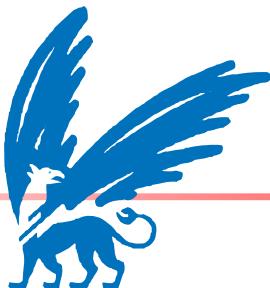
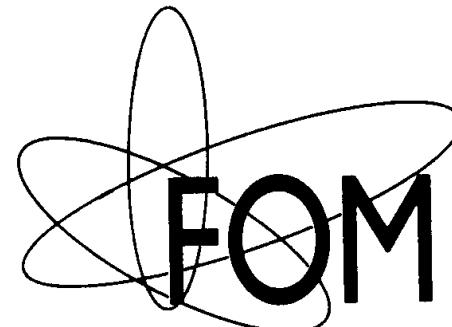


FRAGMENTATION INTO SPIN-1 HADRONS

A. Bacchetta



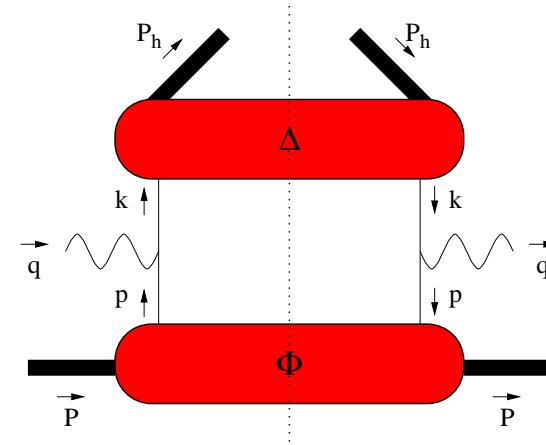
vrije Universiteit *amsterdam*



- ➔ Semi-inclusive DIS
- ➔ Fragmentation to spin-1 hadrons
- ➔ Transversity measurements
- ➔ Positivity bounds and modeling attempts

Semi-inclusive DIS

$$d\sigma(l + H \rightarrow l' + h + X) \propto L_{\mu\nu} W^{\mu\nu}$$



$$2MW^{\mu\nu} \propto \text{Tr} [\Phi(x_B) \gamma^\mu \Delta(z_h) \gamma^\nu]$$

$$x_B = \frac{Q^2}{2P \cdot q} \quad z_h = \frac{2P_h \cdot q}{Q^2}$$

$$\Phi(x) = \frac{1}{2} \int d^2 p_T dp^- \Phi(p, P) \Big|_{p^+ = xP^+}$$

$$\Delta(z) = \frac{z}{4} \int d^2 k_T dk^+ \Delta(k, P_h) \Big|_{k^- = \frac{P_h^-}{z}}$$

Distribution functions for spin-1/2

From decomposition of Φ correlation function.

$$f_1 = \text{blue circle} \rightarrow \text{red dot}$$
$$g_1 = \left(\text{blue circle} \rightarrow \dots \rightarrow \text{red dot} \right) - \left(\text{blue circle} \leftarrow \dots \rightarrow \text{red dot} \right)$$
$$h_1 = \left(\begin{array}{c} \text{blue circle} \\ \uparrow \\ \downarrow \end{array} \rightarrow \text{red dot} \right) - \left(\begin{array}{c} \text{blue circle} \\ \downarrow \\ \uparrow \end{array} \rightarrow \text{red dot} \right)$$

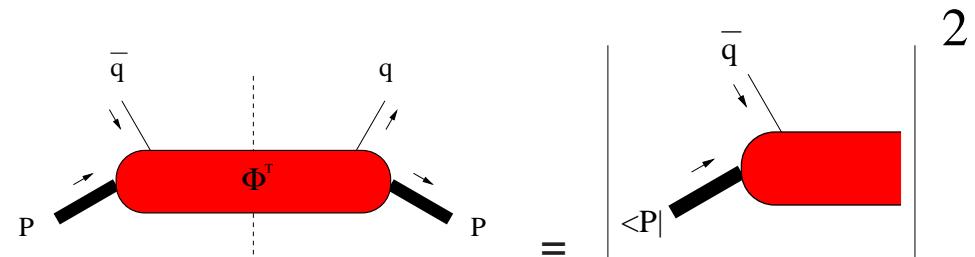
χ -odd

Quark+target spin structure

Full scattering matrix $M = (\Phi \gamma^+)^T$

for $\bar{q} + P \rightarrow X$

in parton \otimes hadron helicity spaces



$$\left(\begin{array}{cccc} f_1 + g_1 & 0 & 0 & 2h_1 \\ 0 & f_1 - g_1 & 0 & 0 \\ 0 & 0 & f_1 - g_1 & 0 \\ 2h_1 & 0 & 0 & f_1 + g_1 \end{array} \right) \quad \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \quad \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array}$$

