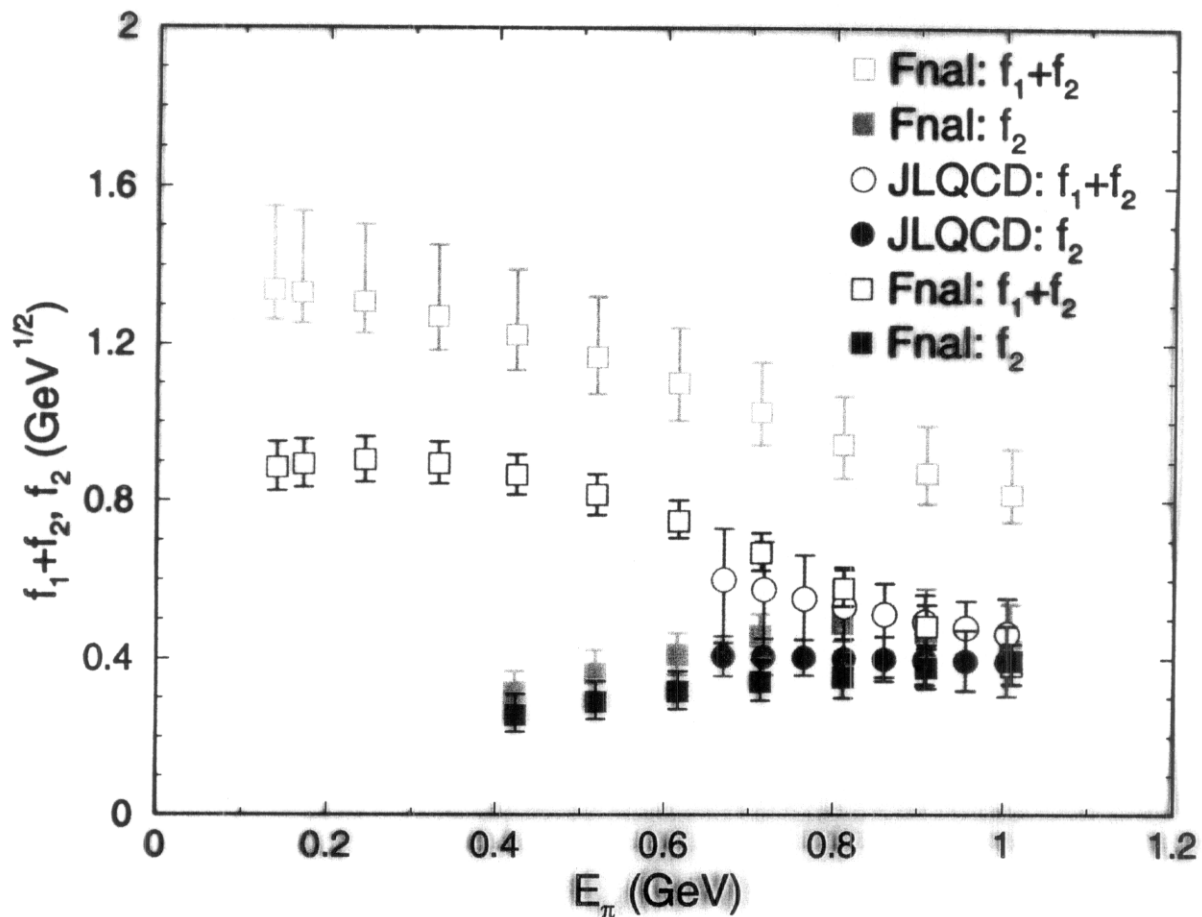




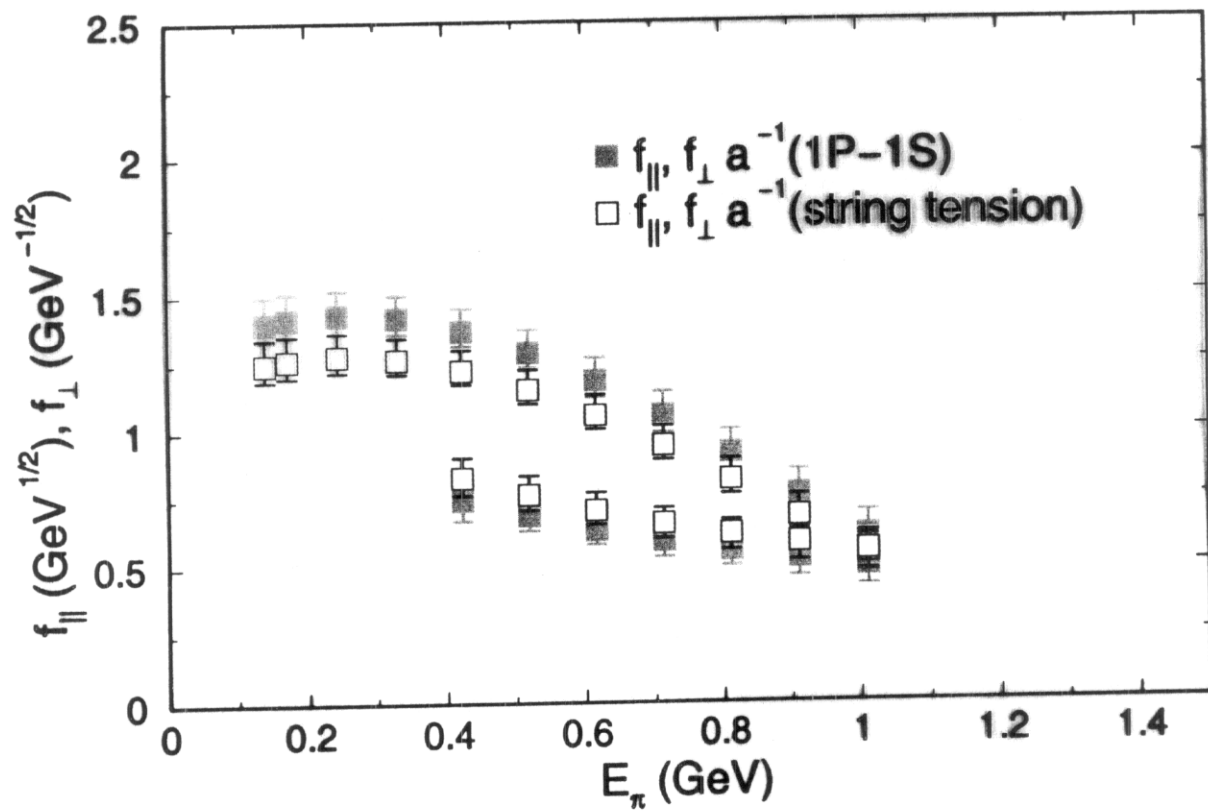
Light Mass Dependence

F_{nal} and JLQCD with the same analysis and range of light quark masses:



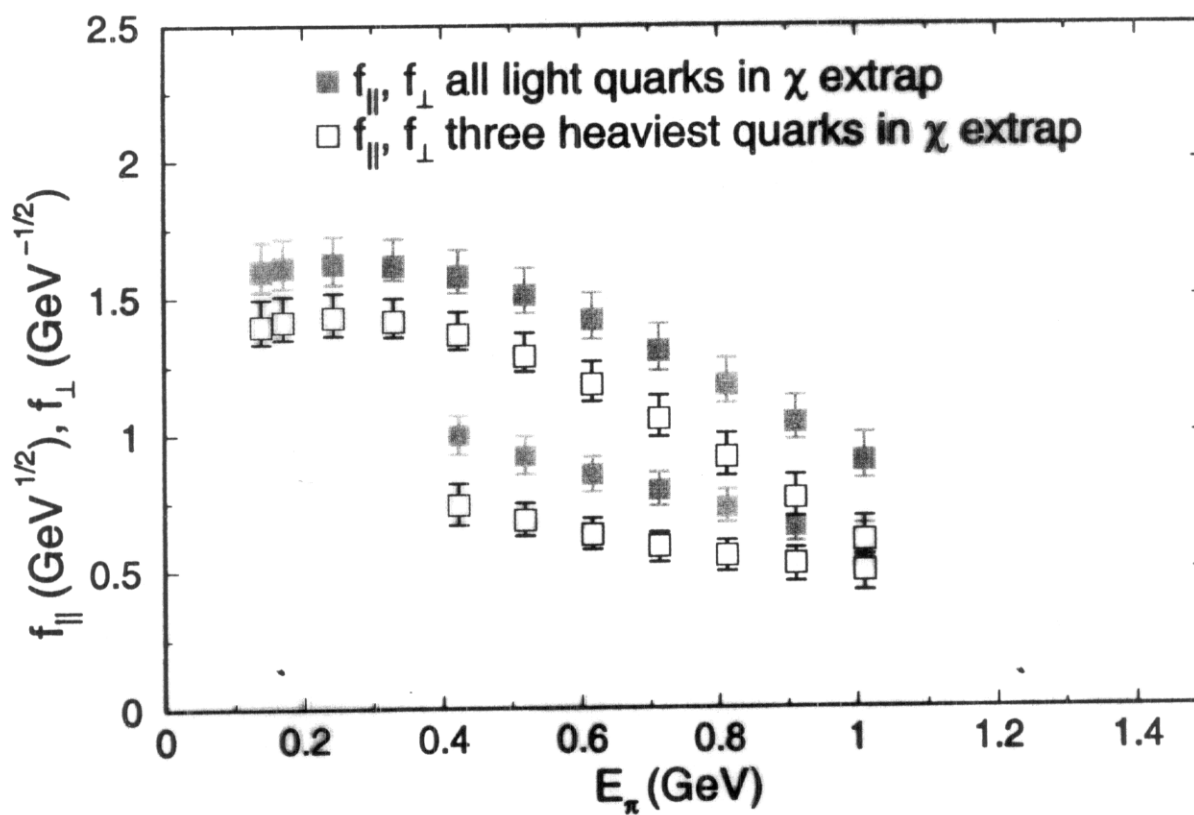
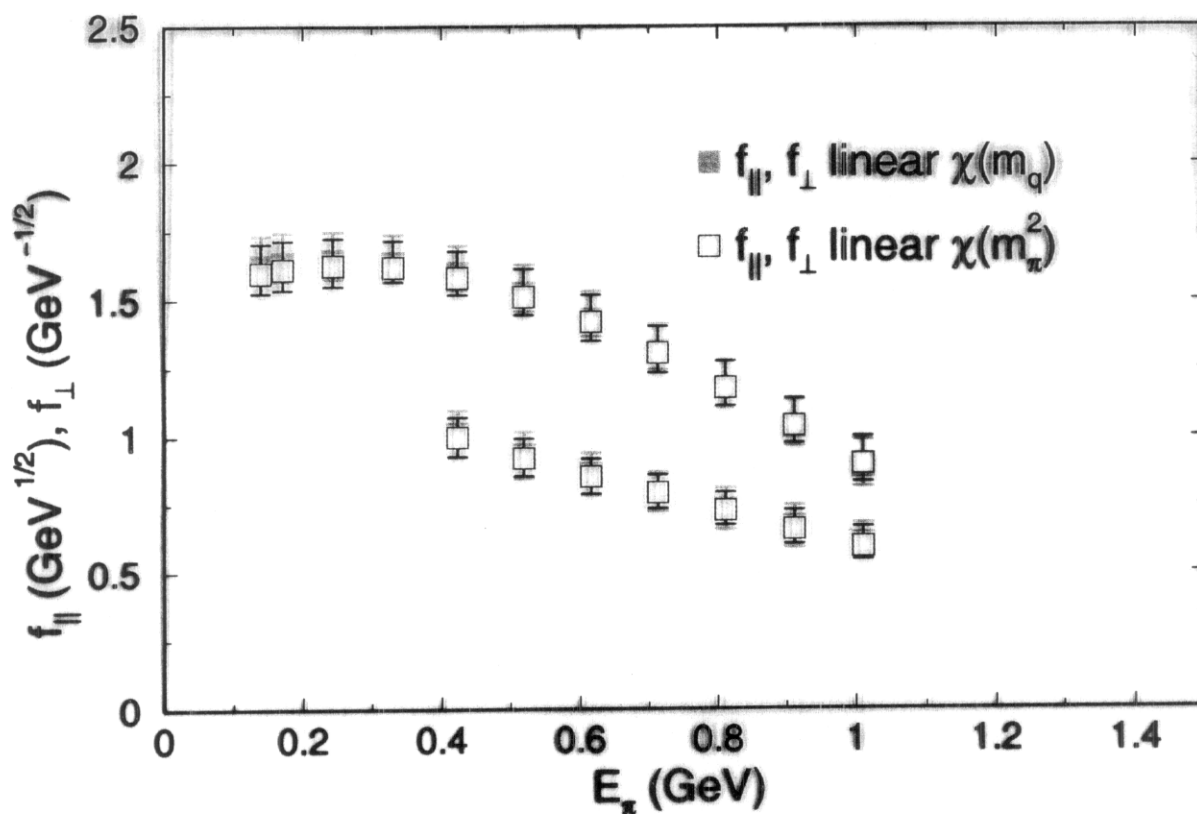
Observations:

