MATRIX ELEMENTS
LATTICE 2001

Theoretical and Numerical Results after Lattice 2000 (only light quarks)

(Special thanks to D. Becirevic, M. Golterman
R. Gupta, D. Lin, R. Mawhinney,
J. Noaki and M. Papinutto, S. Sharpe)

Guido Martinelli

Hadronic matrix elements 1

Chair: **T. Blum** **Mon, Aug 20, 10:50 - 12:30, hall D**

10:50 **Michael, Chris**

C. McNeile and C. Michael

The strangeness content of the nucleon.

11:10 **Gadiyak, Valeriya**

Valeriya Gadiyak Xiangdong Ji Chulwoo Jung

Lattice Calculations of the Magnetic Moment and Electromagnetic Form Factors of the Nucleon.

11:30 **Horsley, Roger**

R. Horsley

A lattice determination of nucleon structure functions

11:50 **Sasaki, Shoichi**

S.Sasaki, T.Blum and S.Ohta

Nucleon axial charge from quenched lattice QCD with domain wall fermions and improved gauge action

12:10 **Goeckeler, Meinulf**

M. Göckeler, R. Horsley, D. Pleiter, P.E.L. Rakow, S. Schaefer, A. Schäfer, G. Schierholz

The spin structure of the Lambda hyperon in quenched lattice QCD

last modified: Mon Aug 13 11:21:00 2001 lattice2001@desy.de

$$\langle N' | Q(0) | N \rangle \quad Q = \bar{q}\,\gamma_\mu q\,,\ \bar{q}\,\gamma_\mu \gamma_5\, q\,,\ \bar{q}\, q,$$
$$\bar{q}\,\gamma_\mu D_\nu D_\rho D_\sigma \ldots q$$

Hadronic matrix elements 2

Chair: **C. Michael** **Wed, Aug 22, 14:30 - 16:10, hall A**

14:30 **Schierholz, Gerrit**

G. Schierholz, for QCDSF and UKQCD

Determination of the Lambda Parameter from quenched and N_f=2 dynamical QCD

14:50 **Mawhinney, Robert**

R. Mawhinney (RBC Collaboration)

Lattice Values for the Low Energy Constants of $K \rightarrow \pi\pi$ Matrix Elements

15:10 **Cristian, Calin**

T. Blum, P. Chen, N. Christ, C. Cristian, C. Dawson, G. Fleming, R. Mawhinney, S. Ohta, G. Siegert, A. Soni, P. Varanas, M. Wingate, L. Wu, Y. Zhestkov

${\rm Re} A_0$ and ${\rm Re} A_2$ from Quenched Lattice QCD

15:30 **Blum, Tom**

T. Blum (RBC Collaboration)

${\rm Im} A_0$, ${\rm Im} A_2$ and ϵ' from Quenched Lattice QCD

15:50 **Pena, Carlos**

M. Guagnelli J. Heitger C. Pena S. Sint A. Vladikas

$K-\bar{K}$ mixing amplitude with Schrödinger Functional and twisted mass QCD

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$$\langle \bar{K}^0 \,|\, Q^{\Delta S=2} \,|\, K^0 \rangle \quad \& \quad \langle \pi \,|\, Q_i \,|\, K \rangle$$

Hadronic matrix elements 3

Chair: **M. Goeckeler** **Thu, Aug 23, 14:30 - 16:10, hall A**

14:30 **Papinutto, Mauro**

D.Becirevic, Ph.Boucaud, V.Gimenez, C.-J.D.Lin, V.Lubicz, G.Martinelli, M.Papinutto, F.Rapuano, C.T.Sachrajda

K->pi pi matrix elements for epsilon'/epsilon and Delta I = 1/2 rule, with Wilson like fermion.

14:50 **Becirevic, Damir**

D.Becirevic, V.Gimenez, V.Lubicz, G.Martinelli, M.Papinutto

Computing the matrix elements of the four-fermion operators with/without subtractions

15:10 **Lin, C.-J. David** **beamer**

SPQR Collaboration

$K\rightarrow\pi\pi$ matrix elements beyond the leading-order chiral expansion

15:30 **Noaki, Jun-Ichi**

CP-PACS Collaboration: S. Aoki, Y. Aoki, R. Burkhalter, S. Ejiri, M. Fukugita, S. Hashimoto, N. Ishizuka, Y. Iwasaki, T. Izubuchi, K. Kanaya, T. Kaneko, Y. Kuramashi, V. Lesk, K. Nagai, J. Noaki(*), M. Okawa, Y. Taniguchi, A. Ukawa, T. Yoshi\'e

Calculation of $K\to\pi\pi$ decay amplitudes from $K\to\pi$ matrix elements in quenched domain-wall QCD

15:50 **Pallante, Elisabetta**

Maarten Golterman Elisabetta Pallante

Effects of Quenching and Partial Quenching on Penguin Matrix Elements

$\langle\, \bar{K}^{\mathbf{0}} \,|\, Q^{\Delta S=2} \,|\, K^{\mathbf{0}} \,\rangle$, $\langle\, \pi \,|\, Q_{\mathbf{i}} \,|\, K \,\rangle$ &

$\langle\, \pi\, \pi \,|\, Q_{\mathbf{i}} \,|\, K \,\rangle$ +chiral expansion

Hadronic matrix elements 4

Chair: **R. Horsley** **Thu, Aug 23, 16:40 - 18:00, hall A**

16:40 **Herdoiza, Gregorio**

A. Abada, Ph. Boucaud, G. Herdoiza, J.P. Leroy, J. Micheli, O. Pene, J. Rodriguez-Quintero

A partonic signal on the lattice?

17:00 **Dong, Shao-Jing**

Shao-Jing Dong, T. Draper, Keh Fei Liu, University of Kentucky F. X. Lee, George Washington University and J. B. Zhang, University of Adlaide

Pion Decay Constant, Z_A, and quark masses from Overlap Fermions

17:20 **Fiebig, H Rudolf**

H Rudolf Fiebig (for the LHPC)

Spectral density calculations in a heavy-light meson-meson system

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$$\langle 0 \,|\, Q\,(0) | \,\pi \,\rangle$$

$$Q = \bar{q}\,\gamma_\mu \gamma_5\, q\,, \quad \bar{q}\,\gamma_\mu \gamma_5\, D_\nu D_\rho D_\sigma \ldots q$$

Improvement and renormalization 1

Chair: **S. Aoki** **Sun, Aug 19, 14:30 - 16:10, hall B**

14:30 **Bali, Gunnar**

Gunnar Bali, Peter Boyle, Christine Davies

Where do perturbative and non-perturbative Lattice QCD meet?

14:50 **Ide, Kiyotomo**

CP-PACS Collaboration : S.~Aoki, R.~Burkhalter, M.~Fukugita, S.~Hashimoto, K.~Ide, N.~Ishi
Y.~Iwasaki, K.~Kanaya, T.~Kaneko, Y.~Kuramashi, V.~Lesk, M.~Okawa, Y.~Taniguchi, A.~Ul
T.~Yoshi\'e

Non-perturbative renormalization for a renormalization group improved gauge action

15:10 **collins, sara**

S. Collins, C. Davies, G. Lepage, J. Shigemitsu

A nonperturbative determination of c_A and the scaling of f_pi and renormalised quark mass

15:30 **Wittig, Hartmut**

P. Hernandez, K. Jansen, L. Lellouch and H. Wittig

Non-perturbative renormalization of quark masses and the condensate for Ginsparg-Wilson fermions

15:50 **Bhattacharya, Tanmoy**

T. Bhattacharya, R. Gupta, W. Lee

Nonperturbative Renormalization: Fixed gauge vs Ward Identities

Perturbative vs Non-perturbative
vs Ward Identities, Scaling etc.

