TRANSVERSE SPIN TOPICAL WORKSHOP

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"A Calculation of the Nucleon Tensor Charge"

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THE TENSOR CHARGE

- Defined from the forward nucleon matrix element of the tensor current

$$\langle P, S_\perp \left| \overline{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi \right| P, S_\perp \rangle = 2\delta q^a(\mu^2) \left(P^\mu S^\nu - P^\nu S^\mu \right),$$

- To be compared with chiral even nucleon matrix element of axial current

$$\langle P, S_z \left| \overline{\psi} \gamma^\mu \gamma_5 \frac{\lambda^a}{2} \psi \right| P, S_z \rangle = 2\Delta q^a(\mu^2) S^\nu$$

$$\int_0^1 \left(\Delta q^a(x) + \Delta \overline{q}^a(x) \right) dx = \Delta q^a \qquad (1)$$

- This "charge", which is the first moments of the *quark transversity distribution functions* $\delta q^a(x)$ in principle measured in hard scattering processes

- Jaffe&Ji (PRL:1991) relate first moment to flavor components of tensor charge:

$$\int_0^1 \left(\delta q^a(x) - \delta \overline{q}^a(x) \right) dx = \delta q^a$$

Transversity: Understood in Context of Parton Model in the IMF

- Light Cone Rep. of "Good" Dirac Spinors (Kogut, Soper PRD:1970)

$$\psi_+(\xi^-) = \int d^2k_\perp dk^+ \sum_{s=1,2} \left\{ b_s(k)u(s)\mathrm{e}^{-ik^+\xi^-} + d_s^\dagger(k)v(s)\mathrm{e}^{ik^+\xi^-} \right\}$$

where $P_\pm \psi = \psi_\pm$

- Diagonalize W/R to

  ⋆ L/R HELICITY: $P_{L/R} = \frac{1}{2}\left(\mathbf{1} \mp \gamma_5\right)$

  OR

  ⋆ $\uparrow\downarrow$ TRANSVERSITY: $Q_{\uparrow\downarrow} = \frac{1}{2}\left(\mathbf{1} \mp \gamma_5\gamma^\perp\right)$

CHIRAL/HELICITY BASIS

- Helicity Basis: $P_{L/R} = \frac{1}{2}\left(\mathbf{1} \mp \gamma_5\right)$

$$\begin{aligned}
f_1(x) &= \frac{1}{2x}\int d^2k_\perp \langle P|R^\dagger(xP,k_\perp)R(xP,k_\perp) + L^\dagger(xP,k_\perp)L(xP,k_\perp)|P\rangle \\
g_1(x) &= \frac{1}{2x}\int d^2k_\perp \langle P\hat{\mathbf{e}}_z|R^\dagger(xP,k_\perp)R(xP,k_\perp) - L^\dagger(xP,k_\perp)L(xP,k_\perp)|P\hat{\mathbf{e}}_z\rangle \\
h_1(x) &= \frac{1}{x}\int d^2k_\perp \left(\langle P\hat{\mathbf{e}}_\perp|L^\dagger(xP,k_\perp)R(xP,k_\perp)|P\hat{\mathbf{e}}_\perp\rangle\right)
\end{aligned}$$

$f_1(x)$: measures quarks independent of their helicity

$g_1(x)$: measures helicity asymmetry

Meaning of $h_1(x)$ obscure

TRANSVERSITY BASIS

- Transverse Basis: $Q_{\uparrow\downarrow} = \frac{1}{2}\left(\mathbf{1} \mp \gamma_5 \gamma^\perp\right)$

$$
\begin{aligned}
f_1(x) &= \frac{1}{2x}\int d^2k_\perp \langle P|t_\uparrow^\dagger(xP,k_\perp)t_\uparrow(xP,k_\perp) + t_\downarrow^\dagger(xP,k_\perp)t_\downarrow(xP,k_\perp)|P\rangle \\
g_1(x) &= \frac{1}{x}\int d^2k_\perp \left(\langle P\hat{\mathbf{e}}_3|t_\uparrow^\dagger(xP,k_\perp)t_\downarrow xP,k_\perp)|P\hat{\mathbf{e}}_3\rangle\right) \\
h_1(x) &= \frac{1}{2x}\int d^2k_\perp \langle P\hat{\mathbf{e}}_\perp|t_\uparrow^\dagger(xP,k_\perp)t_\uparrow(xP,k_\perp) - t_\downarrow^\dagger(xP,k_\perp)t_\downarrow(xP,k_\perp)|P\hat{\mathbf{e}}_\perp\rangle
\end{aligned}
$$

$h_1(x)$: probability to find quark with spin polarized along

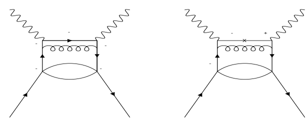
transverse spin direction minus the oppositely polarized case.

