# Dynamics of Hot Gauge Theories

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Lattice '01

## Motivation:

### Early Cosmology

baryogenesis, leptogenesis, magnetogenesis

### Heavy Ion Collisions

central rapidity, small-x physics, ...

### Theoretical Interest

interplay of QFT, stat. mech., kinetics, hydro, ...

generalized RG hierarchies

# Outline:

Qualitative Overview

Known Quantitative Results

Open Questions + Problems

# Relevant Scales

| length | | time |
|---|---|---|
| typical de Broglie wavelength | $T^{-1}$ | typical excitation $(\text{energy})^{-1}$ |
| Debye screening length | $(gT)^{-1}$ | plasmon $(\text{frequency})^{-1}$ |
| color coherence length | $(g^2 T \ln 1/g)^{-1}$ | quasiparticle lifetime |
| non-perturbative magnetic length | $(g^2 T)^{-1}$ | |
| transport mean free path | $(g^4 T \ln 1/g)^{-1}$ | large angle scattering time, non-perturbative magnetic relaxation time |

hard ($p \sim T$) quarks, gluons

= well defined, weakly interacting quasiparticles

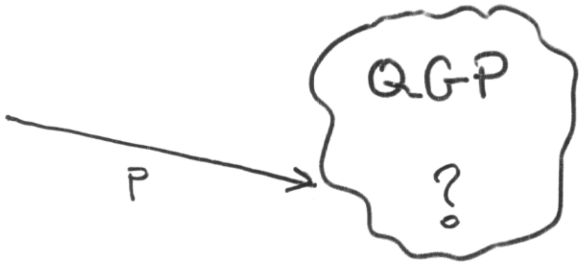
$\Rightarrow$ dominant contribution to bulk thermodynamics

Most physical observables primarily sensitive to properties of hard excitations

pressure, energy density, ...
screening length
equilibration rates
transport coefficients
photo emission rate
⋮

exception: baryon violation rate

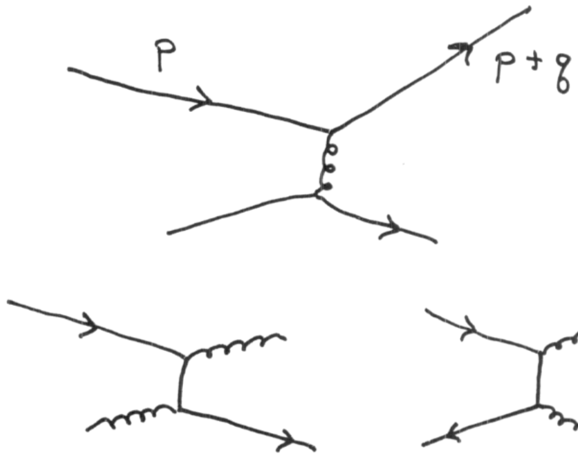
# Quasiparticle Dynamics




hard scattering

$q \sim T$

rate $\sim g^4 T$

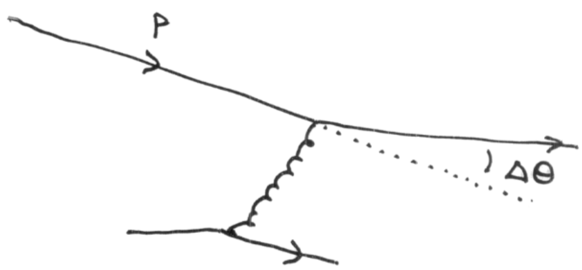



soft scattering

$q \sim gT$

$\Delta\theta \sim g$

rate $\sim g^2 T$




negligible change to quasiparticle momentum

$O(1)$ " " " color

Key assumptions:

Thermal equilibrium

Very hot $\Rightarrow$ weakly coupled

$g(T) \ll 1$

Goal:

Calculate physical quantities probing real-time dynamics:

transport coefficients $\left\{ \begin{array}{l} \text{viscosity} \\ \text{conductivity} \\ \text{diffusion} \end{array} \right.$

photo-emission rate

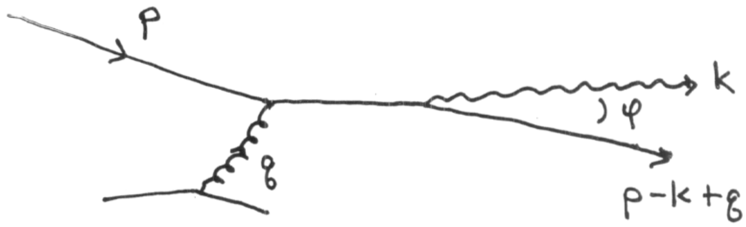
di-lepton emission rate

energy loss $dE/dx$

topological transition rate

$\vdots$

hard bremsstrahlung




$k \sim T$

$q \sim gT$

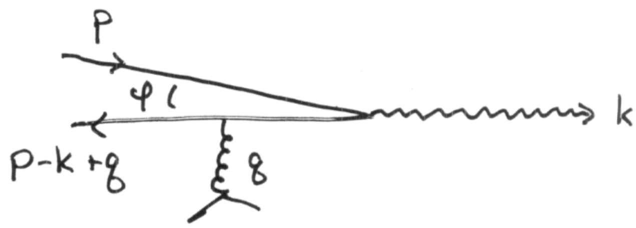
$\varphi \sim g$

$$\text{rate} \sim \underbrace{g^6}_{\text{explicit}} \cdot \underbrace{g^4 T^4}_{\text{phase space}} \cdot \underbrace{\frac{1}{g^4 T^4}}_{\text{soft gluon}} \cdot \underbrace{\frac{1}{g^2 T^2}}_{\text{near on-shell quark}} \cdot T^3$$

$\sim g^4 T$ – same as $2 \leftrightarrow 2$ hard scattering

Aurenche, Gelis, Kobes, Zaraket, ...

inelastic annhilation



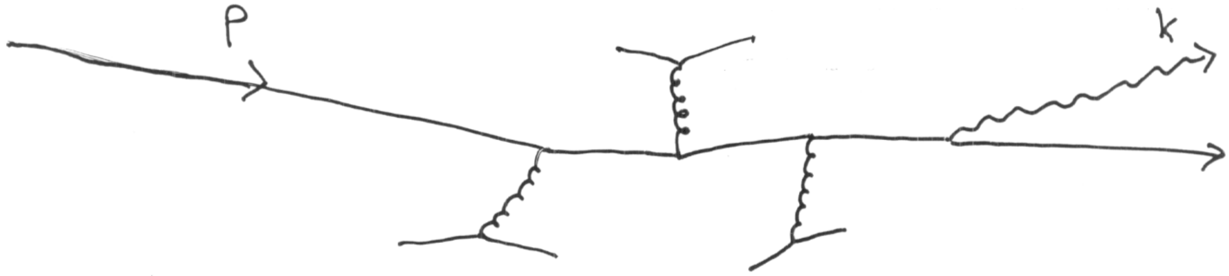

$p, k \sim T$

$q \sim gT$

$\varphi \sim g$

rate $\sim g^4 T$

multiple soft scattering during
bremsstrahlung / annhilation




near on-shell intermediate propagators
⇒ long-lived virtual states
⇒ photon formation time ~ $1/g^2 T$
— same as soft scattering mean free time