The matrix elements correspond to the four possible combinations of quark and gluon helicities. Below the matrix, four diagrams show the quark-gluon vertex for each element. Each diagram consists of a yellow circle with a red dot representing a quark, and two arrows indicating gluon helicity. The first row shows a quark with spin up and a gluon with spin up-right. The second row shows a quark with spin up and a gluon with spin down-left. The third row shows a quark with spin down and a gluon with spin up-right. The fourth row shows a quark with spin down and a gluon with spin down-left.

Soffer bound

Positive definiteness of this matrix requires:

Positivity of diagonal elements \Rightarrow trivial bounds

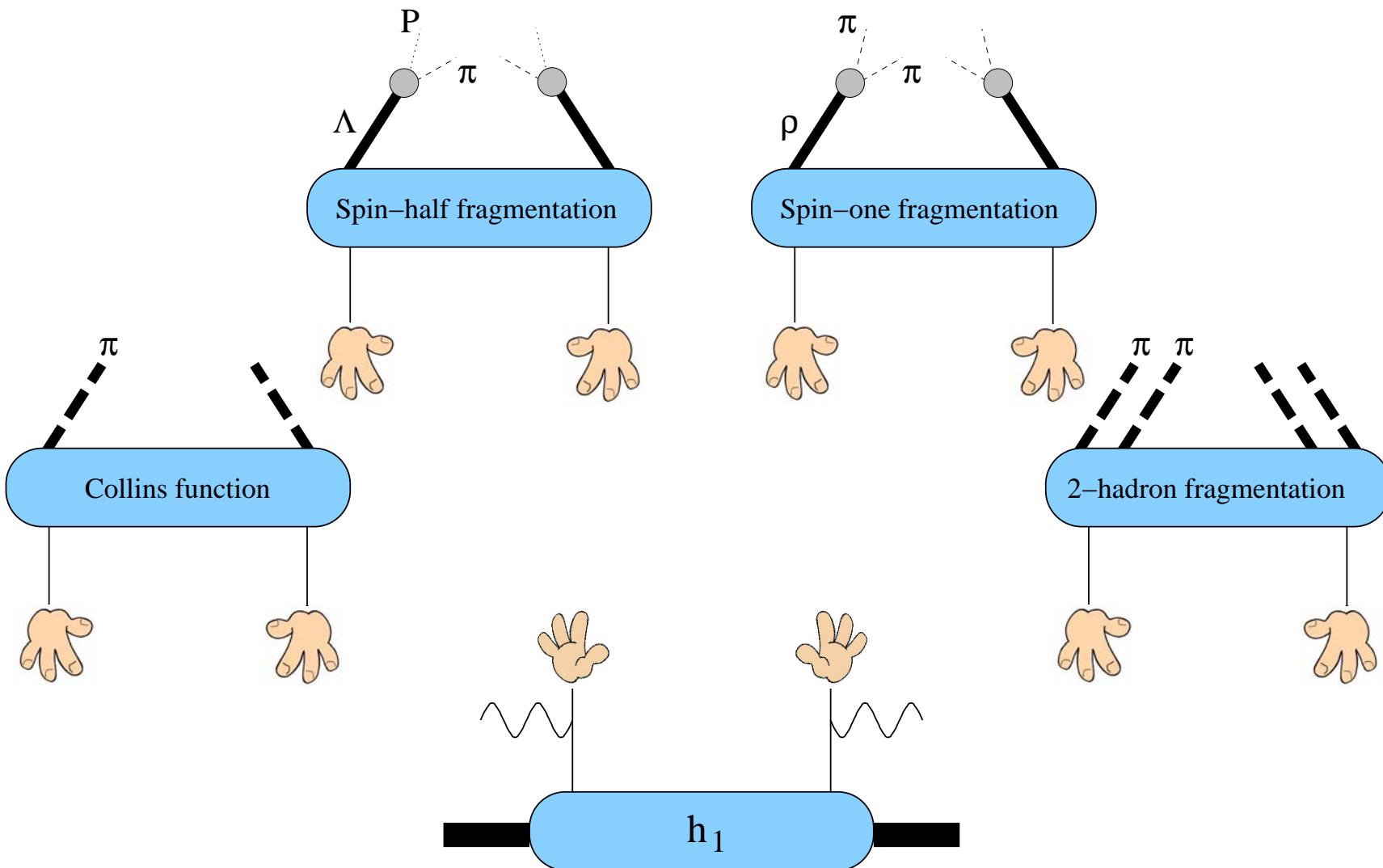
$$\textcolor{red}{f}_1(x) \geq 0$$

$$|\textcolor{red}{g}_1(x)| \leq \textcolor{red}{f}_1(x)$$