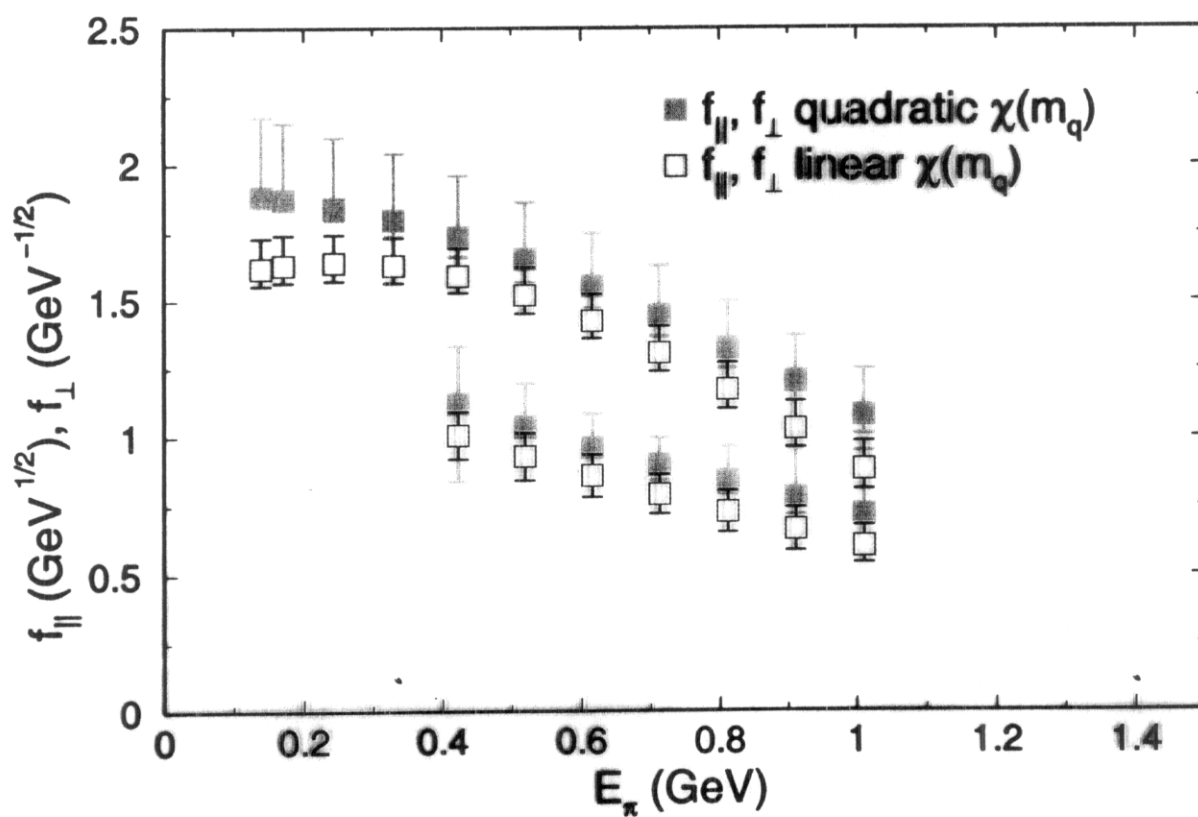
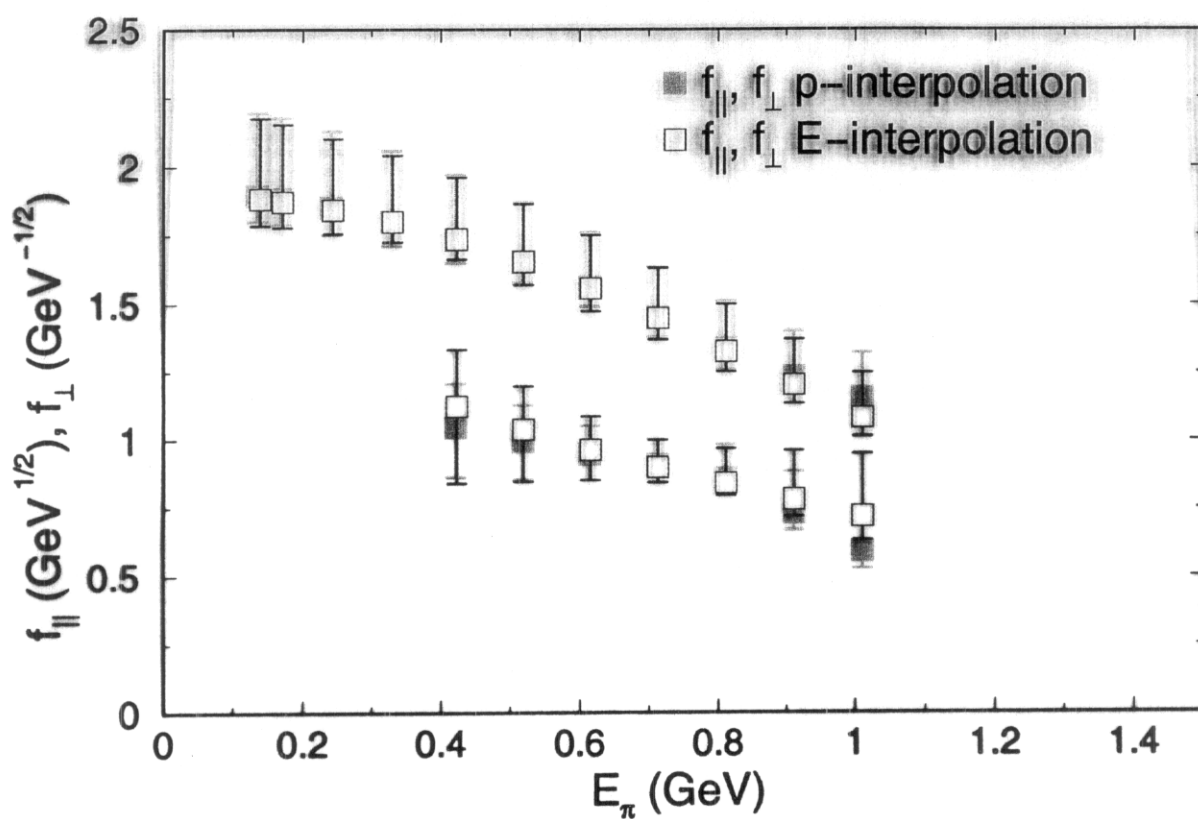
- ◇ $f_1 + f_2$ in much better agreement than previously.
- ◇ f_2 in reasonable agreement $\Rightarrow d\Gamma/dq^2 (\propto f_+)$ agrees.
The slope of $f_2^{\text{fnal}} > f_2^{\text{jlcqd}}$ at small E .
- ◇ JLQCD's larger lattice $\rightarrow$ more reliable ground state but larger statistical errors.



Now, re-plotting $f_1(E) + f_2(E)$ and $f_2(E)$









Transforming the Fnal analysis

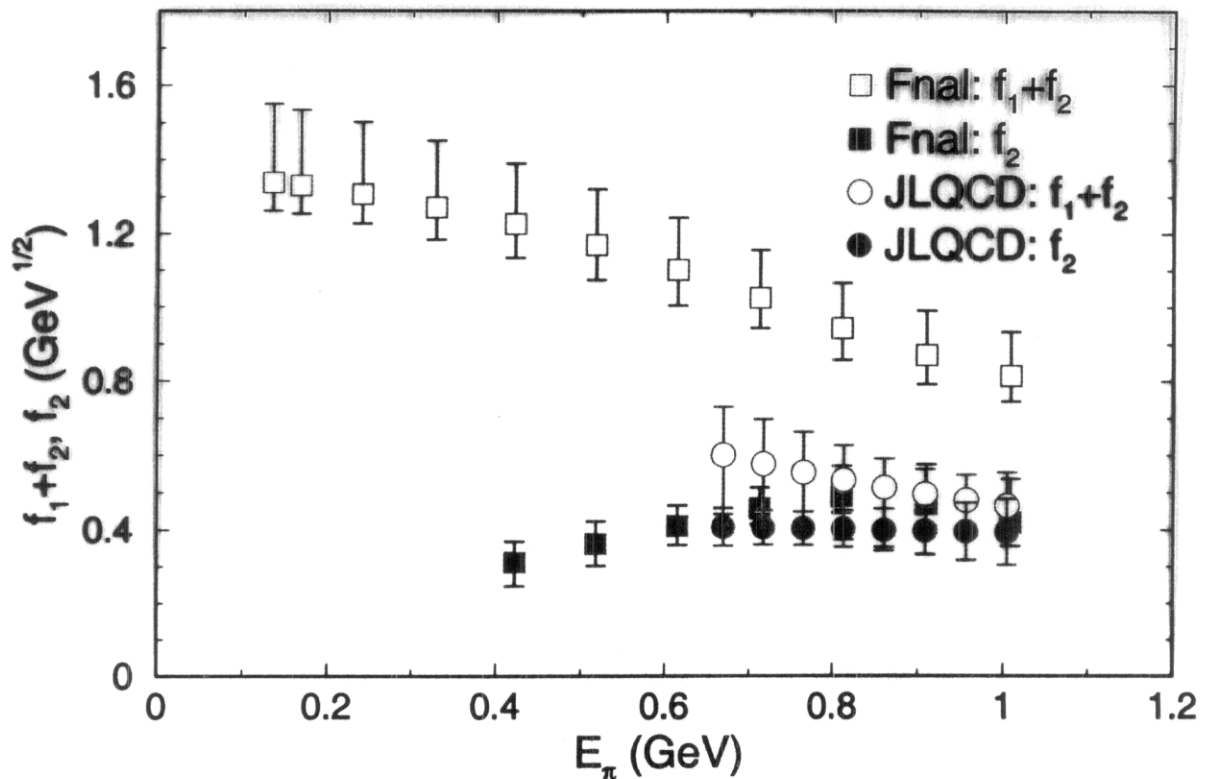
The next 6 plots show the effect on $f_{\parallel}$ and $f_{\perp}$ as the steps in the FNAL analysis are changed to resemble JLQCD.

- p_{π} -interpolation $\longrightarrow$ E_{π} -interpolation
- quadratic χ extrap in $m_q \longrightarrow$ linear χ extrap in m_Q
- Linear χ extrap in $m_Q \longrightarrow$ linear χ extrap in m_{π}^2
- The same range of light quark masses used in this analysis as in JLQCD analysis ie don't include the lightest mass in χ -extrapolations.
- For the purposes of this comparison set the scale, a^{-1} from the string tension. Except for this comparison we take $a^{-1}(1P - 1S)$.



Comparing the form factors ...

JLQCD: hep-lat/0106024, fig.10



◇ Significant difference in f_1+f_2 . Yields f_0 , which does not contribute to the rate. However, the reason(s) for such a difference are important.

Can we understanding the difference by focusing on analyses?

	Fnal (central value)	JLQCD
momentum interpolation	p_π	E
chiral extrapolation	m_q	m_π^2
fit form	quadratic	linear
a^{-1}	$(1P - 1S)$	$\sqrt{\sigma}$



Why focus on $f_{\parallel}$ and $f_{\perp}$?

- ◇ Symmetry considerations: the $m_{\pi}, E \rightarrow 0$ yield

$$f_{\parallel} = \frac{f_B \sqrt{m_B}}{\sqrt{2} f_{\pi}} \quad \dots \text{soft pion relation}$$

$$f_{\perp} = \frac{f_B^* \sqrt{m_B^*}}{\sqrt{2} f_{\pi}} g_{BB^* \pi} \frac{2m_B}{m_{B^*}^2 - q^2} \quad \dots \text{vector meson dominance}$$

through $\mathcal{O}(1/m_Q)$ in the HQ expansion. It is clear that $f_{\parallel}$ and $f_{\perp}$ behave differently as $m_{\pi} \rightarrow m_{\pi}^{\text{phys}}$.

- ◇ $f_{(\parallel, \perp)}$ have simple descriptions in HQET $\Rightarrow$ natural Fermilab or NRQCD approaches.

- ◇ They emerge directly from lattice calculation $\rightarrow$ simpler to analyse separately, form $f_{(+,0)}$ at the end.

Comparing simulation parameters:

	Fnal	JLQCD
β	5.9	5.9
#Cfgs.	350	2150
Heavy Quark	Fermilab	NRQCD
		includes all $1/M$ terms
$a m_{\ell}$, Light Quark	0.024–0.059	0.034–0.072



Fnal and JLQCD Analyses

- ◇ Both use HQET motivated parameterisation

FNAL

$$\langle \pi(p_\pi) | \mathcal{V}^\mu | B(p_B) \rangle = \sqrt{2m_B} [v^\mu f_\parallel(E) + p_\perp^\mu f_\perp(E)]$$

where $v = p_B/m_B$ and $p_\perp = p_\pi - Ev$.

JLQCD

$$\langle \pi(p_\pi) | \mathcal{V}^\mu | B(p_B) \rangle = 2 \left[f_1(E) v^\mu + f_2(E) \frac{p_\pi^\mu}{E} \right]$$

- ◇ In the Fnal parameterisation $f_{+,0}(E)$ are

$$f_+(E) = (2m_B)^{-1/2} [f_\parallel(E) + (m_B - E)f_\perp(E)]$$

$$f_0(E) = \frac{\sqrt{2m_B}}{m_B^2 - m_\pi^2} [(m_B - E)f_\parallel(E) + (E^2 - m_\pi^2)f_\perp(E)]$$

at $q^2 = 0$, $f_+ = f_0$ as required.

- ◇ The JLQCD and FNAL form factors are related as

$$f_1(E) + f_2(E) = \frac{1}{\sqrt{2}} f_\parallel(E)$$

$$f_2(E) = \frac{E}{\sqrt{2}} f_\perp(E)$$

Overview of Analyses

	APE	UKQCD	FNAL	JLQCD
m_b	X	X	✓	✓
m_c	✓	✓	✓	✓
mass defn.	M_{pole}	M_{pole}	M_{kinetic}	M_{kinetic}
χ -Extrap.	✓	✓	✓	✓
Multi- a	X	X	✓	X
Matching	NP	NP	NP+PT	PT
	$m_q = 0$	$m_q = 0$	$\sqrt{Z_{V^{uu}} Z_{V^{bb}}}$ NP	1-loop
	+KKM(m_c)	+KKM(m_c)	rad. corr. (ρ_V) PT	
$p = p_{\pi}^{\text{exptl}}$	X	X	X	X
$N_f > 0$	X	X	X	X



$|V_{ub}|$ from $B \rightarrow \pi l \nu$: theoretical background

Considerable experimental effort to measure differential decay rates.

◇ For $B \rightarrow \pi l \nu$

$$\frac{d\Gamma}{dp} = \frac{G_F^2 |V_{ub}|^2 2m_B p^4 |f_+(E)|^2}{24\pi^3 E}$$

where $E = p_\pi \cdot p_B / m_B$.

◇ $f_+(E)$ parameterises the hadronic matrix element

$$\begin{aligned} \langle \pi(p_\pi) | \mathcal{V}_\mu | B(p_B) \rangle &= f_+(E) \left[p_B + p_\pi - \frac{m_B^2 + m_\pi^2}{q^2} q \right]^\mu \\ &\quad + f_0(E) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \end{aligned}$$

◇ Recent calculations of hadronic matrix elements by

UKQCD – 1999 (extrapolate from D)
 APE – 2000 (extrapolate from D)
 FNAL – 2001 (Fermilab)
 JLQCD – 2001 (NRQCD)

◇ APE and FNAL also report $D \rightarrow \pi(K) l \nu$. ($|V_{ub}| \rightarrow |V_{cd}|$ and $m_B \rightarrow m_D$ in the above).

◇ UKQCD also have $B \rightarrow \rho l \nu$ (PRELIM.)

talk by J. GILL

Summary

Quenched

f_B result is stable, since last year. Good agreement between different methods.

large scaling violations seen by CP-PACS, not seen by other groups.

B_B recent calculations agree with previous ones

JLQCD: including entire $1/m$ corrections for heavy quark

APE: interpolate between charm and static.

Unquenched

f_B $(f_B^{N_f=2})_{\text{AVG}} \sim 20\%$ higher than average quenched result.

B_B unaffected by unquenching (?) (JLQCD)

large discretisation errors in some calculations, not studied in others

χ -extrapolation at $N_f = 2$; for f_B problematic?

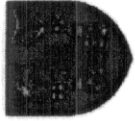


Table of $N_f = 2$ results

$N_f = 2$		
Group	f_B	$f_B^{N_f=2} / f_B^{N_f=0}$
Collins99(NR)	186(5)(19)(9)(13)($^{+50}_{-0}$)	$\simeq 1.26$
MILC'00*	191(6)($^{+24}_{-18}$)($^{+11}_{-0}$)	$\simeq 1.10$
MILC'01*		1.23(3)(3)(2)(2)(8)
CP-PACS'00	208(10)(11)	1.11(6)
CP-PACS'00(NR)	204(8)(29)($^{+44}_{-0}$)	1.10(5)
JLQCD(NR)*	190(14)(7)	$\simeq 1.14$

$N_f = 2$		
Group	f_D	$f_D^{N_f=2} / f_D^{N_f=0}$
MILC'00*	215(5)($^{+17}_{-13}$)($^{+8}_{-0}$)	$\simeq 1.075$
CP-PACS'00	225(14)(14)	1.03(6)

Note: * denotes a preliminary result.



Decay constant and B-parameter averages

◇ Quenched Decay Constants

$$\begin{aligned} f_B &= 173(23)\text{MeV} \\ f_{B_s}/f_B &= 1.15(3) \\ f_D &= 203(14)\text{MeV} \\ f_{D_s}/f_D &= 1.12(2) \end{aligned}$$

◇ Unquenched Decay Constants

$$\begin{aligned} f_B &= 198(30)\text{MeV} \\ f_{B_s}/f_B &= 1.15(5) \\ f_{D_s} &= 250(30)\text{MeV} \\ f_{D_s}/f_D &= 1.16(4) \end{aligned}$$

C. Bernard at Lat00:

$\hat{B}_{B_q}^{\text{quench}} = 1.30(12)$. This includes all recent data except JLQCD NP-improved calculation. Including this does not change the average. I therefore keep this value as the quenched average.

$$\hat{B}_{B_q}^{\text{quench}} = 1.30(12)$$

JLQCD find no significant effect of unquenching on B_B .

$Q\chi PT \simeq 10\%$ effect.

It may be that this 10% will be an upper bound.

