QCD at e^+e^- Experiments

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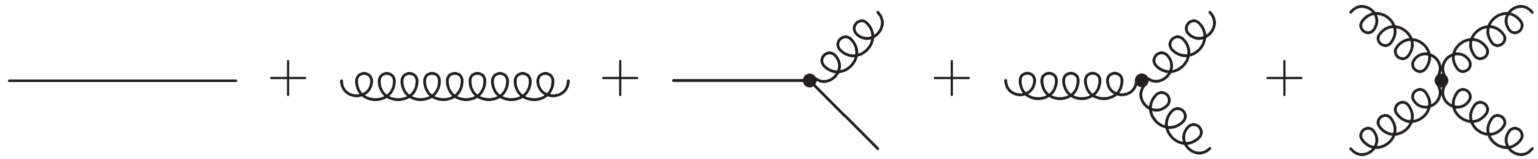
Setting the Scene – The QCD Lagrangian

QCD Lagrangian – determined from $SU(3)_c$ invariance

$\mathcal{L} =$ 

Free parameters: coupling and quark masses

Aim: measure parameters and test properties of the QCD Lagrangian

- verify basic diagrams and their relative coupling strength
  $\Longleftrightarrow$ QCD Colour Factors $\Longleftrightarrow$ verify $SU(3)_c$ gauge group
- test quantum corrections (loops) causing asymptotic freedom & confinement
  $\Longleftrightarrow$ running of α_s (& quark–masses),
- study rich phenomenology of strong interaction physics

Modeling the Fragmentation Process

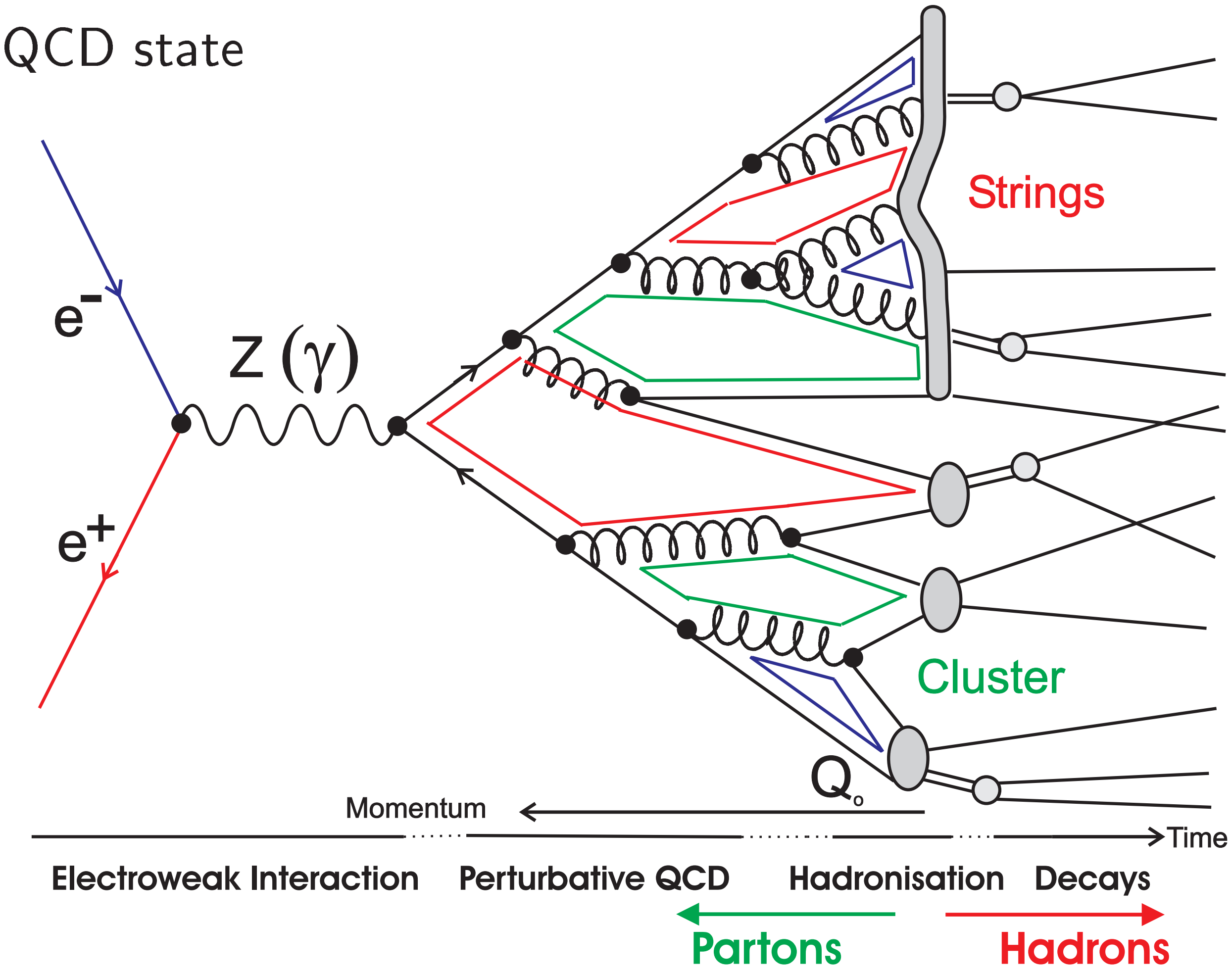
$e^+e^- \to q\bar{q}$: simple initial QCD state

Perturbative QCD phase:

$\mathcal{O}(\alpha_s^2)$
$\mathcal{O}(\alpha_s) + LLA$ ***
$\mathcal{O}(\alpha_s^4)_{Tree} + LLA$ NEW

Hadronisation:

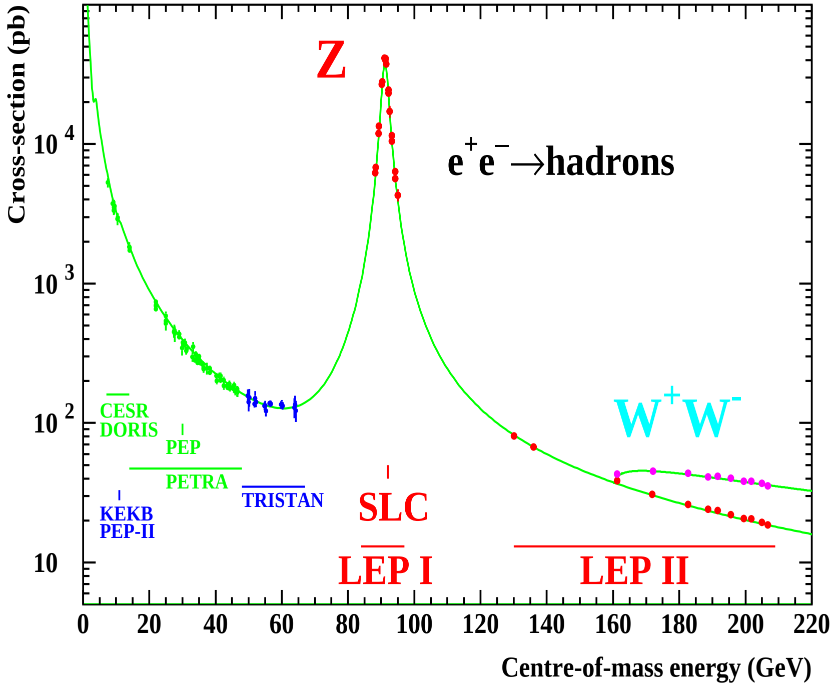
LUND string model ***
Cluster fragmentation



Models: PYTHIA, ARIADNE, HERWIG, APACIC++

Setting the Scene – The e^+e^- Experiments

- $e^+e^- \rightarrow q\bar{q}$ experiments provide simplest strongly interacting initial state
- many generations of accelerators: ADA ... LEPII
- recent results to cover ($\sqrt{s} \gtrsim 10\,\text{GeV}$): BABAR, BELLE, CLEO,(JADE) SLD, ALEPH, DELPHI, L3, OPAL



- B factories: $10^{\mathcal{O}(7)}$ events/experiment
- Z data: $\mathcal{O}(0.5 \rightarrow 4 \cdot 10^6)$ events/experiment
  "background free"; precise QCD studies even for "rare" events
- LEP II: $\mathcal{O}(10^4)$ events/experiment
  background: bremsstrahlung $\rightarrow$ "Z-return" events, WW and ZZ events

Outline

- Momentum spectra and multiplicity (inclusive, b quarks, gluons)
- Heavy quark fragmentation (b, c)
- The mass of the b quark and its running
- 4 jet angular distributions and colour factors
- Critical review of α_s from event shapes
- Measurement of the QCD β function
- Consequences for the gauge group of strong interactions
- Summary

The Charged Multiplicity

Multiplicity increase in $e^+e^- \to q\bar{q}$ due to coherent gluon bremsstrahlung off quarks

$$\left| \text{(quark–gluon vertex)} \right|^2 \propto C_F \cdot \alpha_s$$

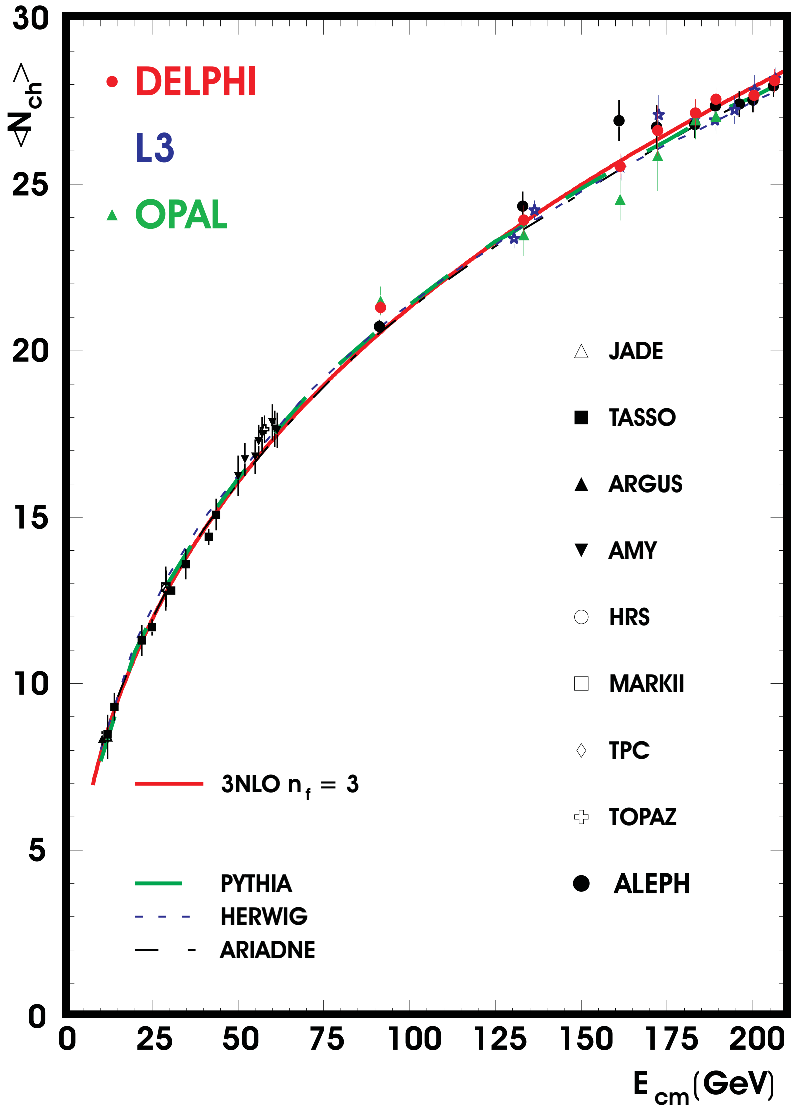
Consistent description of the energy dependence of the multiplicity by:

- fragmentation models
- MLLA (+ LPHD: $\#_{\text{hadrons}} \propto \#_{\text{gluons}}$)

$$\langle N_{ch} \rangle = K_0 \cdot \alpha_s(E_{cm})^{C_1} \cdot e^{\frac{C_2}{\sqrt{\alpha_s(E_{cm})}}}$$

- higher order (3NLO) predictions.

Dependence on flavour composition small.