Positivity of eigenvalues \Rightarrow less trivial (**Soffer**) bound

$$|\textcolor{red}{h}_1(x)| \leq \frac{1}{2} (\textcolor{red}{f}_1(x) + \textcolor{red}{g}_1(x))$$

Chiral-odd partners for the transversity distribution



Fragmentation to spin-1 hadrons

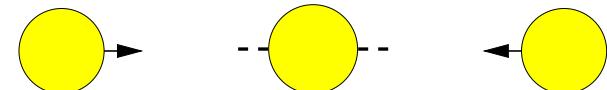
We need to measure the polarization of the outgoing hadron by means of **VECTOR AND TENSOR ANALYZING POWERS** of the decay

$$\Delta_{ij}(k, P_h, A_h^i, A_h^{ij}) = \int \frac{d^4\xi}{(2\pi)^4} e^{-ik\cdot\xi} \langle 0 | \psi_i(0) | P_h, A_h^i, A_h^{ij} \rangle \langle P_h, A_h^i, A_h^{ij} | \bar{\psi}_j(\xi) | 0 \rangle$$

or as a matrix in the hadron's helicity space

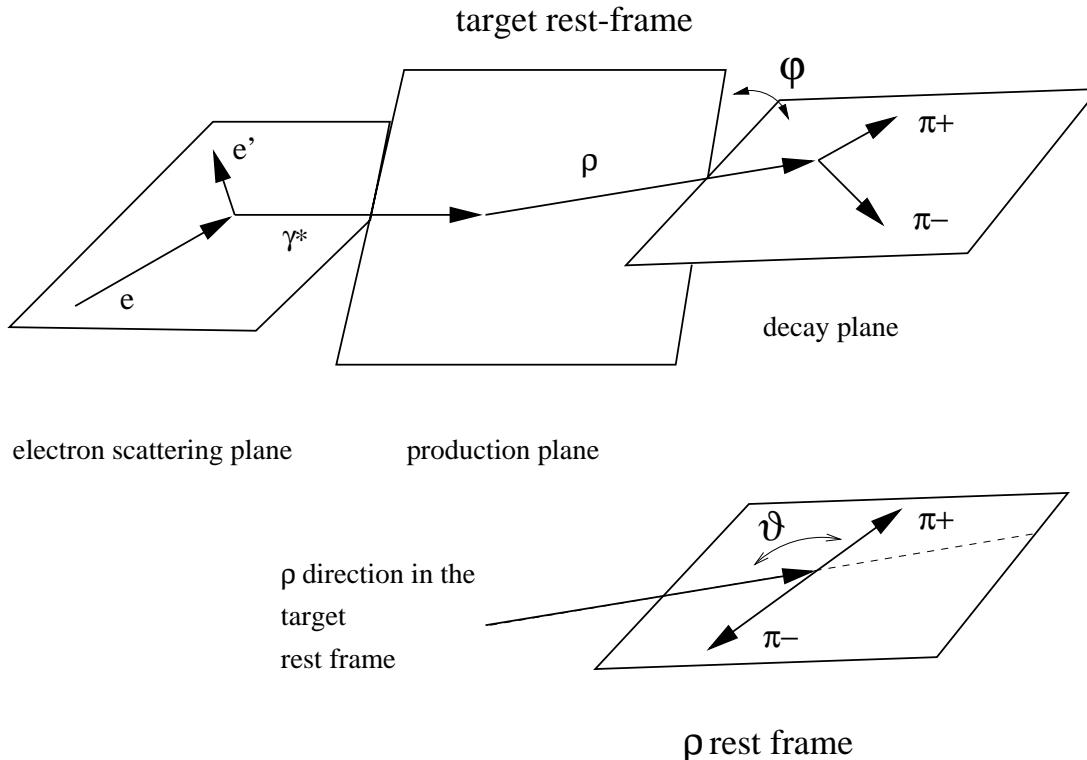
$$\Delta_{ij;\Lambda\Lambda'}(k, P_h) = \int \frac{d^4\xi}{(2\pi)^4} e^{-ik\cdot\xi} \langle 0 | \psi_i(0) | P_h, \Lambda \rangle \langle P_h, \Lambda' | \bar{\psi}_j(\xi) | 0 \rangle$$