∴ multiple scattering $O(1)$ importance,
suppresses rate — LPM effect

N.B. Quantum interference important
on $1/g^2 T$ time scale

# Soft gauge fields

$\underset{\sim}{k}, \omega \ll T$ : relevant d.o.f. ~ classical fields

$$D_\nu F^{\mu\nu} = J^\mu$$

soft gauge field    color current of hard quasiparticles

~ Boltzmann - Vlasov theory

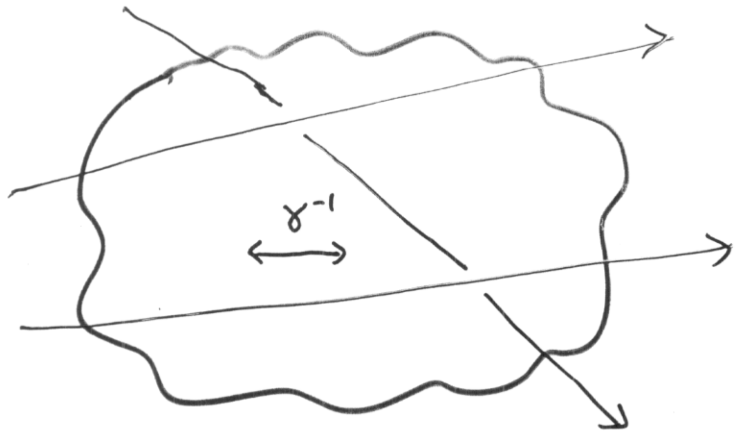
Integrate out hard particles    D. Bodeker

⇒ effective theory of soft gauge field,

non-local on scale $\gamma^{-1}$ = color coherence length

Restrict attention to long distance, $k \ll \gamma$

⇒ local stochastic effective theory




Ampere + Ohm + fluctuation-dissipation $\Rightarrow$

$$D \times B = \sigma E - \zeta$$

noise $\langle \zeta \zeta \rangle = 2\sigma T$

color conductivity $\sigma \sim T / \ln g^{-1}$

characteristic frequency $\omega \sim k^2/\sigma$

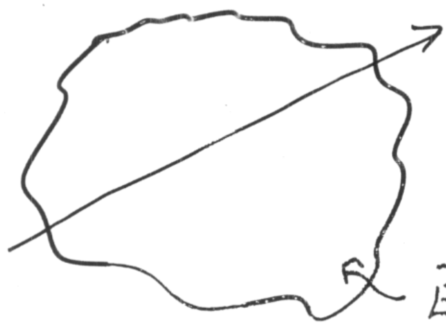
non-perturbative spatial scale $k \sim g^2 T$

$\Rightarrow$ characteristic relaxation rate of
non-perturbative magnetic fields $\omega \sim g^4 T \ln g^{-1}$

$\Rightarrow$ topological transition rate / volume

$$\Gamma \sim \# \, k^3 \omega \sim \# \, \frac{(g^2 T)^5}{\sigma} = O(\alpha_w^5 T^4 \ln 1/\alpha_w)$$

# Color conductivity



$\vec{E} \neq 0$

acceleration $\quad \Delta p \sim g E \Delta t$

induced current $\quad \Delta j \sim g \left(\frac{\Delta p}{p^0}\right) n \sim g^2 \frac{E}{T} T^3 \Delta t$

acceleration time $\quad \Delta t \sim \tau_{mft} \sim 1/g^2 T \ln g^{-1}$

$$\Rightarrow \quad \sigma = \frac{\Delta j}{E} = g^2 T^2 \tau_{mft} \sim T/\ln g^{-1}$$

# Quantitative Results

## I. Transport coefficients

shear viscosity

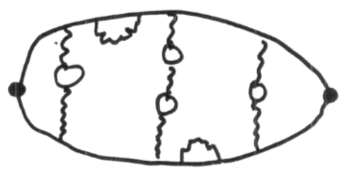
$$\eta = \frac{\beta}{20} \int d^4x \left[ \langle T_{ij}(x)\, T_{ij}(0) \rangle - \tfrac{1}{3} \langle T_{ii}(x)\, T_{jj}(0) \rangle \right]$$

$$= \tfrac{1}{20} \cdot \text{slope of spectral density at } \omega = \underset{\sim}{k} = 0$$

$$\sim \# T^4 \Big/ \left\{ g^4 T \ln\left(\tfrac{1}{g}\right) \left[ 1 + O\left(1/\ln g^{-1}\right) \right] \right\}$$

[ $\eta \propto$ large angle scattering time ]

