Sudakov Suppression of Collins Effect Asymmetries

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No experimental data on h_1 (transversity) exists
What are the realistic scenarios to obtain such data?

The Collins effect offers one such option
Going beyond tree level is necessary, but complicated

Double spin asymmetries

Double transverse spin asymmetries do not seem promising yet to extract the transversity distribution:

- $H_1^{\uparrow} + H_2^{\uparrow} \to \ell + \bar{\ell} + X$ (Drell-Yan)

  A_{TT} is shown (NLO) to be small at RHIC [future upgrade]

  (Martin, Schäfer, Stratmann & Vogelsang, PRD 60 (1999) 117502)

  Problem: h_1 for antiquarks inside a proton presumably much smaller than for quarks

- $e\, p^{\uparrow} \to \Lambda^{\uparrow}\, X$ and $p\, p^{\uparrow} \to \Lambda^{\uparrow}\, X$ [E704, RHIC]: $D_{NN} \propto h_1 H_1$

  H_1 is the transversity fragmentation function, but is another unknown function and difficult to measure:

- $e^+\, e^- \to \Lambda^{\uparrow}\, \overline{\Lambda}^{\uparrow} X$: $\propto H_1 \overline{H}_1$

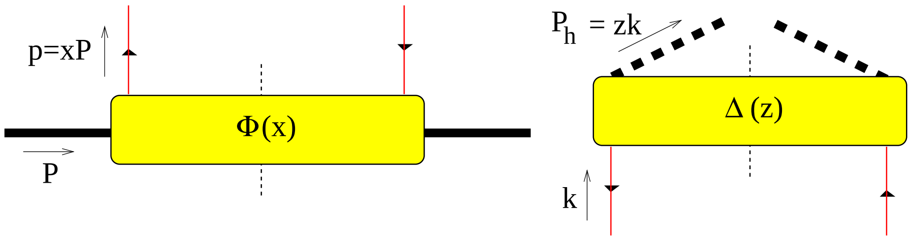
SSA options

Remaining option: single spin asymmetries

Unpolarized final state [pions are easier than hyperons]

- $\Delta(z) \to \Delta(z, \boldsymbol{k}_T)$ (measure transverse momentum of the final state hadron compared to jet)
- Produced hadrons with higher spin, e.g. ρ
  Related to $\pi^+\pi^-$ interference fragmentation functions
- Higher twist SSA ($\propto h_1 \times$ twist-3 function)

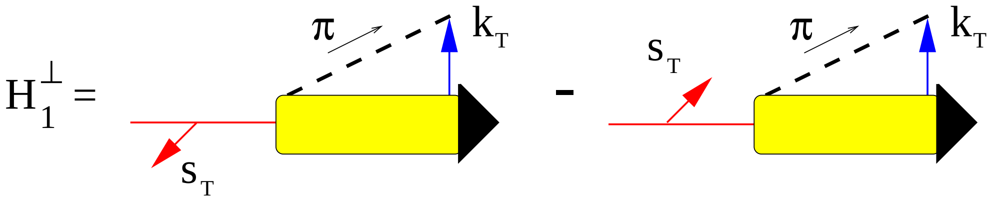
The Collins effect is $\not{k}_T \not{P}_\pi H_1^\perp(z, \boldsymbol{k}_T)/M_\pi^2$ in $\Delta(z, \boldsymbol{k}_T)$ [option 1]



SSA: Collins effect

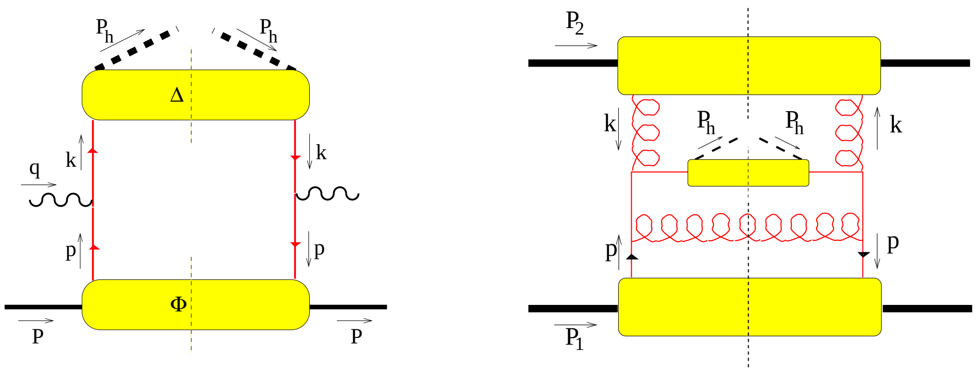
'Collins effect': the transverse spin of quarks can be transformed into orbital angular momentum during the fragmentation process