Improvement and renormalization 2

Chair: **S. Capitani** **Mon, Aug 20, 10:50 - 12:30, hall B**

10:50 **Gupta, Rajan**

T. Bhattacharya, R. Gupta, W. Lee, S. Sharpe

Scaling behavior of improvement and renormalization constants

11:10 **Wolff, Ulli**

Achim Bode, Roberto Frezzotti, Bernd Gehrmann, Martin Hasenbusch, Jochen Heitger, Karl Jansen, Stefan Kurth, Juri Rolf, Hubert Simma, Stefan Sint, Rainer Sommer, Peter Weisz, Hartmut Wittig and Ulli Wolff

First results on the running coupling in QCD with two massless flavours

11:30 **Gehrmann, Bernd**

Bernd Gehrmann, Juri Rolf, Stefan Kurth, Ulli Wolff

Schrödinger functional at negative flavour number

11:50 **Di Pierro, Massimo**

Massimo Di Pierro and Paul Mackenzie

On the non-perturbative tuning of the O(a^2) improved Kogut-Susskind action

12:10 **Hauswirth, Simon**

P. Hasenfratz, S. Hauswirth, K. Holland, T. Jorg, F. Niedermayer

First results from a parametrized Fixed-Point QCD action

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Non-improved, Improved, Twisted, Anisotropic Lattices etc.

Improvement and renormalization 3

Chair: **H. Wittig** **Wed, Aug 22, 14:30 - 16:10, hall B**

14:30 **Harada, Junpei**

Junpei Harada, Shoji Hashimoto, Ken-Ichi Ishikawa, Andreas S. Kronfeld, Tetsuya Onogi, Norikazu Yamada

One-loop renormalization of heavy-light currents

14:50 **UMEDA, TAKASHI**

H.~Matsufuru, T.~Onogi, T.~Umeda

$O(a)$ improved Wilson quark action on anisotropic lattice

15:10 **Matsufuru, Hideo**

H. Matsufuru, T. Onogi, T. Umeda

Anisotropic O(a) improved Wilson quark action and hadron spectroscopy

15:30 **Boyle, Peter**

Peter Boyle, Gunnar Bali

The interquark potential in lattice perturbation theory to O(alpha^2).

15:50 **Sint, Stefan**

R. Frezzotti, S. Sint, P. Weisz

O(a) improved twisted mass lattice QCD

In a fixed gauge, gauge invariant etc.

Improvement and renormalization 4

Chair: S. Caracciolo — Thu, Aug 23, 14:30 - 16:10, hall B

14:30 **Sharpe, Stephen**
Stephen Sharpe
On-shell improved operators using unimproved correlators

14:50 **Bowman, Patrick** — **beamer**
P.O. Bowman, U.M. Heller and A.G. Williams
The Quark Propagator in Landau and Laplacian Gauges

15:10 **Scimia, Roberto**
G. Di Carlo, F. Palumbo, R. Scimia
Larger physical volume with a noncompact lattice regularization

15:30 **Capitani, Stefano**
Stefano Capitani
Perturbative renormalization for overlap fermions

15:50 **Bietenholz, Wolfgang**
W. Bietenholz, N. Eicker, I. Hip, Th. Lippert, K. Schilling
Fast Evaluation of Overlap Fermions

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see the accurate and complete reviews by

S^2 = S. Sint and
L^2 = L. Lellouch
at Lattice 2000

Total = 18 + 20 presentations
+ 13 posters ! → one talk

A Selection has been unavoidable

Apologizes to the authors of submitted contributions and the speakers of talks given in the parallel sessions which I had to omit in my review

Many thanks to all my collegues who have kindly sent material and information for the preparation of this talk

Plan of the talk

- A few physics issues (not on the lattice);
- The UV problem: results on non-perturbative renormalization (heavy-light not covered see talk by S. Ryan); perturbative; here and there
- The IR problem: non-leptonic weak decays and related items;
- Physics issues for the lattice (see also talk by M. Beneke); here and there
- Conclusions and outlook.

Observed Genuine FCNC

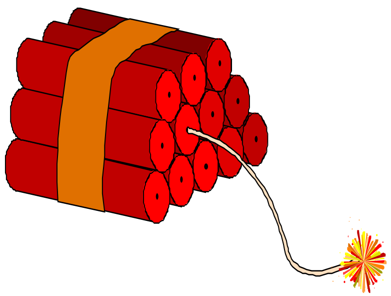
	Exp	Th
ε	$2.271 \pm 0.017\ 10^{-3}$	$\eta\ (1-\rho)\ B_K$
$\varepsilon' / \varepsilon$	$17.2 \pm 1.8\ 10^{-4}$	$\mathbf{-7 \div 30}\ 10^{-4}$
		(**RBC AND CP-PACS** to be discussed in the next slide)
$\Delta M_s / \Delta M_d$	>30 (95%cl)	$[(1-\rho)^2 + \eta^2]^{-1}\ \xi$
		see talk by S. Ryan
$BR(B \rightarrow X_s\ \gamma)$	$3.11 \pm 0.39\ 10^{-4}$	$3.50 \pm 0.50\ 10^{-4}$
$BR(K^+ \rightarrow \pi^+\ \nu\nu)$	$1.5\ {+3.4}{-1.2}\ 10^{-10}$	$0.8 \pm 0.3\ 10^{-10}$
	no lattice QCD needed	figures

Physics Results from RBC and CP-PACS

no lattice details here, they will be discussed below. See talks by

Mawhinney,Calin,Blum and Soni (RBC) and Noaki (CP-PACS)

