomultiplet $\sum_i \sum_j \langle n \rangle_{ij} = (2I+1) \bullet \langle n \rangle = A\gamma^k e^{-bm^2}$
- Mesons: Rates per spin & isospin state $\langle n \rangle_{ij} = \langle n \rangle / (2J+1) = A' \gamma^k e^{-b' m}$
- Same Strangeness (k) suppression; can be factorised out.

# Regularity of Hadron Production Rates

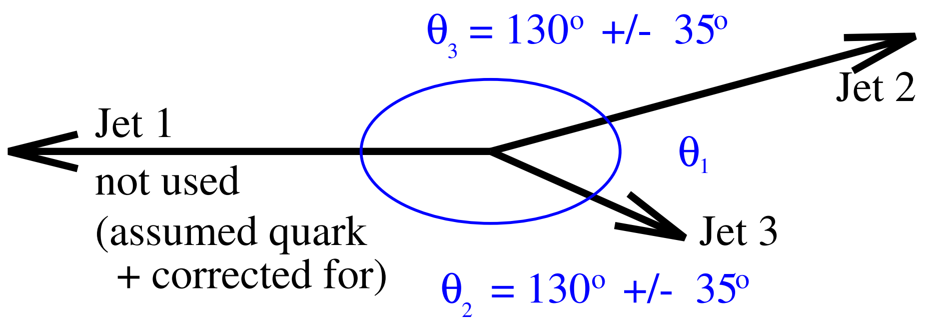
- Baryons: universal mass$^2$ dependence.
- Mesons: Two different mass dependencies for pseudoscalars/others.
- Extrapolating to $m = 0$ find $\sim$ratios expected from quark combinatorics ($\sim$3).

$$\frac{\rho^+}{\pi^+} = 3.6 \pm 0.9 \qquad \frac{\pi^+}{\mathrm{p}} = 2.9 \pm 0.3$$



# Analysing Gluons in 3 Jet Events




- Take quarks and gluons from 3 jet evts.
- Tag $b$-quarks to identify gluons.
- Subtract gluons from udscg jets to obtain quarks.
- DELPHI: aritrary 3 jets give 140k gluon jets.
- DELPHI: symmetric 3 jets (Y, Mercedes topology)
- OPAL: gluon jet recoils vs. quarks. Still 3 jets! Smallest angle $\sim 90°$. Rare events!

- Need evolution scale for jets!
- Energy wrong! Not Lorentz-invariant.
- Look e.g. at NLLA splitting kernels

$$\Gamma_{q/g} = \frac{C_{q/g}}{2\pi}\frac{\alpha_s(y')}{y'}\left(\log\frac{y}{y'} - k_{q/g}\right)$$

$$y \cdot E_{CM} \sim (2E_{jet}\sin\frac{\theta_{\min}}{2})^2$$

$\theta_{\min}$ is the angle to closest jet.

- For convenience use linear scale:

$$\kappa_h = E_{jet}\sin\left(\frac{\theta_{\min}}{2}\right)$$

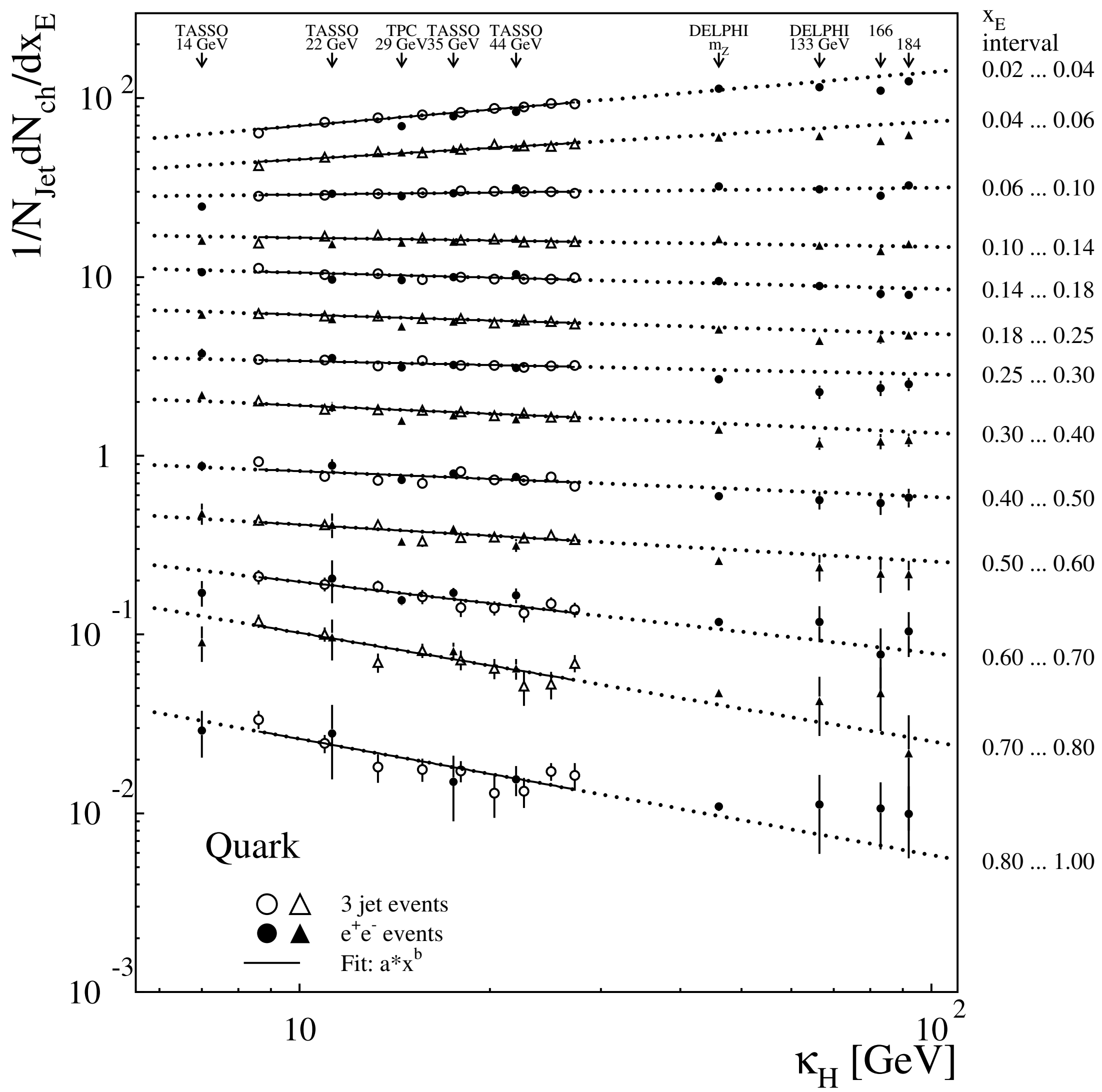
$\kappa_h \simeq p_t$ is Lorentz-invariant, considers coherence effects to LO.