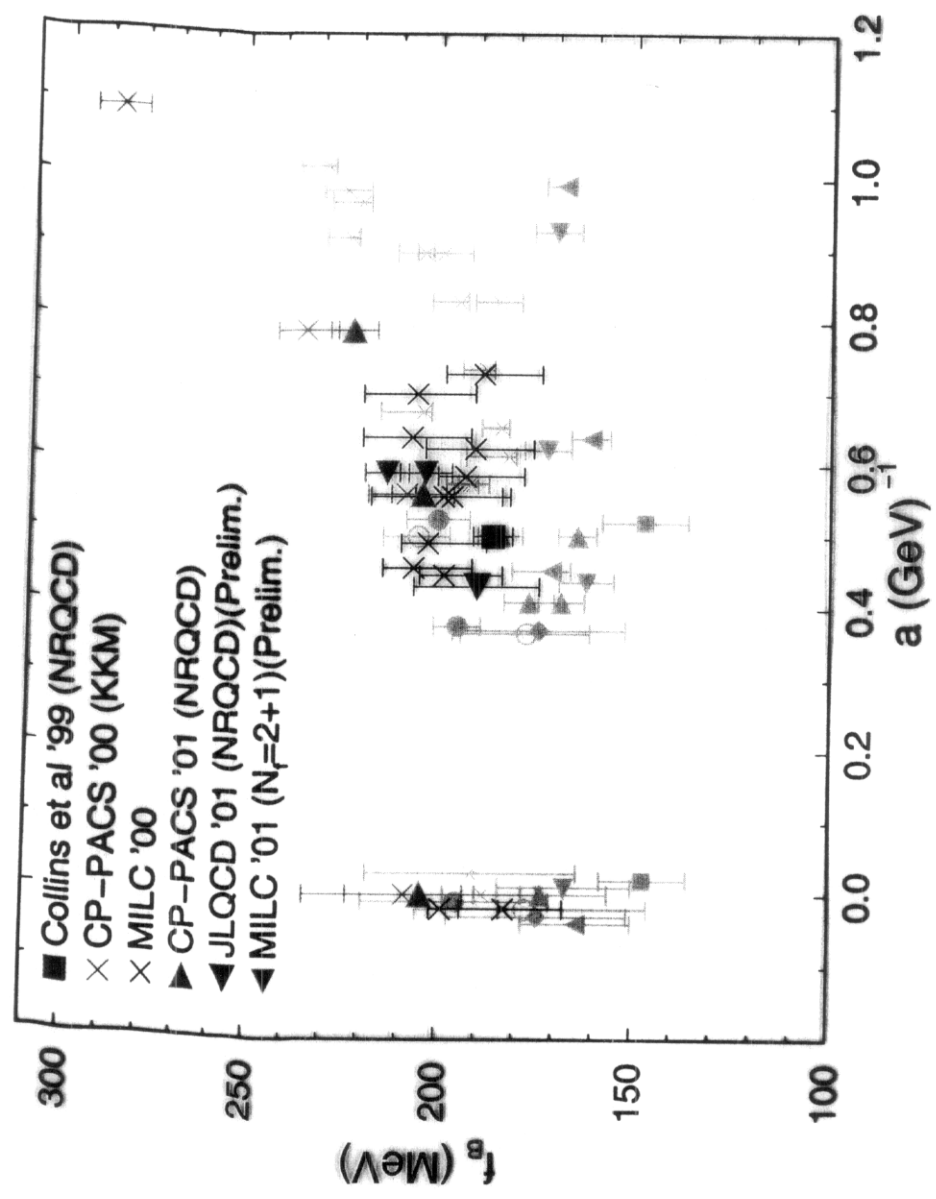
$$\hat{B}_{B_q} = 1.30(12)(< 10\%)$$

The ratio B_{B_s}/B_{B_d} is stable wrt cut-off effects and unquenching. I estimate

$$\left(\frac{B_{B_s}}{B_{B_d}}\right) = 1.01(3)$$



a dependence of unquenched f_B





JLQCD :

[talk by Yamada]

A calculation of f_B and the **B-parameters** with $N_f = 2$.

NRQCD and NP-improved Wilson light quarks.

Includes H-L matching $\rightarrow$ no $\mathcal{O}(\alpha/aM)$ error.

Results extrapolated to static $\rightarrow$ agree with Gimenez-Reyes calculation.

- On 3 lattices, compare QNP and QTI $\Rightarrow$ effect of NP c_{SW} .
- Compare UNP and QNP $\Rightarrow$ sea quark effects.

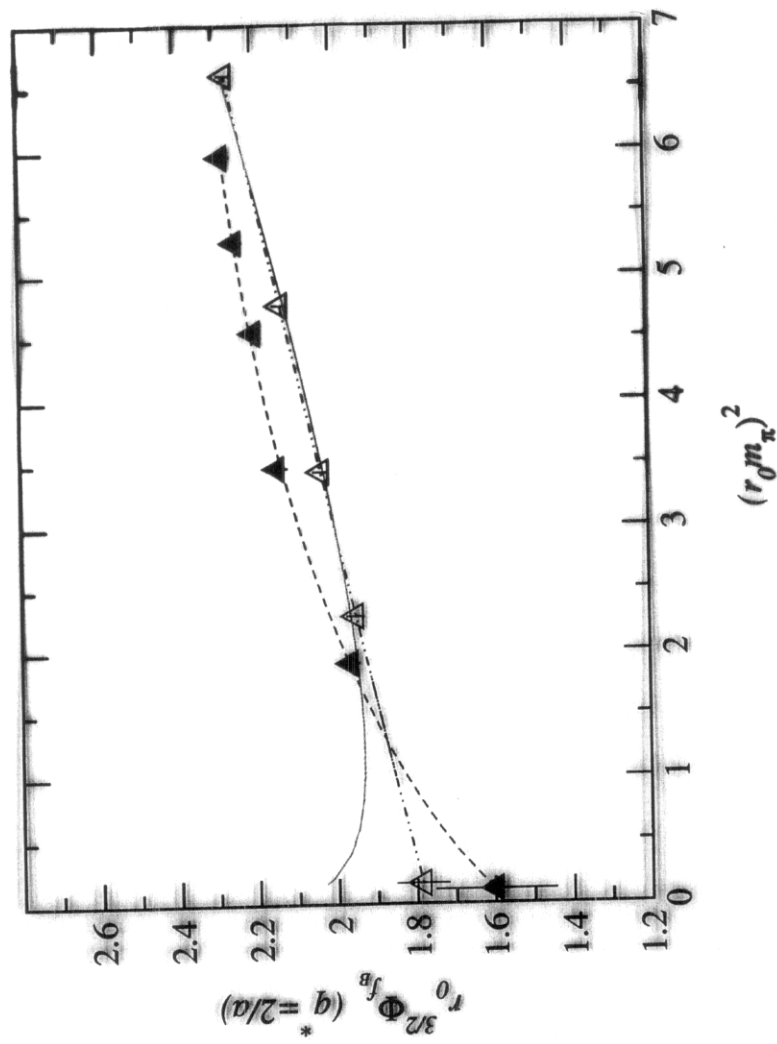
Observations

- The use of NP c_{SW} does not change f_B .
- $f_B^{N_f=2} > f_B^{N_f=0}$. Depends on chiral extrapolation.
- For B_B , no dependence on c_{SW} or cutoff.
- **No significant sea quark effects on B_B .**

Preliminary Results;

$$f_{B_d} = 0.190(14)(7)\text{GeV} \quad , \quad B_{B_d}(m_b) = 0.872(39)(19)(4) \\ f_{B_s}/f_{B_d} = 1.217(26)(20) \quad , \quad \frac{B_{B_s}}{B_{B_d}} = 0.999(12)(1)(4),$$

JLQCD chiral extrapolation of f_B





Chiral Extrapolations with $N_f = 2$

JLQCD try linear, quadratic and ln fits.

In fit motivated by χ PT (Sharpe and Zhang):

$$\Phi_{f_B} = (\Phi_{f_B})^{tree} \left[1 - \frac{3}{4} \frac{1}{(4\pi f_\pi)^2} (1 + 3g_{B^*B\pi}^2) m_\pi^2 \ln\left(\frac{m_\pi^2}{\Lambda^2}\right) + \dots \right]$$

$$B_B = (B_B)^{tree} \left[1 - \frac{1}{2} \frac{1}{(4\pi f_\pi)^2} (1 - 3g_{B^*B\pi}^2) m_\pi^2 \ln\left(\frac{m_\pi^2}{\Lambda^2}\right) + \dots \right]$$

Φ_{f_B} : good χ^2/N_{df} for quad and ln extrapolations.

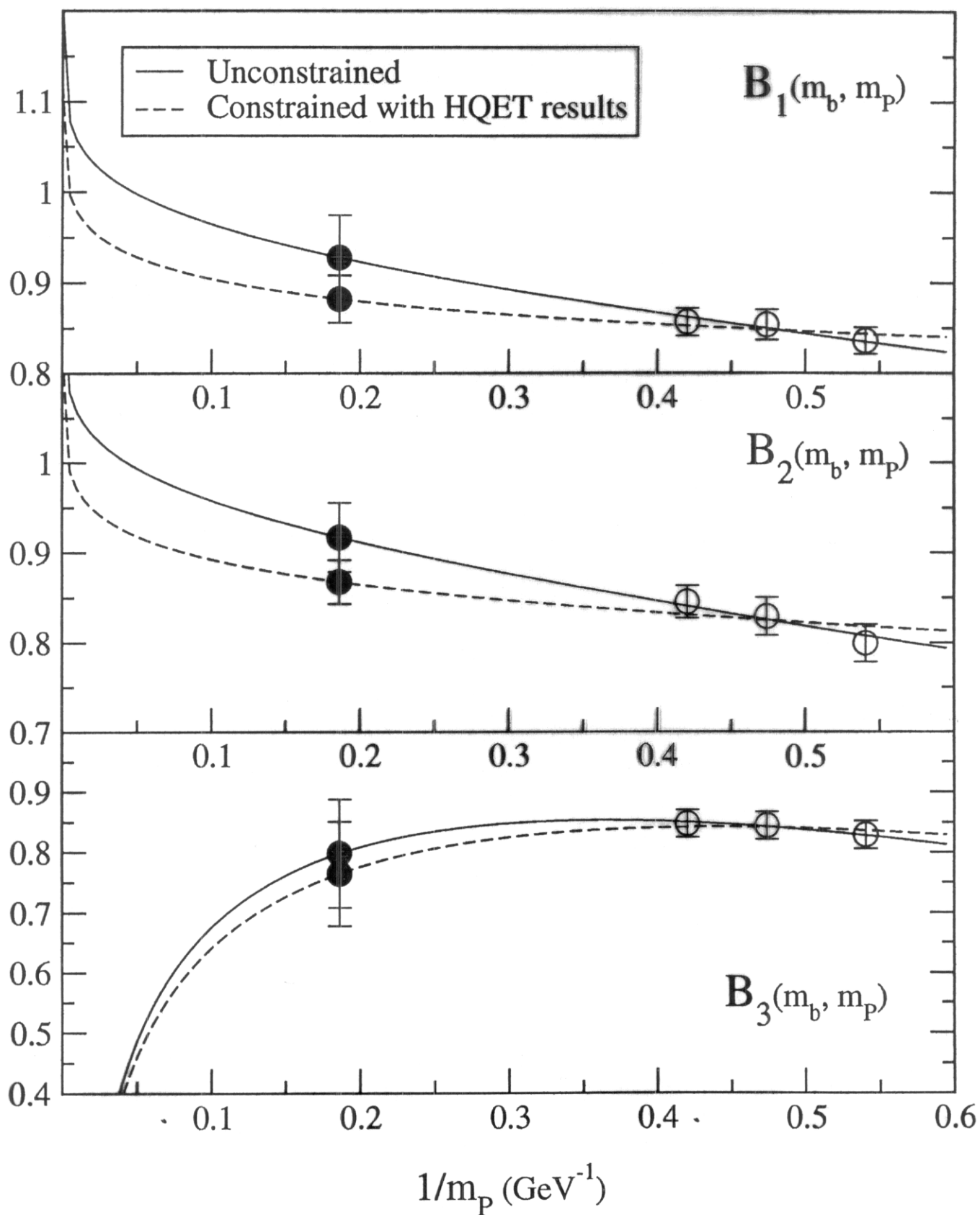
B_B : coefficient of ln fit consistent with zero.

For the quenched case a similar analysis motivated by quenched χ PT is attempted.

→ Data show no curvature.

A careful analysis of the chiral behaviour warranted?

APE :
[talk by Reyes]



B-parameters in $\overline{\text{MS}}$

PRELIMINARY

B-Parameters	Unconstrained	Constrained
$B_1(m_b)$	0.93(5)	0.88(3)
$B_2(m_b)$	0.92(4)	0.87(3)
$B_3(m_b)$	0.80(9)	0.74(10)
$B_4(m_b)$	1.09(3)	1.06(3)
$B_5(m_b)$	1.95(3)	1.99(7)

HQET B-Parameters	HQET results	Fit to dynamical data
$\bar{B}_1(m_b)$	0.98(8)	0.89(5)
$\bar{B}_2(m_b)$	0.93(6)	0.83(4)
$\bar{B}_4(m_b)$	1.10(7)	1.07(5)
$\bar{B}_5(m_b)$	2.26(12)	2.40(13)

B-parameters

$$B_1(\mu) = \frac{\langle \bar{B}^0 | O_1(\mu) | B^0 \rangle}{\frac{8}{3} |\langle 0 | A | B^0 \rangle|^2}$$

$$B_2(\mu) = \frac{\langle \bar{B}^0 | O_2(\mu) | B^0 \rangle}{-\frac{5}{3} |\langle 0 | P(\mu) | B^0 \rangle|^2}$$

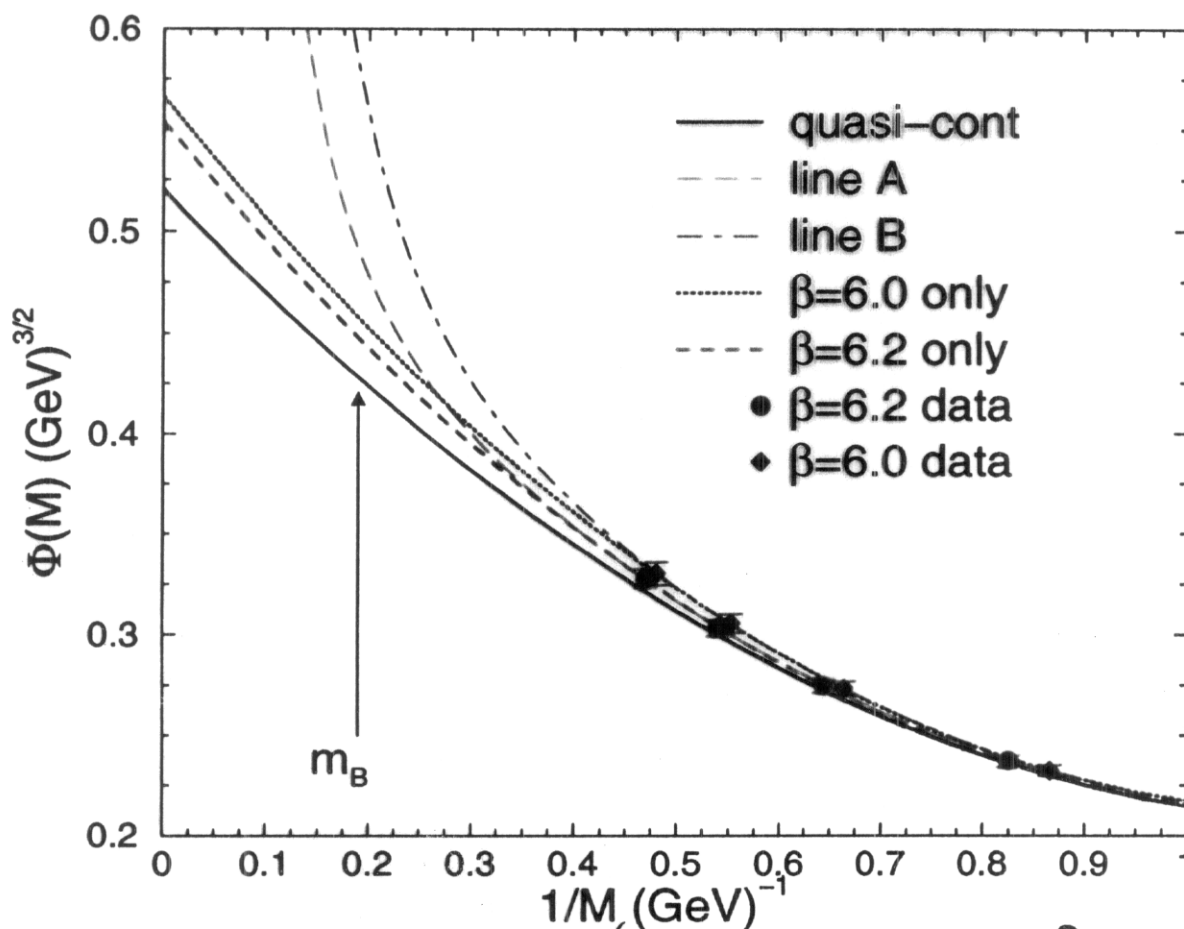
$$B_3(\mu) = \frac{\langle \bar{B}^0 | O_3(\mu) | B^0 \rangle}{\frac{1}{3} |\langle 0 | P(\mu) | B^0 \rangle|^2}$$

$$B_4(\mu) = \frac{\langle \bar{B}^0 | O_4(\mu) | B^0 \rangle}{2 |\langle 0 | P(\mu) | B^0 \rangle|^2}$$

$$B_5(\mu) = \frac{\langle \bar{B}^0 | O_5(\mu) | B^0 \rangle}{\frac{2}{3} |\langle 0 | P(\mu) | B^0 \rangle|^2}$$