Produces particles with transverse momentum orthogonal to the spin direction of the fragmenting quark



Expected to arise due to final state interactions between π and X

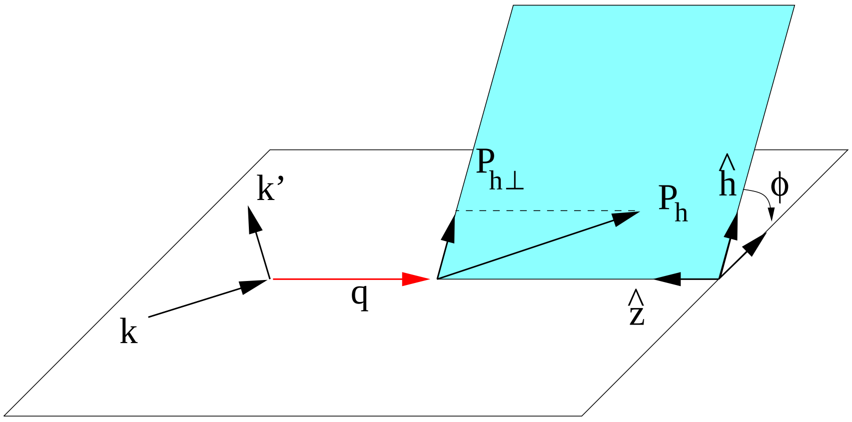
Can lead to $e\,p^\uparrow \to e'\pi\,X$ and $p\,p^\uparrow \to \pi\,X$ single spin asymmetries:



$$A_T \sim h_1(x_1)(\otimes f_1(x_2) \text{ or } g(x_2)) \otimes H_1^\perp(z, \boldsymbol{k}_T)$$

Details of Collins SSA

Collins '93: semi-inclusive DIS $e + p^{\uparrow} \to e' + \pi + X$



$$A_T = \frac{\sigma(e\, p^{\uparrow} \to e'\, \pi\, X) - \sigma(e\, p^{\downarrow} \to e'\, \pi\, X)}{\sigma(e\, p^{\uparrow} \to e'\, \pi\, X) + \sigma(e\, p^{\downarrow} \to e'\, \pi\, X)}$$

where the spin of the proton is orthogonal to the direction of γ^*

$$A_T \propto \sin(\phi_\pi^e + \phi_S^e)\, |\boldsymbol{S}_T|\; h_1\; H_1^{\perp}$$

Some experimental indications that the Collins effect is nonzero:

- SSA at low energies [HERMES, SMC]
- SSA in $p\, p^{\uparrow} \to \pi\, X$ [E704, AGS]

A. Bravar (for SMC), NPB (Proc. Suppl.) 79 (1999) 520
$A_N(\pi^+) = 11\% \pm 6\%$, $A_N(\pi^-) = -2\% \pm 6\%$ [preliminary]

Projecting out asymmetries

Consider the cross sections integrated, but weighted with a function of the transverse momentum of π:

$$\langle W \rangle \equiv \int d^2 \boldsymbol{P}_\perp^\pi \; W \, \frac{d\sigma_{P_e P_p}^{[e\, p^\uparrow \to e\, \pi\, X]}}{dx\, dy\, dz\, d\phi^e\, d^2 \boldsymbol{P}_\perp^\pi}$$

where $W = W(|\boldsymbol{P}_\perp^\pi|, \phi_\pi^e)$ (Restrict to the case of $\boldsymbol{P}_\perp^{\pi\,2} \ll Q^2$)

For $W = 1$ one obtains the integrated cross section

$$\frac{\langle 1 \rangle}{\left[4\pi\, \alpha^2\, s/Q^4\right]} = (1-y+\tfrac{1}{2}y^2) \sum_{a,\bar{a}} e_a^2\, x\, f_1^a(x) D_1^a(z)$$