- Still an approximation, in general one needs 2 scales!

# Check of Evolution Scale: Quark Frag. Function

- Quark fragment. function from 3 jet events (open points/lines) compared to overall $e^+e^-$ result (full points).
- Scaling violation observed.
- 3 jet result extrapolates to $e^+e^-$ results at higher/lower energies.

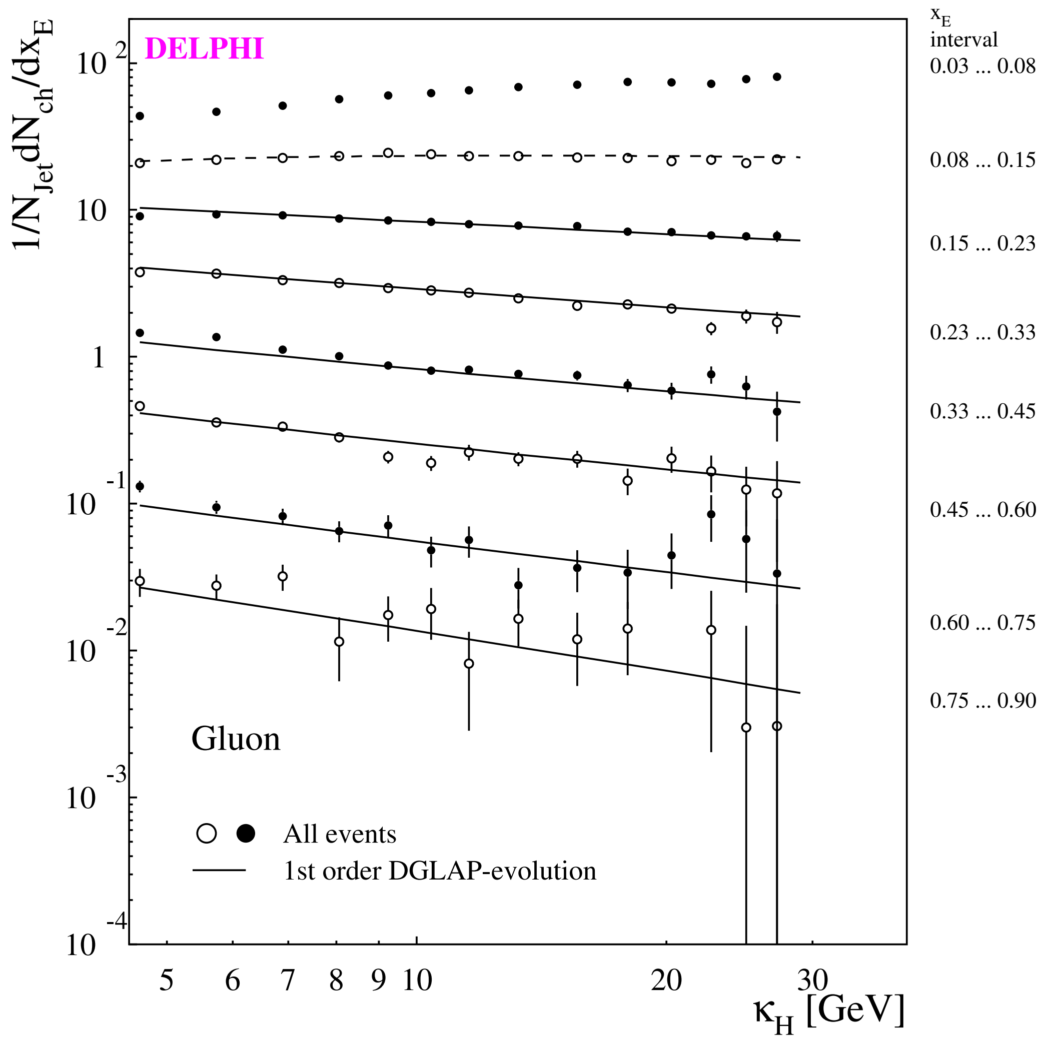
$\rightarrow$ $\kappa_H$ suitable scale.
$\rightarrow$ Environmental independence.



# Scaling violation: gluon jets

- Gluon fragmentation function $D_g^h(z)(\kappa_h)$ from 3 jet events.
- Scaling violation of gluon fragmentation function directly observed.
- Lines: LO GDLAP-evolution.

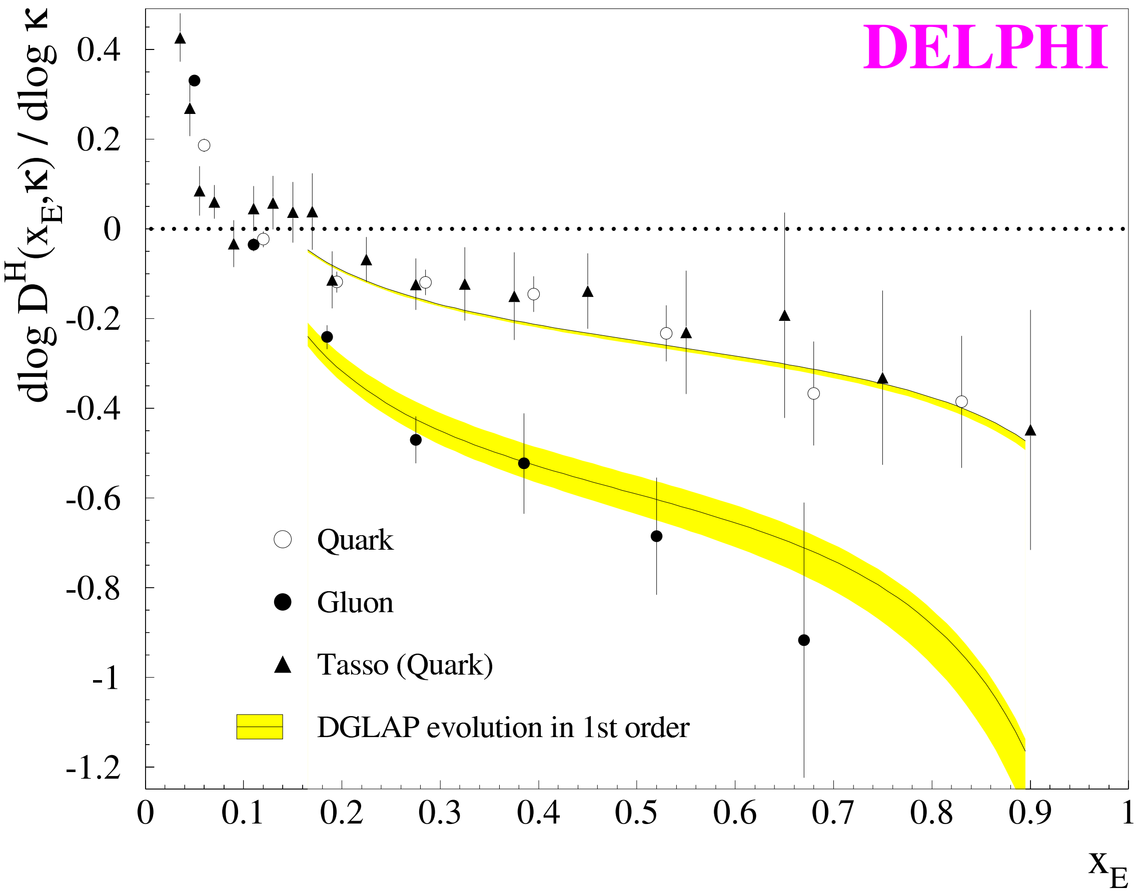
  $\rightarrow$ Describes data well
- Simple parameterisations of $D_g^h(z)(\kappa_h)$ exist (DELPHI).