Momentum Spectra

Colour coherence limits gluon emission
at small energies / large

$$\xi = -\ln x = -\ln \frac{2E_h}{E_{cm}}$$

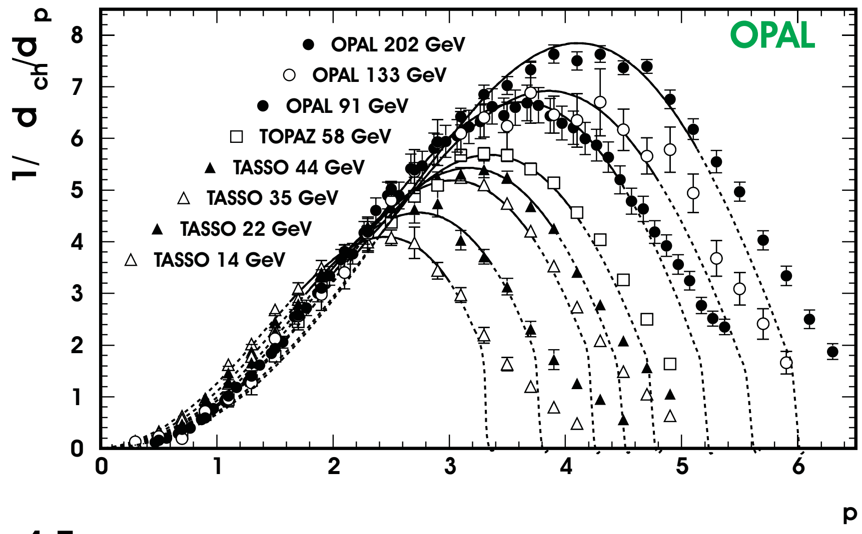
Predicted by MLLA "limiting spectrum"

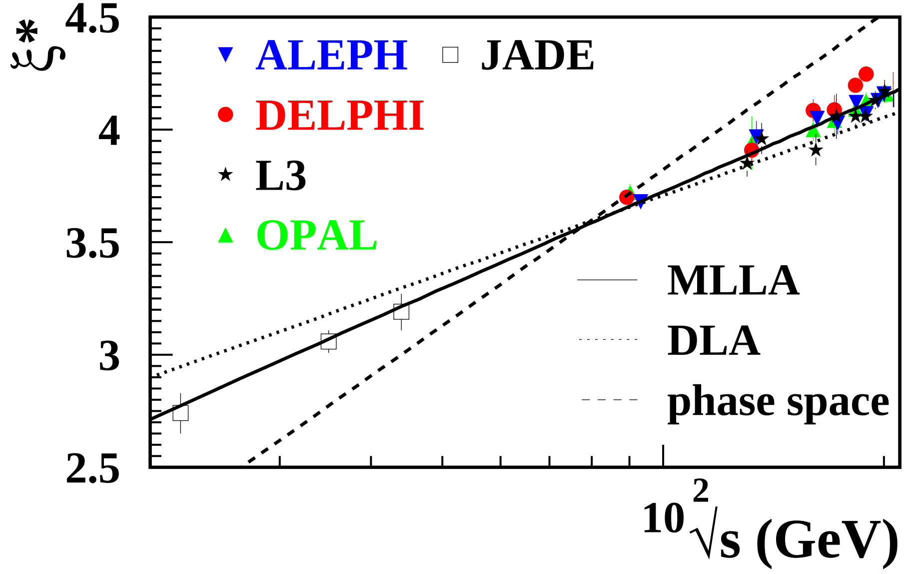
Coherence $\rightarrow$ change of peak position

$$\xi^*(E) \sim \sqrt{\log E}$$

slower than expected from phase space

$$\xi^*(E) \sim \log E$$





Multiplicity Difference of b and Light Quark Events

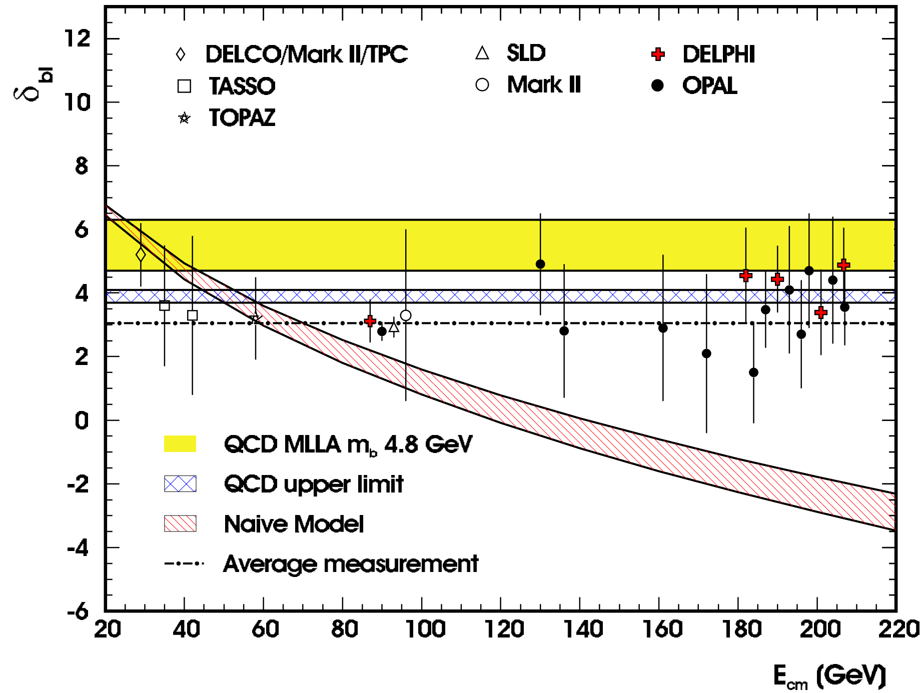
- Compare multiplicity of tagged heavy and light quark events
- Study energy dependence of multiplicity difference $\delta_{bl}(s) = N^b_{ch}(s) - N^{uds}_{ch}(s)$

- Two predictions:

  QCD (MLLA+LPHD): $\delta_{bl} \sim$ const.
  due to coherence of gluon radiation

  Naive incoherent model: $\delta_{bl} \sim 2\langle N_B \rangle + N_{q\bar{q}}((1 - \langle x_B \rangle)^2 s) - N_{q\bar{q}}(s)$

  $\rightarrow$ δ_{bl} decreasing with energy !

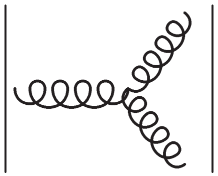


Z: $\langle \delta_{bl} \rangle = 2.98 \pm 0.21$ $\sqrt{s} > 130 GeV$: $\langle \delta_{bl} \rangle = 3.99 \pm 0.56$

Measurements support QCD expectation $\rightarrow$ coherence of gluon radiation

Multiplicity in Gluon Jets

Multiplicity increase in gluon jets due to coherent gluon bremsstrahlung off gluons.



$$\left| \quad \right|^2 \propto C_A \cdot \alpha_s$$

Expect increased multiplicity in gluon jets

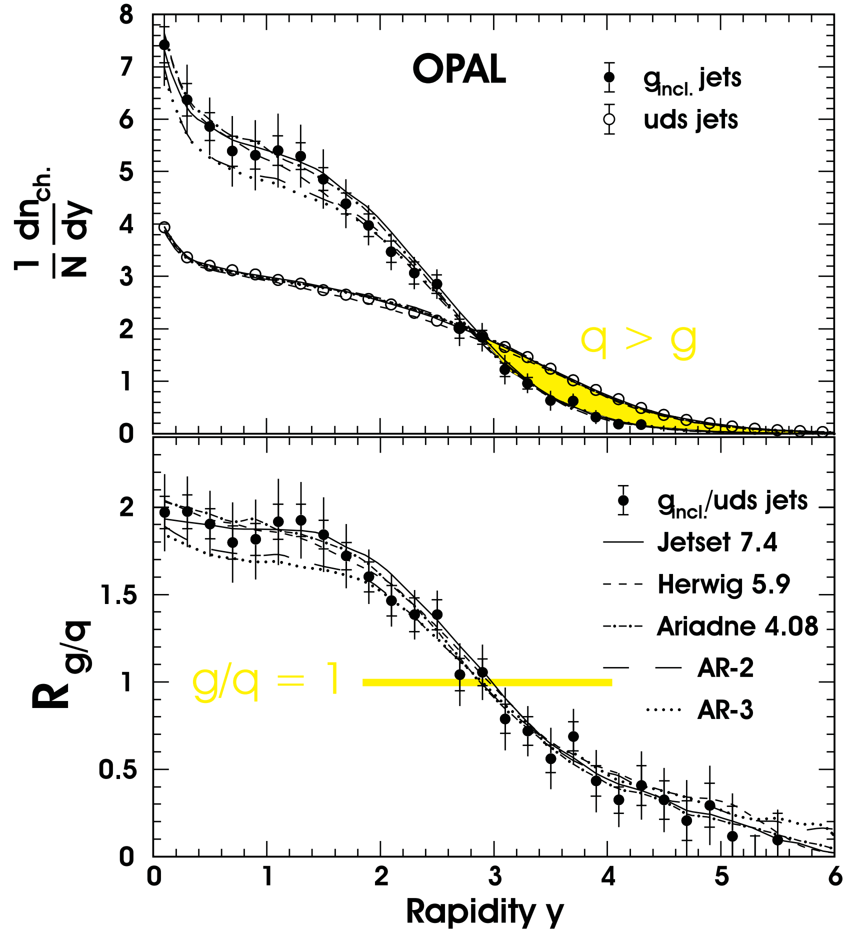
$$\frac{N_{ch}^{\text{gluon jet}}}{N_{ch}^{\text{quark jet}}} = \frac{C_A}{C_F} = \frac{3}{4/3} = 2.25$$

Find in $q\bar{q}g$ events at PETRA ... LEP

$$\frac{N_{ch}^{\text{gluon jet}}}{N_{ch}^{\text{quark jet}}} \sim 1.1 \rightarrow 1.4$$

Small ratio is understood:

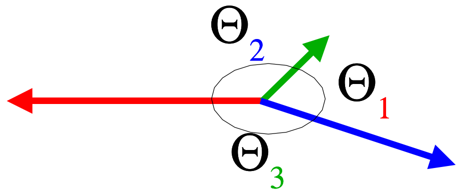
Coherence $\rightarrow$ use "$p_\perp$ like" scales,
Biases due to 3 jet selection,
Non perturbative / finite E effects



Multiplicity of Three Jet Events

Measure topology dependence of the multiplicity of 3 jet $\mathrm{q\bar{q}g}$ events at the Z

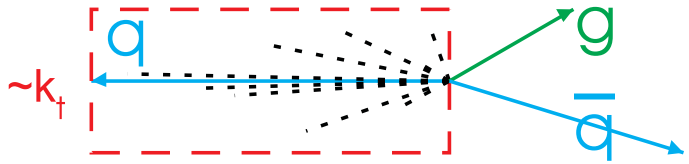
Theory considers:

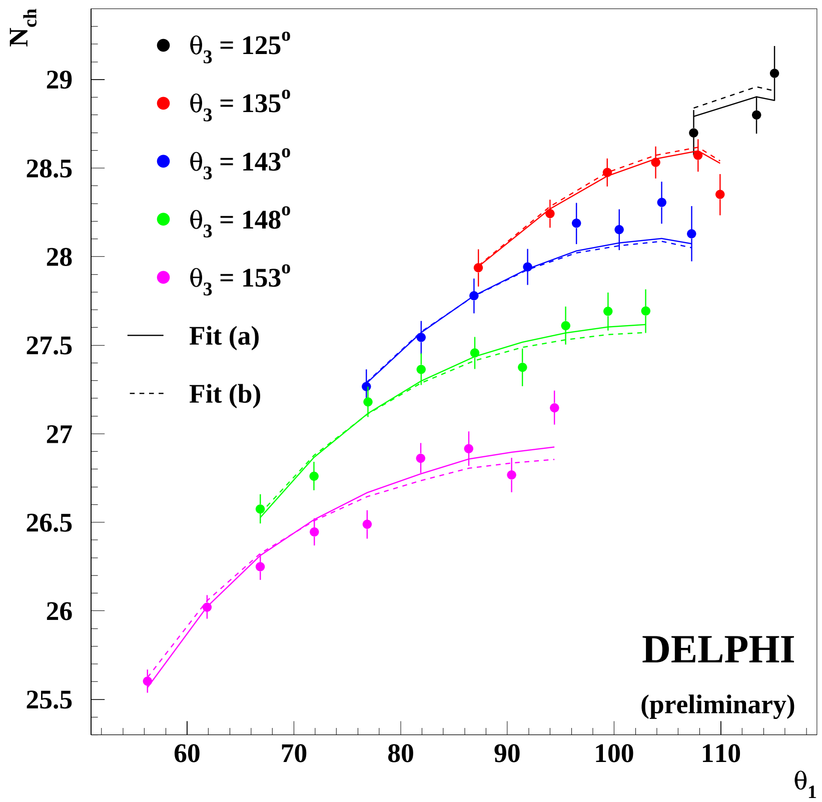


- coherence ($p_\perp^2$ scales)

  $$N^{ch}_{\mathrm{q\bar{q}g}} = N^{ch}_{\mathrm{q\bar{q}}}(s_{\mathrm{q\bar{q}}}, y_{cut}) + \frac{1}{2} \cdot N^{ch}_{g}(p^2_{\perp g})$$

- phase space (y_{cut}) restriction of $\mathrm{q\bar{q}}$ system due to g jet





- Differential eqn. for multiplicity slopes

  $$\frac{dN_{gg}(L')}{dL'} = \frac{C_A}{C_F}\left(1 - \frac{\alpha_0 c_r}{L}\right)\frac{d}{dL}N_{q\bar{q}}(L)$$

  $L = \log s/\Lambda^2 \ , \quad c_i = \text{const.}$

  $\rightarrow$ non-pert. integration const. for N_g^{ch}

Multiplicity of Colour Singlet gg Systems

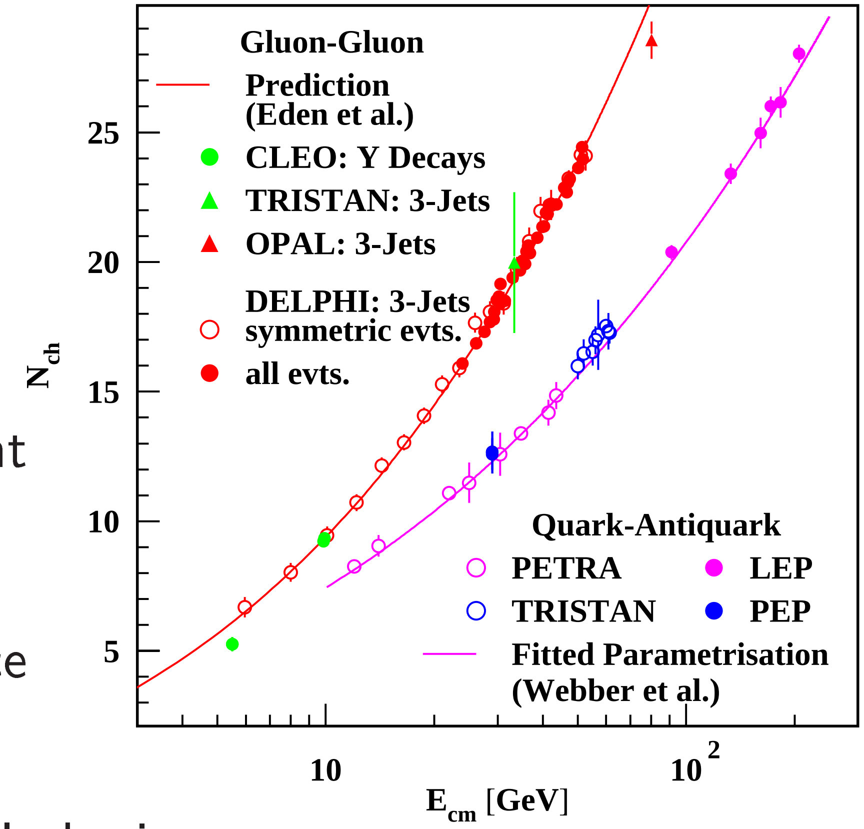
- Extract multiplicity of gg events:

  $$N_g^{ch} = 2 \cdot (N_{q\bar{q}g}^{ch} - N_{q\bar{q}}^{ch})$$

- Good agreement of 3 jet analyses with direct gluons (χ_b decays CLEO)

- Similar OPAL analysis uses different scheme

- Multiplicity for gluons increases $\sim$ twice as fast as for quarks $\Longrightarrow C_A/C_F$!