(C. R. Allton et al., Phys.Lett.**B453**(99), 30)

Lattice Artefacts

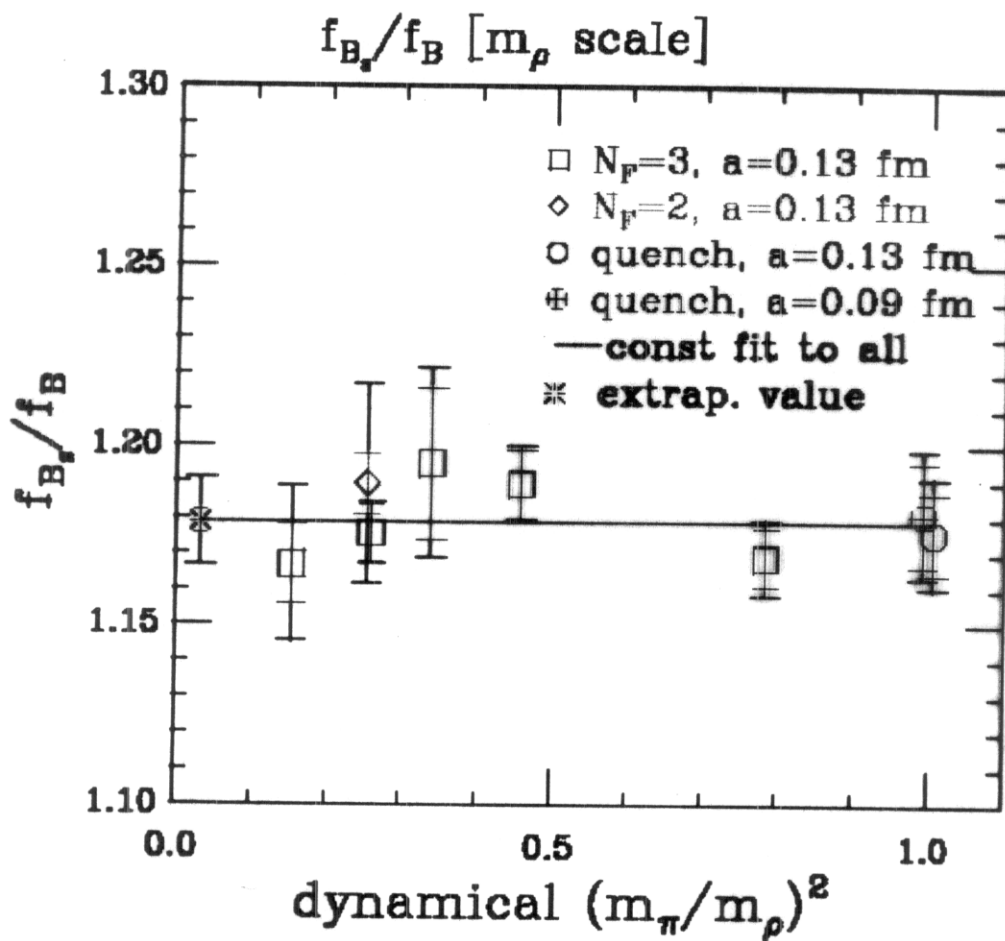


$$\Phi(M, a) = \gamma \left(1 + \delta/M + \eta/M^2 + \varepsilon(aM)^2 + \zeta(aM)^3 \right)$$

- Simultaneously fit to both lattice spacings
- lines A & B show lattice artefacts blowing
- Quasi-Continuum line is close to simple HQS fits



◇ Systematics cancel in ratios
⇒ mild dependence on N_f or a^{-1} . $f_{B_s}/f_B \approx 1.18$





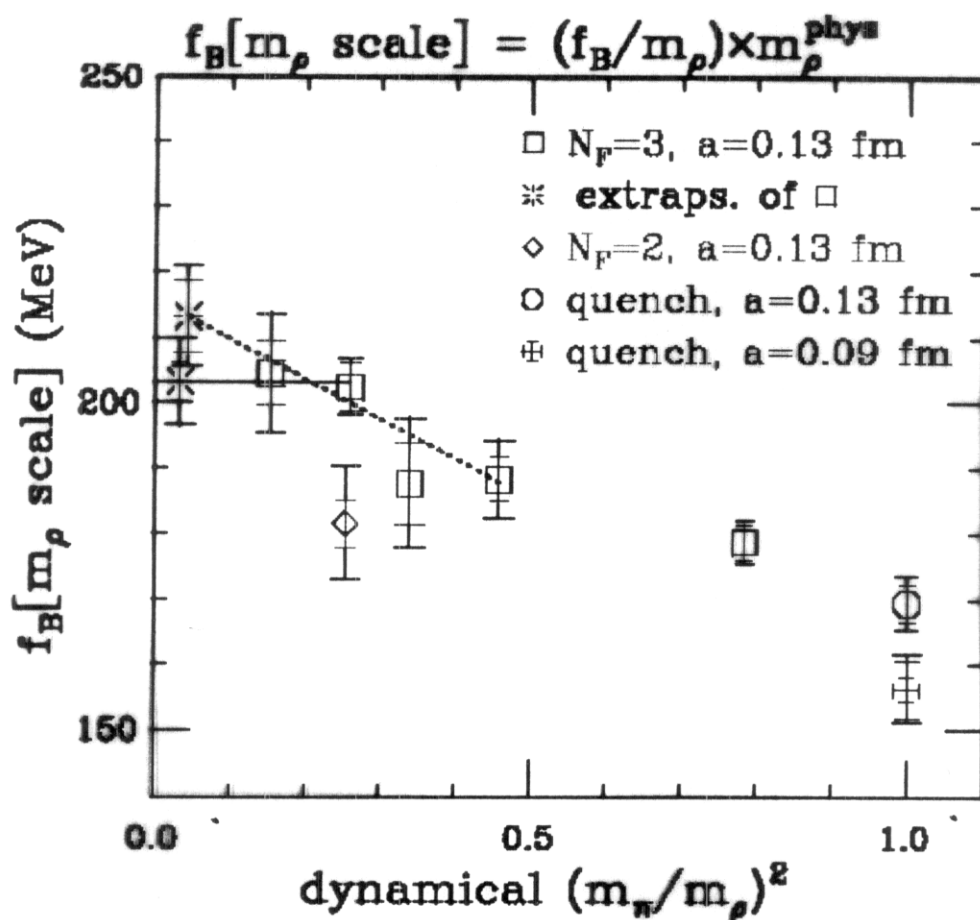
Milc:

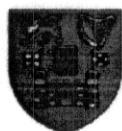
[poster by Bernard]

f_B with $N_f = 2 + 1$ using improved KS action ("Asqtad"), tadpole-improved clover and Fnal formalism.

Preliminary results at $a = 0.13\text{fm}$. (No renormalisation constants for HL axial currents).

- ◇ Dependence on $(m_\pi/m_\rho)^2_{\text{dyn}}$ with const and lin fits.
- ◇ Valence χ -extrap well controlled.
- ◇ Dependence on lattice scale remains.





New this year

C. Davies *et al*:

[talk by Davies]

Improved smearings to analyse f_B at finite momentum.

Results at $\beta = 5.7$ and $ma = 8$ (twice the B mass) show

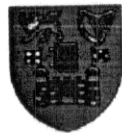
- Heavy quark approx local better than smeared heavy at finite momentum.
- All A_k currents, ie $k = 0, 1, 2, 3, 4$ important for f_B at finite momentum. Eg. the contribution from A_1 is $\approx$ that of A_0 .

Solves a discrepancy described by Jim Simone at Lat96?

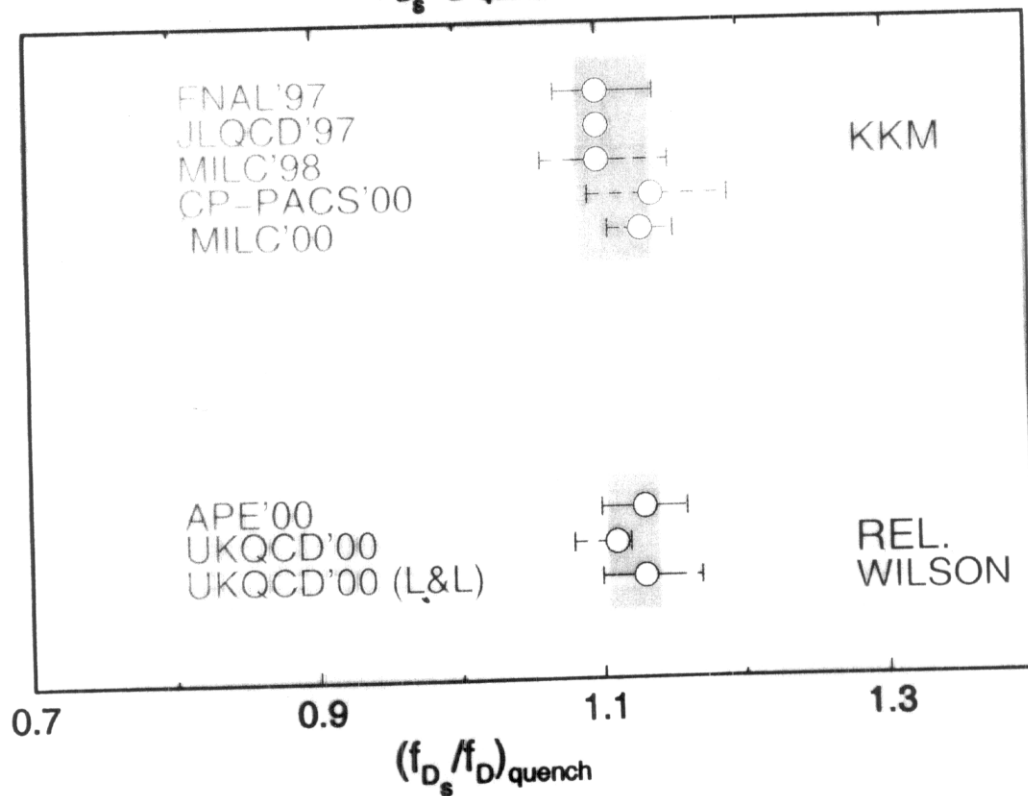
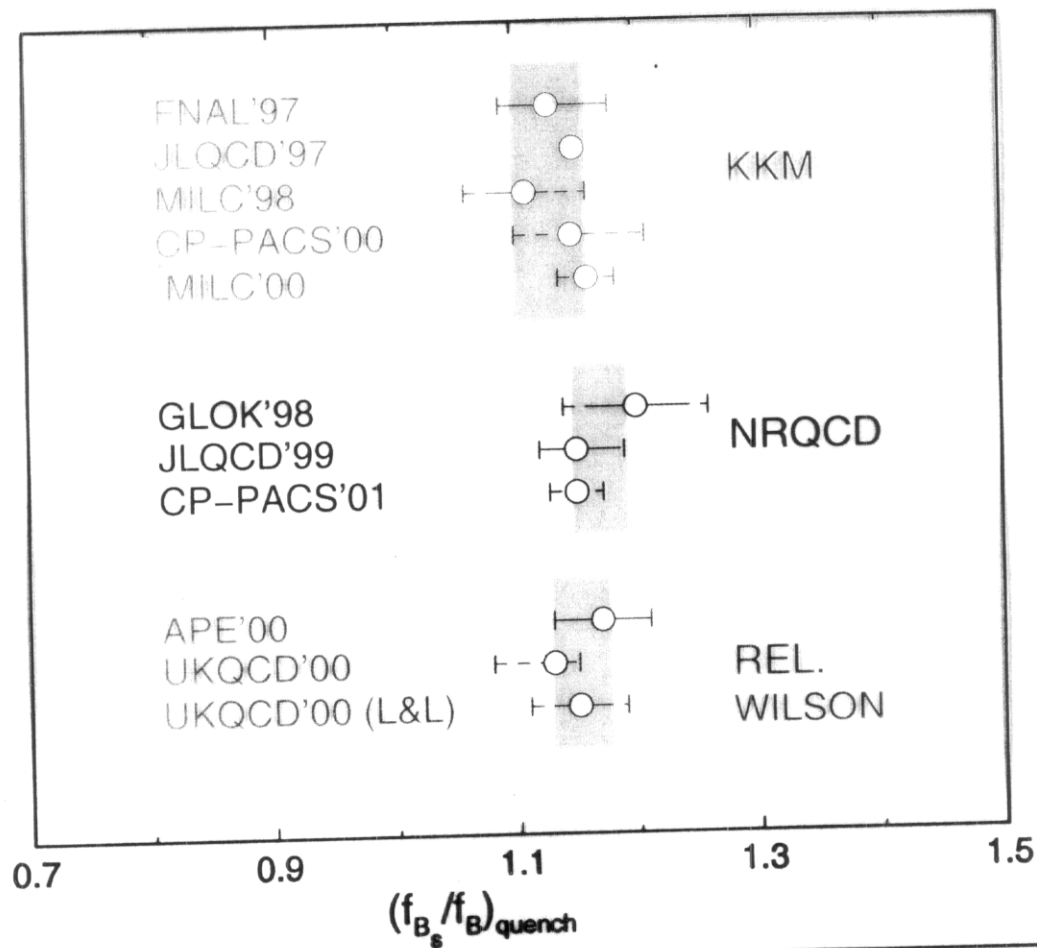
UKQCD: Preliminary results ($f_{D(s)}$) in talk by Maynard

Calculation of $f_{B(s)}$ and $f_{D(s)}$ with UKQCD dynamical configs. Heavy quarks treated in : Fermilab, NRQCD and relativistic+HQET extrapolation.

The aim is to compare results from different approaches, understand systematics and the size of the unquenching effect.