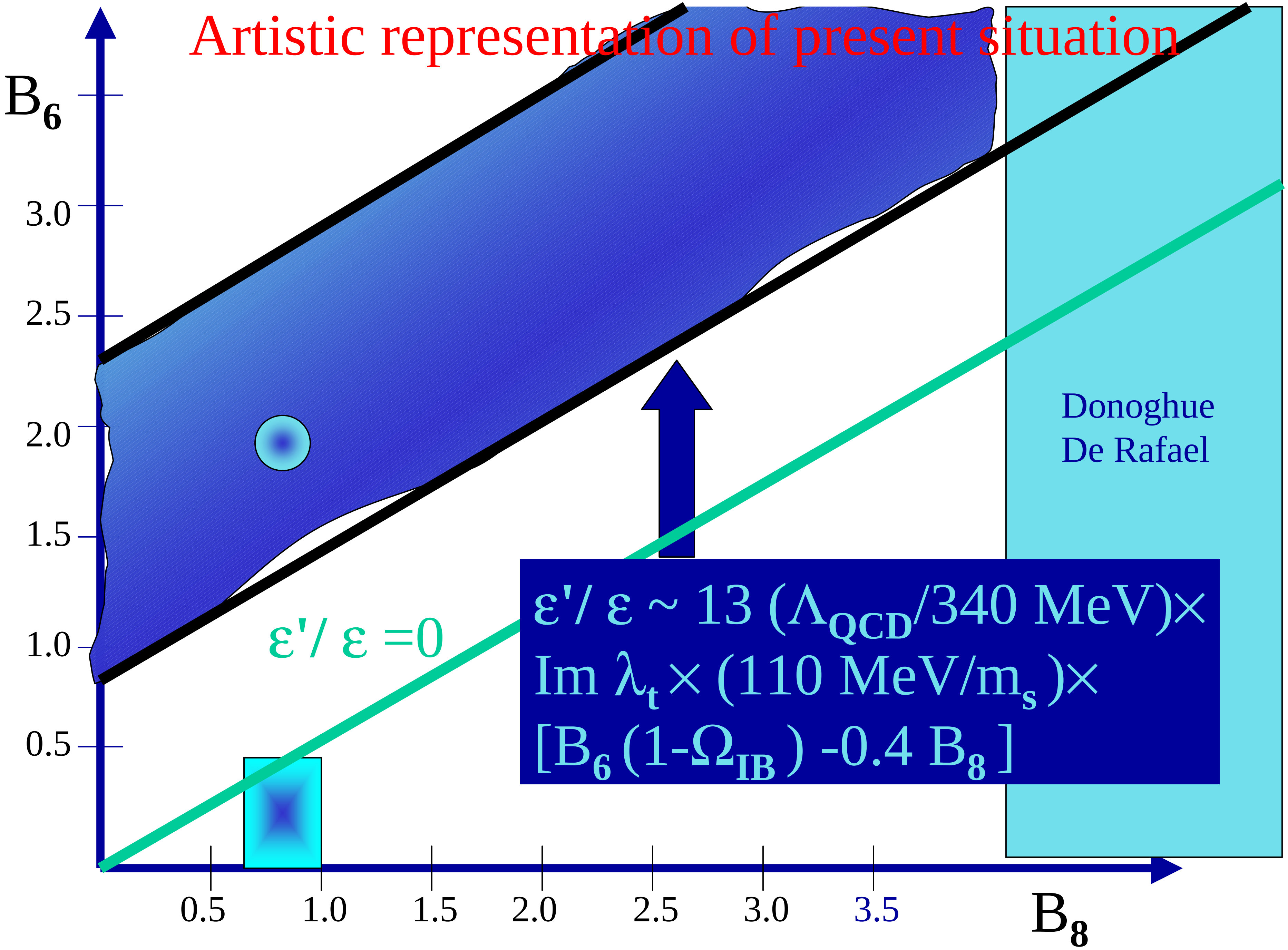
	$Re(A_0)$	$Re(A_2)$	$Re(A_0)/Re(A_2)$	ε'/ε
RBC	$29 \div 31\ 10^{-8}$	$1.1 \div 1.2\ 10^{-8}$	$24 \div 27$	$-4 \div -8\ 10^{-4}$
CP PACS	$16 \div 21\ 10^{-8}$	$1.3 \div 1.5\ 10^{-8}$	$9 \div 12$	$-2 \div -7\ 10^{-4}$
EXP	$33.3\ 10^{-8}$	$1.5\ 10^{-8}$	22.2	$17.2 \pm 1.8\ 10^{-4}$



Total Disagreement with experiments ! (and other th. determinations)

Opposite sign !

New Physics?



Artistic representation of present situation
B_6
3.0
2.5
2.0
1.5
1.0
0.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
B_8
Donoghue
De Rafael
$\varepsilon'/\varepsilon = 0$
$\varepsilon'/\varepsilon \sim 13\,(\Lambda_{QCD}/340\text{ MeV})\times$
$\text{Im}\,\lambda_t \times (110\text{ MeV}/m_s)\times$
$[B_6\,(1-\Omega_{IB})\,-0.4\,B_8\,]$

Chromomagnetic operators vs ε'/ε and ε

$$H_{\mathbf{g}} = C^+_{\mathbf{g}} O^+_{\mathbf{g}} + C^-_{\mathbf{g}} O^-_{\mathbf{g}}$$

$$O^{\pm}_{\mathbf{g}} = \frac{g}{16\pi^2} \left(s_{\mathbf{L}}\, \sigma^{\mu\nu} t^{\mathbf{a}} d_{\mathbf{R}}\, G_{\mu\nu}^{\mathbf{a}} \pm s_{\mathbf{R}}\, \sigma^{\mu\nu} t^{\mathbf{a}} d_{\mathbf{L}}\, G_{\mu\nu}^{\mathbf{a}} \right)$$

- It contributes also in the Standard Model (but it is chirally supressed $\propto m_{\mathbf{K}}^{\mathbf{4}}$)
- Beyond the SM can give important contributions to ε' (Masiero and Murayama)
- It is potentially dangerous for ε (Murayama et. al., D'Ambrosio, Isidori and G.M.)
- It enhances CP violation in $K \rightarrow \pi\,\pi\,\pi$ decays (D'Ambrosio, Isidori and G.M.)
- Its cousin $O^{\pm}_{\gamma}$ gives important effects in $K_L \rightarrow \pi^{\mathbf{0}}\, e^+ e^-$

($\langle \pi^0 \,|\, Q_{\gamma}^{+} \,|\, K^0 \rangle$ computed by D. Becirevic et al. , The SPQ$_{cd}$R Collaboration, Phys.Lett. B501 (2001) 98)

$$L^{\not{CP}} = L^{\Delta F=0} + L^{\Delta F=1} + L^{\Delta F=2}$$

ΔF=0 $d_e < 1.5\ 10^{-27}$ e cm $d_N < 6.3\ 10^{-26}$ e cm

ΔF=1 $\varepsilon' / \varepsilon$

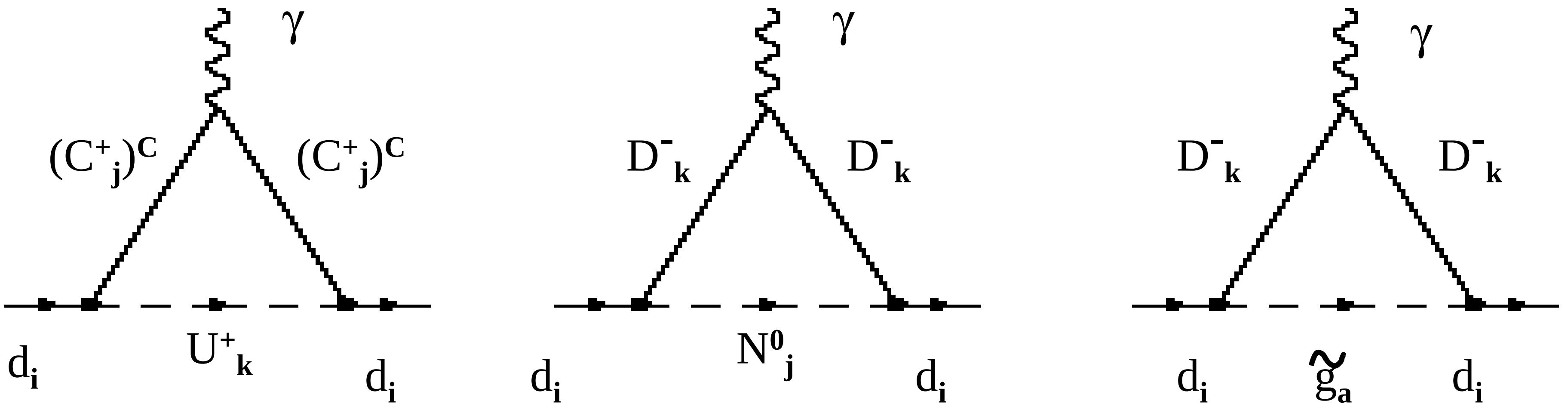
ΔF=2 ε and $B \rightarrow J/\psi\, K_s$

After the first attempts at the end of the 80s (Aoki, Manohar, Sharpe, Gocksch) the calculation of the matrix element of the **neutron electric dipole moment** has been abandoned. Renormalization of this operator and calculation of disconnected diagrams with stocastic sources is now a common practice