Leading log approx.: Compute overall #, ignore $O(1/\ln g^{-1})$ effects

Requires summing 

$\Rightarrow$ linearized Boltzmann equation in $l=2$ channel

$$I(p) = \int d^3k \; \mathcal{C}(p,k)\, f(k)$$

linearized collision operator for soft scattering

Appropriate effective theory = kinetic theory of hard excitations

P. Arnold, G. Moore, L.Y. :

shear viscosity $\eta = A T^4 / [g^4 T \ln g^{-1}] \, (1 + O(1/\ln g^{-1}))$

$$A = \begin{cases} 86.5 & N_f = 2 \quad \text{QCD} \\ 106.7 & N_f = 3 \quad '' \end{cases}$$

flavor diffusion $D = A_F / [g^4 T \ln g^{-1}] \, (1 + O(1/\ln g^{-1}))$

$$A_F = \begin{cases} 13.0 & N_f = 2 \\ 11.9 & N_f = 3 \end{cases}$$

$$\simeq \frac{2^4 \, 3^6 \, \zeta(3)^2 \, \pi^{-3}}{24 + 4N_f + \pi^2}$$

electrical conductivity $\sigma_{EM} \sim C e^2 T^2 / [e^4 T \ln e^{-1}] \, (1 + O(1/\ln e))$

$$C = \begin{cases} 15.7 & e \\ 20.7 & e, \mu \\ 12.3 & e, \mu, u, d, s \end{cases}$$

$$\simeq \frac{12^4 \, \zeta(3)^2 \, \pi^{-3} \, N_{\text{leptons}}}{3\pi^2 + 32 \sum_i (q_i)^2}$$

earlier attempts : Baym, Monien, Pethick, Ravenhall; Heiselberg; Joyce, Prokopec, Turok, Moore

# II. Color conductivity

N.B. No gauge invariant Kubo formula

Determine by matching long distance effective theory to underlying (HTL) hot QCD

$$\sigma^{-1} = \underbrace{\frac{3 N_c \alpha T}{m_D^2}}_{\text{Bodeker; Selikov + Gyulassy}} \left[ \ln\left(\frac{m}{\gamma(\mu)}\right) + C \right] \left[ 1 + O\left(1/\ln^2(1/g)\right) \right]$$

$C = 3.041$ P. Arnold, L.Y.

# III. Topological transition rate

$\Gamma$ = diffusion constant/volume for Chern-Simons number

$\propto$ baryon violation rate in electroweak theory

$$\Gamma = \kappa \left(\frac{2\pi}{3\sigma}\right) (\alpha T)^5$$

$\kappa = 10.8 \pm 0.7$ from real-time simulation of stochastic effective theory G. Moore

N.B. long distance effective theory

= stochastic quantization of 3-d Y.M.

superrenormalizable, UV finite

# IV. Photoemission rate

$$\frac{dn_\gamma}{dk\, dV\, dt} = \left(3 \sum_s g_s^2\right) \frac{4\pi k}{e^{k/T}+1} \frac{m_a^2 \alpha_{EM}}{2\pi^3}\left[\ln\frac{T}{m_a} + \frac{1}{2}\ln\frac{k}{T} + C\left(\frac{k}{T}\right)\right]$$