# Strength of scaling violation for Gluons and Quarks

Plot:
logarithmic slopes for gluons and quarks.

- Points: power-law fits to data.
- Lines: LO GDLAP-evolution.
- Good agreement between quarks from 3 jets and $e^+e^-$.




Colour factor ratio from GDLAP-fit:

$$\frac{C_A}{C_F} = 2.25 \quad \pm 0.09 \text{ (stat.)} \quad \pm 0.06 \text{ (syst.)} \quad \pm 0.11 \text{ (clus.)}$$

# Gluon and Quark Multiplicities

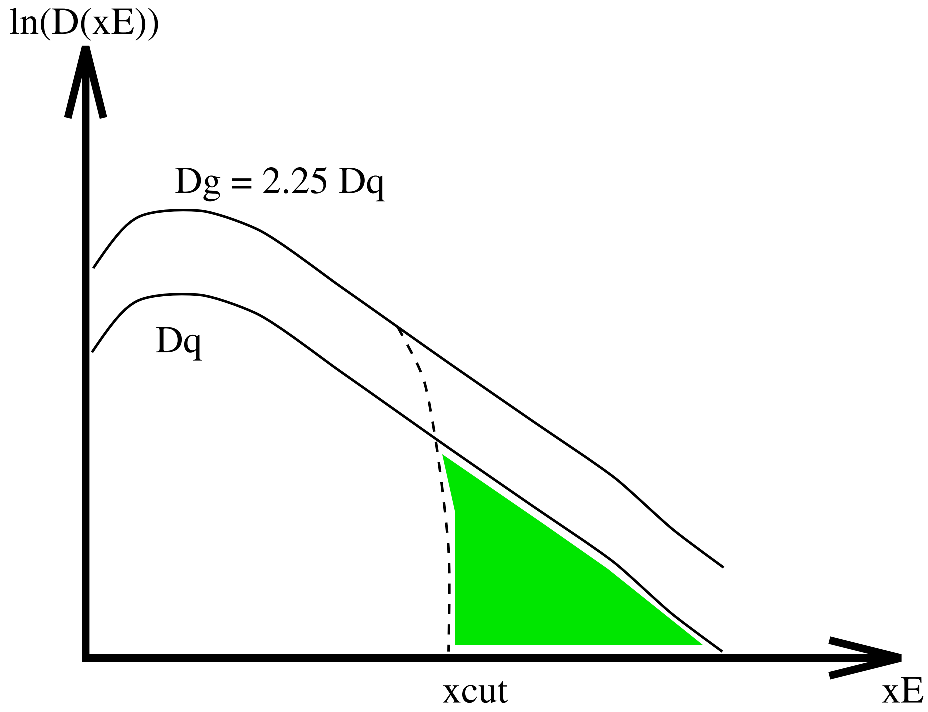
- Multiplicity "sumrule" $N_{ch} = \int D(x)\, dx$.
- Naive expectation from splitting functions: $r_n = N_{gg}/N_{q\bar{q}} = C_A/C_F$
- Experimentally: counting tracks.
- For single jets: Assignment of soft particles to jets is important!
- Measurements of $r_n$ indicate energy dependence:

| | | |
|---|---|---|
| CLEO | $E_{\text{Jet}} < 3.5\text{GeV}$ | $r_n = 1.04 \pm 0.02 \pm 0.05$ |
| HRS | $E_{\text{Jet}} = 9.7\text{GeV}$ | $r_n = 1.29 \pm 0.2(\text{stat.})^{+0.21}_{-0.20}(\text{syst.})$ |
| TASSO | $E_{\text{Jet}} = 11\text{GeV}$ | $r_n \simeq 1.$ |
| OPAL | $E_{\text{Jet}} = 24.5\text{GeV}$ | $r_n = 1.02 \pm 0.04^{+0.06}_{-0.00}$ |
| OPAL | $E_{\text{Jet}} = 24\text{GeV}$ | $r_n = 1.25 \pm 0.02 \pm 0.03$ |
| ALEPH | $E_{\text{Jet}} = 24\text{GeV}$ | $r_n = 1.249 \pm 0.084 \pm 0.022$ |
| DELPHI | $\overline{E_{\text{Jet}}} = 24\text{GeV}$ | $r_n = 1.241 \pm 0.015 \pm 0.025$ |
| OPAL | $E_{\text{Jet}} = 39\text{GeV}$ | $r_n = 1.552 \pm 0.041 \pm 0.061$ |