If one weights with powers of the observed transverse momentum one obtains for instance the following leading order expression

$$\frac{\left\langle \sin(\phi_\pi^e + \phi_S^e)\, |\boldsymbol{P}_\perp^\pi|/M_\pi \right\rangle}{\left[4\pi\, \alpha^2\, s/Q^4\right]} = |\boldsymbol{S}_T|\, (1-y) \sum_{a,\bar{a}} e_a^2\, x\, h_1^a(x) z H_1^{\perp(1)a}(z)$$

$$H_1^{\perp(1)}(z) \equiv \int d^2\boldsymbol{k}_T \, \frac{\boldsymbol{k}_T^2}{2z^2 M_\pi^2} H_1^\perp(z, \boldsymbol{k}_T^2)$$

D.B. & Mulders, PRD 57 (1998) 5780

Evolution of h_1

There is no gluon analogue that might mix with h_1 under evolution

LO evolution of h_1:

$$\begin{aligned}\frac{\partial\, h_1(x;Q^2)}{\partial \ln Q^2} &= \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\,\left[P(x/y)\; h_1(y;Q^2)\right]\\ P(\beta) &= C_F\left[\frac{3}{2}\,\delta(1-\beta)+\frac{2\beta}{(1-\beta)_+}\right]\end{aligned}$$

Evolution known to NLO:

A. Hayashigaki, Y. Kanazawa and Y. Koike, PRD 56 (1997) 7350
S. Kumano and M. Miyama, PRD 56 (1997) 2504
W. Vogelsang, PRD 57 (1998) 1886

$h_1 \to 0$ as $Q^2 \to \infty$ (asymptotically)
$h_1(x)$ will actually first grow in magnitude towards small x

Evolution of $H_1^{\perp(1)}$

Since $H_1^{\perp(1)}$ is not of definite twist, one must perform a twist-3 evolution calculation, which is more involved than for twist-2

In the large N_c limit the evolution simplifies (DGLAP-like):

$$
\begin{aligned}
\frac{\partial\, zH_1^{\perp(1)}(z)}{\partial \ln Q^2} &= \frac{\alpha_s}{2\pi}\int_z^1 \frac{dy}{y}\left[K(z/y)\, yH_1^{\perp(1)}(y)\right] \\
K(\beta) &= \frac{N_c}{2}\left[\frac{1}{2}\,\delta(1-\beta) + \frac{3-\beta}{(1-\beta)_+}\right]
\end{aligned}
$$

(Henneman, D.B., Mulders, hep-ph/0104271; based on Belitsky, PLB 405 (1997) 312)

Both h_1 and $H_1^{\perp(1)}$ (in the large N_c limit) evolve autonomously and vanish asymptotically

<u>Side remark</u>:
The lowest anomalous dimension of $zH_1^{\perp(1)}(z)$ is positive, but this is not relevant because of the Schäfer-Teryaev sum rule

$$\int dz z H_1^{\perp(1)}(z) = 0$$

Leading order evolution

So far all studies of the Collins effect have been done at tree level

- We have the *tree level* observable

$$\mathcal{A} \equiv \frac{\left\langle \sin(\phi_\pi^e + \phi_S^e)\, |\boldsymbol{P}_\perp^\pi|/M_\pi \right\rangle}{[4\pi\,\alpha^2\, s/Q^4]}$$

$$= |\boldsymbol{S}_T|\,(1-y) \sum_{a,\bar{a}} e_a^2\, x\, h_1^a(x) z H_1^{\perp(1)}(z)$$

- LO evolution equations are known for both h_1 (NLO even) and $H_1^{\perp(1)}$ (at least in the large N_c limit; Henneman, D.B., Mulders)
Both evolve autonomously

Question: If one measures $\mathcal{A}(Q_0^2)$ and $\mathcal{A}(Q_1^2)$, can one relate them via LO evolution ?