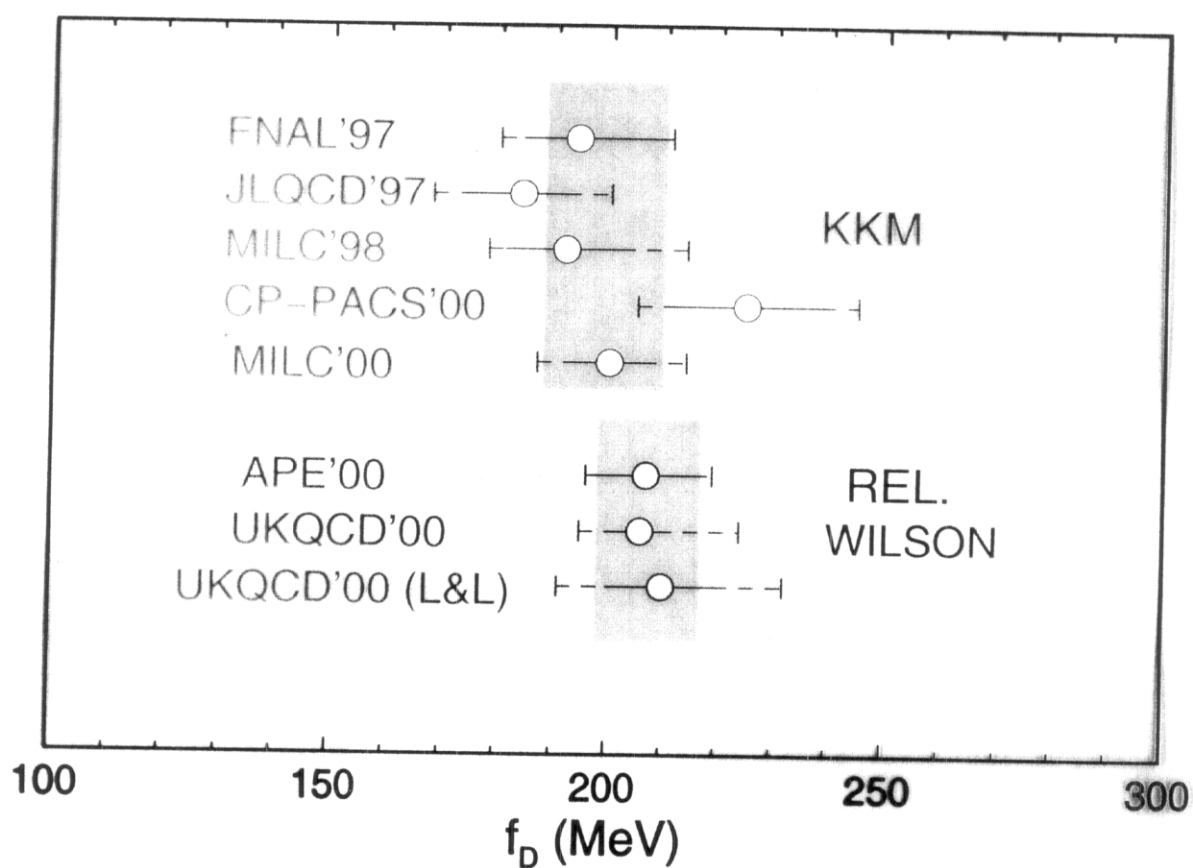


Ratios



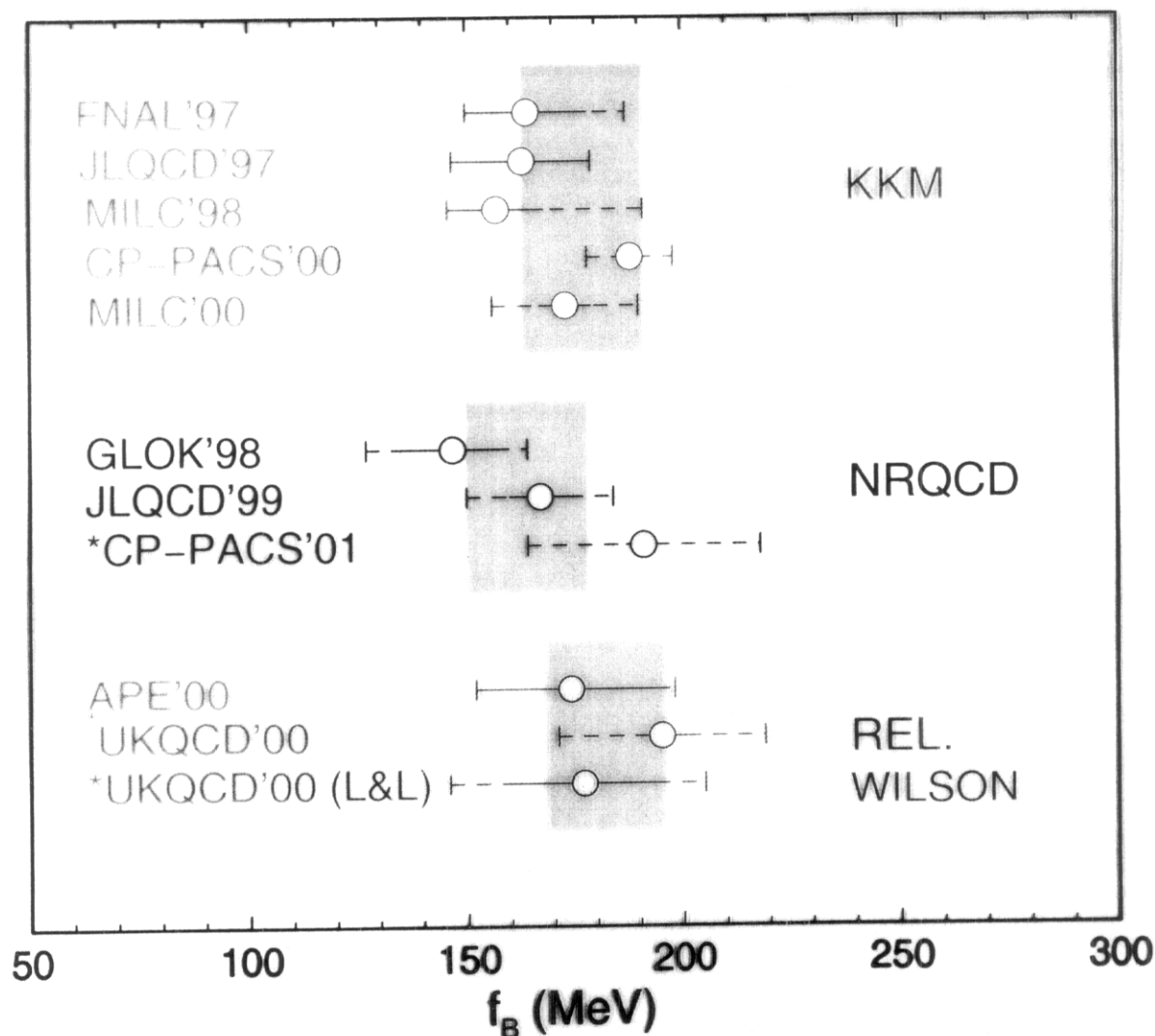


Current Status of quenched f_D





Current Status of quenched f_B



* indicates a result has changed from Lat'00.

CP-PACS

Systematic error increased : represents discretisation error. Additional $\approx 30\%$ uncertainty from a^{-1} .

UKQCD

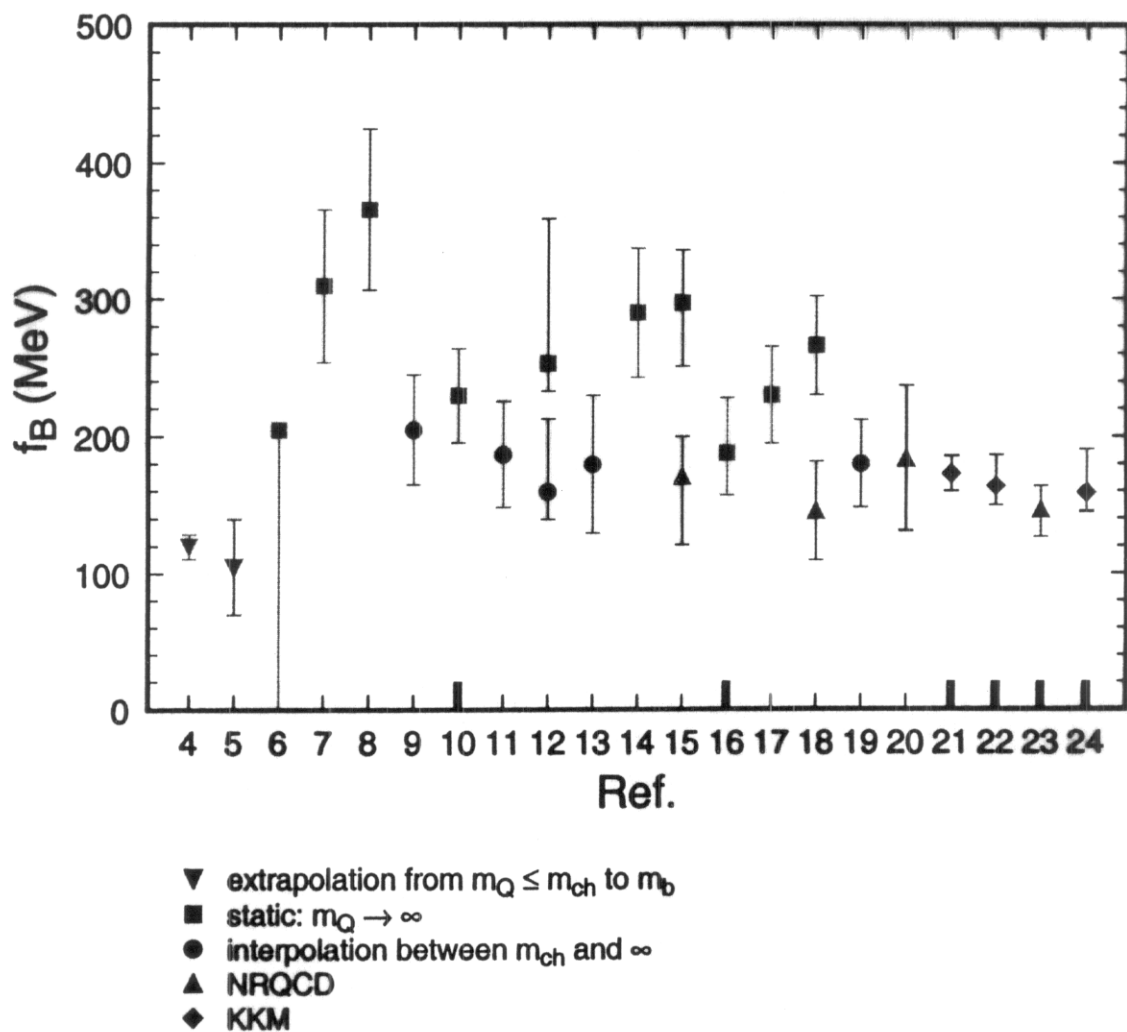
- L&L small change to errors.
- a^{-1} from m_ρ rather than r_0 . New analysis of the heavy mass extrapolation.

[talk by Maynard]

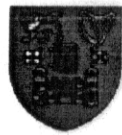


Timeline for f_B

A. Kronfeld, HQ99



After initial uncertainty there is **general agreement** between groups and methods



Leptonic Decay constants and B-parameters

◇ B-meson parameters

Important phenomenological quantities. Dominant source of uncertainty in CKM matrix elements V_{td}, V_{ts} , calculated from $B_q^0 - \bar{B}_q^0$ mixing.

$$\Delta m_q = \frac{G_F^2}{6\pi^2} M_W^2 S_0(x_t) \eta_B M_{B_q} f_{B_q}^2 \hat{B}_{B_q} |V_{tq} V_{tb}^*|^2$$

Δm_d known to $\simeq 4\%$.

In addition, B_B and B_s input parameters in theoretical prediction of $\Delta\Gamma_{B_s}$.

◇ D-meson parameters

f_{D_s} measured experimentally with $\simeq 30\%$ accuracy.
 f_D is not measured.

CLEO-c will determine these to $\simeq 2\%$.

Also measure $D - \bar{D}$ mixing, semileptonic decays etc.

This will provide a test of LQCD heavy quark physics.

◇ Leptonic decay constants and B -parameters

◇ Semileptonic decays :

$$B(D) \rightarrow \pi l \nu$$

$$B \rightarrow D^{(*)} l \nu$$

◇ The bottom quark mass

◇ Spectroscopy

◇ Some new developments

◇ Summary and conclusions



Trinity College,
Dublin.

Heavy Quark Physics from Lattice QCD

Sinéad Ryan
Trinity College Dublin

Lattice 2001, 22nd August, Berlin

... continued.

Spectroscopy

HFS remains a puzzle : for mesons, not baryons.

New Developments :

a new method to calculate m_b

study of difference between static and full QCD in continuum limit.

new look at relativistic heavy quarks.

NP determination of heavy-light current renormalisation.

Conclusions

Leptonic Decays and B -parameters

f_B : error remains $\sim 15-20\%$. Main uncertainties are discretisation, perturbation theory (matching), mass extrapolation and (maybe) χ -extrapolation.

B_B : results from different methods show good agreement. little effect of unquenching (JLQCD).

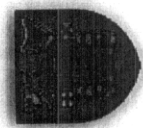
Semileptonic Decays

$B \rightarrow \pi l \nu$: recent calculations agree, within errors, on $\frac{1}{|V_{ub}|^2} \frac{d\Gamma}{dq^2}$. remaining uncertainties / improvements needed:

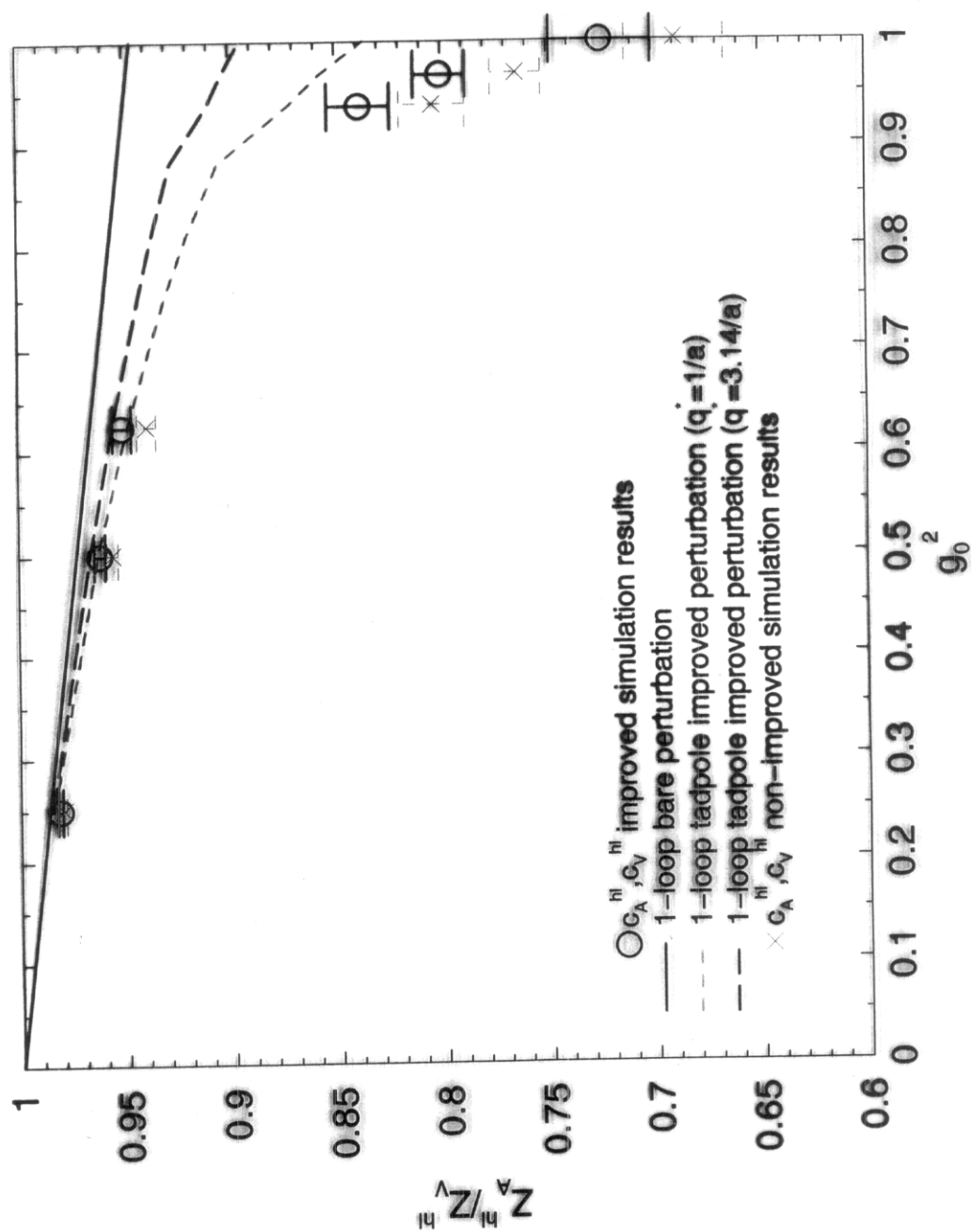
- extending the range of q^2 (anisotropic lattices)
- unquenching

$D \rightarrow \pi l \nu$: Some discrepancy between results here.
good news : entire kinematic range covered.
this decay will be measured to a few % by CLEO-c, soon!

$B \rightarrow D^* l \nu$: precision lattice calculations of semi-leptonic decays is possible! Novel ratio method from FNAL allows determination of form factor at zero recoil.



Evaluation of results



◆ "Relativistic Heavy Quarks on the lattice"

Aoki et al hep-lat/0107009 + talk by Kuramashi

The authors use a somewhat different choice of operators than El-Khadra, Kronfeld, Mackenzie
(PRD 55, 3933 (1997)).

They find operators in L_{HQ} :

$$\bar{\psi} \gamma \cdot \bar{D} \psi \quad \text{and} \quad \bar{\psi} \bar{D}^2 \psi$$

... believed to be interchangeable by a field transformation (and therefore 1 is redundant)
(this is the case considered by El-Khadra et al)
must be separately normalised.

→ still under discussion.



◇ Hashimoto, Ishikawa, Onogi: [talk by Onogi]

A calculation of the non-perturbative renormalisation of heavy-light currents.