# Why is $r_n = N_{gg}/N_{q\bar{q}} << C_A/C_F$?

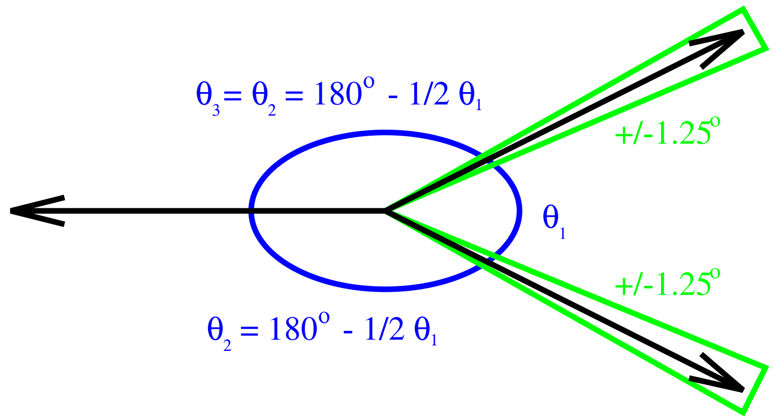
- Suppose gluon to quark fragmentation function ratio equals to $C_A/C_F$ at small $x_E$.
- Cannot hold for all $x_E$; violates energy conservation:

$$2.25 \int_0^1 D_q(x_E)\, x_E\, dx_E = 2.25 > 1$$




- "Restore" energy conservation: cut off $D_g$ at some $x_{cut}$.
- Contributions to $D_q$ at higher $x_E$ reduce multiplicity ratio.
- This contribution vanishes logarithmically with energy.
- Way out for Colour Factor measurements at lower energies: look at ratio of energy slopes rather than ratio of absolute values.

# Analysis of Complete Three-Jet-Events




- Analysis of complete one-fold symmetric events
- All events are clustered to three jets (no $y_{\mathrm{cut}}$)
- Cambridge-algorithm
- $\theta_2 = \theta_3$ with small tolerance
- Compare to theoretical predictions for three jet events.
- Scale variables can be determined from event-topology (from $\theta_1$).
- No tagging of jets.
- No assignment of particles to jets.

**High statistics $\simeq 162.000$ events**

# 3 Jet Event Multiplicity (Step 1)

- Prediction by P.Eden et al. connects energy slopes for colour singlet gg and $q\bar{q}$ events:

$$\boxed{\left.\frac{dN_{gg}(L')}{dL'}\right|_{L'=L+c_g-c_q} = \frac{C_A}{C_F}\left(1-\frac{\alpha_0 c_r}{L}\right)\frac{d}{dL}N_{q\bar{q}}(L)}$$

$$\text{with} \quad L = \log\frac{s}{\Lambda^2} \quad , \quad c_g = \frac{11}{6} \quad , \quad c_q = \frac{3}{2} \quad , \quad c_r = \frac{10}{27}\pi^2 - \frac{3}{2}$$

- Constant of integration considers non perturbative differences!
  $\rightarrow$ Can be determined from measurement of $N_{gg}$
  $\rightarrow$ Here: CLEO-measurement of $N_{gg}$ in the (colour singlet) decay $\chi_b'(J=2) \rightarrow gg$ at $E_{cm} = 9.9132$GeV

- $N_{q\bar{q}}$ taken from $e^+e^-$ measurements at various $\sqrt{s}$

# 3 Jet Event Multiplicity (Step 2)

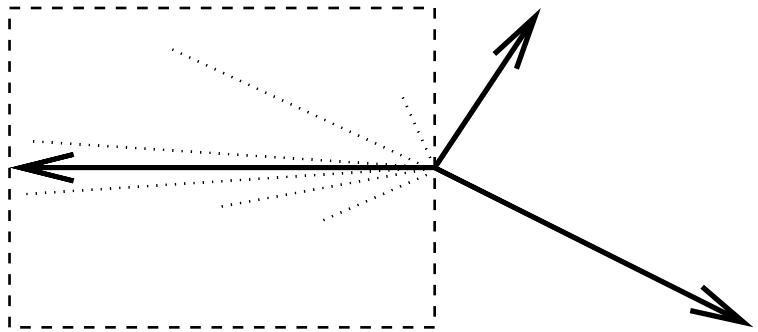
- Multiplicity of three-jet-events looks like an incoherent sum:

$$N_{q\bar{q}g}(L_{q\bar{q}}, \kappa_{Lu}, \kappa_{Le}) = \quad N_{q\bar{q}}(L_{q\bar{q}}, \kappa_{Lu}) + \tfrac{1}{2} N_{gg}(\kappa_{Le}) \qquad (A)$$

$$N_{q\bar{q}g}(L_{q\bar{q}}, \kappa_{Lu}, \kappa_{Lu}) = \quad N_{q\bar{q}}(L, \kappa_{Lu}) + \tfrac{1}{2} N_{gg}(\kappa_{Lu}) \qquad (B)$$

with $L = \ln\left(\frac{s}{\Lambda^2}\right), \quad L_{q\bar{q}} = \ln\left(\frac{s_{q\bar{q}}}{\Lambda^2}\right),$

$\kappa_{Lu} = \ln\left(\frac{s_{qg}s_{\bar{q}g}}{s\Lambda^2}\right), \quad \kappa_{Le} = \ln\left(\frac{s_{qg}s_{\bar{q}g}}{s_{q\bar{q}}\Lambda^2}\right)$



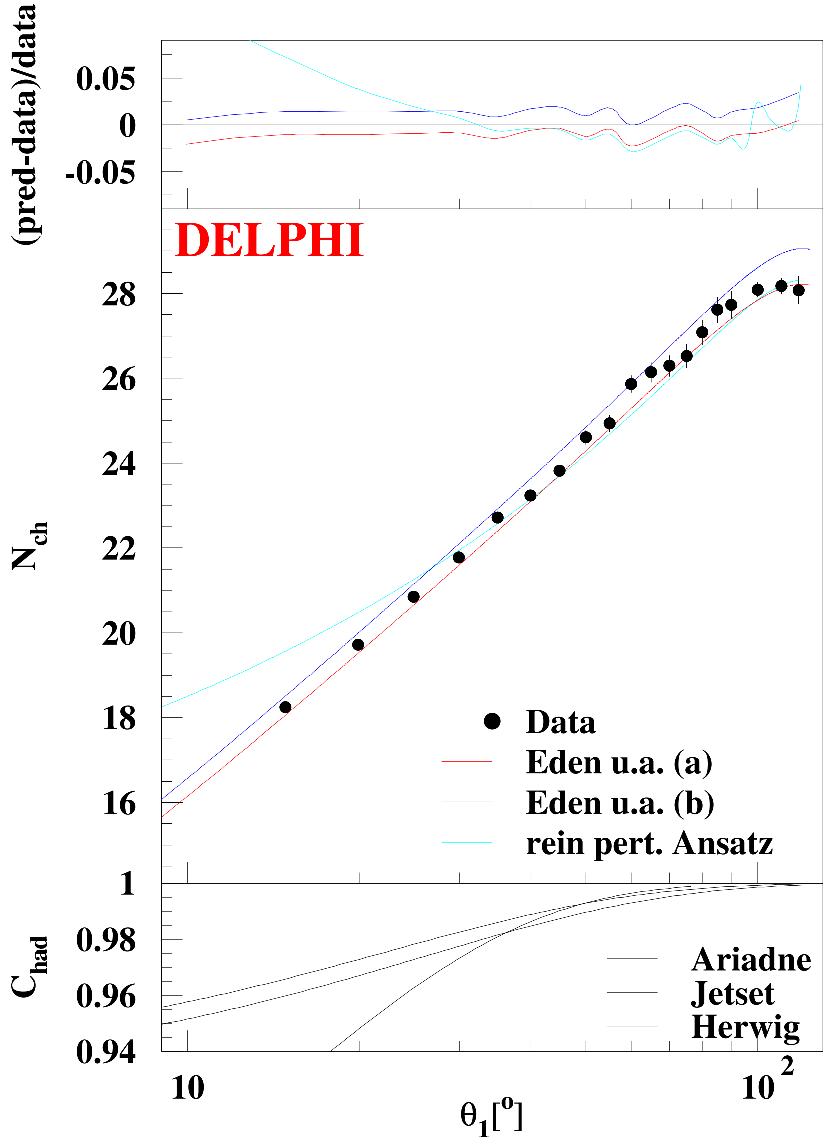
- Coherence effects absorbed by the choice of scale variables.
- Phasespace of the $q\bar{q}$ pair is restricted by the transverse momentum of the gluon jet.