Meaning of $g_1(x)$ obscure

Remarks

- δq essential to specify *spin* structure of nucleon along with $q^a, \Delta q^a$

- However transversity distribution (and thus the tensor charge) decouple at leading twist in deep inelastic scattering

- Must be a helicity flip of struck quark in order to probe transverse spin polarization of nucleon (higher twist effect) 

- No such suppression appears in Drell-Yan where Ralston and Soper (NPB:1979) encountered transversity distribution entering transverse spin asymmetries

$$A_{TT}^{DY} = f(\theta, \phi) \frac{\sum_a e_a^2 \delta q^a(x) \overline{\delta q}^a(x)}{\sum_a e_a^2 q^a(x) \overline{q^a}(x)}$$

- However, charge is difficult to measure

- *Non-conservation* Artru Mekhfi(ZPC:1990) makes it difficult to predict

Theoretical Status of Tensor Charge

- While bounds placed on the leading twist quark distributions through positivity constraints suggest that they satisfy Soffer's inequality (PRL:1995)

$$|2\delta q^a| \leq q^a + \Delta q^a,$$

- Aside from model and lattice estimates, there $\nexists$ no definitive theoretical predictions of the tensor charge

- Unlike the axial vector charges, no sum rule has been written that enables a clear calculation that can be parameterized by data

Incomplete List of Model Calculations

- **BAG MODEL:** Jaffe & Ji (PRL:91,NPB:92)

- **QCD SUM RULES:** He & Ji (PRD:95,96), Jin & Tang (PRD:97)

- **LATTICE:** Fukugita *et al.*(PRL:95), Aoki *et al.*(PRD:97)

- **Chiral Quark Models w/Goldstone Boson:** Suzuki, Shigetani (NPA:97)

- **Spectator Quark-Diquark Models:** Jakob& Mulders, Rodrigues (NPA:97)

- **Chiral Soliton Models:**

  1) **Tensor Charge : Chiral Soliton Model**

     w/ $1/N_c$ effect, sea cal., Kim, Polyakov, Goeke(PRD:96)

  2) **Distributions and Tensor Charge: NJL Chiral Soliton Model**

     w/ $1/N_c$ effects, sea estim., Gamberg, Reinhardt, Weigel (PRD:98)

  3) **Distributions and Tensor Charge: Instanton Liquid Model**

     w/ $1/N_c$ effect, sea calc., Wakamatsu, Kubota(PRD:99,01)

     Lg.N_c limit, sea calc.,Schweitzer, Urbano, Polyakov, Weiss, Pobylitsa, Goeke (PRD:01)

MOTIVATION

1) Tensor charge doesn't mix with glue under QCD evolution

$\longrightarrow$ NON-SINGLET MATRIX ELEMENT

2) Tensor current CHARGE CONJUGATE ODD

$\star$ Vacuum being even, does not couple to quark anti-quark pair excitations

$\star$ Sea quark effects small: $\Longrightarrow \delta s \approx 0$

3) SUGGESTS: Tensor Charge is amenable to a VALENCE QUARK ANALYSIS

4) Low energy, Chiral Odd property, and valence analysis implies tensor current

dominantly couples to parity and charge conjugation odd, vector mesons.

$$\langle 0 \left| \overline{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi \right| \mathcal{M} \rangle = i f_{\mathcal{M}} \left(\epsilon_\mu k_\nu - \epsilon_\nu k_\mu \right)$$

Tensor Charge and Axial Pole Dominance:

- Realized by determining tensor charge at scale where matrix element of tensor current dominated by lowest lying axial vector mesons (Weingberg:PRL:1967)