Large 1-loop coefficients of static-light currents. Perturbative error $\simeq \mathcal{O}(\alpha^2)$.

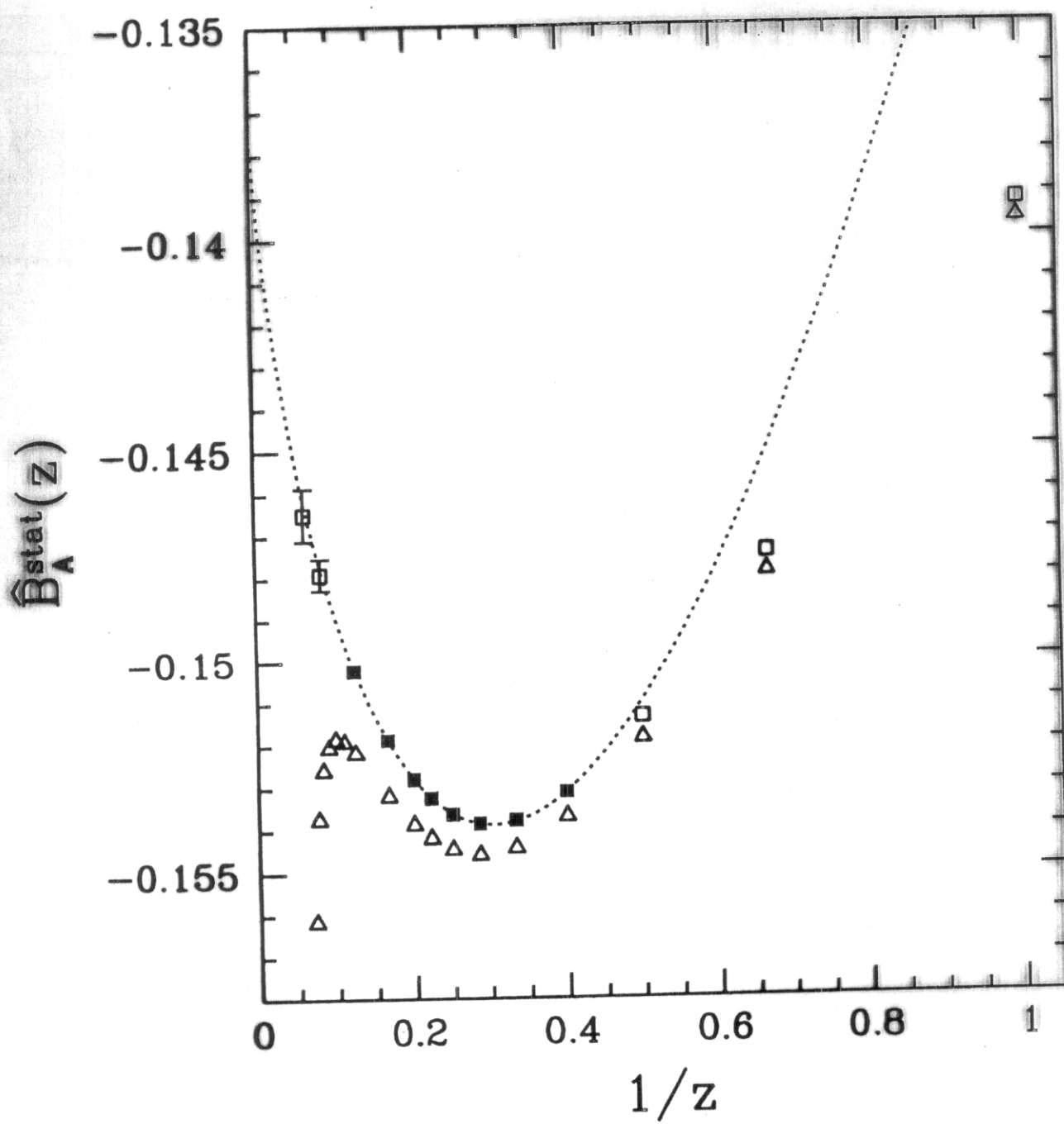
NP renormalisation would reduce this uncertainty and hence the error in $f_B, f_{+,0}$ etc.

Method:

Ward-Takahashi identity to extract $Z_{A_4}^{\text{hl}}/Z_{V_4}^{\text{hl}}$.

Questions

- NP $\mathcal{O}(\alpha)$ improvement reduce discrepancy further?
- Specify to static or a feature of finite mass quarks?





- Investigate cutoff effects, by defining

$$\delta(z, a\overline{m}_{2,\overline{MS}}) = \frac{Y_{CA}(z, a/L) - Y_{CA}(z, 0)}{Y_{CA}(z, 0)}$$

and expanding $\delta = \delta^{(0)} + \delta^{(1)}g_{\overline{MS}}^2 + \mathcal{O}(g_{\overline{MS}}^4)$

→ Disc errors $\simeq (a\overline{m}_{2,\overline{MS}})^2$.

⇒ Problem for heavy quark extrapolation at finite a as the slope of extrapolation may have sizeable contribution from disc errors.

⇒ Perform heavy quark extrapolations in the continuum limit.



New developments

◇ Kurth and Sommer:

[talk by Kurth]

NBP597(2001) NP renormⁿ of HL axial current in static.

hep-lat/0108018 studies the difference between static and full QCD in *continuum limit* for correlation functions at tree level and 1-loop of PT.

Key Findings

- Explicitly demonstrate gauge-invariant, IR finite corrlⁿ funcs are described by HQET in $m \rightarrow \infty$ limit.
- Match QCD and static and extract finite mass effects.

This is achieved by imposing the matching condition

$$X_{\text{match}}(\overline{m}_2, \overline{MS}) = Y_{CA}(z, a/L) + \mathcal{O}(1/z) + \mathcal{O}((a/L)^2)$$

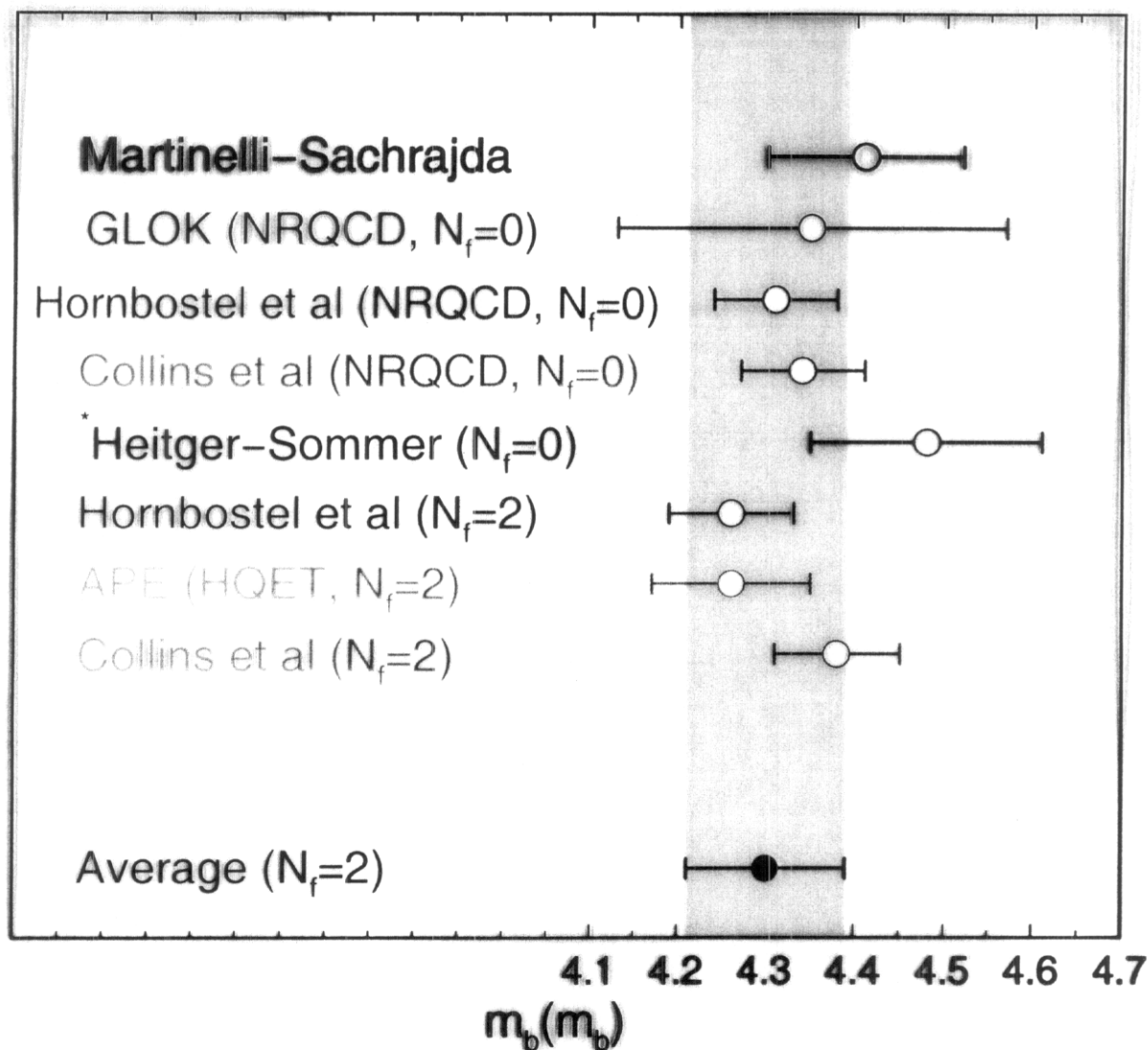
with

$$\begin{aligned} X_{\text{match}}(\mu) &= (\chi_A^{\text{stat}}) X_{\text{lat}}(\mu) + \mathcal{O}((a/L)^2) \\ (\chi_A^{\text{stat}}) &= 1 + B_A^{\text{stat}} g_{\text{lat}}^2 + \dots \end{aligned}$$

The constant B_A^{stat} is calculated and found to agree with a previous calculation.



The b quark mass



$$\overline{m}_b(\overline{m}_b) = 4.30(10) \text{ GeV}$$

In very good agreement with other methods and experiment.

◇ H-S result is larger than others. NP effect? Discretisation? A continuum extrapolation will be interesting.



◇ Lewis et al:

[talks by Lewis & Mathur]

Charmed baryon spectrum (Qqq and QQq).
Anisotropic and NRQCD actions.
Worse stats but no suppression of HFS observed
(maybe overestimated?).

◇ JLQCD:

[talk by Tsutsui]

HQ expansion parameters, μ_π^2 and μ_G^2 are calculated.

The mass dependence of these parameters is compared to HFS

$$M_{B^*} - M_B = -\frac{\Delta\mu_G^2}{2m_Q}$$

$$M_{\Lambda_b} - M_{\bar{B}} = \bar{\Lambda}(\Lambda_b) - \bar{\Lambda}(B) + \frac{1}{2m_Q} [-\mu_\pi^2(\Lambda_b) + \mu_\pi^2(B)]$$

The meson HFS is underestimated. The baryon HFS agrees with experiment.



Spectroscopy of Mesons and Baryons

$$M_V^2 - M_P^2 \approx \text{constant} \quad \forall \text{ flavours.}$$

In practice $(D^*(B^*) - D(B))_{\text{latt}} < (D^*(B^*) - D(B))_{\text{expt}}$
by 20 – 40% respectively. Quenching effect since:

$$\Delta m \propto \alpha_s(m) |\psi(0)|^2$$

◇ QCDTARO:

[talk by Garcia-Perez]

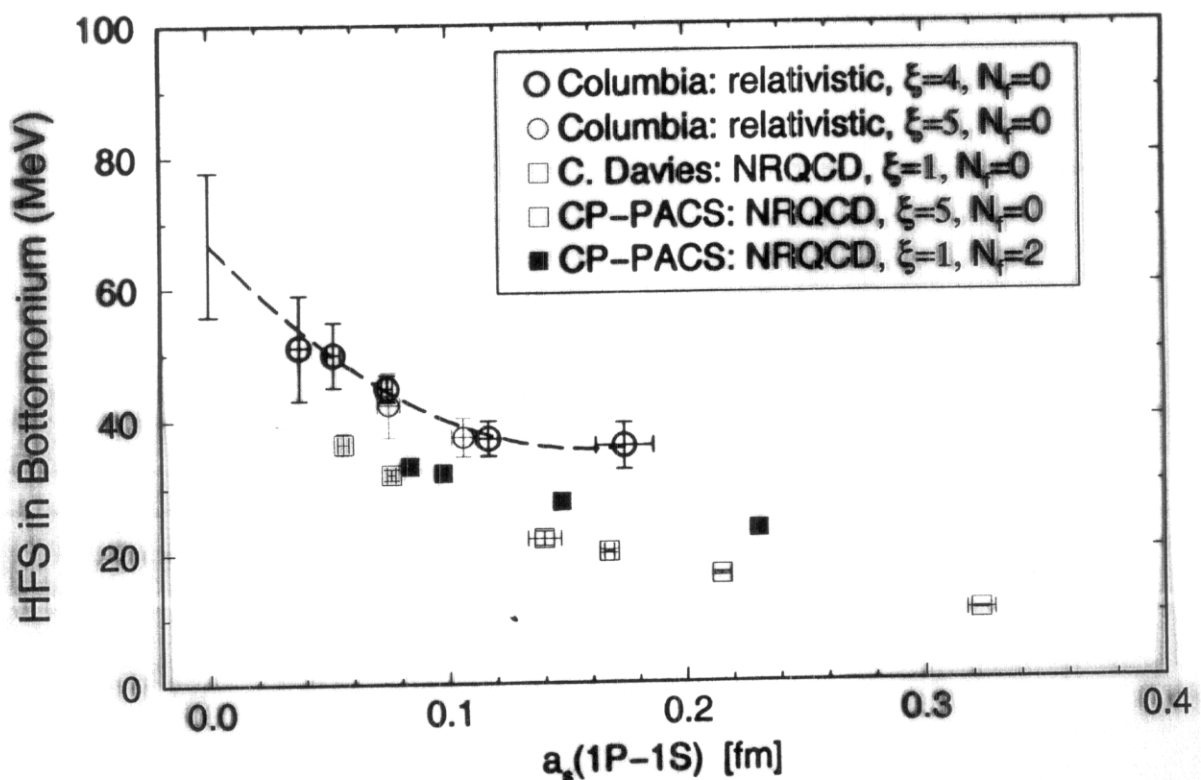
$\bar{c}c$ HFS at $\beta = 6.6$. Use UKQCD data ($\beta = 6.0, 6.2$)
to take $a \rightarrow 0$ and find an increased HFS.

⇒ Quenching effect not as large as expected?
Controlled quenched continuum limit feasible.

◇ Columbia

[talk by Manke]

$b\bar{b}$ HFS also shows scaling violations.





The b quark mass

An important input parameter for many phenomenologically interesting quantities.

Renormalon ambiguities in $M_{\text{pole}} \Rightarrow$ require short distance mass definition:

$$\bar{m}_b(\mu) = Z_{\text{cont}}(\mu) M_{\text{pole}}$$

◇ Martinelli and Sachrajda: 2-loop coefficient for continuum and lattice HQET matching. Subsequent results are stable against a .

◇ m_b determined with NRQCD for $N_f = \{0, 2\}$ with 1-loop matching between M_{kinetic} and the $\overline{MS}$ mass. Ali Khan et al and Hornbostel et al.

◇ Collins et al extrapolate high statistics NRQCD results to static and use $M_{\text{pole}} = M_B^{\text{expt}} - (E_{\text{sim}}^\infty - E_0^\infty)$ to determine m_b . $\mathcal{O}(\alpha^3)$ coefficient of E_0^∞ (quenched) and Z_{cont} known.

◇ Heitger and Sommer:

[talk by Sommer]

A strategy to compute the b -quark mass with non-perturbative accuracy.

• Using Schrödinger functional approach and renormalisation of the static approx. as discussed in K&S hep-lat/0108018 the b -quark mass is computed.

This allows the continuum limit to be taken.