Answer: Yes, LO corrections to the *tree level* observable $\mathcal{A}$ arise only from the evolution of h_1 and $H_1^{\perp(1)}$

Warning: nontrivial result, since not a case of collinear factorization

$d\sigma/d^2\boldsymbol{P}_\perp^\pi$ beyond tree level: soft gluon corrections do not cancel; Sudakov factors need to be taken into account

Factorization and Sudakov factors

The semi-inclusive DIS process has three dimensionful quantities:

$$M, |\boldsymbol{P}_\perp^\pi| \text{ and } Q$$

Need to consider factorization for this type of multiscale processes

$$\gamma^*(\boldsymbol{q}_T)\, p \to \pi\, X \qquad\qquad \boldsymbol{q}_T \leftrightarrow -\boldsymbol{P}_\perp^\pi/z$$

The virtual photon has a transverse momentum $\boldsymbol{q}_T$ compared to the two hadrons (p and π)

Beyond tree level: include gluon emission corrections; if $\boldsymbol{q}_T^2 \ll Q^2$, perform resummation of large logarithms, yields Sudakov factors

Collins & Soper, NPB 193 (1981) 381 [e^+e^-]
Collins, Soper & Sterman, NPB 250 (1985) 199 [DY]
Collins, NPB 396 (1993) 161 [SIDIS]

Example: Collins asymmetry

Precise nonperturbative input is unknown

- Collins function unknown; assume Gaussian form
- S_{NP} not determined from SIDIS data (despite HERA)
  Take generic S_{NP}, e.g. Ladinsky-Yuan '94

$$\frac{d\sigma(e\,p \to e'\pi X)}{dx dz dy d\phi_e d^2\boldsymbol{q}_T} \propto \{1 + |\boldsymbol{S}_T| \sin(\phi_\pi^e + \phi_S^e)\, A(\boldsymbol{q}_T)\}$$

The asymmetry analyzing power is given by

$$A(\boldsymbol{q}_T) = \frac{\sum_a \; e_a^2 \; B(y)\; h_1^a(x) H_1^{\perp(1)a}(z)}{\sum_b \; e_b^2 \; A(y)\; f_1^b(x) D_1^b(z)} \mathcal{A}(Q_T)$$

$$\mathcal{A}(Q_T) = M_\pi \frac{\int db\, b^2\, J_1(bQ_T)\, \exp\left(-S(b_*) - S_{NP}(b)\right)}{\int db\, b\, J_0(bQ_T)\, \exp\left(-S(b_*) - S_{NP}(b)\right)}$$

$b_* = b/\sqrt{1+b^2/b_{\max}^2}$, $A(y) = (1-y+\frac{1}{2}y^2)$, $B(y) = (1-y)$

At tree level: $\mathcal{A}^{(0)}(Q_T) = M_\pi Q_T \dfrac{R^4}{R_u^2}\, e^{-(R^2-R_u^2)Q_T^2/2}$

D.B., Nucl. Phys. B 603 (2001) 195 [hep-ph/0102071]