Important for :

- Strong CP problem $\bar{u}\,\gamma_5\, u + \bar{d}\,\gamma_5\, d$ or $\tilde{G}^{\mu\nu a}\, G^{a}{}_{\mu\nu}$

- SUSY extention of the Standard Model



$$\mathcal{L}^{\Delta F=0} = -i/2\, C_e\, \bar{\psi}\sigma_{\mu\nu}\gamma_5\psi\, F^{\mu\nu}$$
$$-i/2\, C_C\, \bar{\psi}\sigma_{\mu\nu}\gamma_5\, t^a\psi\, G^{\mu\nu a}$$
$$-1/6\, C_g\, f_{abc}\, G^a{}_{\mu\rho}\, G^{b\rho}{}_{\nu}\, G^c{}_{\lambda\sigma}\, \varepsilon^{\mu\nu\lambda\sigma}$$

$C_{e,C,g}$ can be computed perturbatively

$$L_{SM}^{\Delta F=2} = \Sigma_{ij=d,s,b} (V_{td_i} V^*_{td_j})^2 \, C \, [\bar{d}_i \gamma_\mu (1 - \gamma_5) d_j]^2$$

$$L^{\Delta F=2}_{general} = \Sigma_\alpha \Sigma_{ij=d,s,b} (V_{td_i} V^*_{td_j})^2 \, C^{ij}_\alpha \, Q^{ij}_\alpha$$

α = different Lorentz structures $L\times L$, $L\times R$ etc.

C^{ij}_α =complex coefficients from perturbation theory

$\langle \bar{K} | Q^{ij}_\alpha | K \rangle$ from lattice QCD (Donini et al. Phys. Lett. B470 (1999) 233; phenomenological analyses Ciuchini et al.; Ali and London; Ali and Lunghi; Buras et al.; Bartl et al.)

With/Without subtractions presented at this Conference by Becirevic, $SPQ_{cd}R$ Collaboration (also $\langle \bar{B} | Q^{ij}_\alpha | B \rangle$)

NEW RESULTS FOR B_K slide I

	$B^{NDR}_K(2\ \mathrm{GeV})$	$\hat{B}_K$
World Average by L.Lellouch at Lattice 2000	0.63 ± 0.04 ±0.10	0.86 ±0.06 ±0.14
CP-PACS perturbative renorm. (quenched) DWF	0.575 ±0.006 0.5746(61)(191)	0.787 ±0.008
RBC non-perturbative renorm. (quenched) DWF	0.538 ±0.008	0.737 ±0.011
$SPQ_{cd}R$ (preliminary) (quenched) Improved with subtractions	0.71 ± 0.13	0.97 ±0.14
without subtractions	0.70 ± 0.10	0.96 ±0.14

β=6.2 non-perturbatively improved actiojn

Some questions on B_K

B_K computed by De Rafael and Peris in the chiral limit is very small $B_K = 0.38 \pm 0.11$
Is this due to chiral corrections ?
B_K/B_K^{chiral} = 1.10(8) (with subtractions)
1.11(10) (without subtractions)
Becirevic quenched

A large value of $B_K \sim 0.85$ corresponds to a too large value of $K^+ \otimes \pi^+ \pi^0$ if SU(3) symmetry and soft pion theorems at lowest order are used (J. Donoghue 82) . Even 0.75 is too large.
RBC and CP-PACS seems to find instead the physical $K^+ \otimes \pi^+ \pi^0$ amplitude. What is their value for B_K/Bk^{chiral} ?