where Λ and Λ' are hadron's helicities (1,0,-1)



$$\Delta(k, P_h, A_h^i, A_h^{ij}) = \text{Tr} \left[R_h^{(\text{decay})}(A_h^i, A_h^{ij}) \Delta(k, P_h) \right]$$

Analyzing powers



$$A_{LL} = \frac{1}{3} (\cos^2 \theta + \cos 2\theta)$$

$$A_{LT}^x = -\sin 2\theta \cos \varphi$$

$$A_{TT}^{xx} = -\sin^2 \theta \cos 2\varphi$$

$$A_{LT}^y = -\sin 2\theta \sin \varphi$$

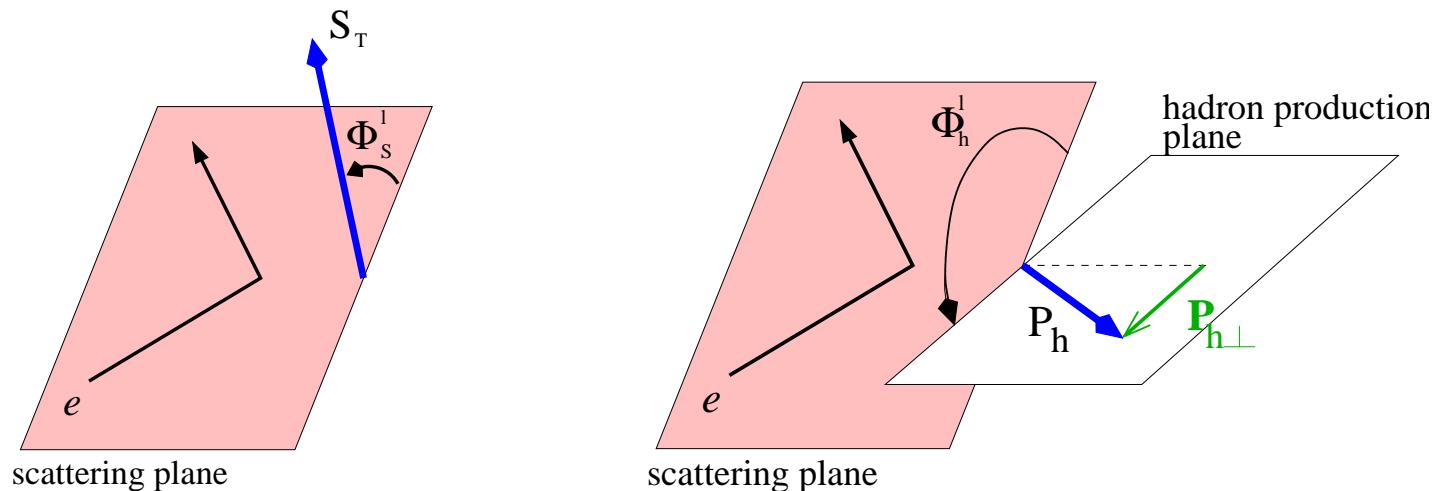
$$A_{TT}^{xy} = -\sin^2 \theta \sin 2\varphi$$

Transversity and asymmetries

$$d\sigma^{\uparrow} - d\sigma^{\downarrow} = f(\phi_S^\ell, |P_{h\perp}|, \phi_h^\ell, x_B, z_h)$$

$$\left\langle \begin{array}{c} \text{Weight} \\ \text{Function} \end{array} \right\rangle_T (x_B, z_h) = \int d\phi_S^\ell \, d|P_{h\perp}| \, d\phi_h^\ell \, \text{Weight Function} (d\sigma^{\uparrow} - d\sigma^{\downarrow})$$