$$N_{q\bar{q}}(L, \kappa_{\mathrm{cut}}) = N_{q\bar{q}}(\kappa_{\mathrm{cut}} + c_q) + (L - \kappa_{\mathrm{cut}} - c_q) \left.\frac{dN_{q\bar{q}}(L')}{dL'}\right|_{L'=\kappa_{\mathrm{cut}}+c_q}$$

# Compare Predictions to Data (Y Topologies)

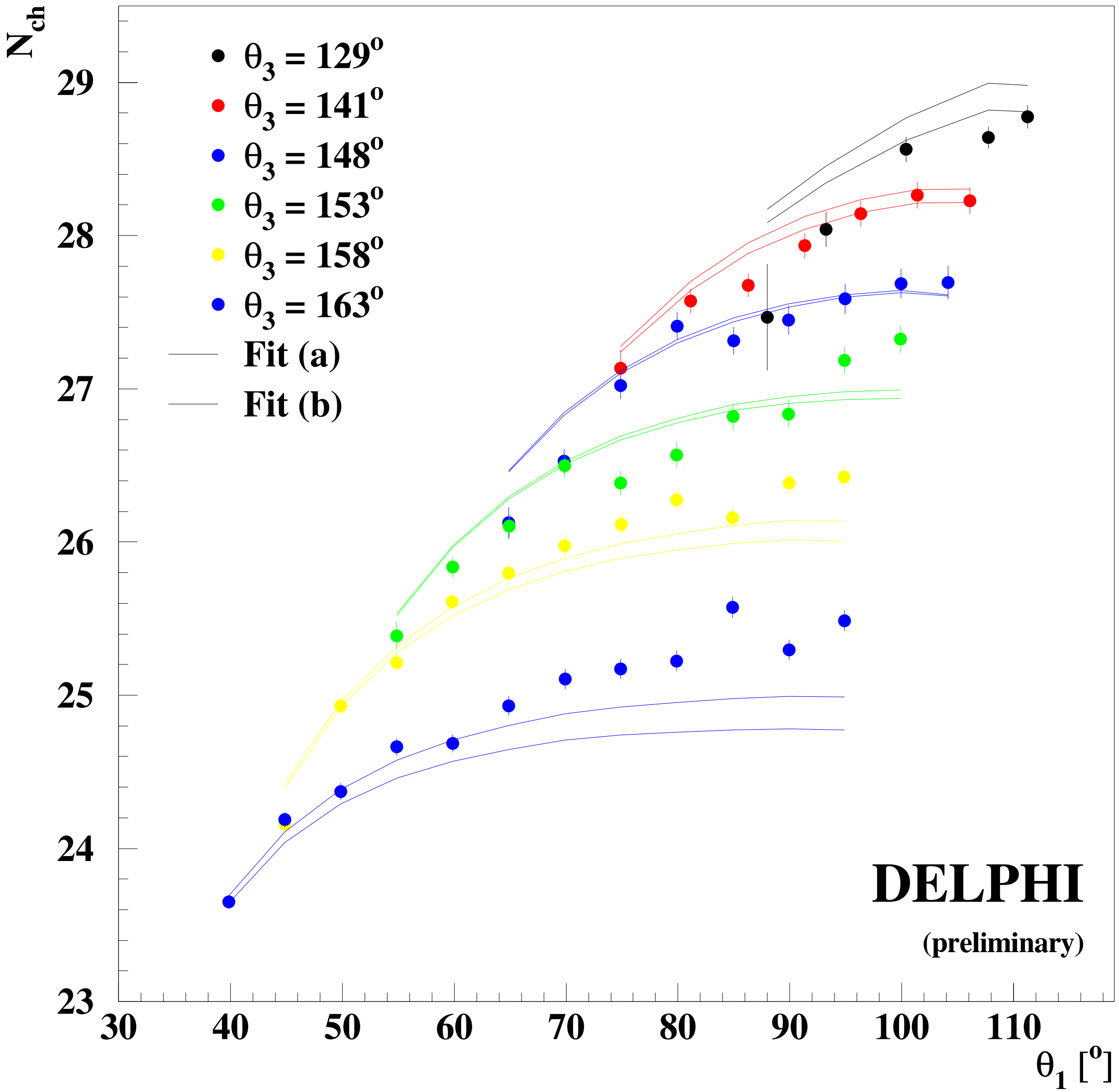
- Data: Anti-$b$-tagged
- Eden et al. (A): Outstanding consistency with data
- Eden et al. (B): Too high at large $\theta_1$, good agreement at small $\theta_1$.
- The constraint on the phase space by the gluon gives a relevant contribution
- Hadronisation-correction less than 5%