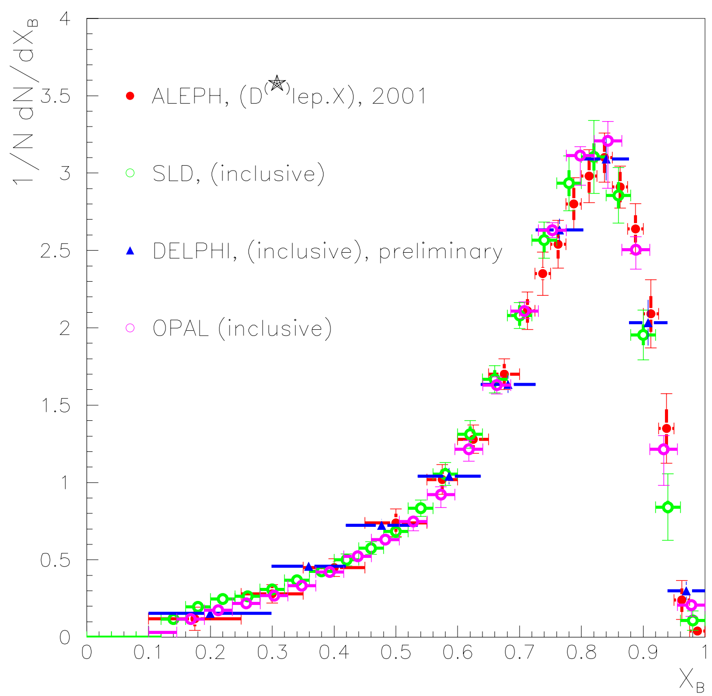
From symmetric events $\Theta_1 = \Theta_2$ DELPHI obtains:

$$\frac{C_A}{C_F} = 2.221 \pm 0.032(stat.) \pm 0.047(exp.) \pm 0.058(hadr.c.) \pm 0.075(theo.)$$

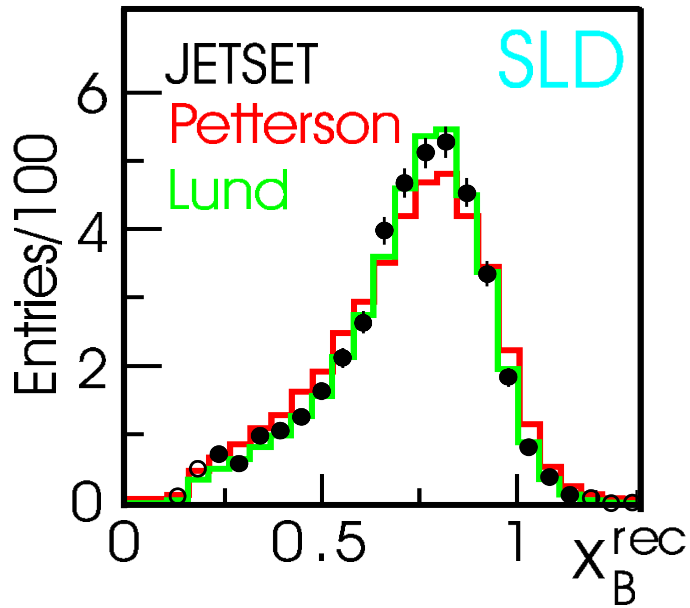
Heavy Quark Fragmentation

- Fragmentation functions specify energy transfer: partons $\to$ hadrons
- Simple case heavy quarks
- Recent b fragmentation measurements

$$D_b^B(x) = \frac{1}{\sigma_{tot}} \frac{d\sigma_B}{dx} \qquad x = \frac{2E_B}{E_{CM}}$$



- partial/inclusive B reconstruction: high stat. precision $\sim 0.3\%$ good energy resolution $\mathcal{O}(10)\%$
- New average (weakly decaying B's): $\langle x_B \rangle = 0.715 \pm 0.003$
- Lund/Bowler ansatz for FF favoured Peterson ansatz disagrees.



Interpretation of Heavy Quark Fragmentation

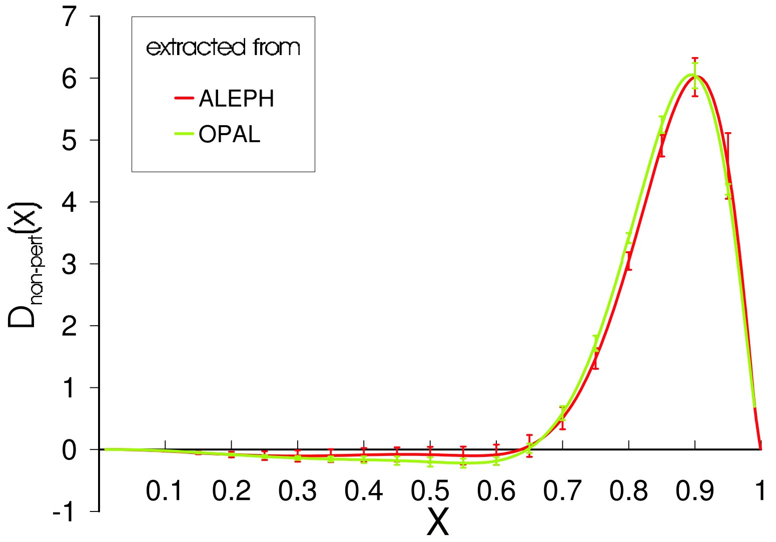
Observed FF is described by a convolution of a perturbative and a non-pert. part.

$$D_b^B(x) = \int_0^1 D_{pert}(z) \cdot D_{non-pert}(\frac{x}{z})\frac{dz}{z}$$

a Mellin transform ($\int dx x^{N-1} D(x)$) yields a simple product equation

$\tilde{D}_b^B(N) = \tilde{D}_{pert}(N) \cdot \tilde{D}_{non-pert}(N) \rightarrow$ solve for $\tilde{D}_{non-pert}(N)$

Backtransformation $\rightarrow$ model-independent $D_{non-pert}(x)$



$D_{non-pert}(x)$ can be universally applied

Use similar assumptions for perturbative calculations !

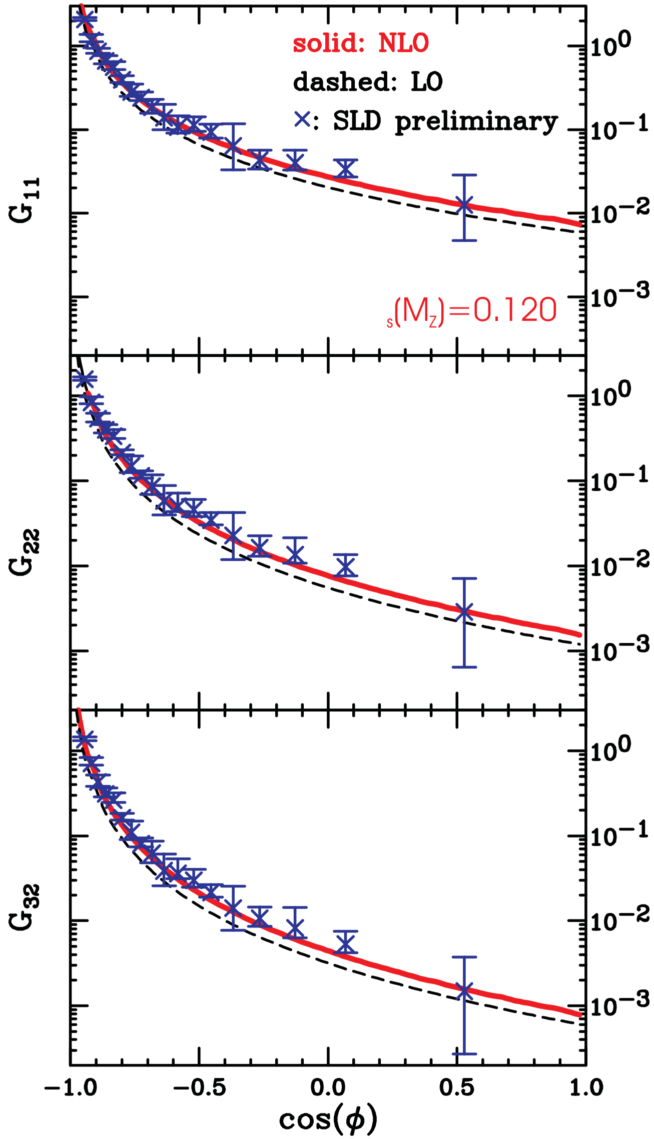
Double Inclusive b Fragmentation

- Study both B hadrons, measure double moments (SLD, Brandenburg et al.)

$$\tilde{D}_{ij}(\phi) = \frac{1}{\sigma_B} \int x_{B_1}^{i-1} x_{B_2}^{j-1} \frac{d^3\sigma}{dx_{B_1} dx_{B_2} d\cos\phi} dx_{B_1} dx_{B_2}$$

$$G_{ij} = \frac{\tilde{D}_{ij}}{\tilde{D}_i \tilde{D}_j}$$

ϕ is $\angle$ between b's

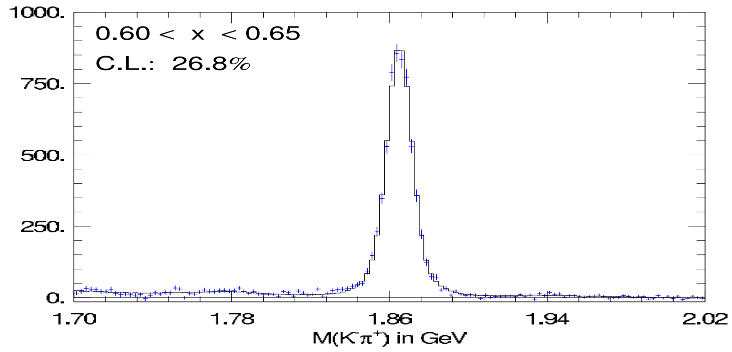


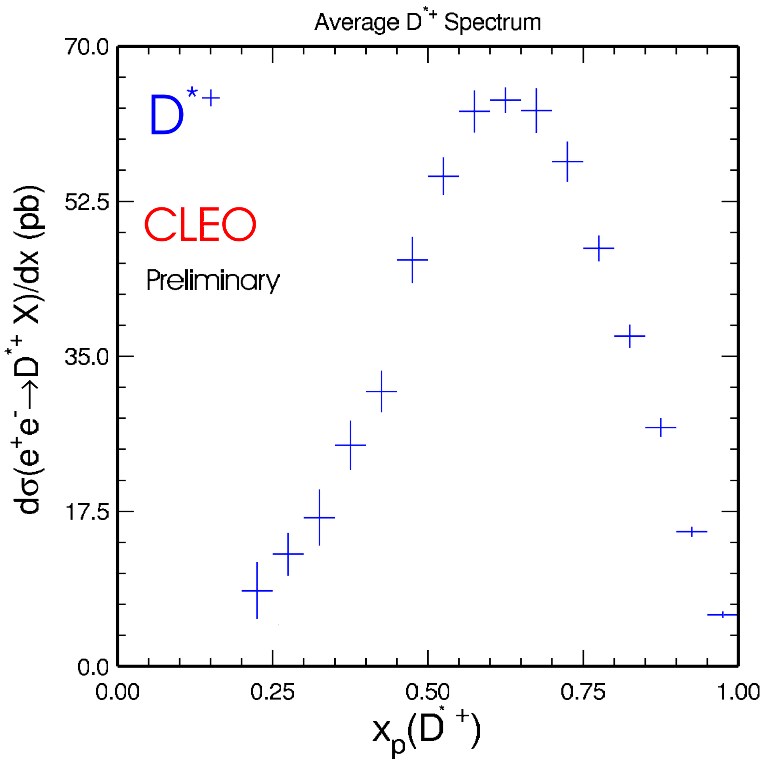
- b mass regulates collinear singularity, save pert. prediction for G_{ij}
- Non-perturbative and large $\log E_{CM}/m_b$ terms cancel to all orders
- Measurement implies test of factorisation theorem (SLD)
- Allows new measurement of α_s (Brandenburg et al.)