$\propto h_1(x_B) \times ?(z_h)$

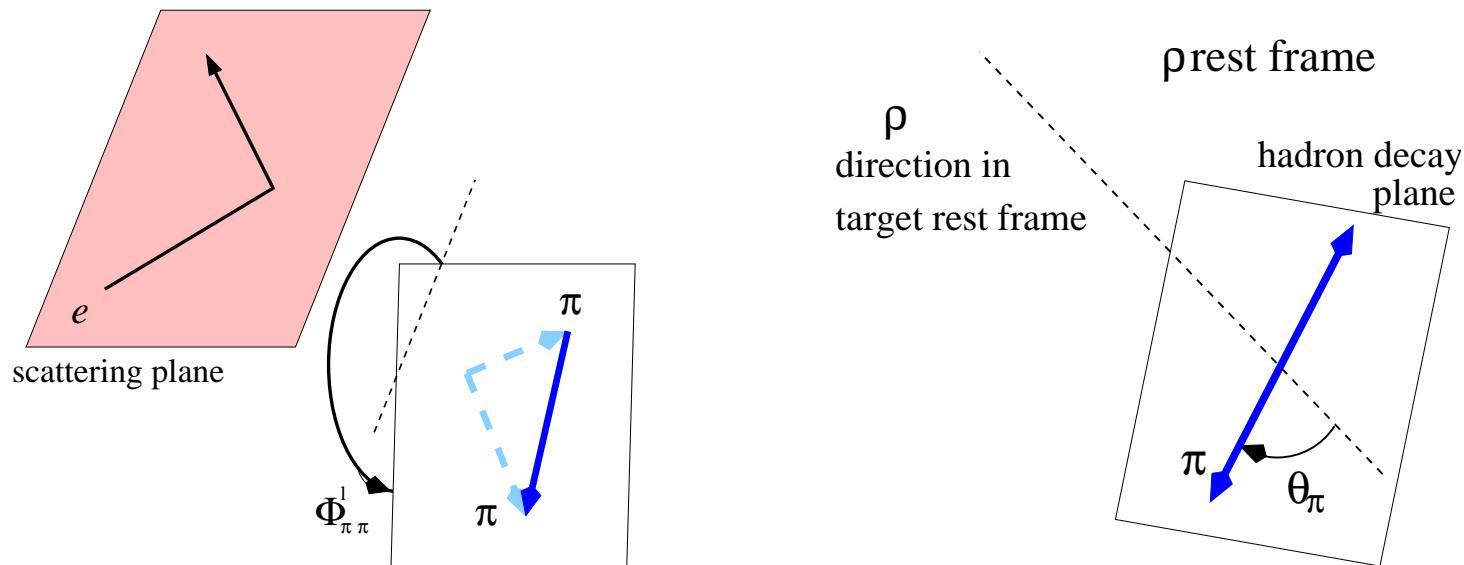


Asymmetries in spin-1 production

$$d\sigma^{\uparrow} - d\sigma^{\downarrow} = f(\theta_{\pi}, \phi_{\pi\pi}^{\ell}, \phi_S^{\ell}, |P_{\rho\perp}|, \phi_{\rho}^{\ell}, x_B, z_{\rho})$$

$$\left\langle \begin{array}{c} \text{Weight} \\ \text{Function} \end{array} \right\rangle_T (x_B, z_{\rho}) = \int d\theta_{\pi} d\phi_{\pi\pi}^{\ell} d\phi_S^{\ell} d|P_{\rho\perp}| d\phi_{\rho}^{\ell} \begin{array}{c} \text{Weight} \\ \text{Function} \end{array} (d\sigma^{\uparrow} - d\sigma^{\downarrow})$$

$\propto h_1(x_B) \times \boxed{?}(z_{\rho})$



Asymmetries containing the transversity distribution

$$\left\langle \sin(\phi_{\pi\pi}^\ell + \phi_S^\ell) \right\rangle_T \propto h_1(x_B) H_{1LT}(z_\rho)$$

$\chi\text{-odd}$
 $T\text{-odd}$

- Requires measurement of $\phi_{\pi\pi}^\ell, \phi_S^\ell, x_B, z_\rho$
- No complications due to presence of hadron transverse momentum