$\Rightarrow$ Use Eden et al. (A) to extract $N_{gg}$



# Compare Predictions to Data (Arbitrary Topologies)

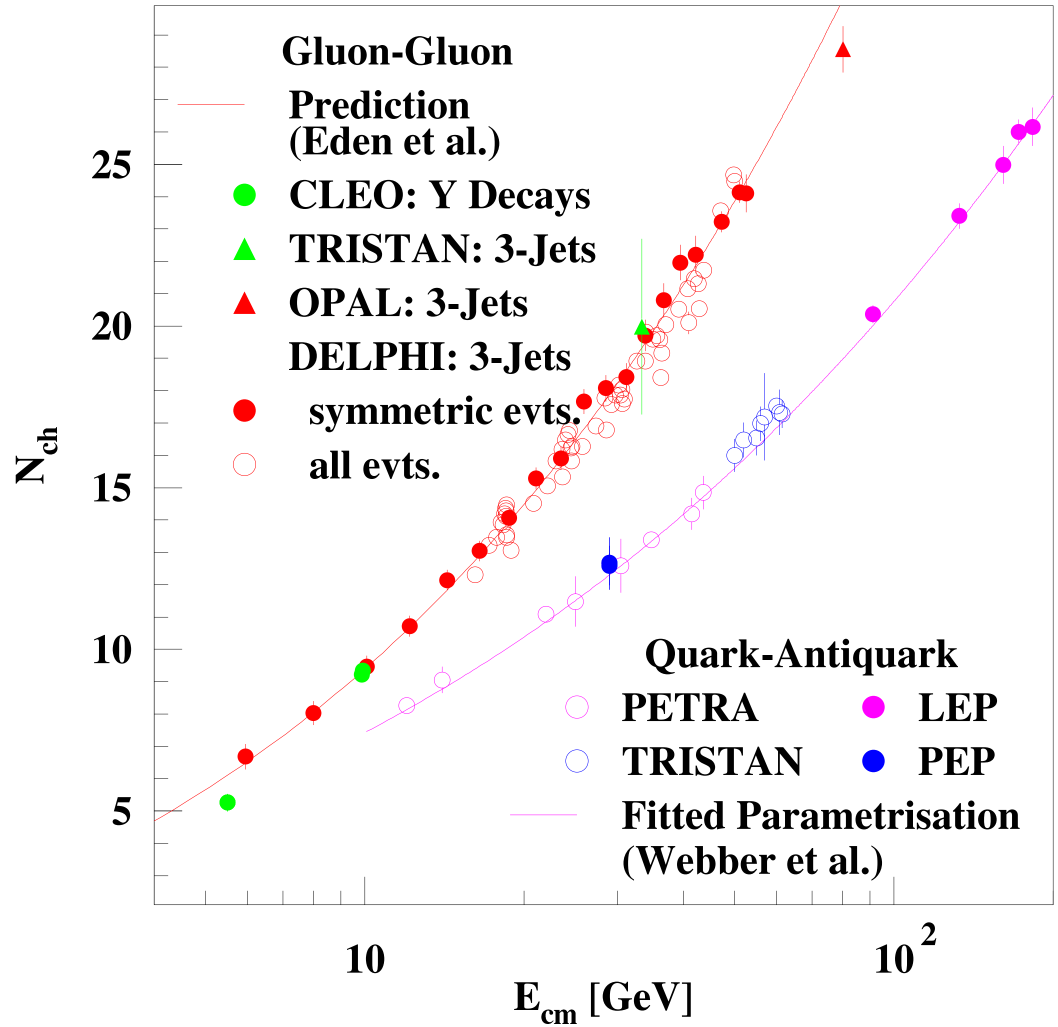
- Analysis also extended to arbitrary 3 jet topologies.
- So far only very preliminary result.



# Extraction of the Gluon Multiplicity

$$N_{gg}(\kappa_{Le}) = 2 \cdot [N_{q\bar{q}g}(\theta_1) - N_{q\bar{q}}(L_{q\bar{q}}, \kappa_{Lu})]$$

- No $b$-tag $\rightarrow$ avoid systematics.
- Additional mulitplicity due to $b$-decays has been subtracted.
- Agreement between symmetric and un-symmetric topologies and with measurements by CLEO, TRISTAN and OPAL.
- The two times ($\sim C_A/C_F$) larger slope of $N_{gg}$ is evident.
- $N_{gg}$ determined over a large variation of scale with one experiment at fixed $\sqrt{s}$!



# Identified particle Rates in g/q

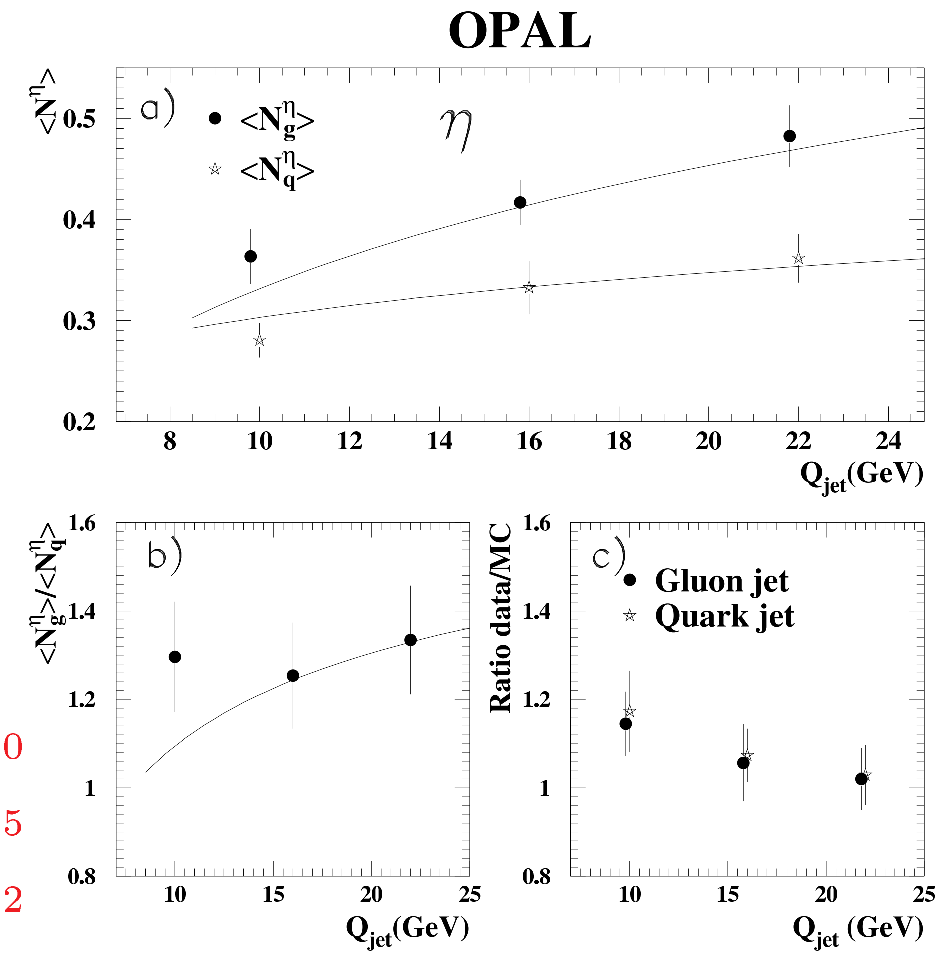
Is there extra production of specific particles in gluon jets?