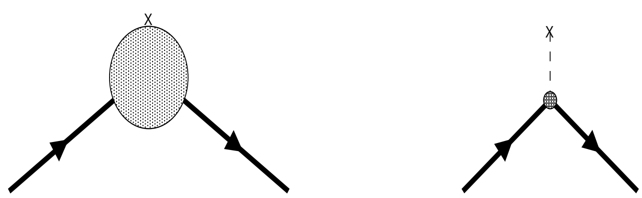
$$\sum_{\mathcal{M}} \frac{\langle 0 \left|\overline{\psi}\sigma^{\mu\nu}\gamma_5 \frac{\lambda^a}{2}\psi\right| \mathcal{M}\rangle\langle \mathcal{M}, P, S_\perp | P, S_\perp\rangle}{M_{\mathcal{M}}^2 - k^2}.$$

- Summation over those mesons with quantum numbers,

  $J^{PC} = 1^{+-}$ that couple to the nucleon via the tensor current:

  Namely, the charge conjugation odd axial vector mesons:

  $\star$ Isoscalar $h_1(1170)$, Isovector $b_1(1235)$



REQUIRE:

⋆ Vertex functions for nucleon coupling to the h_1 and b_1 mesons

$$\langle \mathcal{M} P | P \rangle = i \frac{g_{\mathcal{M}NN}}{2M_N} \overline{u} \left(P, S_\perp \right) \sigma^{\mu\nu} \gamma_5 u \left(P, S_\perp \right) \varepsilon_\mu P_\nu$$

⋆ Corresponding matrix elements of meson decay amplitudes: related to coupling of quark tensor current to mesons

$$\langle 0 \left| \overline{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi \right| \mathcal{M} \rangle = i f_{\mathcal{M}} \left(\epsilon_\mu k_\nu - \epsilon_\nu k_\mu \right)$$

- Former yield nucleon coupling constants $g_{\mathcal{M}NN}$
- Latter yield meson decay constant, $f_{\mathcal{M}}$
- For transverse polarized particles, $S^\mu = (0, S_T)$ project tensor charge using constraint on vector meson, $k \cdot \varepsilon_{\mathcal{M}} = 0$

$$\delta q^a(\mu^2) = \frac{f^a_{\mathcal{M}} g_{\mathcal{M}NN} (S_T \cdot k)^2}{2 M_N M^2_{\mathcal{M}}}$$

REMARKS:

- To determine the isoscalar and isovector meson coupling constants take hint from the valence interpretation of the tensor charge

- Exploit phenomenological mass symmetry among the lowest lying axial vector mesons ($a_1(1260), b_1(1235)$ and $h_1(1170)$)

- Approximate by adopting spin-flavor symmetry characterized by an $\mathrm{SU}(6)_{\mathrm{W}} \otimes \mathrm{O}(3)$ (Sakita,Gursey,Close) multiplet structure

- The 1^{+-}, h_1 and b_1 mesons fall into $(35 \otimes L = 1)$ multiplet containing $J^{PC} = 1^{+-}, 0^{++}, 1^{++}, 2^{++}$ **(PDG)**:

$$\begin{array}{llll} f_0(980), & a_0(980) & & \\ h_1(1170), & b_1(1235), & a_(1260), & f_1(1285), \\ f_2(1270), & a_2(1320), & \ldots & \end{array}$$

SYMMETRY RELATIONS

- Analysis enables us relate the a_1 meson decay constant measured in

  $\tau^- \to a_1^- + \nu_\tau$ decay W.Tsai:PRD(71) and g_{a_1NN} coupling constant

$$f_{a_1} = (0.19 \pm 0.03)\mathrm{GeV}^2 \,, \quad g_{a_1NN} = 7.49 \pm 1.0 \tag{2}$$

  to meson decay constants, f_{b_1} , f_{h_1} and coupling constants, g_{b_1NN} and g_{h_1NN}.

- We find

$$f_{b_1} = \frac{\sqrt{2}}{M_{b_1}} f_{a_1} = 0.20 \pm 0.03 \,, \quad g_{b_1NN} = \frac{5}{3\sqrt{2}} g_{a_1NN}$$

- The h_1 couplings related to b_1 couplings via $SU(3)$ and $SU(6)$ F/D value

$$f_{b_1} = \sqrt{3} f_{h_1} \,, \quad g_{b_1NN} = \frac{5}{\sqrt{3}} g_{h_1NN}.$$

- In turn enables determination of isovector and isoscalar contributions

$$\delta q^v = \frac{f_{b_1} g_{b_1NN} k_\perp^2}{\sqrt{2} M_N M_{b_1}^2} \,, \quad \delta q^s = \frac{f_{h_1} g_{h_1NN} k_\perp^2}{\sqrt{2} M_N M_{h_1}^2}$$