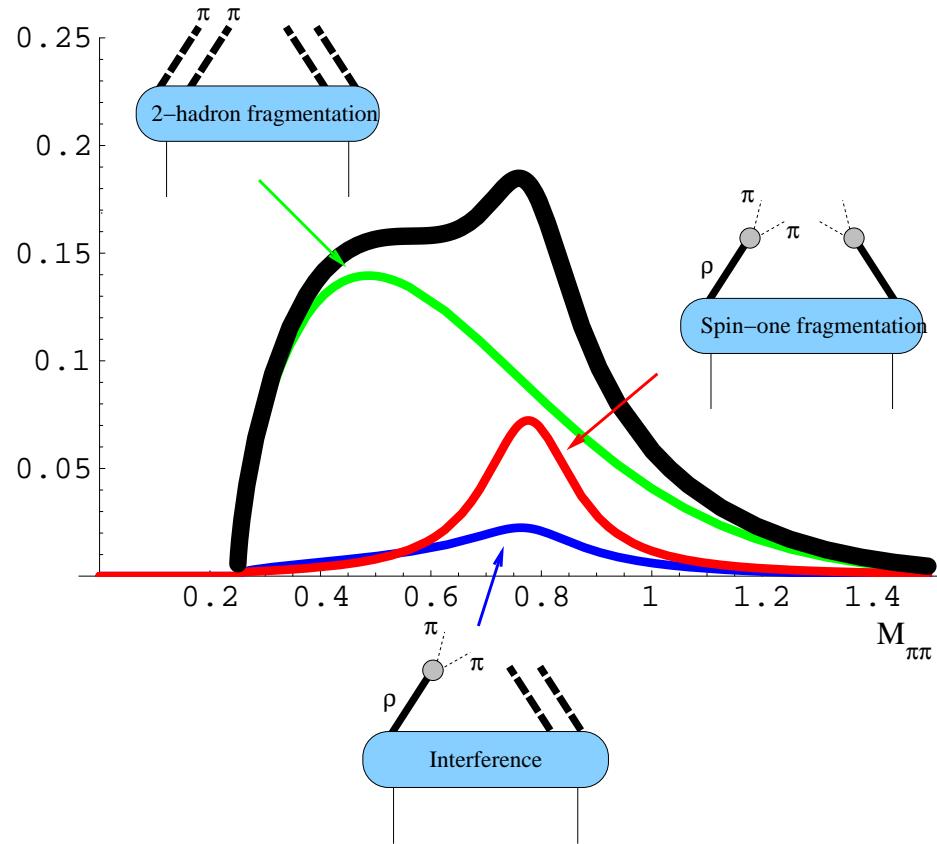
$$\left\langle \frac{|P_{\rho\perp}|}{2 z M_h} \sin(2\phi_{\pi\pi}^\ell + \phi_S^\ell - \phi_\rho^\ell) \right\rangle_T \propto h_1(x_B) H_{1TT}^{(1)}(z_\rho)$$

$$\left\langle \frac{|P_{\rho\perp}|}{2 z M_h} \sin(\phi_\rho^\ell + \phi_S^\ell) \right\rangle_T \propto h_1(x_B) H_{1LL}^{\perp(1)}(z_\rho)$$

$$\left\langle \frac{|P_{\rho\perp}|^2}{2 z^2 M_h^2} \sin(\phi_{\pi\pi}^\ell - \phi_S^\ell - 2\phi_\rho^\ell) \right\rangle_T \propto h_1(x_B) H_{1LT}^{\perp(2)}(z_\rho)$$