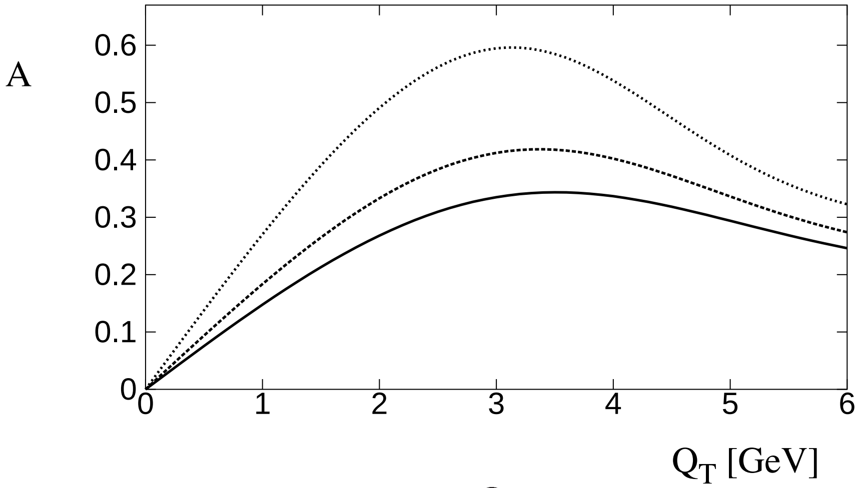
Numerical estimates

$\mathcal{A}(Q_T)$

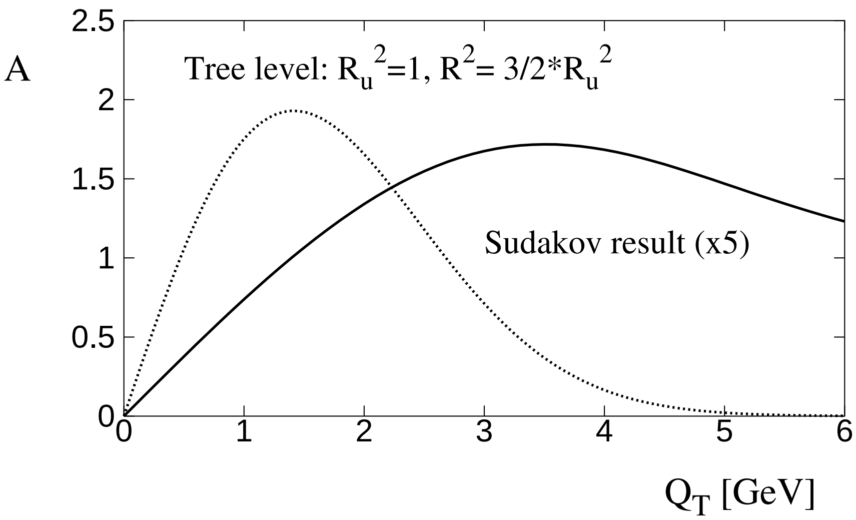
$Q = 30$ GeV (upper)

$Q = 60$ GeV (middle)

$Q = M_Z$ (lower)

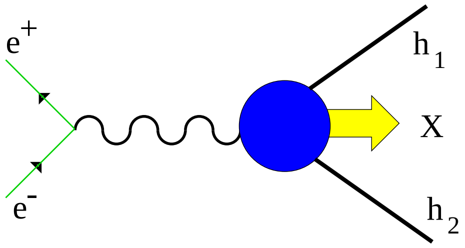


The asymmetry factor decreases with increasing Gaussian width
$\max(\mathcal{A}(Q_T, Q)) \sim Q^{-0.5} - Q^{-0.6}$ (partial power suppression)



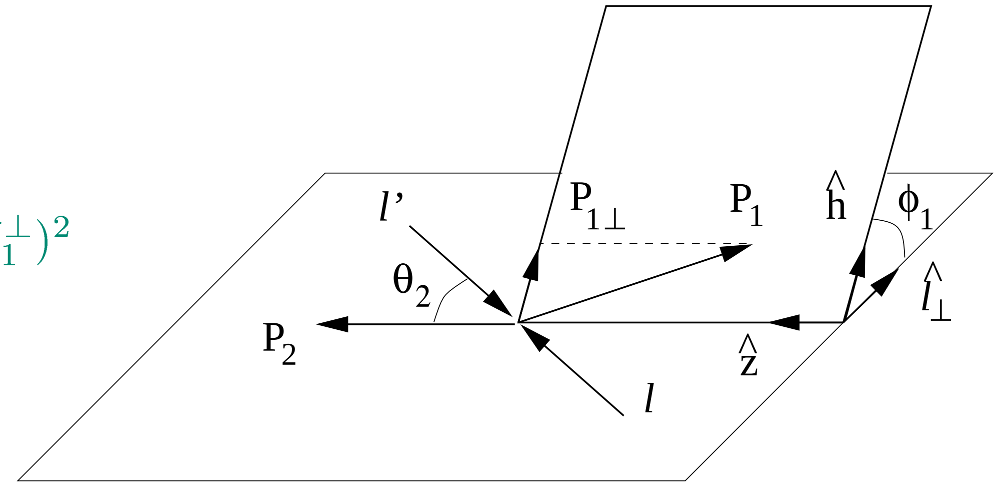
$\mathcal{A}(Q_T, Q = 90)$ is considerably smaller and broader than $\mathcal{A}^{(0)}$
Hence, tree level estimates tend to overestimate transverse momentum dependent azimuthal spin asymmetries

Collins effect in $e^+ e^- \to \pi^+ \pi^- X$



$$e^+ + e^- \to \pi^+ + \pi^- + X$$

$$\langle \cos(2\phi_1) \rangle \propto (H_1^\perp)^2$$



D.B., Jakob & Mulders, NPB 504 (97) 345; PLB 424 (98) 143

A first indication of a nonzero correlation comes from a preliminary analysis of the 91-95 LEP1 data (DELPHI) by Efremov, Smirnova & Tkatchev, NPB (Proc. Suppl.) 79 (1999) 554