Charm Fragmentation

- New precise charm frag. function CLEO (preliminary)
  $\sim 3fb^{-1}$ below $b\bar{b}$ threshold,
  $\sim 6fb^{-1}$ of $\Upsilon(4S)$ data ($x_p^D > 0.5$)
- Decays: $D^+ \to K\pi\pi$,
  $D^0 \to K\pi, K3\pi$, $D^{*+} \to D^0\pi$
- Excellent resolution $\Delta x \sim 0.006$,
  high efficiency $\epsilon \sim 0.77$
- Pure charm fragmentation –
  not influenced by B decays

- C fragmentation softer than b
- To be analysed like b data
- Slightly different x spectra for D^0, D^+, D^{*+}
  partly $\to$ mass difference
  More important: resonance decays





Measurement of the b Quark Mass $m_b(M_Z)$

- Quarks bound inside hadrons $\rightarrow$ masses known only to limited precision
- Assess masses via dynamical relations
- m_b: reduction of gluon bremsstrahlung $\quad \propto \frac{m_b^2}{p_{\perp g}^2} = \frac{m_b^2}{M_Z^2} \cdot \frac{1}{y_{cut}} \sim 0.3\% \cdot \mathcal{O}(10)$
- Besides pert. pole mass M_b define renormalised mass $m_b(Q)$ in $\overline{\mathrm{MS}}$ scheme. This running mass absorbs loop corrections like $\alpha_s(Q)$.

Measurement:

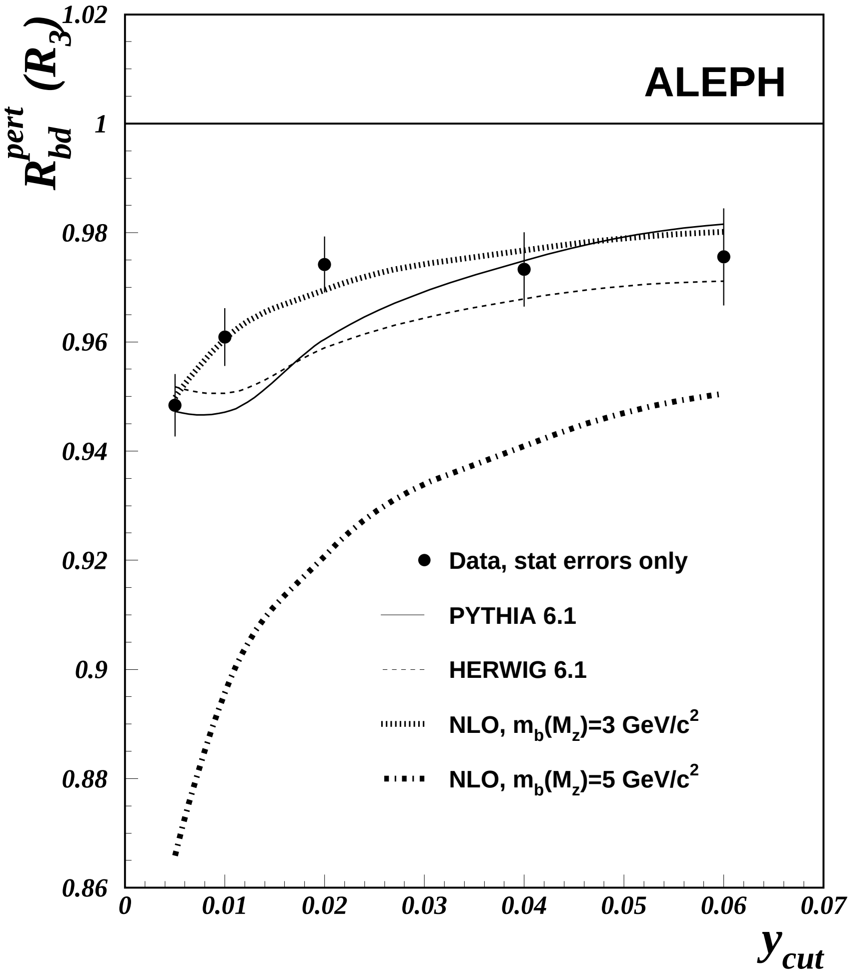
- Compare b/inclusive ratio of jet rates/event shapes obs.
- b-identificaton using impact param. tag. purity 65-90%
- Corrections: acceptance, tag bias, hadronisation (partly neglected)

- ALEPH: several shapes use R_3, $\langle y \rangle$
- Brandenburg et al (SLD): several jet algos (assign syst. for discr.)
- Opal: T, M_h, B_w, y_3, C expoiting correlation btw. observables
- DELPHI 2003: R_3 Durham, Cambridge

Theory: four independent calculations

Measurement of the b Quark Mass $m_b(M_Z)$

- Observe expected y_{cut} dependence.
- Recent improvements:
  understanding of hadronisation correction
  new important error: b mass in model
  overall $\rightarrow$ reduced uncertainty
- Theory error substantial:
  Mass ambiguity, renormalisation scale, $\Delta\alpha_s$
- Combined ADO/SLD result
  $m_b(M_Z) = 2.95 \pm 0.15 \pm 0.24_{hadc./theo}$ GeV
- Alternatively: flavour independence of α_s
  $\frac{\alpha_s^b}{\alpha_s^l} = 0.996 \pm 0.009$



The Running of $m_b(Q)$

- Compare LEP/SLD average $\Rightarrow$ to b-mass from bound states:

  $m_b(m_b) = 4.24 \pm 0.11$ GeV

  $\Rightarrow$ running of the b-mass

- Soon improved measurements of m_b from leptonic B decays

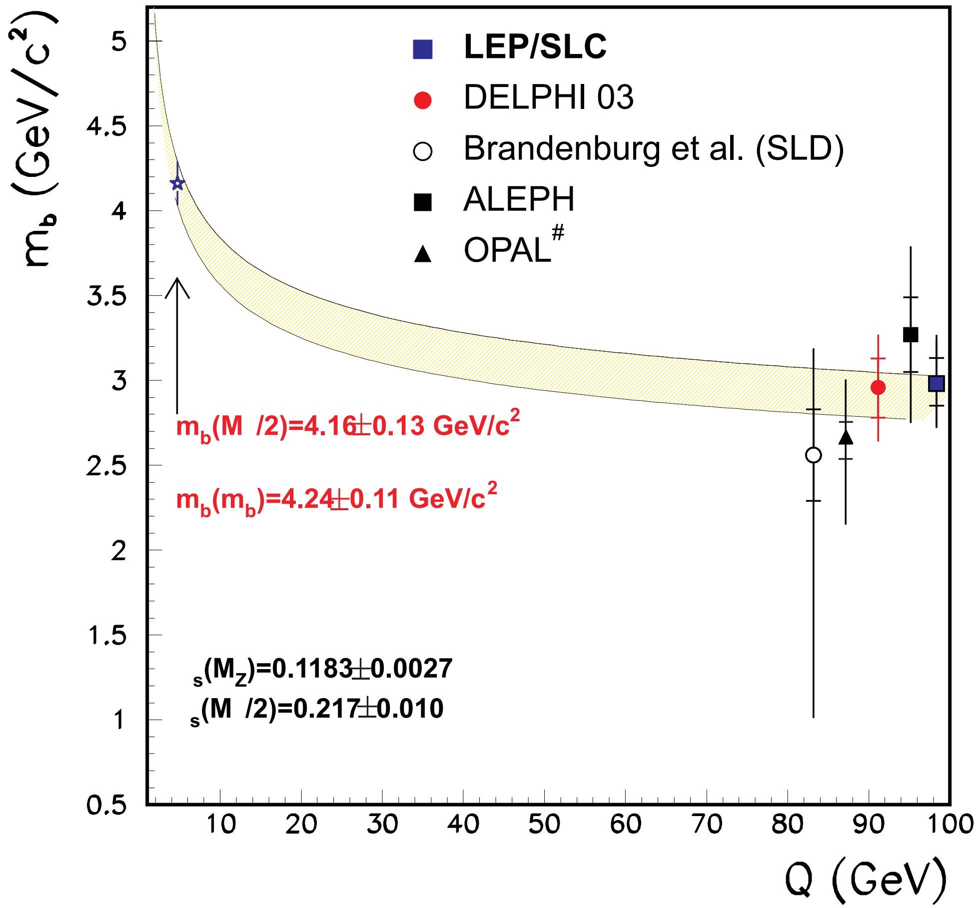
- Tests RGE for the mass

  $$\frac{\partial m}{\partial \ln Q^2} = -\gamma_m(\alpha_s) \cdot m(Q^2)$$

  mass anom. dimension

  γ_m 

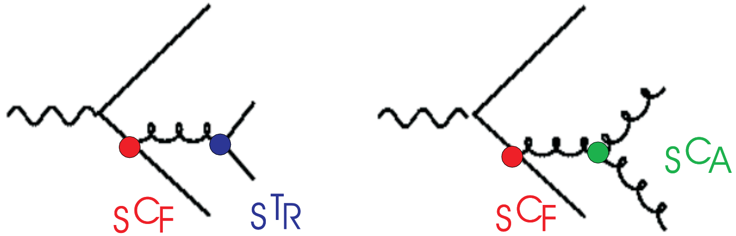
  contains loop corrections to b quark propagator

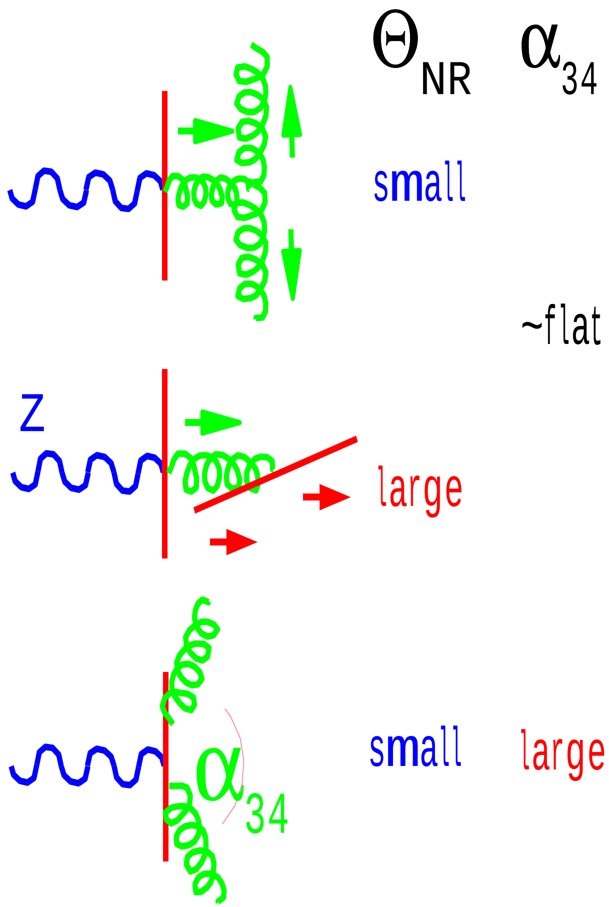


$m_b(M_Z)$ measurement implies a successful test of QCD loop structure!

Colour Factors from Four Jet Events

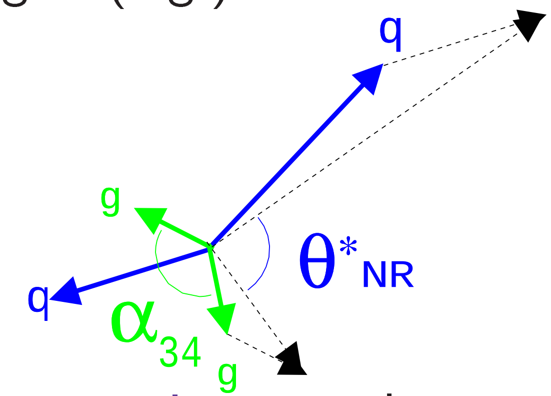
4 jet events give access to full QCD gauge structure





Connect colour to helicity structure.

Sensitive angles (e.g.):



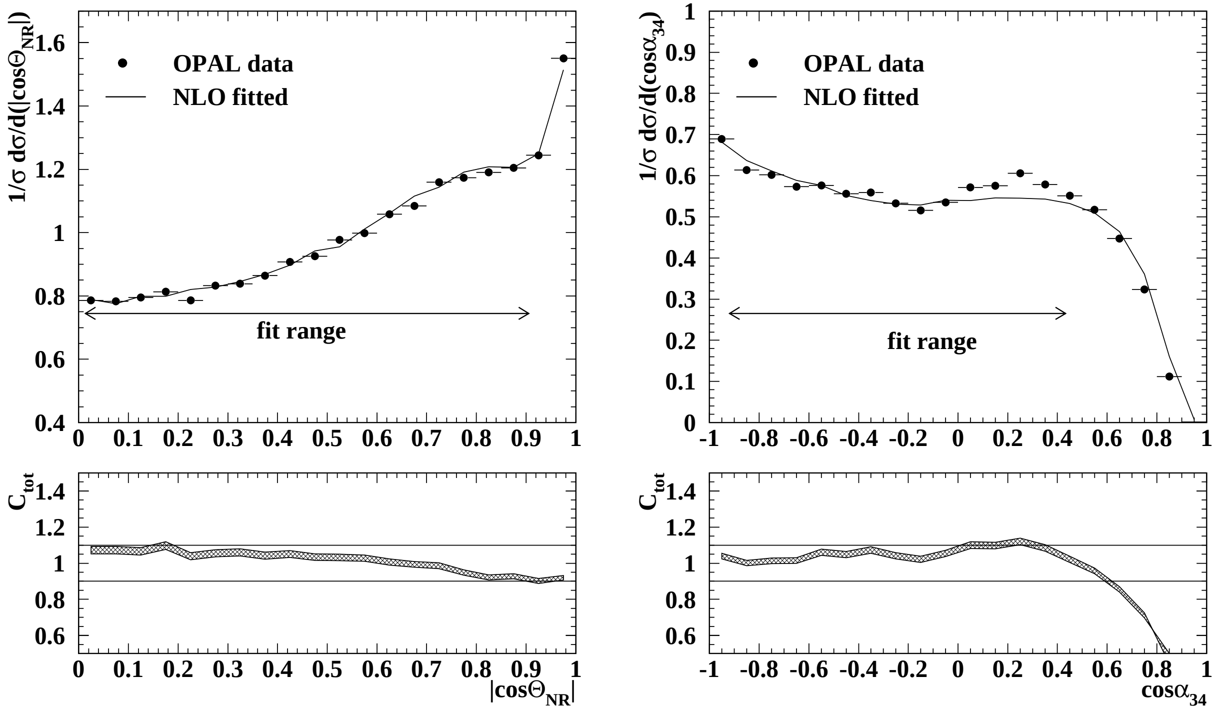
$\Theta^*_{NR} \sim \angle$(betw. primary and secondary partons)
$\alpha_{34} \sim \angle$(betw. secondary partons)

Initial gluon polarised, like any vector particle. Spin 0 or 2 case different.