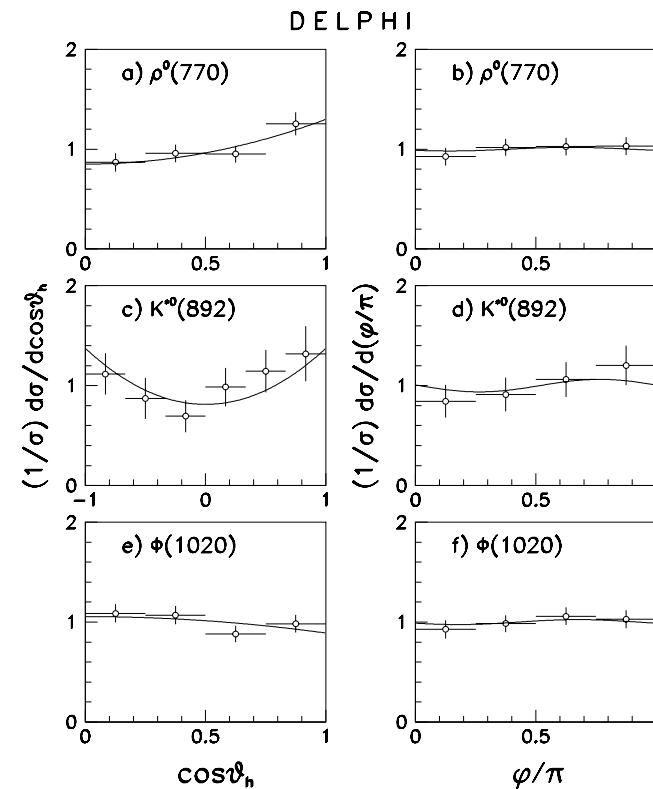
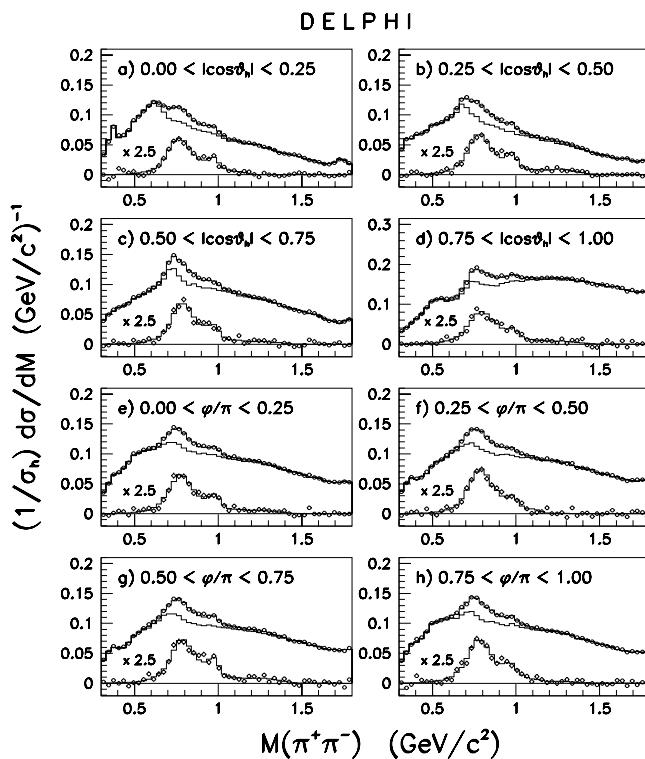
Where to observe these functions?

Mass spectrum of pion couples



Measurements in $e^+ e^-$ annihilation

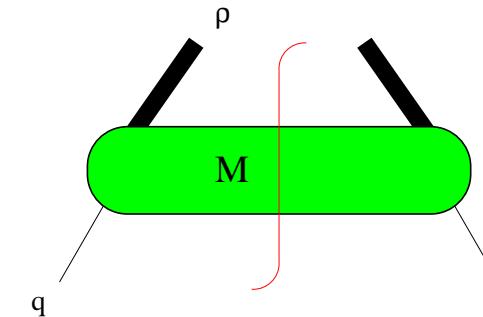
Delphi collaboration Phys. Lett. B 406 (1997) 271



Measurement of decay distribution seems to be feasible.

Quark+hadron spin structure

Full scattering matrix M for $q \rightarrow \rho + X$



$$\left(\begin{array}{cccccc} D_1 + G_1 - \frac{D_{1LL}}{3} & 0 & 0 & 0 & \sqrt{2}(H_1 + iH_{1LT}) & 0 \\ 0 & D_1 + \frac{2D_{1LL}}{3} & 0 & 0 & 0 & \sqrt{2}(H_1 - iH_{1LT}) \\ 0 & 0 & D_1 - G_1 - \frac{D_{1LL}}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_1 - G_1 - \frac{D_{1LL}}{3} & 0 & 0 \\ \sqrt{2}(H_1 - iH_{1LT}) & 0 & 0 & 0 & D_1 + \frac{2D_{1LL}}{3} & 0 \\ 0 & \sqrt{2}(H_1 + iH_{1LT}) & 0 & 0 & 0 & D_1 + G_1 - \frac{D_{1LL}}{3} \end{array} \right) \quad \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array}$$

Below the matrix, six small diagrams show the initial quark state (spin up-right) and final hadron states (rho meson and two spin-1 hadrons X) corresponding to each row.