Possibilities at BELLE are currently being examined

Asymmetric background from hard gluon radiation ($Q_T \sim Q$) and weak decays

Sudakov suppression

Warning: tree level asymmetry expression not sufficient

$$\frac{d\sigma(e^+e^- \to \pi^+\pi^- X)}{d\Omega dz_1 dz_2 d^2\boldsymbol{q}_T} \propto \{1 + \cos(2\phi_1) A(\boldsymbol{q}_T)\}$$

with ($\boldsymbol{q}_T^2 \equiv Q_T^2 \ll Q^2$)

Including Sudakov factors:

$$A(\boldsymbol{q}_T) = \frac{\sum_a \; c_2^a \; B(y) \; H_1^{\perp(1)a}(z_1) \, \overline{H}_1^{\perp(1)a}(z_2)}{\sum_a \; c_1^a \; A(y) \;\; D_1^a(z_1) \, \overline{D}_1^a(z_2)} \; \mathcal{A}(Q_T)$$

where

$$\mathcal{A}(Q_T) \equiv M^2 \frac{\int db\, b^3\, J_2(bQ_T) \,\exp\left(-S(b_*) - S_{NP}(b)\right)}{\int db\, b\, J_0(bQ_T) \,\exp\left(-S(b_*) - S_{NP}(b)\right)}$$

Surprisingly there is also no determination of $S_{NP}(b)$ for $e^+ \, e^- \to A \; B \; X$ (despite LEP), so for illustration purposes again Ladinsky-Yuan's $S_{NP}(b)$

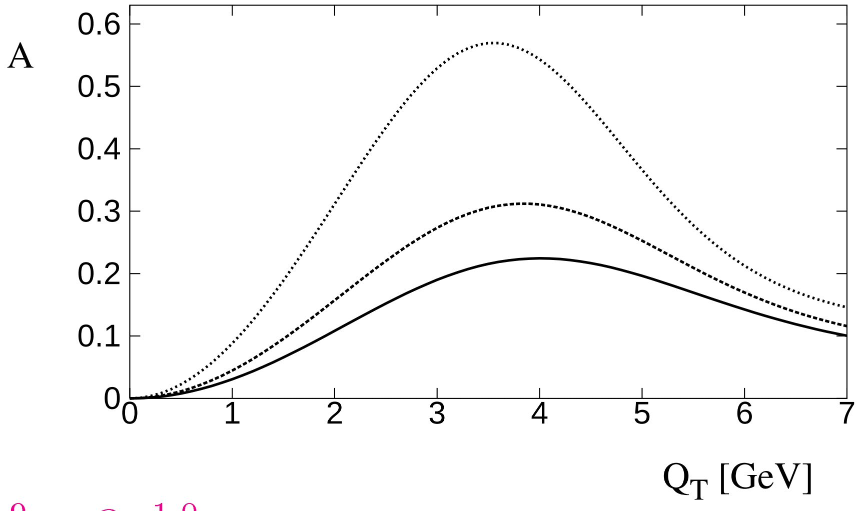
Sudakov suppression

$\mathcal{A}(Q_T)$

$Q = 30$ GeV (upper)

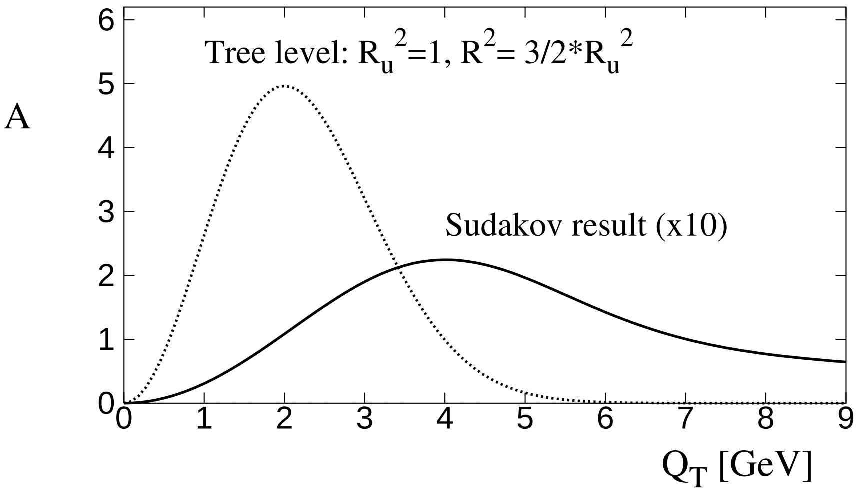
$Q = 60$ GeV (middle)