- Consider higher gluon multiplicity: study double ratio:

$$R_D = \frac{N_g^x/N_q^x}{N_g^{ch}/N_q^{ch}}$$

- No excess seen for $\pi^{\pm 0}$, $\mathrm{K}^{\pm 0}$
- No excess of isoscalars $\Phi$, $\eta$, $\eta'$
- But increased baryon (p) production!

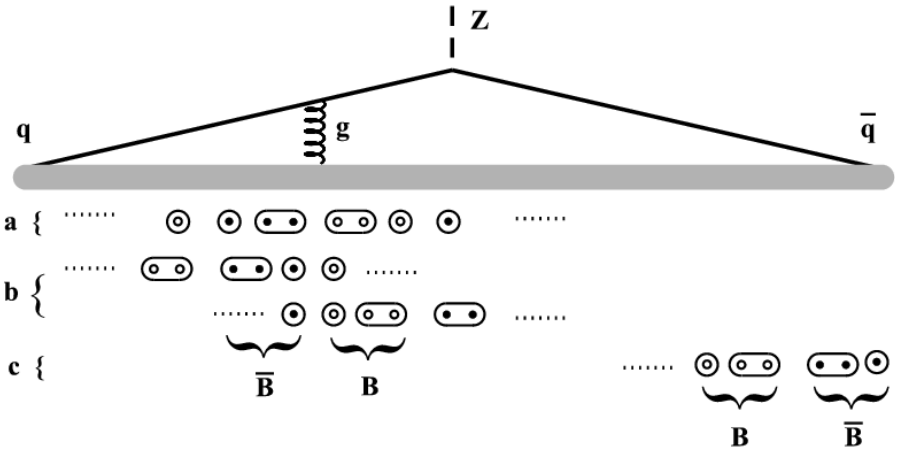
| | | |
|---|---|---|
| ARGUS | $E \sim 10\mathrm{GeV}$ | $R_D = 1.58 \pm 0.10$ |
| DELPHI Y | $E \sim 22\mathrm{GeV}$ | $R_D = 1.21 \pm 0.05$ |
| DELPHI Merc, | $E \sim 30\mathrm{GeV}$ | $R_D = 1.27 \pm 0.12$ |
| OPAL 3 Jets | | $R_D = 1.1 \pm 0.014$ |

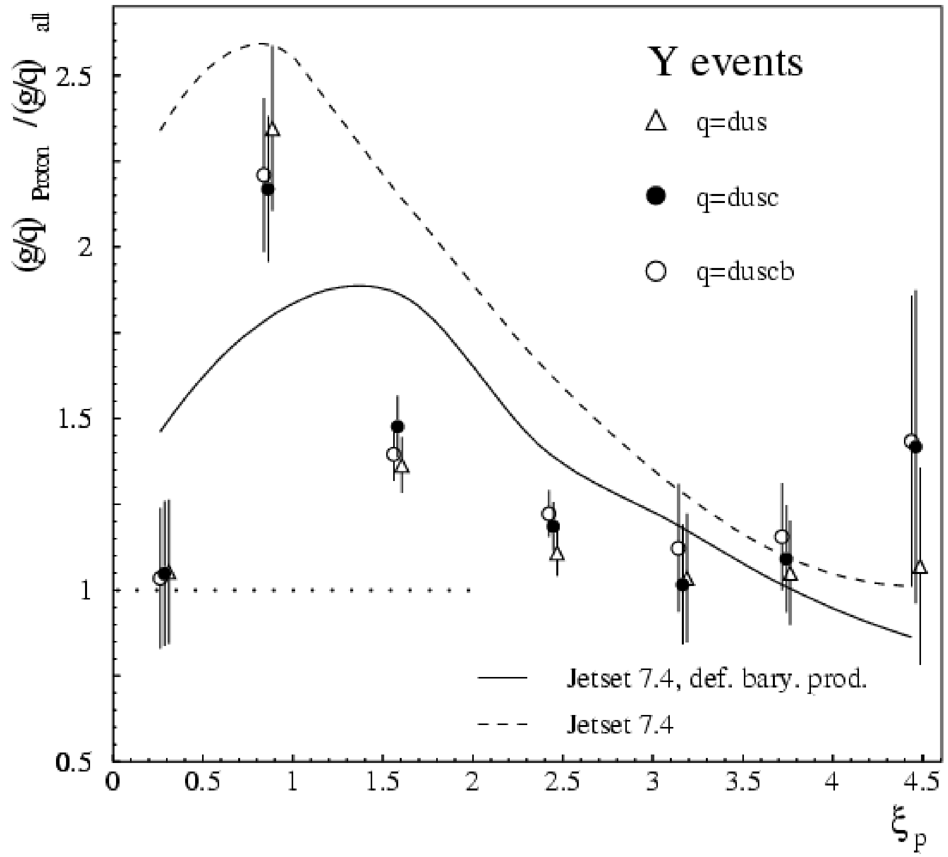



Indication of energy dependence.

# Proton Double Ratio as Function of Log Scaled Momentum

- Find extra production of leading protons (small $\xi_p$).
- Understand extra production based on "golden rule". Extra "channel" in gluon jet.
- "Leading" explains observed energy decrease of relative production rate.




Shows that hadrons are directly produced from coloured partons!