$m_a$ = asymptotic thermal quark mass $= g_s^2 T^2/6$

$2 \leftrightarrow 2$ scattering $\Rightarrow$ leading log + part of $C(k/T)$

Kapusta, Lichard, Seibert; Baier, Nakkagawa

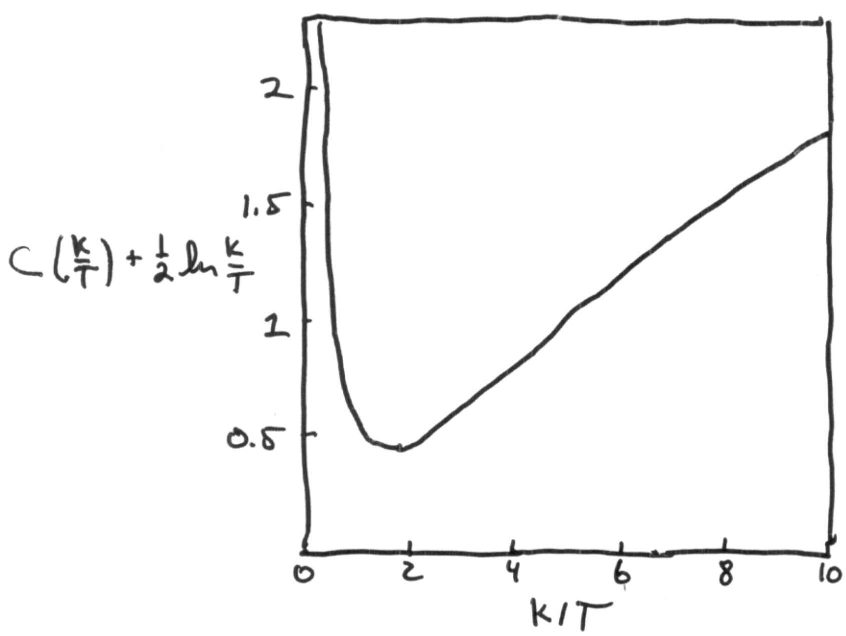
Complete leading order : G. Moore, P. Arnold, L.Y.

derive :

$$C(k/T) = C_{2\leftrightarrow 2} + \int_{-\infty}^{\infty} dp_\parallel \, d^2p_\perp \, [\text{stuff}] \, \vec{p}_\perp \cdot \vec{f}(\vec{p}_\perp, p_\parallel; k)$$

$$2p_\perp = i\,\delta E \, f(\vec{p}_\perp) + \int d^2q \, \mathcal{C}(q, p_\perp)\left[f(p_\perp) - f(p_\perp + q_\perp)\right]$$

evaluate:



# Open Questions + Problems

## Perturbative

1. Generalize photoemission analysis to gluon emission. Compute gluon emission beyond leading log.

2. Compute any QCD transport coefficient beyond leading log. Requires inserting hard gluon LPM result into Boltzmann collision term.

3. Compute QCD bulk viscosity — even in leading log approx.

4. Compute any transport coefficient beyond leading order. Is kinetic thy. still adequate effective theory?

## Non-Perturbative

1. Test practical domain of ~~validity~~ utility of leading order, or leading log, results.

2. Extract transport coefficients from Euclidean LGT.

3. Extract more physics from real-time, classical lattice simulations. Hydrodynamic fluctuations, long time tails.

4. Improve classical Hamiltonian lattice gauge theory.

# Utility of weak coupling results

1. Special case: $N_f \to \infty$ limit G. Moore

   Bremsstrahlung + LPM irrelevant.

   NLLO results for transport coefficients quite good provided $\cancel{m_D/T < 1}$. $\mu \frac{d}{d\mu} \ln g^2 \lesssim 1/4$.

2. Topological transition rate also extracted from real time simulations of more microscopic ("less effective") classical lattice theory.

   Moore, Rummukainen

   Excellent agreement with Bodeker's effective theory w. NLLO color conductivity (given $\approx 20\%$ systematic errors)

3. Overdamped $\omega \sim k^2/\sigma$ behavior visible in other observables? Ambjorn, Anagnostopolous, Krasnitz

   Examined $\langle B^2(x) B^2(0) \rangle$, $\langle D \times B(x)\, \mathcal{U}^{adj}(0,t;x)\, D \times B(x,t) \rangle$, ... in real-time classical lattice gauge theory at $\approx$ electroweak scale.

   No clear sign of time scales other than $a$, $\omega_{plasma}^{-1}$.

   But: IR sensitivity of chosen observables unclear. Needs more study ...

# Conclusion:

Much recent progress in hot gauge field dynamics, but many interesting problems remain.

While waiting for your 100 TFlop PC, escape Euclidean space and do hot dynamics!