figure

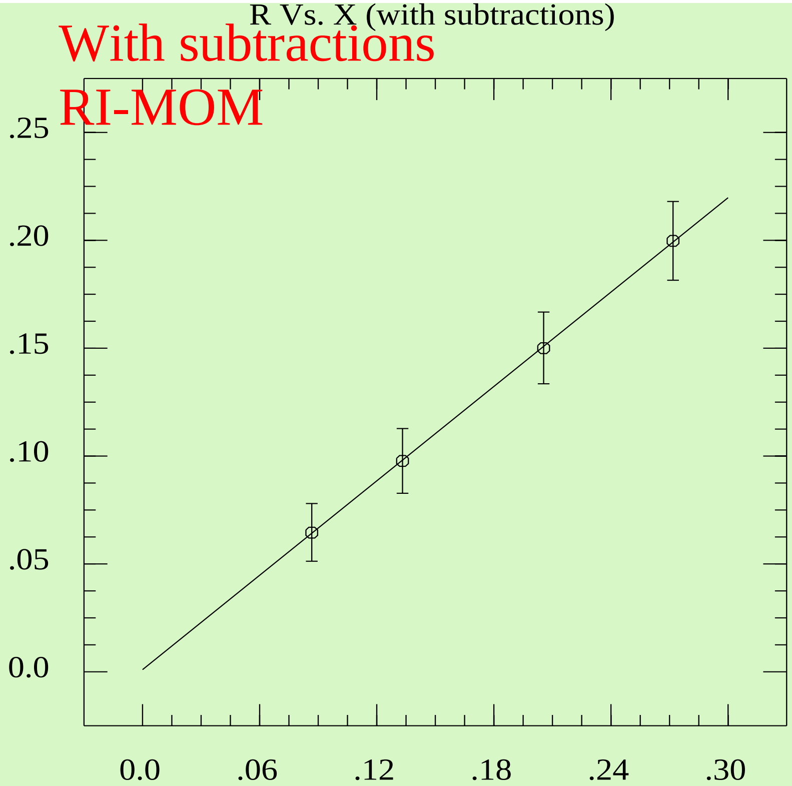
NEW RESULTS FOR B_K slide II

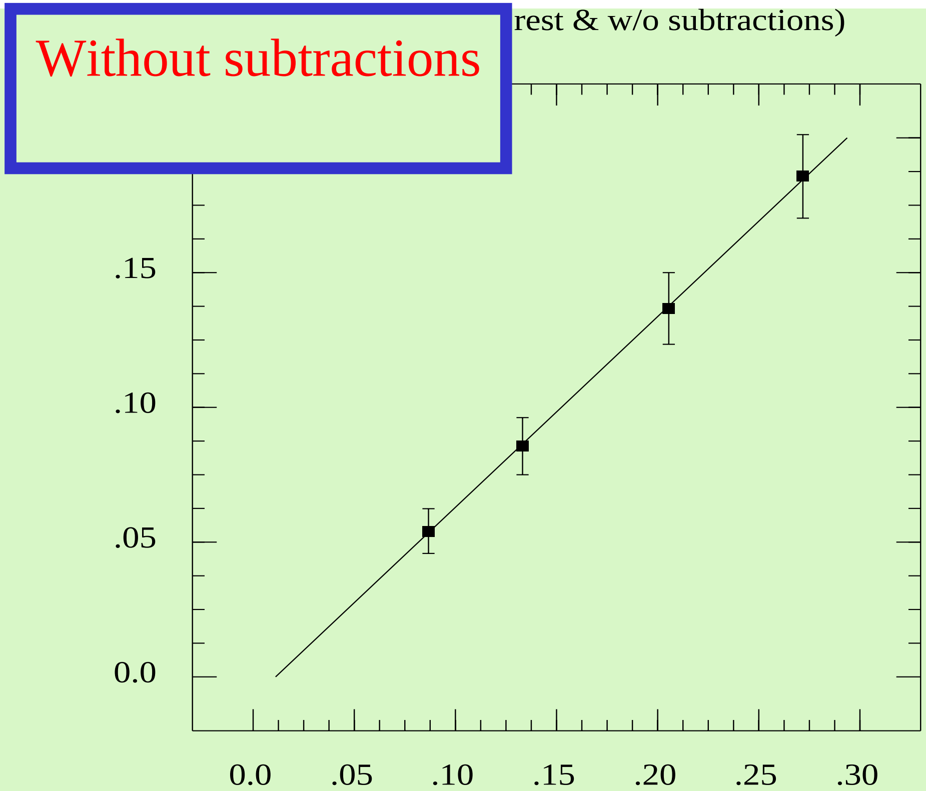
$\langle \overline{K}^0 | Q^{S=2} | K^0 \rangle$ with Wilson-Like Fermions without subtractions of wrong chirality operators. Two proposals with the same physical idea

$Q_{VV+AA} \rightarrow Q_{VA}$ which cannot mix because of CPS

Talk by D. Becirevic at this Conference (D. Becirevic et al. **$SPQ_{cd}R$**)
Use CPS and Ward ids, only exploratory results at Latt2000 $\rightarrow$
New numerical results (with NPR on quark states taking into account the Goldstone Boson Pole, see the talk by C. Dawson at Latt2000, see also Pittori and Le Yaouanc)

Talk by C. Pena at this Conference (Guagnelli et al., ALPHA Collaboration)
Use tmQCD and Schrödinger Functional Renormalization, only a proposal at Latt2000 $\rightarrow$ numerical results for the bare operator
$B_K(a)=$ 0.94(2) on $V=16^3\times 32$ and 0.96(2) (at a given value of the quark masses) on $V=16^3\times 48$
The SF renormalization is underway





Fit lineaire y = a * x + b
a = 7.2928 E-1
b = 9.3602 E-4 chi2/DOF = 1.9775 E-3

entre xmin = 0.00
et xmax = 3.0 E-1

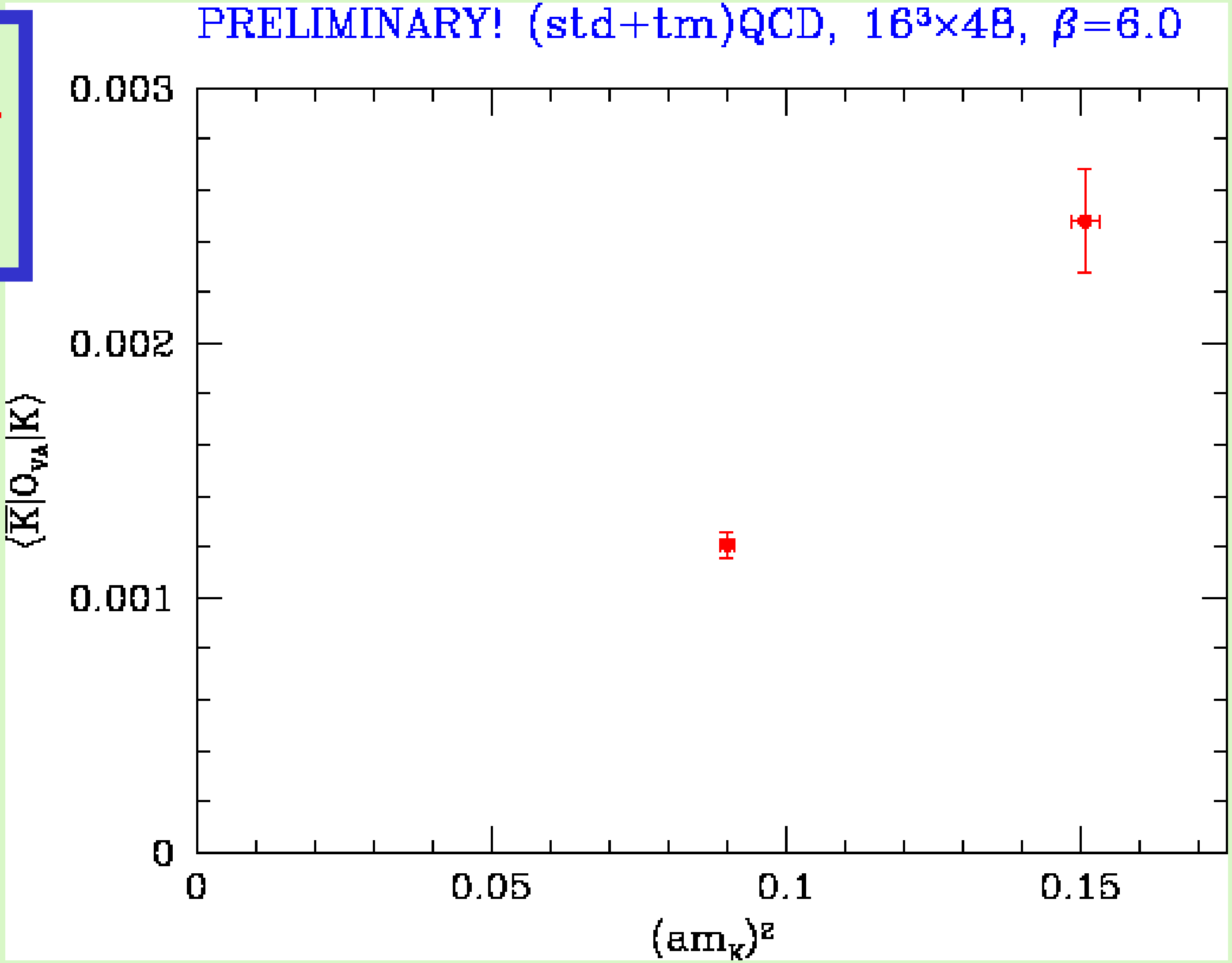
Fit lineaire y = a * x + b
a = 7.0811 E-1
b = -7.8747 E-3 chi2/DOF = 9.8345 E-3

entre xmin = 0.00
et xmax = 3.0 E-1

Chiral behaviour of $\langle\, \mathbf{\overline{K}^0} \mid \mathbf{Q}^{\,\mathbf{S=2}} \mid \mathbf{K^0} \,\rangle$ talk by D. Becirevic **SPQ$_{\mathbf{cd}}$R**

β= 6.2 V=24^3×64 200 configurations

Tm Fermions
Chiral Behaviour
needs further
study



Chiral behaviour of $\langle\, \overline{\mathbf{K}^0}\, |\, \mathbf{Q}^{\,S=2}\, |\, \mathbf{K}^0\, \rangle$ talk by Pena

The renormalisation condition

- The correlator for the renormalisation condition has the form:

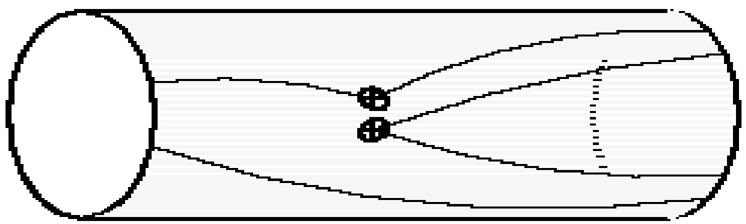
$$h_{VA}(x_0) = \langle \mathcal{W}' \mathbf{O}_{VA}(x) \mathcal{W} \rangle$$

  with some conveniently chosen boundary operators $\mathcal{W}$ and $\mathcal{W}'$.