RESULTS I

- Using a Gaussian momentum distribution, and letting $\langle k_\perp \rangle$ range from $\left(0.76 \text{ to } 1.0 \text{ GeV}^2\right)$ results in the u and d quark transversity

$$\delta u(\mu^2) = (0.43 \text{ to } 0.74) \pm 0.20$$

$$\delta d(\mu^2) = -(0.26 \text{ to } 0.47) \pm 0.20$$

- u-quark tensor charge lie slightly lower than most estimates while d-quark charge is negative and of comparable magnitude

RESULTS II

- In relating $b_1(1235)$ and $h_1(1170)$ couplings, assumed that latter isoscalar was pure octet element, $h_1(8)$
- Experimentally, higher mass $h_1(1380)$ seen in the $K + \bar{K} + \pi's$ decay channel (PDG 2000) while $h_1(1170)$ was detected in the multi-pion channel
- Decay pattern indicates that higher mass state is strangeonium and decouples from lighter quarks
- Well known mixing pattern of vector meson nonet elements ω and ϕ
- If h_1 states mixed states of $SU(3)$ octet $h_1(8)$ and singlet $h_1(1)$ analogously then

$$\begin{aligned} h_1(1170) &= \sqrt{\frac{2}{3}} h_1(8) - \frac{1}{\sqrt{3}} h_1(1) \\ h_1(1380) &= \frac{1}{\sqrt{3}} h_1(8) + \sqrt{\frac{2}{3}} h_1(1) \end{aligned}$$

- Follows

$$f_{h_1(1170)} = f_{b_1} \quad \text{and} \quad g_{h_1(1170)NN} = \frac{3}{5} g_{b_1NN}$$

with $h_1(1380)$ not coupling to nucleon (for $g_{h_1(1)NN} = \sqrt{2} g_{h_1(8)NN}$)

- These symmetry relations alter results to

$$\delta u(\mu^2) = (0.58 \text{ to } 1.01) \pm 0.20$$

$$\delta d(\mu^2) = -(0.11 \text{ to } 0.20) \pm 0.20.$$

	aoki(97)	he&ji(96)	jaffe&ji(92)	suzuki& shigetani(97)	kim *et al*(95)	gamberg *et al*(98)
$\mu^2(\mathrm{GeV}^2)$	LAT(1.4)	SR(1.0)	BAG(0.9)	CQ(0.2)	QS(0.6)	NJL(0.4)
$\delta q^u(\mu^2)$	0.84	1.33	1.09	0.92	1.07	0.82
$\delta q^d(\mu^2)$	-0.23	0.04	-0.27	-0.15	-0.38	-0.07

RESULTS III

- The symmetry relations connecting b_1 couplings to a_1 couplings can be used relate isovector tensor charge to axial vector coupling g_A.

- Accomplished through a_1 dominance expression for isovector longitudinal charges derived by Birkel and Fritzsch PRD(96)

$$\Delta u - \Delta d = \frac{g_A}{g_V} = \frac{\sqrt{2} f_{a_1} g_{a_1 NN}}{M_{a_1}^2}.$$

- Thus δq^v

$$\delta u - \delta d = \frac{5}{6} \frac{g_A}{g_V} \frac{M_{a_1}^2}{M_{b_1}^2} \frac{\langle k_\perp^2 \rangle}{M_N M_{b_1}} ,$$

- Using relation between h_1 and b_1 couplings in same $SU(3)$ multiplet

$$\delta u + \delta d = \frac{3}{5} \frac{M_{b_1}^2}{M_{h_1}^2} \delta q^v ,$$

for ideally mixed singlet-octet $h_1(1170)$.

CONCLUSIONS

- Axial vector dominance model with $SU(6)_W \otimes O(3)$ coupling relations provide simple formulae for tensor charges.
- Simplicity belies the considerable subtlety of (non-perturbative) hadronic physics that is summarized in those formulae.
- Relation between δq^V and g_A/g_V is surprising. At the core of an argument relating the axial currents to the tensor currents.
- We obtain the same order of magnitude as most other calculation schemes.
- Results support view that underlying hadronic physics, while quite difficult to formulate from first principles, is essentially a 1^{+-} meson exchange process.
- Forthcoming experiments will begin to test this notion.