Positivity bounds

Positive definiteness of this matrix requires:

A.B., P.J. Mulders, hep-ph/0104176

Positivity of diagonal elements \Rightarrow trivial bounds

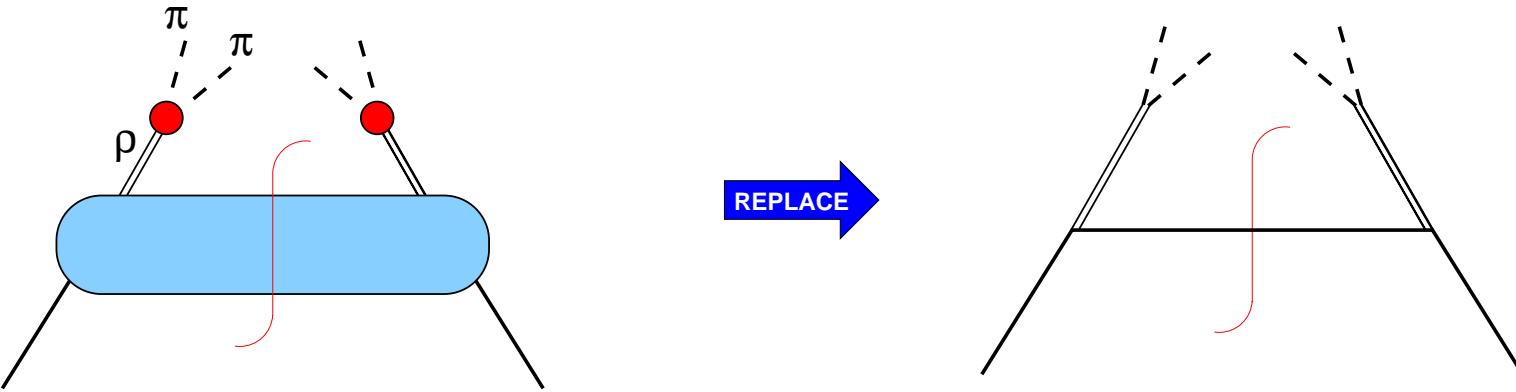
$$\begin{aligned} D_1(z) &\geq 0 \\ -\frac{3}{2} D_1(z) &\leq D_{1LL}(z) \leq 3 D_1(z) \\ |G_1(z)| &\leq D_1(z) - \frac{1}{3} D_{1LL}(z) \leq \frac{3}{2} D_1(z) \end{aligned}$$

Positivity of 2-dimensional minors \Rightarrow (Soffer-like) bound

$$[H_{1LT}(z)]^2 \leq \left[D_1(z) + \frac{2}{3} D_{1LL}(z) \right] \left[D_1(z) - \frac{1}{3} D_{1LL}(z) \right] \leq \frac{9}{2} [D_1(z)]^2.$$

Generating non-zero T-odd functions

Take a **simple model** for the fragmentation process



$$\frac{i \left(-\gamma^{\mu\nu} + \frac{p^\mu p^\nu}{p^2} \right)}{p^2 - m_\rho^2 + i m_\rho \Gamma_\rho}$$

A Feynman diagram for the fragmentation of a ρ meson into two pions, π_1 and π_2 . The incoming ρ meson line is labeled p and the outgoing pion lines are labeled π_1 and π_2 . A horizontal black line labeled q represents the fragmentation source. Red arrows indicate loop corrections: one arrow goes from the ρ meson line to the q line, and another goes from the q line to the pion lines. The loop correction to the fragmentation function is given by $f_{\rho\pi\pi}(P_1 - P_2)^\mu$. The loop correction to the source is given by $f_{q\rho q} \gamma^\mu$.

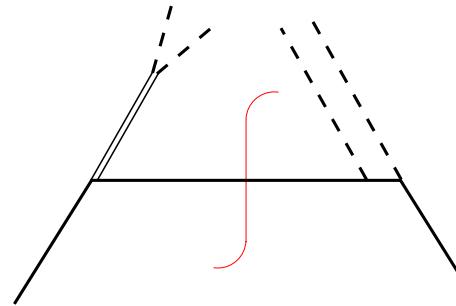
Generating non-zero T-odd functions

Add one-loop corrections (see also talk by Rajen Kundu)



T-odd functions $\neq 0$

“Competing” or “complementary” mechanism: interference fragmentation functions (see talk by Marco Radici)



Conclusions

- Fragmentation to spin-1 hadron is a viable option for transversity measurements, at the same level as Collins function and interference fragmentation functions.
- Since polarimetry on the final hadron requires the observation of two decay products, it represents a specific contribution to the more general case of two hadron production.
- Information on the unknown fragmentation functions can be extracted from positivity bounds and simple models.