Results

$$h_{A_1}(1) = 0.929^{+11}_{-10}$$

Additional uncertainties that must be folded into this result include:

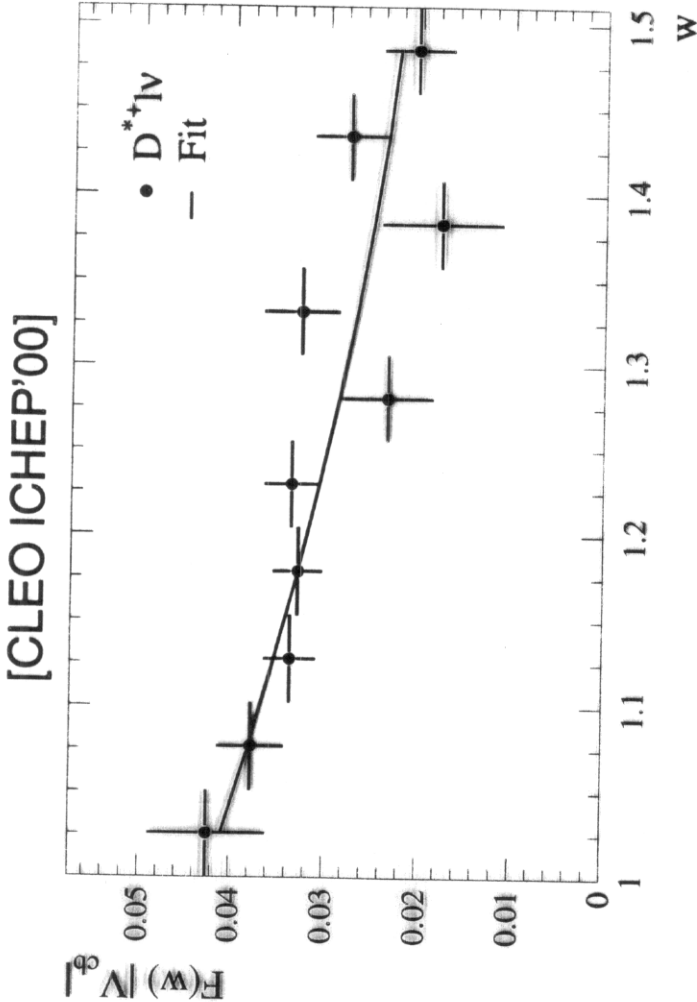
<u>source of uncertainty</u>	<u>uncertainty in $h_{A_1}(1)$</u>
spectator quark mass dependence	less than the stat. error
terms of order $\alpha_s(\Lambda/m_c)^2$	± 0.010
tuning of m_c and m_b	± 0.003
radiative corrections $\geq$ 2-loop order	± 0.002
term of order $1/(m_c^2 m_b)$	± 0.005
residual α dependence	$+0.001$ -0.021

The error due to the quenched approximation affects the **deviation of $h_{A_1}(1)/\eta_A$ from unity**. Guided by the quenched uncertainty in f_B , this error may be as large as ± 0.011 in $h_{A_1}(1)$ or a percent in $|V_{cb}|$.

$|V_{cb}|$ and the $B \rightarrow D^* \ell \nu$ Decay Rate

In the zero recoil limit,

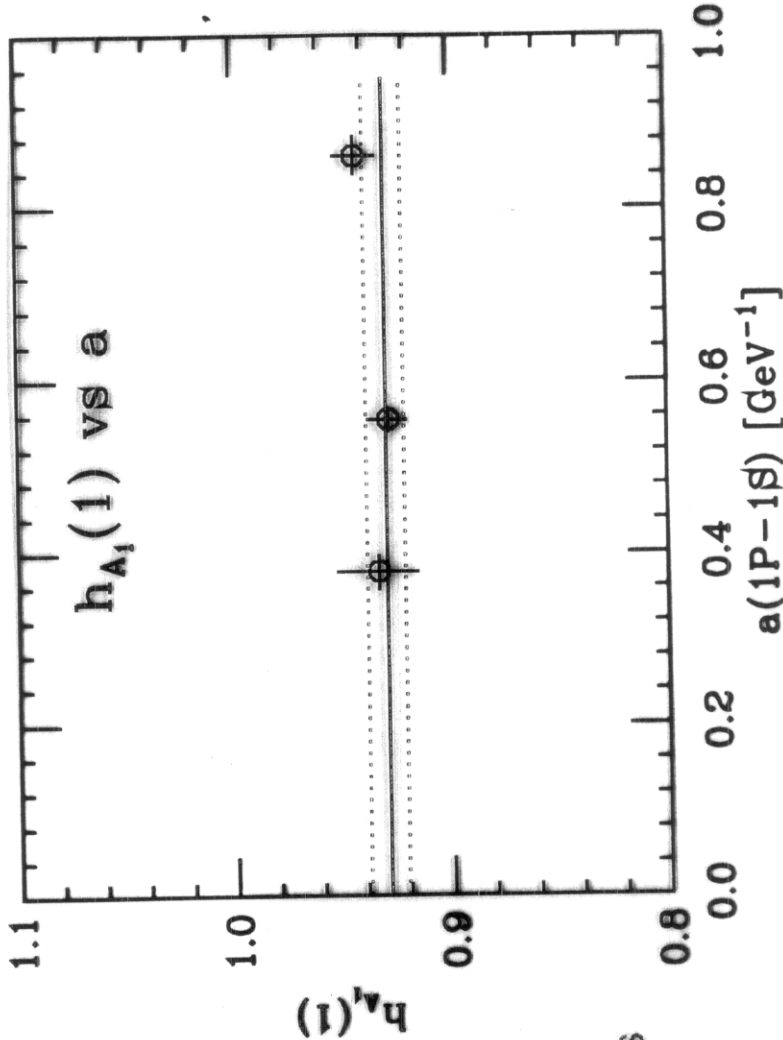
$$\lim_{\omega \rightarrow 1} \frac{1}{(\omega^2 - 1)^{1/2}} \frac{d\Gamma(B \rightarrow D^* \ell \nu)}{d\omega} = \left(\begin{matrix} \text{known} \\ \text{factors} \end{matrix} \right) |V_{cb}|^2 |h_{A_1}(1)|^2$$



$$|V_{cb}| h_{A_1}(1) = \begin{matrix} (4.24 \pm 0.18 \pm 0.19) \times 10^{-2} & [\text{CLEO ICHEP'00}] \\ (3.56 \pm 0.17) \times 10^{-2} & [\text{2001 LEP average}] \end{matrix}$$

h_{A_1} Cutoff Dependence

- Mild cutoff dependence relative to statistical errors.
- Our best estimate is the average of the $\beta = 6.1$ and 5.9 determinations.
- $h_{A_1}(1) = 0.929^{+11}_{-10}$ with combined stat., fit & excited state uncertainties.



β	$h_{A_1}(1)$
6.1	0.933^{+20}_{-19}
5.9	0.928^{+12}_{-11}
5.7	0.943^{+11}_{-11}

Statistical and fit uncertainties added in quadrature.

Determining $h_{A_1}(1)$

Recall,

$$h_{A_1}(1) = \eta_A \left[1 - \frac{\ell_V}{(2m_c)^2} + \frac{2\ell_A}{4m_c m_b} - \frac{\ell_P}{(2m_b)^2} + \mathcal{O}(1/m_Q^3) \right]$$

- HQ spin symmetry relates $B \rightarrow D^*$ to $B^* \rightarrow D^*$ in the limit $m_b \rightarrow \infty$.
- Conclude that the replacement $\ell_V^{eff} = \ell_V - \ell_V^{[3]}/(2m_c)$ in the formula reproduces the $1/(m_c^3)$ correction to $h_{A_1}(1)$.
- We neglect a similar $1/(m_b^3)$ correction from $\ell_P^{[3]}$.
- Remaining leading uncertainty is a $1/(m_c^2 m_b)$ correction to $h_{A_1}(1)$.



Summary

I conclude this section with a table of the systematic errors as budgeted by the Fermilab group in the calculation of $B \rightarrow \pi l \nu$.

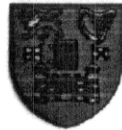
T_B is defined as

$$T_B(p_{\min}, p_{\max}) = \int_{p_{\min}}^{p_{\max}} dp p^4 |f_+(E)|^2 / E$$

and similarly for T_D . The range $0.4\text{GeV} < 1.0\text{GeV}$ is considered to be where systematic errors are minimised.

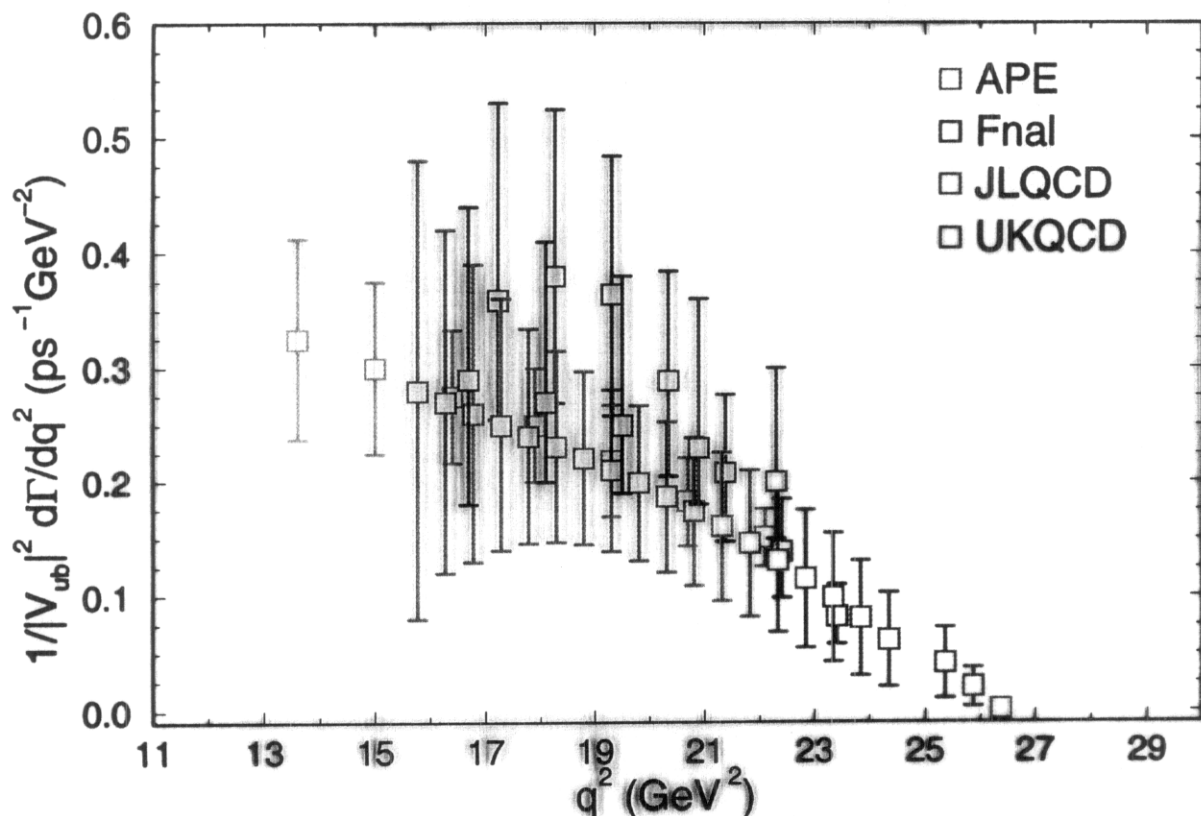
Error budget (in %) for this calculation

Error	T_B	$ V_{ub} $	T_D	$ V_{cd} $
statistical	+27 -9	+14 -5	+17 -8	+9 -5
excited state	± 6	± 3	± 6	± 3
$\vec{p}$ extrapol ⁿ	± 10	± 5	± 9	± 5
m_q extrapol ⁿ	+16 -22	+8 -11	+3 -18	+2 -9
adjusting m_Q	± 6	± 3	± 2	± 1
HQET matching	± 10	± 5	± 10	± 5
a dependence	+16 -3	+8 -2	+23 -6	+11 -3
definition of a	± 11	± 6	± 4	± 2
total systematic	30	15	+28 -24	+14 -12
total (stat $\otimes$ sys)	+40 -31	+20 -16	+32 -26	+16 -13



All groups: $\frac{1}{V_{ub}^2} \frac{d\Gamma}{dq^2}$

Errors are $\sqrt{(\text{statistical})^2 + (\text{systematic})^2}$, as quoted by groups.



- ◇ Broad general agreement between different groups
- ◇ Treatment of heavy quarks is different → systematics are different
- ◇ In more detail, FNAL and JLQCD both simulate at the b -quark mass. Why are the results different?



SPT continued

JLQCD observe that a $\sqrt{m_q}$ term in the χ extrapolation raises the value of the soft pion limit. Thus the problem is dependent on the form of the extrapolation and the interpolation to fixed $E(q^2)$.

FNAL data do not satisfy the relation. However, FNAL note that the experimental rate is $\rightarrow 0$ and the lattice systematics (mostly due to χ extrapolation) are increasing at the soft pion limit. Cuts are imposed on the range of E considered such that $E \geq 0.424\text{GeV}$.

◇ An observation ...

The RHS of $\dagger$ is a ratio of decay constants in which quenching effects are thought to cancel.

The LHS is the form factor f^0 for which no such cancellation can occur and for which the size of the quenching effects are unknown.

Given the size (and subtlety) of the systematic error in this calculation perhaps it is not surprising that the soft pion relation is a difficult problem.



Soft Pion Theorem

In the limit $m_\pi, E \rightarrow 0$

$$f^0(m_B^2) = \frac{f_B}{f_\pi} \quad \ddagger$$

- ◇ APE $\leadsto$ the relation holds
- ◇ UKQCD $\leadsto$ the relation holds
- ◇ JLQCD $\leadsto$ the relation doesn't hold
- ◇ FNAL $\leadsto$ no comment.

In more detail:

APE use the **SPT** to choose the functional form of the heavy mass extrapolation.

UKQCD also choose a quadratic fit for the heavy mass extrapolation. There is mild dependence on the pole form introduced when interpolating to a common set of q^2 .



Significance of this discrepancy

◇ This is a region of quark mass where all three groups should agree.

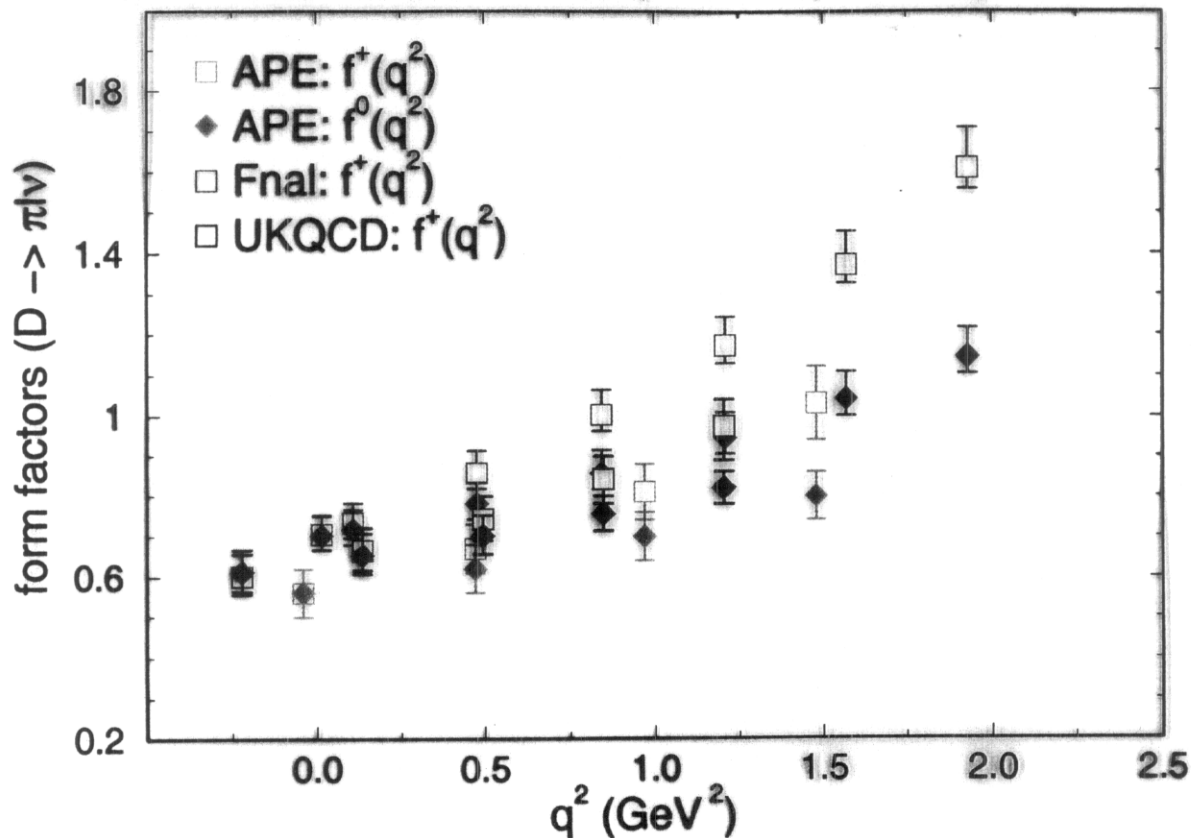
◇ Note that this discrepancy is doubled in the rate since $d\Gamma/dq^2 \propto |f_+(E)|^2$.