- Restriction: the renormalisation condition has to take into account the full symmetry present with all masses equal to 0.

$$\downarrow$$

  - Simplest naive possibility $\mathcal{W} = \mathcal{O}_K$, $\mathcal{W}' = \mathcal{O}'_K$ forbidden: would be killed by parity
  - More generally, the use of two one-meson operators is either forbidden or technically inconvenient.
  - Solution: change one of the one-meson pseudoscalar states of the matrix element by a state with two pseudoscalar mesons e.g. $\mathcal{W} = \sum (\bar{\zeta}_u \gamma_5 \zeta_s)$, $\mathcal{W}' = \sum (\bar{\zeta}'_d \gamma_5 \zeta'_u)\, (\bar{\zeta}'_d \gamma_5 \zeta'_s)$.
    Traces look like:



NEW RESULTS FOR B_K slide III

Sinya Aoki at this Conference presented preliminary results on the renormalization of bilinear quark operators with the SF method and Domain Wall Fermions. The plan is to extend these calculations to $\Delta S=2$ operators

A forgotten method: Rossi, Sachrajda, Sharpe, Talevi, Testa and G.M., Phys. Lett. B411 (1997) 41 (easy to implement, gauge invariant, no contact terms see also G.M. NPB(PS) 73 (1999)58.)

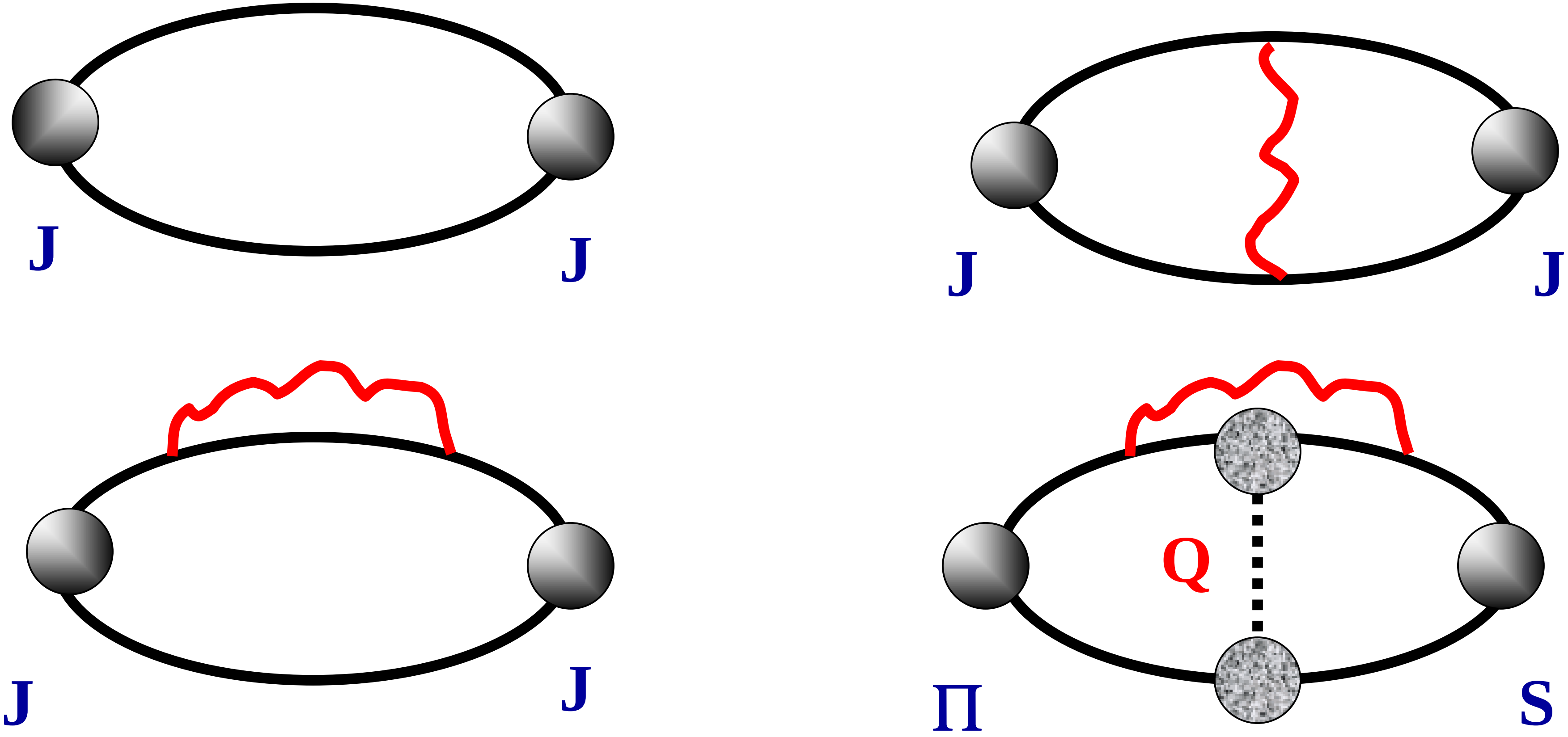
$$Z^2_J \langle T[J(x)J^\dagger(0)] \rangle\Big|_{|x|=1/\mu \ll 1/\Lambda_{QCD}} = \langle T[J(x)J^\dagger(0)] \rangle_{tree}$$

$1/a \ll 1/|x| \sim \mu \ll 1/\Lambda_{QCD}$ window avoided by iterative matching of the renormalization scale at different values of β

The necessary two loop calculations for bilinear operators have been completed (Del Bello and G.M.).

The extension two four Fermion operators is straightforward

$$\langle\, T[J(x)\, J^{\dagger}(0)]\,\rangle\Big|_{|x| \ll 1/\Lambda_{QCD},\ \text{perturbative}} =$$



Renormalization of Four Fermion Operators

Perturbative calculations with overlap fermions by S. Capitani and L. Giusti
hep-lat/0011070 v2 see also S. Capitani and L. Giusti Phys. Rev. D62 (2000) 114506
See talk by Capitani in the parallel session

Perturbative calculations for DWF by S. Aoki et al. , Phys. Rev. D59 (1999) 094505,
Phys. Rev. D60 (1999) 114504, Phys. Rev. D63 (2001) 054504 and in preparation

Renormalization & Improvement

Very nice and complete analysis of improvement constants by Bhattacharya
Gupta Lee and Sharpe c_A, $b_A - b_V$, c_T etc. at β=6.0,6.2 and 6.4
figure

IMPROVEMENT OF FOUR FERMION OPERATORS ?