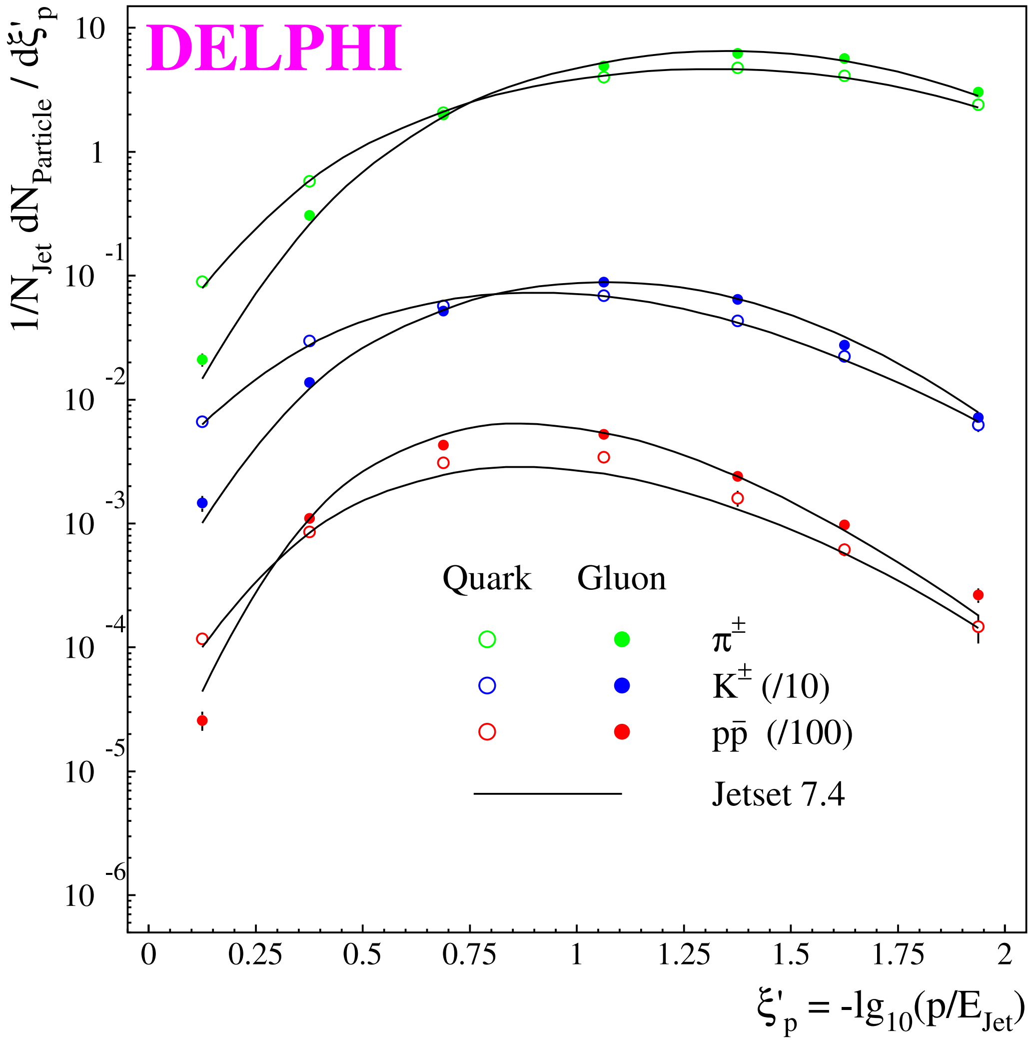
# Identified Particles: $\pi^{\pm}$,$K^{\pm}$, $p$ for Gluons and Quarks

- Here unusual: $\xi'_p$-distributions

- $\xi'_p = log_{10}(p/E_{Jet})$

  $= \xi \cdot 1/\ln(10)$

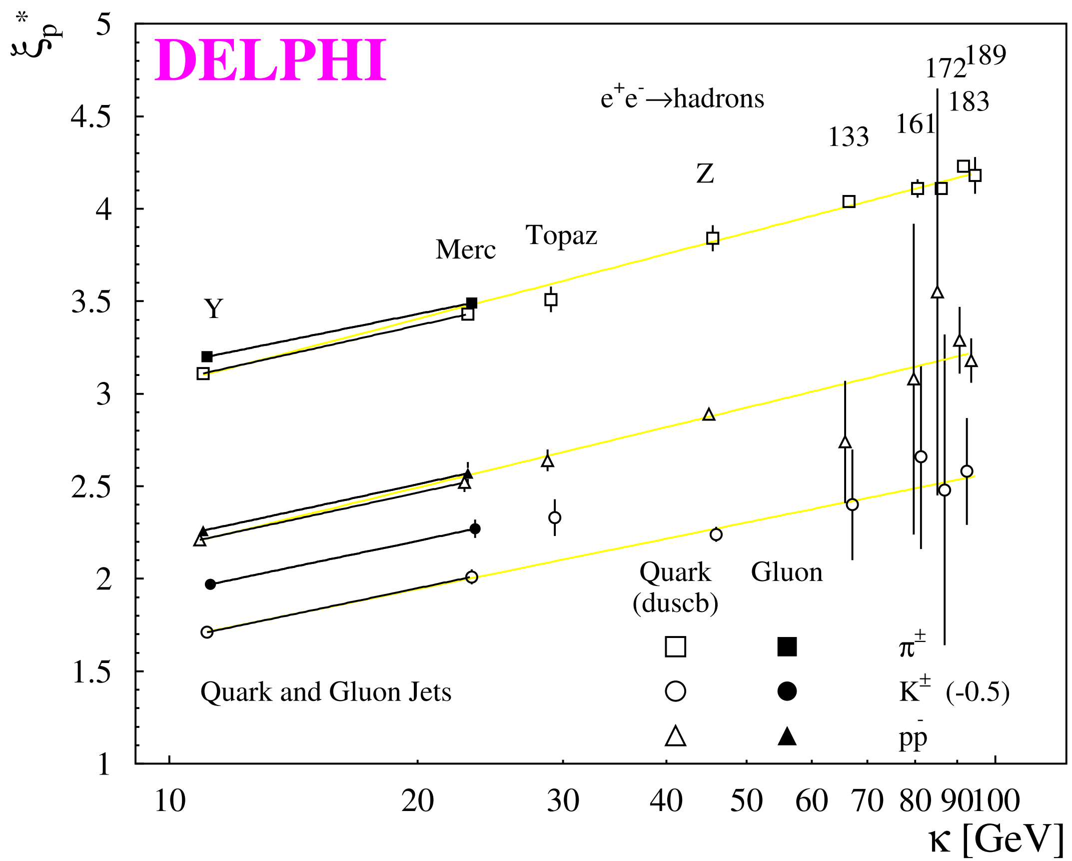
- Determine maxima; $\xi^*$ as function of $\kappa_h$.



# Maxima of $\xi$ Distributions for $g/q/e^+e^-$

- Compare g/q $\xi^*$ as function of $\kappa_H$ to $e^+e^-$.
- Naive expectation: equal for quarks and gluons.
- $\sim$ ok $\pi^\pm$ and p.
- Deviations for $K^\pm$.

→ Leading particle effect "violates" LPHD.



# Summary / Messages

- Must consider colour coherence in jet fragmentation.
- Understand dynamical variations in some detail.
- Identified particle rates show simple regularity.
- "Understand" leading particle effects, flavour dependence, models do good job.
- Very precise measurements of b-fragmentation. Topic not fully exploited!
- Gluons: Coherence mandatory - determines scales.
- Gluons: Understand scaling violations, multiplicity $N_{gg}$.
- Gluons: Extra production of leading baryons.
  Hadrons directly come from coloured partons.