$Q = M_Z$ (lower)



$\max(\mathcal{A}(Q_T, Q)) \sim Q^{-0.9} - Q^{-1.0}$

Conclusions: (Collins effect)^2 decreases faster with Q^2



Sudakov factors produce order of magnitude suppression at $Q = M_Z$

SSA: Interference Fragmentation Functions

Jaffe, Jin, Tang ('98): Collins effect possibly averages to zero

$\sum_X$ averages final state interaction effects to zero

Use $|(\pi^+\pi^-)_{\text{out}}X\rangle$ instead (π^+ and π^- belong to same jet)

See also: Ji ('94); Collins, Heppelmann, Ladinsky ('94)

Phase shift of the $(\pi^+\pi^-)$ system remains after summation over X
Interference between different partial waves, hence the name

Parameterize the fragmentation of quarks into two pions, then the transversity analogue is called $\delta\hat{q}_I$

Again this would lead to single spin asymmetries:

$$
\begin{array}{rl}
e\,p^\uparrow \to e'\,(\pi^+\pi^-)\,X & \qquad h_1 \otimes \delta\hat{q}_I \\
p\,p^\uparrow \to (\pi^+\pi^-)\,X & \qquad f_1/g \otimes h_1 \otimes \delta\hat{q}_I
\end{array}
$$

Unlike Collins effect: collinear factorization. Very promising!

SSA: Interference Fragmentation Functions

Evolution of $\delta\hat{q}_I(z)$ same as for $H_1(z)$; LO evolution:

$$\begin{aligned} \frac{\partial\, z\delta\hat{q}_I(z)}{\partial \ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi}\int_z^1 \frac{dy}{y}\,\left[P(z/y)\, y\delta\hat{q}_I(y)\right] \\ P(\beta) &= C_F\left[\frac{3}{2}\,\delta(1-\beta)+\frac{2\beta}{(1-\beta)_+}\right] \end{aligned}$$

Collinear factorization, hence soft gluon contributions cancel:
no Sudakov factors, only functions of lightcone momentum fractions

Theoretically very clean; LO analysis ($\mathcal{O}(\alpha_s)$) can be done already
NLO analysis very similar to that of A_{TT}^{DY} ($zR_T = z^+k^- - z^-k^+$):

$$\begin{aligned} A_{TT}^{DY} &\propto \cos(\phi^e_{S_{1T}}+\phi^e_{S_{2T}})\; h_1\,\bar{h}_1 \\ A_T^{\pi^+\pi^-} &\propto \cos(\phi^e_{S_T}+\phi^e_{R_T})\; h_1\,\delta\hat{q}_I \end{aligned}$$

For extraction of the interference fragmentation functions use

$$e^+\,e^- \;\to\; (\pi^+\,\pi^-)_{\text{jet}\,1}\,(\pi^+\,\pi^-)_{\text{jet}\,2}\,X \propto \cos(\phi^e_{R_{1T}}+\phi^e_{R_{2T}})\,(\delta\hat{q}_I)^2$$

Artru & Collins, ZPC 69 (1996) 277

Again possible at BELLE; no expected asymmetric background

Conclusions

- Double transverse spin asymmetries not promising as yet
- Single transverse spin asymmetries combined with electron-positron data (off-resonance at B-factories) are the most realistic scenarios
- Going beyond tree level is necessary and possible too
- Collins effect asymmetries are quite involved beyond tree level (not collinear factorization; Sudakov factors; background issues)
  $\mathcal{O}(\alpha_s)$ is already difficult (large N_c doable though)
- Interference fragmentation functions are most promising, appears to be possible (collinear factorization, no background issues)
  $\mathcal{O}(\alpha_s^2)$ is doable, analogous to A_{TT}^{DY}
- Transversity measurements are possible [COMPASS,HERMES,RHIC,...] maybe even a flavor separation