$\Delta I=1/2$ and ε'/ε

- K ⌫ $\pi\,\pi$ from K ⌫ π and K ⌫

- Direct K ⌫ $\pi\,\pi$ calculation

- $\Delta I=1/2$ decays (Q_1 and Q_2)
- ε'/ε electropenguins (Q_7 and Q_8)
- ε'/ε strong penguins (Q_6)

Theoretical Novelties

- Chiral Perturbation Theory for $\langle Q_{+,1,2,7,8} \rangle$ **V. Cirigliano and E. Golowich Phys. Lett. B475 (2000) 351; M. Golterman and E. Pallante JHEP 0008 (2000) 023; talks by D. Lin and E. Pallante at the parallel session.** ★
- FSI and extrapolation to the physical point **Truong, E. Pallante and A. Pich (PP) Phys. Rev. Lett. 84 (2000) 2568; see also A. Buras at al. Phys. Lett. B480 (2000) 80 talk by G. Colangelo**
- $\langle \pi\pi | Q_i | K \rangle$ on finite volumes **L. Lellouch & M. Luscher Commun. Math. Phys. 219 (2001) 31 (LL) and D.Lin, G.M., C. Sachrajda and M. Testa hep-lat/0104006 (LMST)** ★

Only the subjects with a ★ will be discussed

New Numerical Results

$\langle \pi \mid Q_i \mid K \rangle$ for $\Delta I=1/2$ and ε'/ε with domain wall fermions

★

Vicente Gimenez

CP-PACS **talk by Noaki**

RBC talks by Mawhinney,Calin,Blum and poster by Soni

Chiral behaviour of $\langle \pi \pi \mid Q_4 \mid K \rangle$;

First determination of $\langle \pi \pi \mid Q_{7,8} \mid K \rangle$ and of their chiral behaviour;

★

First signal for $\langle \pi \pi \mid Q_{1,2} \mid K \rangle$ and $\langle \pi \pi \mid Q_6 \mid K \rangle$;

Gladiator The SPQ$_{cd}$R Collaboration

(Southapmton, Paris, Rome,Valencia)

results presented by D. Lin and M. Papinutto

GENERAL FRAMEWORK

$$H^{\Delta S=1} = G_F/\sqrt{2}\, V_{ud}\, V_{us}^{*}\Big[\, (1-\tau)\, \Sigma_{i=1,2}\, z_i\, (Q_i - Q^c_i) + \tau\, \Sigma_{i=1,10}\, (z_i + y_i)\, Q_i \,\Big]$$

Where y_i and z_i are short distance coefficients, which are known In perturbation theory at the NLO (Buras et al. + Ciuchini et al.)

$$\tau = -V_{ts}^{*} V_{td} / V_{us}^{*} V_{ud}$$

We have to compute $A^{I=0,2}_i = \langle (\pi\,\pi)_{I=0,2}\, |Q_i|\, K \rangle$ with a non perturbative technique (lattice, QCD sum rules, 1/N expansion etc.)

$$A^{I=0,2}_{i}(\mu) = \langle (\pi\,\pi)_{I=0,2} | Q_i(\mu) | K \rangle$$

$$= Z_{ik}(\mu\, a) \langle (\pi\,\pi)_{I=0,2} | Q_k(a) | K \rangle$$

Where $Q_i(a)$ is the bare lattice operator

And a the lattice spacing.

Two main roads to the calculation:

- K ⌫ π π from K ⌫ π and K ⌫ 0
- Direct K ⌫ π π calculation

So far onl[illegible]alitative [illegible]ni-quantitative) results for

$\langle (\pi\,\pi)_{I=0} | Q_{1,2,6} | K \rangle$

from Lattice QCD

Main sources of systematic errors from the UV and IR behaviour of the theory

UV: In order to obtain the physical amplitude we need $Z_{ik}(\mu\, a)$.

The construction of finite matrix elements of renormalized operators from the bare lattice ones is in principle fully solved

C. Bernard et al. Phys. Rev. D32 (1985) 2343.
M. Bochicchio et al. Nucl. Phys. B262 (1985) 331;
L. Maiani et al. Nucl. Phys. B289 (1987) 505;
C. Bernard et al. Nucl. Phys. B (Proc. Suppl.) 4 (1988) 483;
C. Dawson et al. Nucl. Phys. B514 (1998) 313.
S. Capitani and L. Giusti hep-lat/0011070.

Several non-perturbative techniques have been developed in order to determine $Z_{ik}(\mu a)$. The systematic errors can be as small as 1% for quark bilinears and typically (so far) 10% for four fermion operators.

For $Q_{1,2,6}$ only perturbative calculations (error 20-25%) so far (but see RBC Collaboration, C. Dawson et al. Nucl.Phys.Proc.Suppl.94:613-616,2001) .

More work is needed !!

Discretization errors are usually of $O(a)$, $O(m_q a)$ or $O(|p|a)$, but can become of $O(a^2)$ with Domain Wall Fermions or Non-perturbatively improved actions and operators. Similar problems encountered in effective theories with a cutoff (see V. Cirigliano, J. Donoghue, E. Golowich JHEP 0010:048,2000 hep-ph/0007196)

The IR problem arises from two sources:

- The (unavoidable) continuation of the theory to Euclidean space-time (Maiani-Testa theorem)
- The use of a finite volume in numerical simulations

An important step towards the solution of the IR problem has been achieved by L. Lellouch and M. Lüscher (LL), who derived a relation between the K ⌫ π π matrix elements in a finite volume and the physical amplitudes

presented by L. Lellouch at Latt2000

Commun.Math.Phys.219:31-44,2001
e-Print Archive: **hep-lat/0003023**

Here I discuss an alternative derivation based on the behaviour of correlators of local operator when $V \to \infty$

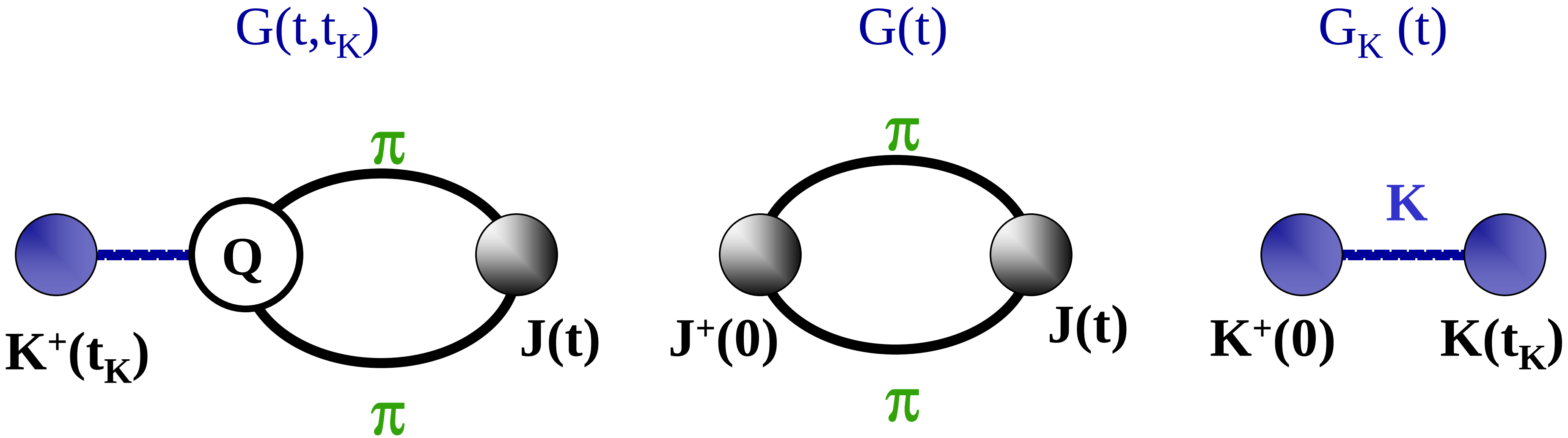
D. Lin, G.M., C. Sachrajda and M. Testa hep-lat/0104006 (LMST)