◇ In the near future we can expect an increase in the experimental data for semileptonic decay rates.

- CLEO-c will determine $D \rightarrow \pi(K)l\nu$ very accurately
- FOCUS will measure ratios of decay rates eg $\frac{D \rightarrow Kl\nu}{D \rightarrow \pi l\nu}$ to determine the ratio $\frac{|V_{cs}|}{|V_{cd}|}$

⇒ Important to know why the results are different.

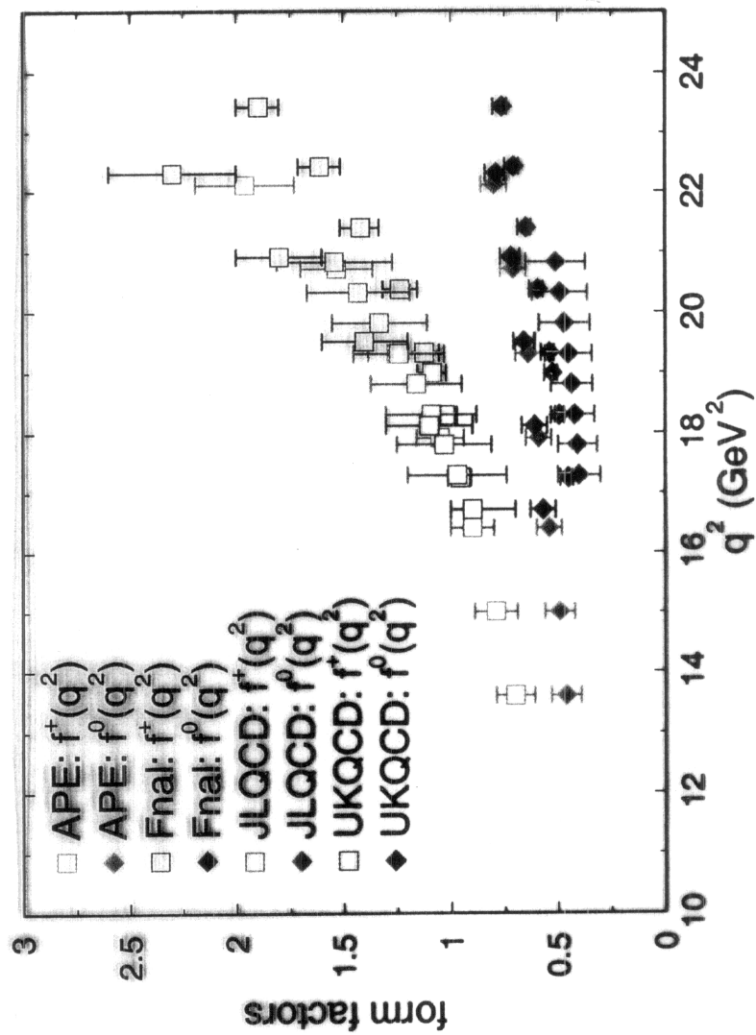
$D \rightarrow \pi l \nu$ from Fnal, APE, UKQCD



Differences: philosophical and analytical

- Fermilab use kinetic mass. Ape use pole mass. This difference in mass definition effects the form factors.
- Different lattice spacings, maybe residual lattice spacing dependence
- The UKQCD data (with new NP coefficients) lies between the APE-FNAL data.
- Renormalisation
- Size of the rotation of quark fields.

- ◇ This difference in ϕ_0 shows up as lower f^0 for JLQCD compared to other results.

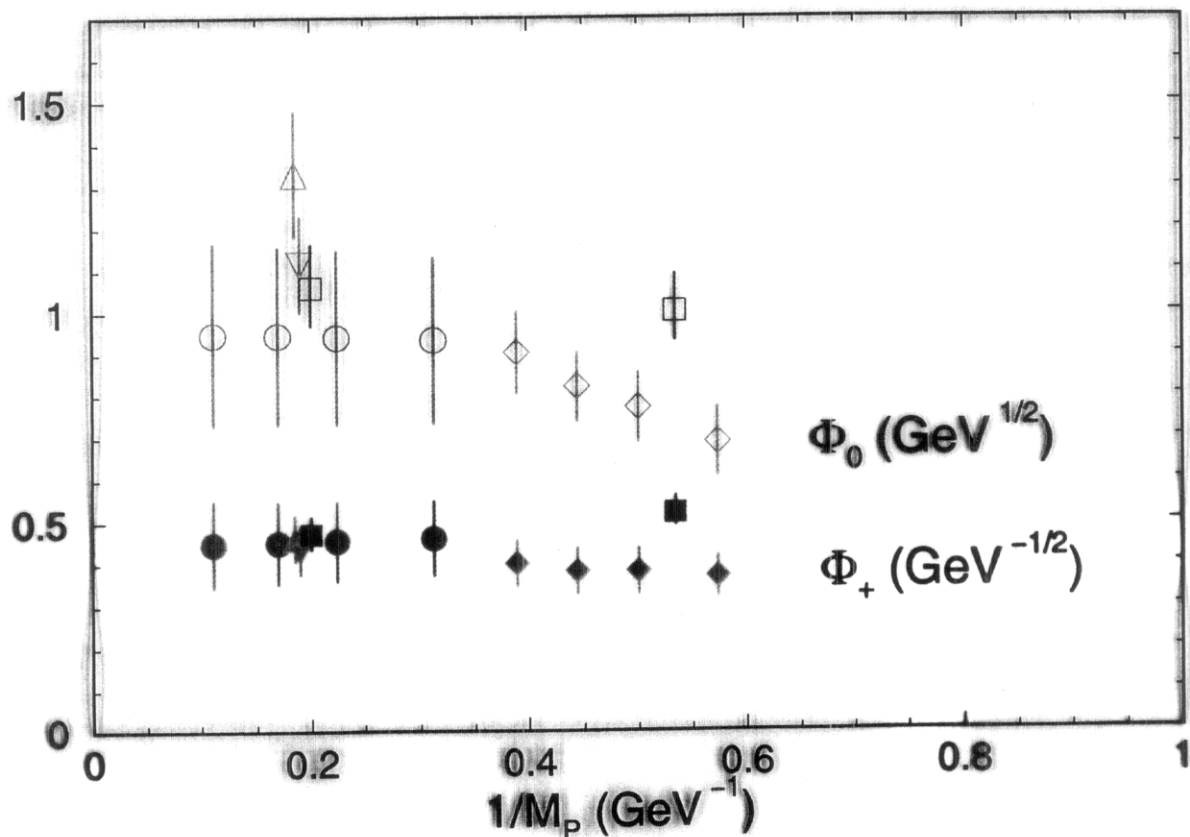




◇ Fnal approach is valid at all quark masses.

Study the mass dependence from m_{ch} to m_b .

◇ Back to ϕ_+ and ϕ_0 and including Fnal data.



○ JLQCD △ APE □ Fnal

◇ Data agree with JLQCD at M_B but show slope as $1/M_P \rightarrow 1/M_D$

◇ Not as much mass dependence as APE data. Agrees better with linear extrapolation to M_B .

◇ Indicates APE and Fnal disagree at M_D .

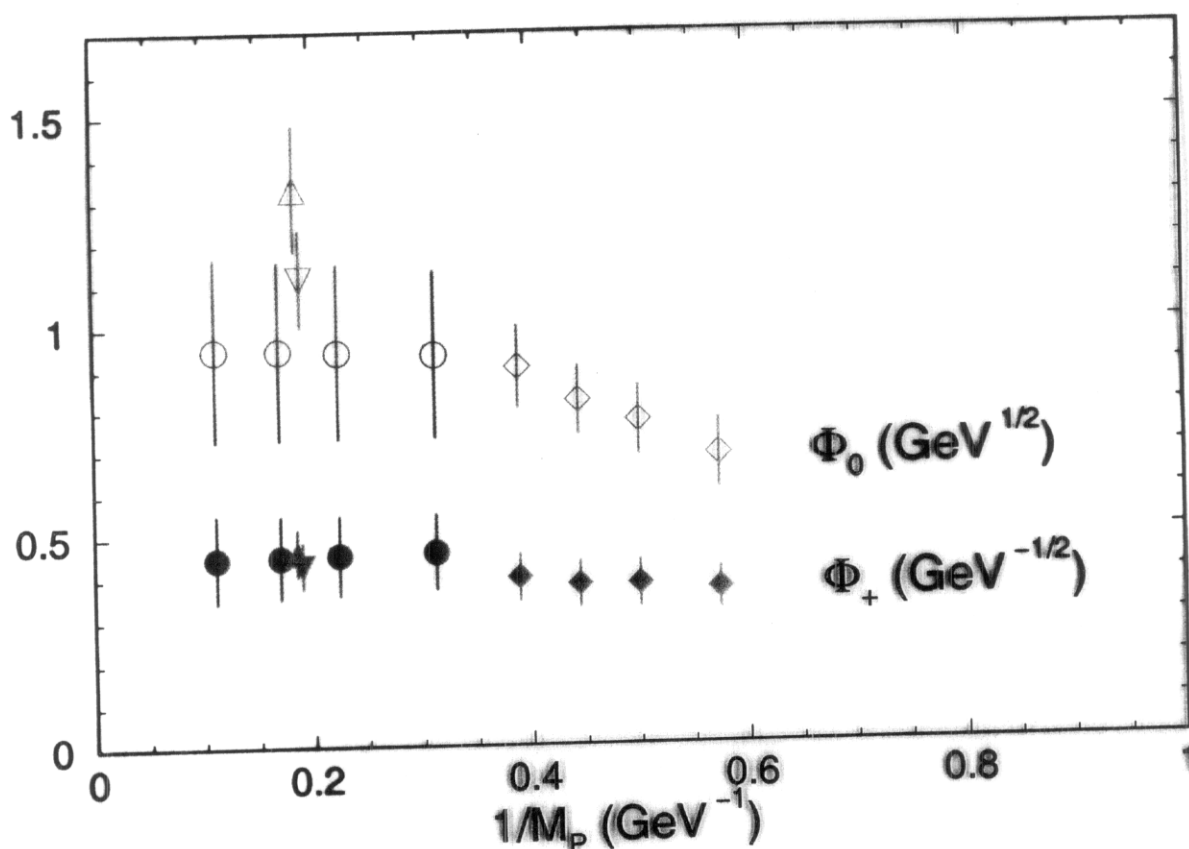


Heavy Mass Dependence

Consider

and $\phi_+ \equiv (\alpha_s(M_P)/\alpha_s(M_B))^{-2/11} f_+/\sqrt{M_P}$
 $\phi_0 \equiv (\alpha_s(M_P)/\alpha_s(M_B))^{-2/11} f_0\sqrt{M_P}$ at fixed E .

◇ Compare JLQCD and APE



JLQCD: **flat slope**. But NRQCD becomes unreliable as $1/M_P \rightarrow 1/M_D$

APE: **negative slope**. But extrapolation $1/M_P \rightarrow 1/M_B$ increases discretisation error. Distorting heavy mass dependence?

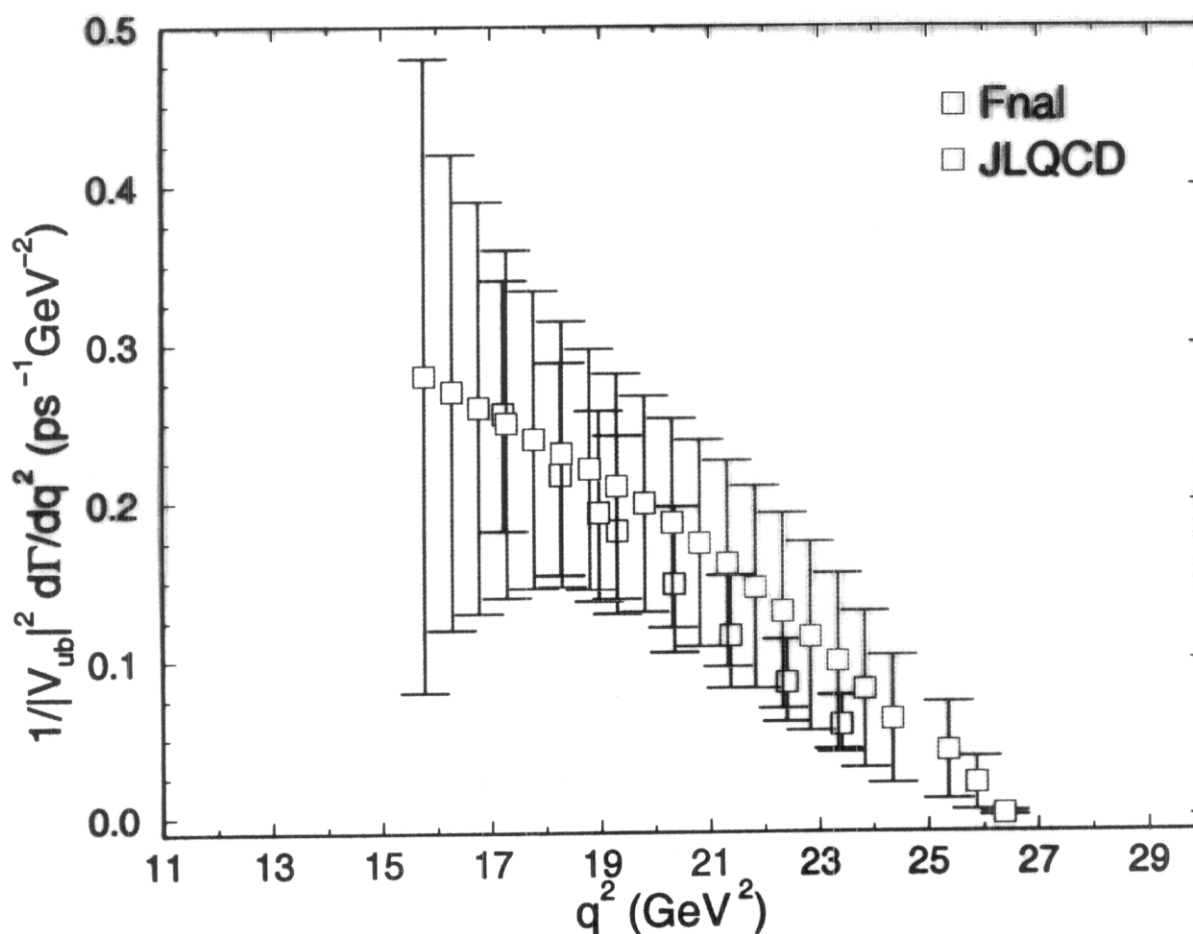
Summary of this comparison.

- ◆ Data are sensitive to the chiral extrapolation
 - range of light quark masses and the functional form.
- ◆ Lighter quark masses are important (and higher statistics)
- ◆ The $\begin{pmatrix} +14\% \\ -22\% \end{pmatrix}$ uncertainty due to χ -extrapolation assigned to dm/dp in the Fermilab analysis seems prudent.
- ◆ The methodology used in this comparison with JLQCD is equivalent to that of UKQCD and APE.

Note that while extrapolation of the matrix elements in m_π^2 at fixed E is more reliable when $p=0$ ($p \ll m_\pi$) in practice, FNAL introduce a cut and consider $p \geq 0.4$ GeV.



$\frac{d\Gamma}{dq^2}$ after "new" analysis



◇ Outstanding differences

- Lattice action and parameters.
- Lattice size, smearing functions and fitting.
- Renormalisation of the heavy-light current.

Note that for the remainder of this discussion I will use $a^{-1}(1P - 1S)$ in the Fermilab data.