Initial analyses used LO theory; recent years NLO calculation.

Colour Factors from Four Jet Events

OPAL & ALEPH use (matched) NLO 4 jet calculation, inc. also 3 jet rate.



Reconstruct jets, unfold detector and hadronisation (implies QCD models)

Primary fit yields: $\frac{C_A}{C_F}$, $\frac{n_f \cdot T_F}{C_F}$ and $C_F \alpha_s$.

Solve for individual colour factors in NLO.

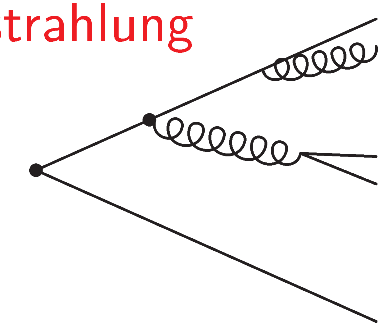
A & O:	$C_A = 2.97 \pm 0.50$	$C_F = 1.34 \pm 0.23$
QCD:	$C_A = 3$	$C_F = 1.33$

But : Correlations between C_A and C_F are large!

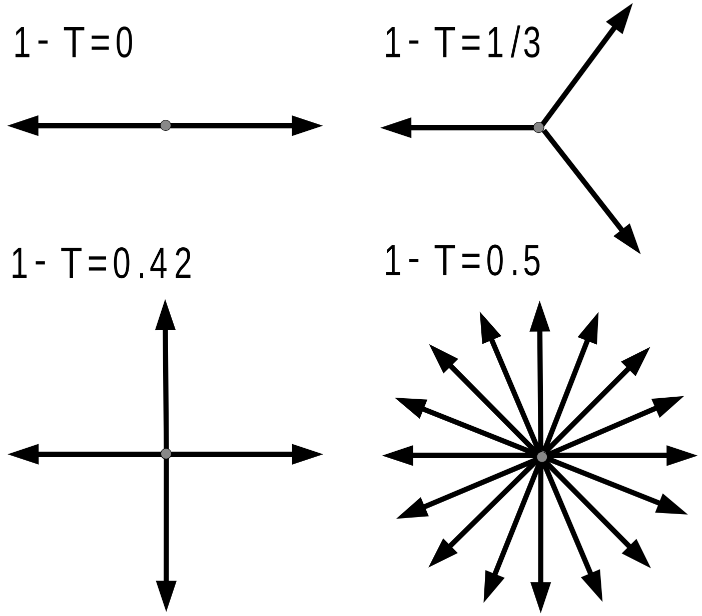
Measuring α_s

Event Shape Observables

Event shapes observables measure the amount of gluon bremsstrahlung
Sensitive to α_s (and QCD colour factors)
Inensitive to electroweak physics.



- Infrared safe
- Collinear safe
- insensitive to non-perturbative effects.



Example Thrust:

$$y = 1 - T := 1 - \max_{|\vec{n}|=1} \sum_{i=1}^{n} |\vec{p_i} \cdot \vec{n}| \Big/ \sum_{i=1}^{n} |\vec{p_i}|$$

Furtermore:

Jet-Masses, Jet-Broadenings, y_{cut}, . . .

The Procedure – Guide through the α_s Maze

Experiment

- Measure event shape distribution
- Correct
  detector acceptance/resolution,
  ISR, backgrounds ...

Theory

- Calculate perturbative predictions
  - NLO α_s M.E.
  - NLLA
  - matched NLO α_s/NLLA
- Correct for non-pert. hadronisation by
  - hadronisation models
  - power corrections

Compare common level

- Fit α_s
- Determine experimental uncertainties
- Determine theoretical uncertainties

Next to Leading (Fixed) Order pQCD

- $\mathcal{O}(\alpha_s^2)$ prediction:

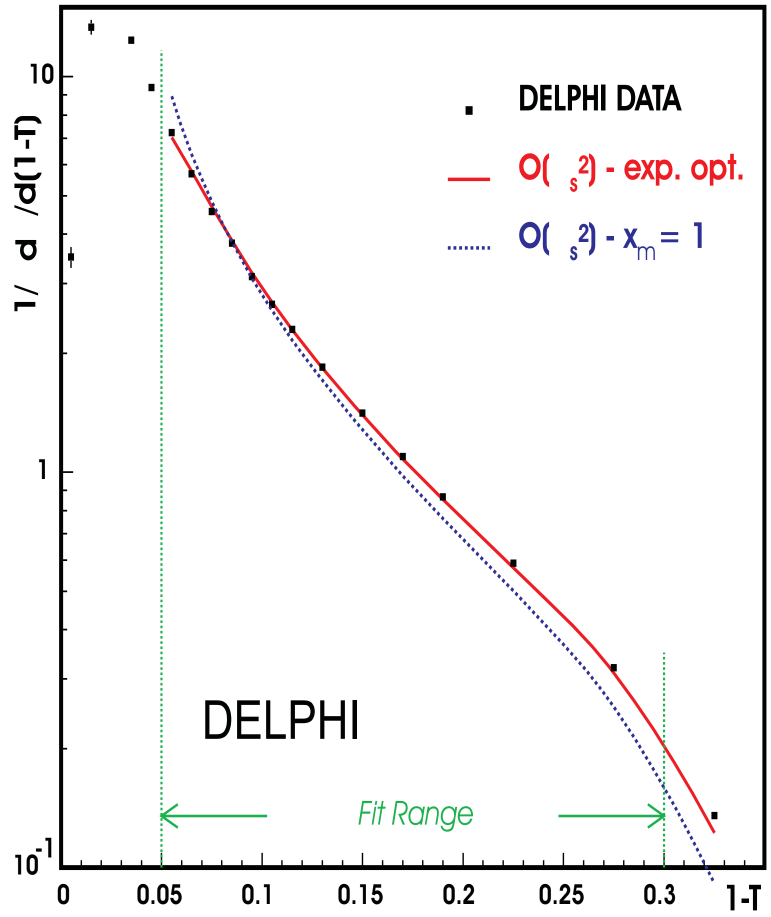
$$\mathcal{D}(y)_{\rm pert} = \frac{1}{\sigma}\frac{d\sigma}{dy} =$$

$$A_y \cdot \bar{\alpha}_s(\mu) + (A_y \cdot \beta_0 \log x_\mu + B_y)\, \bar{\alpha}_s(\mu)^2$$

- Uncalculated higher order terms $\rightarrow$ (unphysical) $x_\mu = \mu^2/s$ renormalisation scale (factor) dependence
- Valid for 3 jet region – large y HO terms important for $y \rightarrow 0$
- Apply MC hadronisation corrections

$$C = \frac{\mathcal{D}_{parton}(y)}{\mathcal{D}_{hadron}(y)}$$

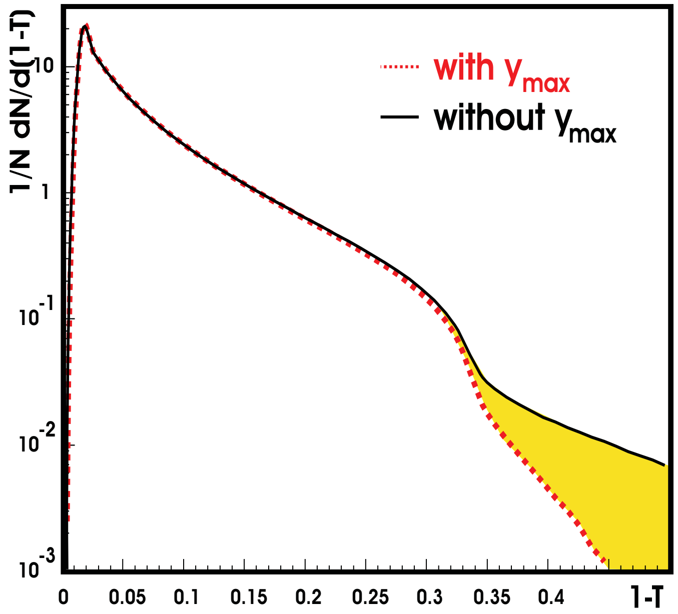
- NLO predictions in general show different slope of $\mathcal{D}_{hadron}(y)$ than data!
- Except x_μ is optimised!



All arguments also apply for the $\mathcal{O}(\alpha_s^3)$ calculation (4 jets)

Next to Leading Log Approximation (NLLA)

$$R_y(y) = \int_0^y dy\, \mathcal{D}_y(y) = e^{(Lg_1(\alpha_s L) + g_2(\alpha_s L))} \qquad L = \log(1/y)$$



- Resums leading & next to leading log. terms
- Use for small y, collinear rad., “2-jet region”
- Problem: $R_y(y_{\max}) \neq 1$, $R'_y(y_{\max}) \neq 0$
- Solution:

$$L \to L' = \frac{1}{p} \ln \left[\frac{1}{(x_L \cdot y)^p} - \frac{1}{(x_L \cdot y_{max})^p} + 1 \right]$$

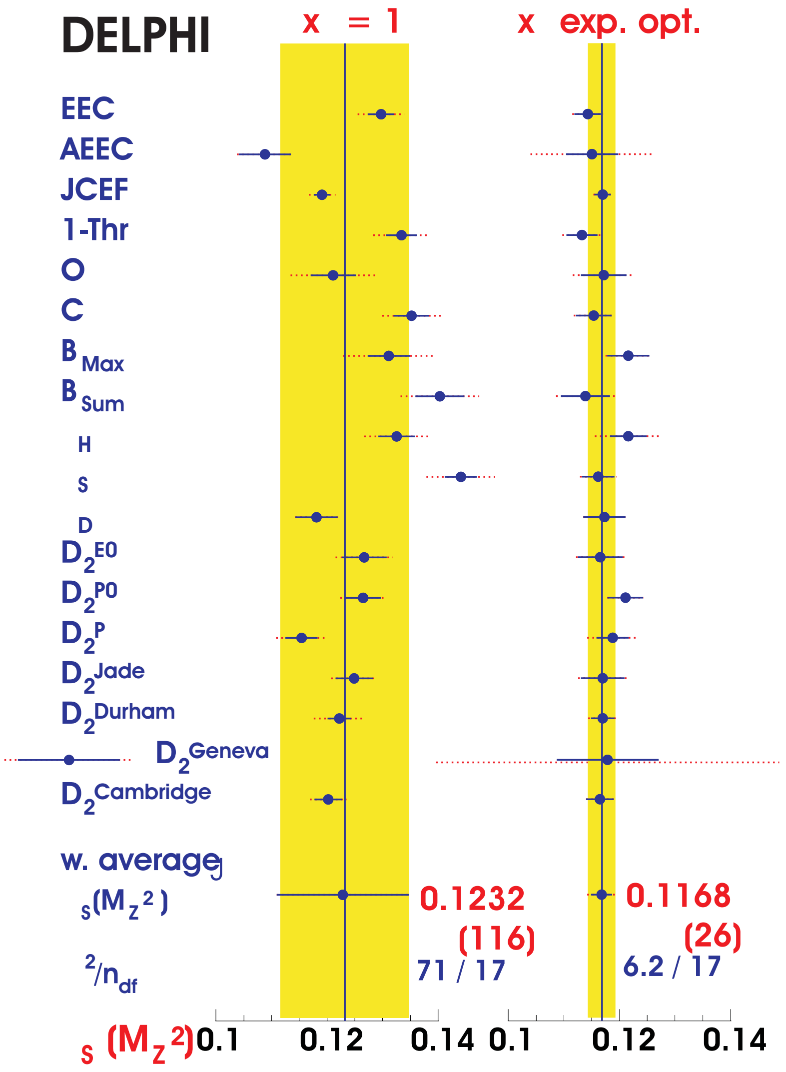
Matching Fixed Order and NLLA

Aim: combine NLLA and $\mathcal{O}(\alpha_s^2)$ ($\overline{\mathrm{MS}}$ scheme, $x_\mu = 1$) calculations.
Avoid double counting:

$$R_{\mathcal{O}(\alpha_s^2)+NLLA} = R_{NLLA} + A \cdot \alpha_s + B \cdot \alpha_s^2 - \text{double counting}$$

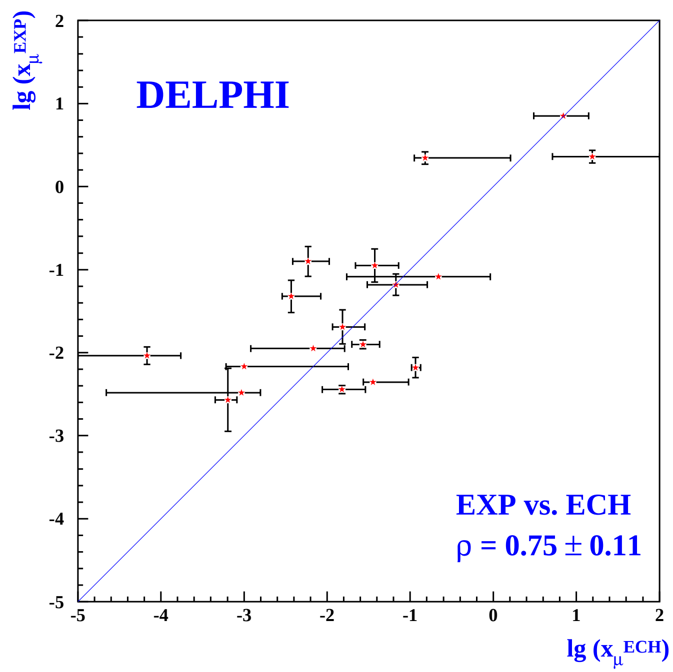
Problem: ambiguity $\to$ perform matching in $R(y)$ or $\log R(y)$.

$\mathcal{O}(\alpha_s^2)$ Results

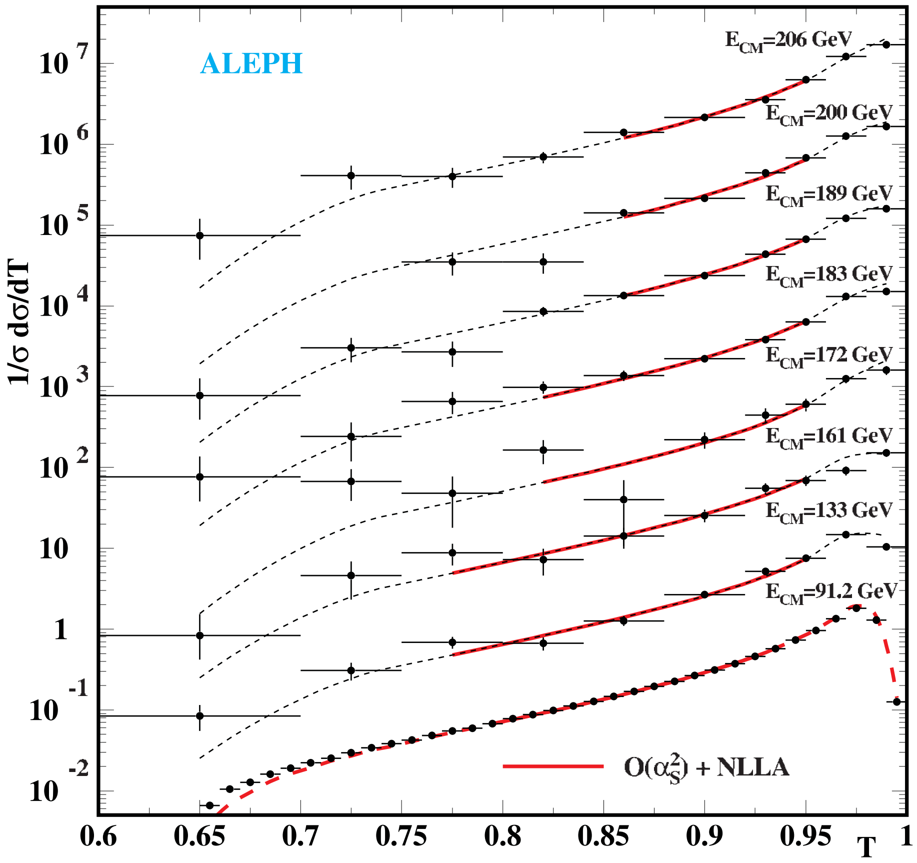


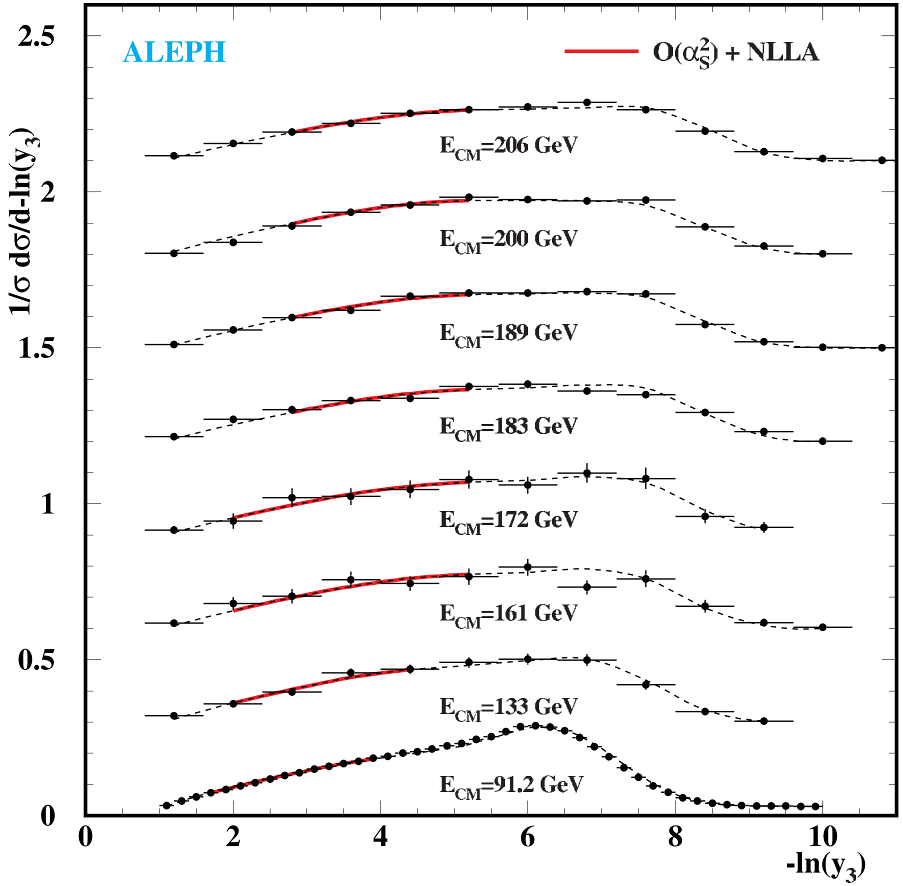
For 18 observables,

- $\overline{\mathrm{MS}}$ scheme: $\sim 10\%$ scatter
  $\rightarrow$ require additional theory error
- Using experimentally optimised scales:
  Consistent results – $\sim 2\%$ scatter
- Observe correlation with theoretically motivated scales (ECH, PMS)



Matched LLA/$\mathcal{O}(\alpha_s^2)$ Fits





Sucessfully describes small y "2jet" region

Fit range: solid lines

Residual slope (?) between data and theory for Thrust

α_s Combination

LEP QCD Working Group:

Aim: average ADLO α_s & running (high energy data)

Input: α_s values (using $\log R$ matching) + exp. uncertainties

Work: Compare theory implementation, hadronisation corrections
Define theory uncertainty
Treat correlations

Output: $\langle\alpha_s(E)\rangle \pm \Delta_{exp.} \pm \Delta_{theo.}$

Problems observed:

- Z: α_s often depends on fit range ($\mathcal{O}(\alpha_s^2 | x_\mu = 1)$ remnant ?)
- Badly described observables obtain smallest error
- α_s downwards biased as $\Delta_{theo}\, \alpha_s \propto \alpha_s^3$
- negative weights $\leftrightarrow$ correlations inaccurate

The Uncertainty Band

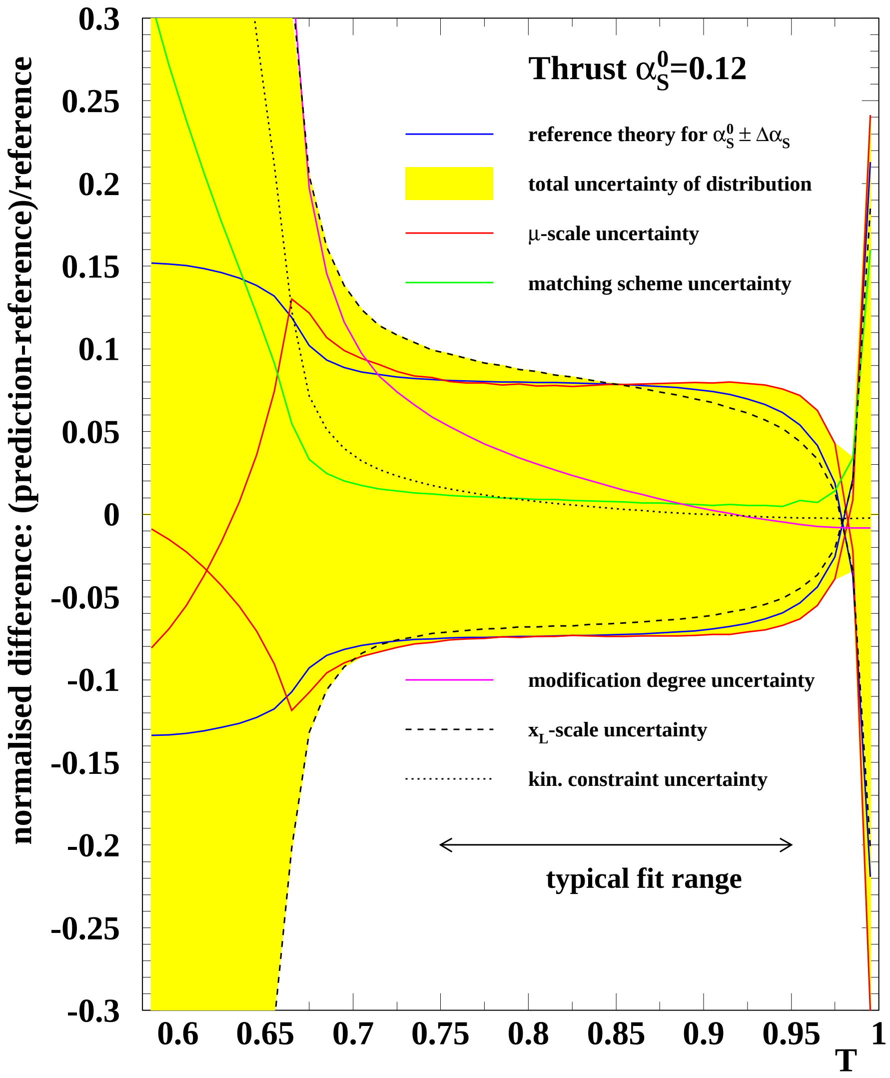
- Take reference theory with fixed $\alpha_s(M_Z)$

- Vary reference theory:
  Renormalisation scale $1/2 < x_\mu < 2$
  Apply (or not) phase space condition:
  $2/3 < x_L < 3/2$, $p = 1; 2$
  Exchange $\log R$ vs. R matching

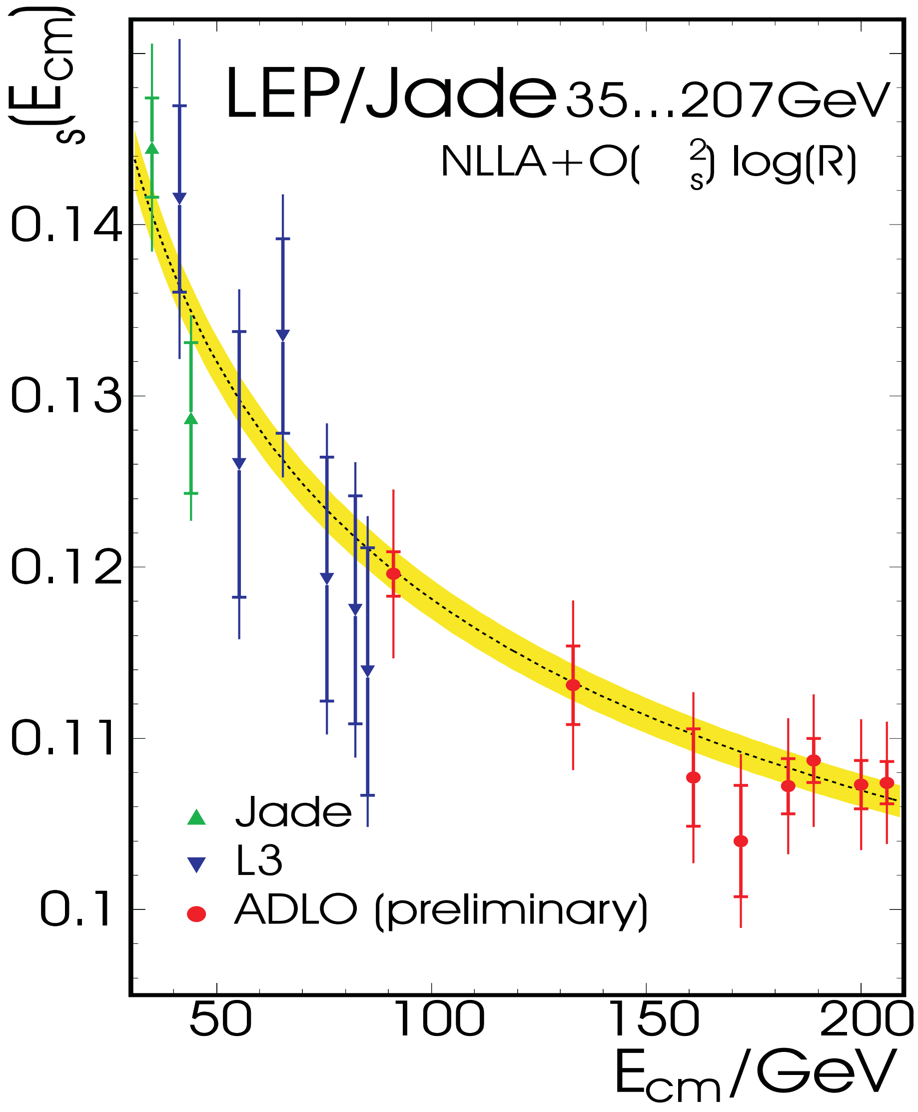
- Envelope defines uncertainty band

- allowed $\alpha_s(M_Z)$ variation in the band determines $\pm\Delta\alpha_s$

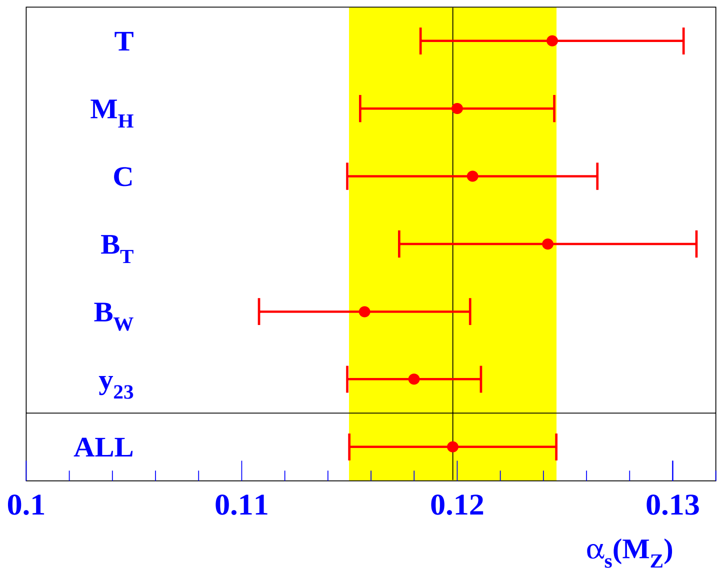
- Determine $\langle\alpha_s(M_Z)\rangle$ and iterate



α_s Combination of the LEP QCD Working Group



	LEP I	LEP II
α_s	0.1197	0.1196
$\pm$stat.	0.0002	0.0005
$\pm$sys. ex.	0.0008	0.0010
$\pm$had.	0.0010	0.0007
$\pm$theo.	0.0048	0.0044
$\pm$tot.	0.0049	0.0046
$\pm$tot.rel	$\sim 4\%$	



Power Corrections – The Dokshitzer-Webber-Ansatz

Parameterise unknown (analytical) behaviour of the physical strong coupling below the IR-matching scale μ_I by its universal mean value.

$$\alpha_0(\mu_I) = \frac{1}{\mu_I}\int_0^{\mu_I} \mathrm{d}k\, \alpha_s(k)$$

Leads to a power term $\mathcal{P} \propto 1/E_{\mathrm{cm}}$

increasing the mean values

$$\langle y \rangle = \langle y_{\mathrm{pert}} \rangle + c_y \mathcal{P}$$

shifting the distributions

$$\mathcal{D}_y(y) = \mathcal{D}_{\mathrm{pert}}(y - c_y \mathcal{P})$$

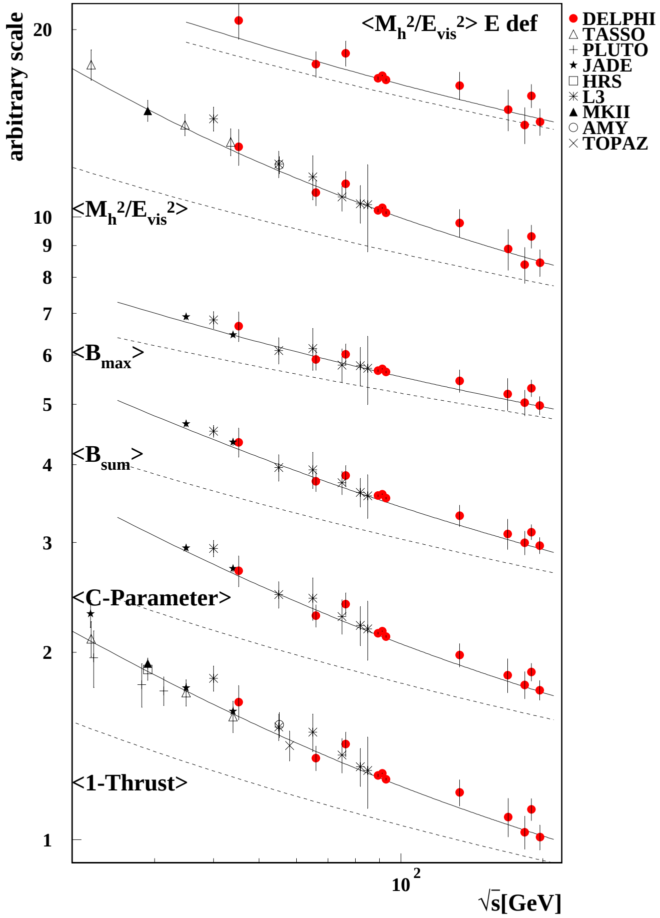
$$\mathcal{P} = \frac{4C_F}{\pi^2}\mathcal{M}\frac{\mu_I}{E_{\mathrm{cm}}}\left[\alpha_0(\mu_I) - \alpha_s(\mu) - \left(b_0 \cdot \log\frac{\mu^2}{\mu_I^2} + \frac{K}{2\pi} + 2b_0\right)\alpha_s^2(\mu)\right]$$

Observable specifics are absorbed in c_y: $\quad c_{1-T} = 2 \qquad c_{M_{\mathrm{h}}^2/E_{\mathrm{vis}}^2} = 1 \qquad \ldots$

Power terms (i.e. $\alpha_0(\mu_I)$) depend on renormalisation scheme (in general $\overline{\mathrm{MS}}$).

Fit $\alpha_0(\mu_I)$ and α_s to mean values and distributions of event shapes

Power Corrections à la Dokshitzer–Webber



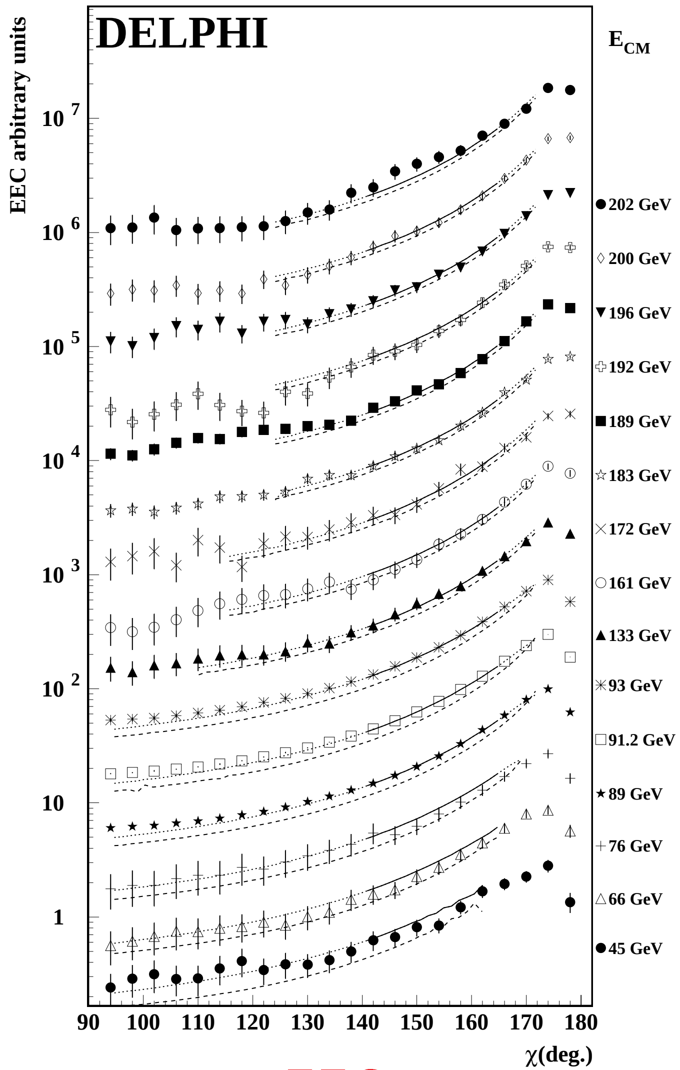
Typically 10% power correction

Successful description of the energy dependence of

⟸ mean values

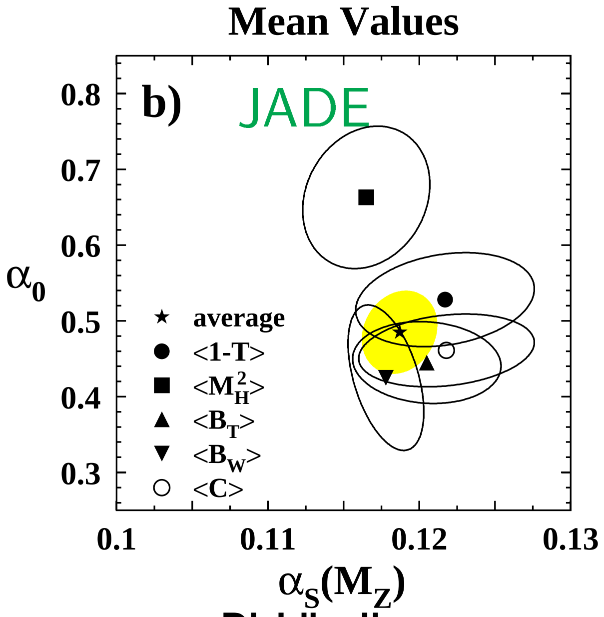
and

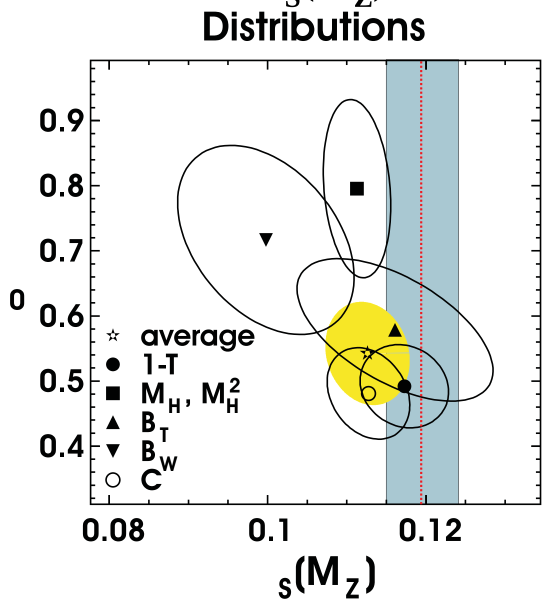
distributions ⟹



EEC

Power Corrections à la Dokshitzer–Webber (α_s&α_0 Results)





- JADE fits use data from $E_{\rm cm} = 14$ to 189 GeV:
- Means:

$$\alpha_s(M_Z) = 0.1187 \pm 0.0014 \pm 0.0001\,^{+0.0028}_{-0.0015}$$

$$\alpha_0(2\,\text{GeV}) = 0.485 \pm 0.013_{\rm fit} \pm 0.001_{\rm sys}\,^{+0.065}_{-0.043\,\rm th}$$

- Distributions:

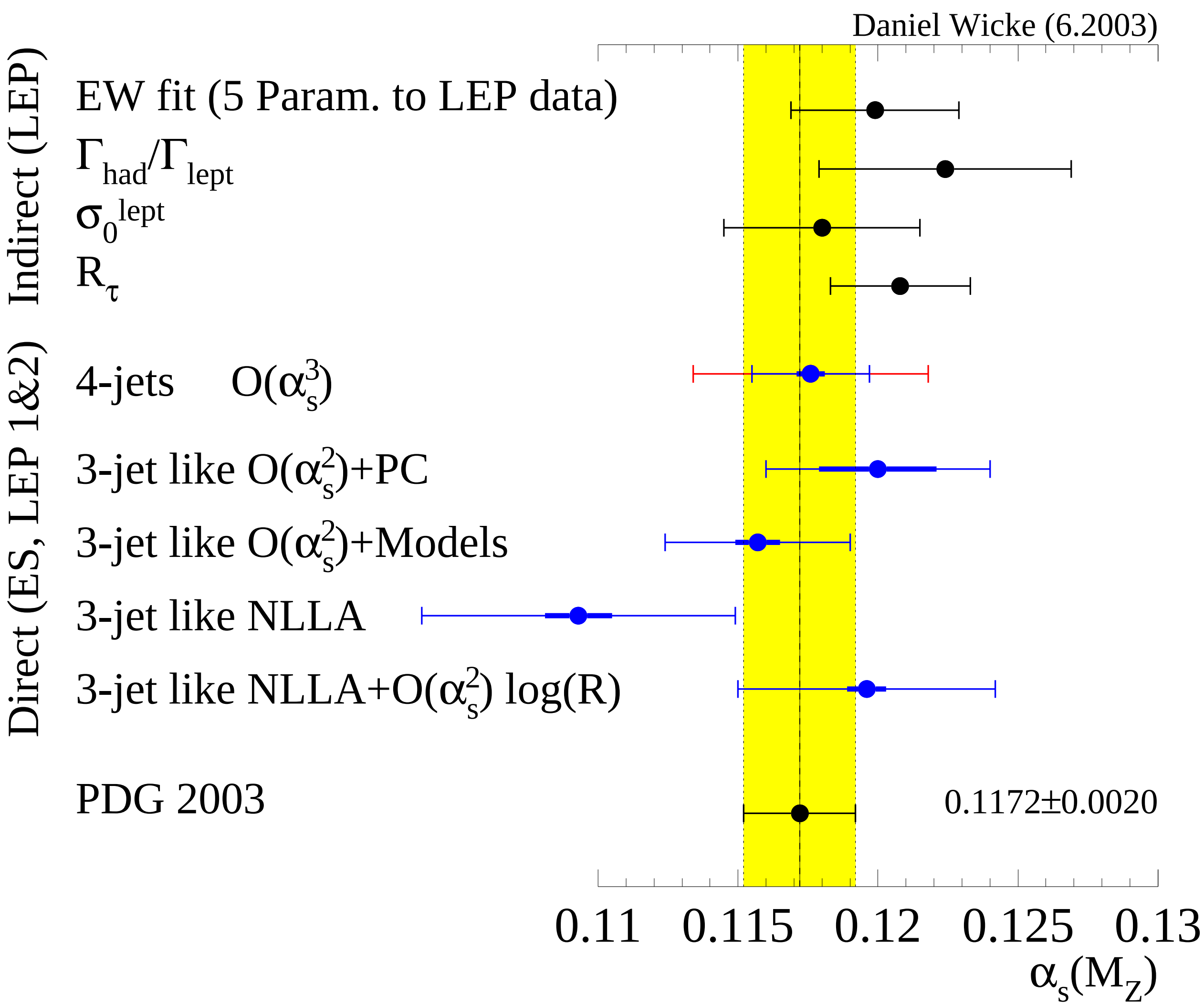
$$\alpha_s(M_Z) = 0.1126 \pm 0.0005 \pm 0.0037\,^{+0.0044}_{-0.0030}$$

$$\alpha_0(2\,\text{GeV}) = 0.542 \pm 0.005_{\rm fit} \pm 0.032_{\rm sys}\,^{+0.084}_{-0.060\,\rm th}$$

- α_0 results from means and distributions only consistent within total error.

- $\alpha_s^{\rm (PowerCorrections)} < \alpha_s^{\rm (MonteCarlo)}$!
  $\leftarrow$ light blue band: current LEP average

Comparison of α_s Results from LEP



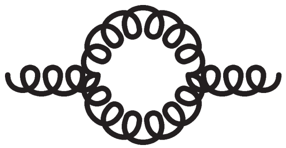
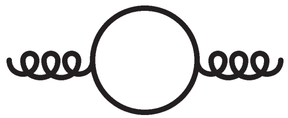
Running of α_s

Asymptotic freedom & confinement $\Longleftrightarrow$ running of the QCD coupling $\alpha_s(E)$

Renormalisation Group Equation controls the running:

$$\frac{\partial \alpha_s}{\partial \ln Q^2} = \beta(\alpha_s) = -\alpha_s^2 \cdot \frac{\beta_0}{4\pi}\left(1 + \frac{\beta_1}{2\beta_0}\alpha_s + \cdots\right)$$

$$\frac{\partial \alpha_s^{-1}}{\partial \ln Q^2} = \frac{\beta_0}{4\pi}\left(1 + \frac{\beta_1}{2\beta_0}\alpha_s + \cdots\right)$$

β function $\Longleftrightarrow$   $\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f$

$\Longrightarrow$ is influenced by all strongly interacting particles.

A measurement of β function implies model independent limits on hypothetical particles.

Renormalisation group invariant perturbation theory

Use the observable itself as expansion parameter

$\longrightarrow$ No dependence on renormalisation scale.

For a mean event shape $R \propto \langle y \rangle / A_f \sim \alpha_s$ require RGE:

$$\frac{\partial R^{-1}}{\partial \ln Q^2} = \frac{\beta_0}{4\pi}\left(1 + \frac{\beta_1}{2\pi\beta_0} R + \rho_2 R^2 + \ldots\right)$$

$\longrightarrow$ Allows to measure $\beta_R \approx \beta$ directly from a mean event shape, e.g. $\langle 1 - T \rangle$;

Solve this RGE like the one for α_s, determine integration const. $\Lambda_R \Longleftrightarrow \Lambda^{\mathrm{QCD}}_{\overline{\mathrm{MS}}}$

- Allows measurements of $\Lambda^{\mathrm{QCD}}_{\overline{\mathrm{MS}}}$ without freedom of renormalisation scale.
- RGI is numerically equivalent to ECH; RGI resums UV divergencies.
- Power corrections can be included in RGI theory.

RGI with power corrections

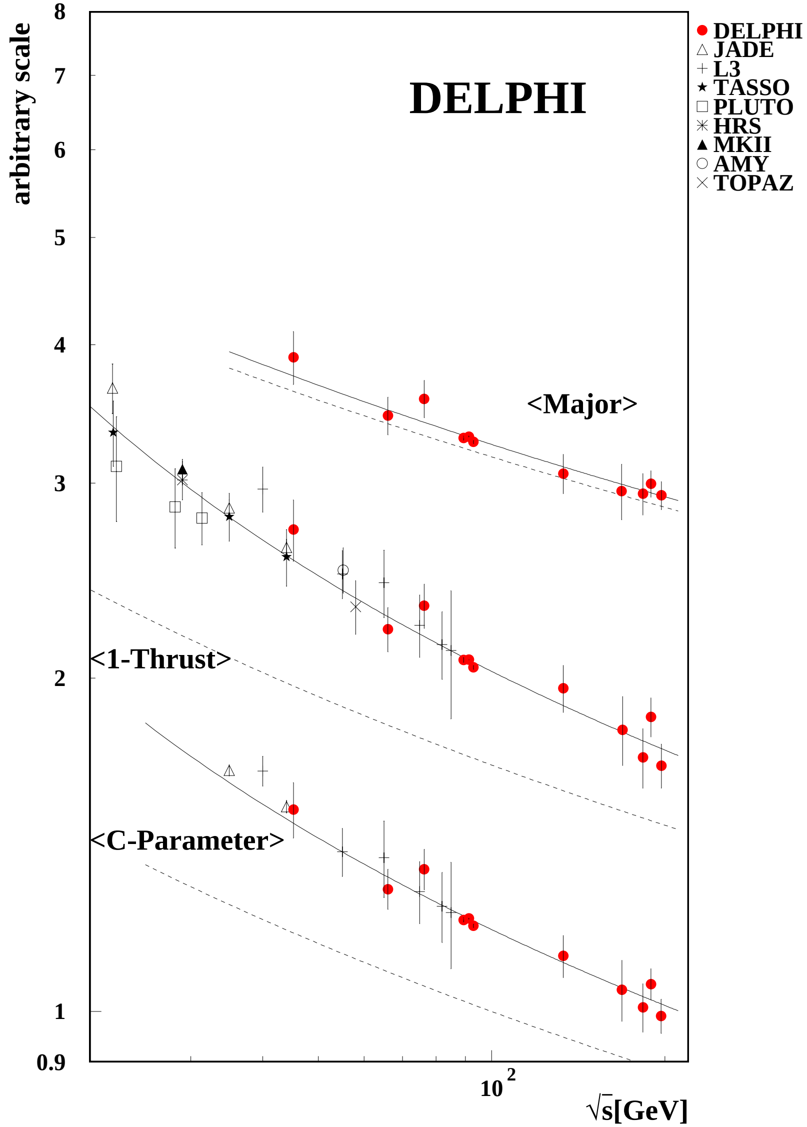
- Gives good description of the data
  small spread from 7 observables

- $\alpha_s(m_Z) = 0.1179$; spread 0.0020
- Power terms are compatible with zero!

Comparing Pure RGI to data

- Gives still a good description of the data

- $\alpha_s(m_Z) = 0.1201$; spread 0.0020
- Mean values can be described without hadronisation correction at 2% level using a theory without renormalisation scale freedom.



Power corrections are to large part missing higher order terms in $\overline{\mathrm{MS}}$ scheme

The measurement of the β-Function

β_R is slope of $1/R$ vs $\log s$

From DELPHI $\langle 1-T \rangle$:

$$\frac{\partial R^{-1}}{\partial \ln Q} = 1.38 \pm 0.05 \qquad n_f = 4.7 \pm 1.2$$

Uncertainty due to power terms small!
Include low energy data, extract β_0:

$$\beta_0 = 7.86 \pm 0.32$$

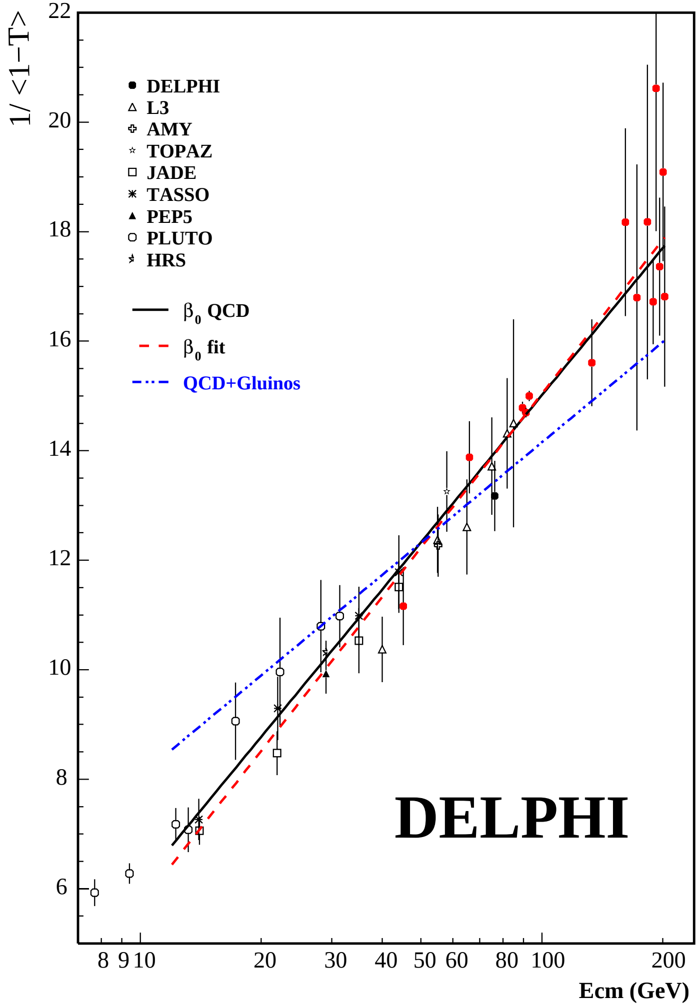
$$n_f = 4.75 \pm 0.44 \text{ (using QCD expression)}$$

Compare indirect measurement (via α_s):
LEP event shapes:

$$\beta_0 = 7.67 \pm 1.63$$

R_τ, F_2, F_3, R_Z, event shapes ... world data:

$$\beta_0 = 7.76 \pm 0.44$$



QCD + light gluinos excluded

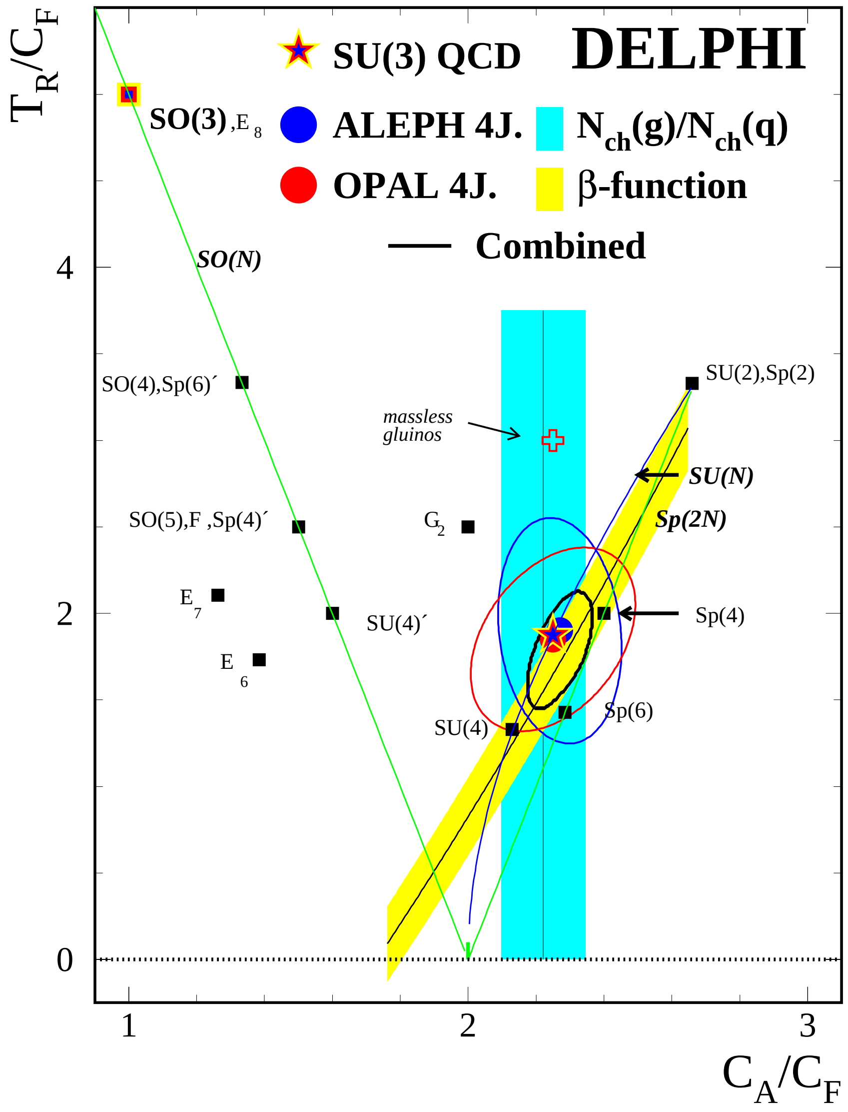
The Gauge Group of Strong Interactions

the measurements of

- the QCD β-function
- the multiplicity of gluon and quark jets
- the 4–jet angular distributions
- ...

strongly restrict the gauge group of the strong interaction to

$SU(3)_c$



Summary

e^+e^- experiments provide extensive precision tests of QCD

- Heavy quark fragmentation $\rightarrow$ input for hadron colliders

- Consistent picture of the energy evolution from b factories to LEPII
  α_s & β-function, m_b, inclusive distributions

- Absolute measurements of α_s to be improved $\rightarrow$ NNLO calculation

- $SU(3)_c$ is experimentally identified as the gauge group of strong interactions