Consider the following Euclidean T-products (correlation functions)

$$G(t,t_K) = \langle 0 \,|\, T\,[J(t)\, Q(0)\, K^+(t_K)]\, |0\rangle,$$

$$G(t) = \langle 0 \,|\, T\,[J(t)\, J(0)\,]\, |0\rangle, \qquad G_K(t) = \langle 0 \,|\, T\,[K(t)\, K^+(0)\,]\, |0\rangle,$$

where J is a scalar operator which excites (annhilates) zero angular momentum ($\pi\pi$) states from (to) the vacuum and K is a pseudoscalar source which excites a Kaon from the vacuum ($t > 0$; $t_K < 0$)



At large time distances:

$$G(t,t_K) \to V\, \Sigma_{\mathbf{n}} \langle 0\; |J\,|\pi\pi\; n\rangle_{\mathbf{V}} \langle \pi\pi\; n\,|Q(0)\,|K\,\rangle_{\mathbf{V}} \langle K\,|K^+\,|0\,\rangle_{\mathbf{V}}$$

$$\exp[\,-(W_n\, t\;+m_K|\,t_K\,|)\,]$$

$$G(t) \;\; = V\, \Sigma_{\mathbf{n}} \langle 0\; |J\,|\pi\pi\; n\rangle_{\mathbf{V}} \langle \pi\pi\; n|J\,|\,0\rangle_{\mathbf{V}}\;\; \exp[\,-W_n\, t\,]$$

From the study of the time dependence of $G(t,t_K)$, $G(t)$ and $G_K\,(t)$ we extract

- the mass of the Kaon m_K
- the two-π energies W_n
- the relevant matrix elements in the finite volume

$$\langle K\,|K^+\,|0\,\rangle_{\mathbf{V}}\;,\; \langle 0\;\;|J\,|\pi\pi\; n\rangle_{\mathbf{V}}\;,\; \text{and} \;\langle \pi\pi\; n\,|Q(0)\,|K\,\rangle_{\mathbf{V}}$$

We may also match the kaon mass and the two pion energy, namely to work with $m_K \;= W_{n^*}$

Necessary to obtain a finite $\Delta I=1/2$ matrix element

The fundamental point is that it is possible to relate the finite-volume Euclidean matrix element with the absolute value of the **Physical Amplitude** $|\langle \pi\pi\, E\, |Q(0)|K\rangle|$

by comparing, at large values of V, the finite volume correlators to the infinite volume ones

$$|\langle \pi\pi\, E\, |Q(0)|K\rangle| = \sqrt{F}\ \langle \pi\pi\, n\, |Q(0)|K\rangle_V$$

$F = 32\,\pi^2 V^2 \rho_V(E)\, E\, m_K/k(E)$ where $k(E) = \sqrt{E^2/4 - m^2_\pi}$ and

$\rho_V(E) = (q\,\phi'(q) + k\,\delta'(k))/4\,\pi\, k^2$ is the expression which one would heuristically derive by interpreting $\rho_V(E)$ as the density of states in a finite volume (D. Lin, G.M., C. Sachrajda and M. Testa)

On the other hand the phase shift can be extracted from the two-pion energy according to (Lüscher):

$$W_n = 2\sqrt{m^2_\pi + k^2} \qquad n\,\pi - \delta(k) = \phi(q)$$

Main differences between LL and LMST:

- The LL formula is derived at fixed finite volume ($n < 8$) whereas the LMST derivation holds for $V \to \infty$ at fixed energy E;
- It is possible to extract the matrix elements even when $m_K \neq W_{n*}$ this is very useful to study the chiral behaviour of $\langle \pi\pi \, | Q(0) | K \rangle$
  - In the near future, in practice, it will only be possible to work with a few states below the inelastic threshold

$$G(t,t_K) = V \, \Sigma_{\mathbf{n}} \langle 0 \, | J \, | \pi\pi \; n \rangle_{\mathbf{V}} \langle \pi\pi \; n \, | Q(0) \, | K \rangle_{\mathbf{V}} \langle K \, | K^+ \, | 0 \rangle_{\mathbf{V}}$$
$$\exp[-(W_n \, t + m_K | \, t_K \, |)]$$

For the validity of the derivation, inelasticity at W_{n*} must be small (which is realized for $\pi\pi$ states with $W_{n*} = m_K$);

- If one uses $G(t_{\mathbf{1}}, t_{\mathbf{2}}, t_K) = \langle 0 \, | \, T \, [\pi \, (t_{\mathbf{1}}) \, \pi \, (t_{\mathbf{2}}) \, Q(0) \, K^+ \, (t_K)] \, | 0 \rangle$, no correcting factor is necessary; in this case we get the real part of the amplitude $R = | \langle \pi\pi \; E \, | Q(0) \, | K \rangle | \cos \delta(E) + O(1/L)$

W_n is determined from the time dependence of the correlation functions

$$
\begin{aligned}
G(t,t_K) &= V \langle K | K^+ | 0 \rangle_V \exp(-m_K |t_K|) \\
&\quad \Sigma_{\mathbf{n}} \langle 0 | J | \pi\pi\, n \rangle_V \langle \pi\pi\, n | Q(0) | K \rangle_V \exp(-W_n t) \\
&= \Sigma_{\mathbf{n}}\ \mathbf{A}_{\mathbf{n}} \exp(-W_n t)
\end{aligned}
$$

From W_n it is possible to extract the FSI phase
(for a different method to obtain $\delta(E) = \delta(k)$ see LMST)

$$W_n = 2 \sqrt{m^2_\pi + k^2} \qquad n\pi - \delta(k) = \phi(q)$$

1) IT IS VERY DIFFICULT TO ISOLATE W_n WHEN n IS LARGE !

2) THIS METHOD HAS BEEN USED FOR THE $\Delta I=3/2$ AND 1/2 TRANSITIONS DISCUSSED IN THE FOLLOWING and talks by Lin and Papinutto

Example:

$$-i\,M_{\mathbf{4}} = \alpha\,[m_{\mathbf{K}}(m_{\pi} + E_{\pi})/2 + m_{\pi}\,E_{\pi}]$$

$$+ 4\,\beta_{\mathbf{2}}\,m_{\pi}^{\mathbf{2}}\,(m_{\pi}^{\mathbf{2}} - m_{\mathbf{K}}^{\mathbf{2}}) + \beta_{\mathbf{4}} m_{\mathbf{K}} m_{\pi}\,(m_{\pi}^{\mathbf{2}} + m_{\mathbf{K}}^{\mathbf{2}})$$

$$+ \ldots.$$

$$(E_{\pi} = \sqrt{m_{\pi}^{\mathbf{2}} + \vec{p}_{\pi}^{\,\mathbf{2}}})$$

In general $M_{\mathbf{4}} = M_{\mathbf{4}}(m_{\mathbf{K}}, m_{\pi}, E_{\pi})$

We can work with $W_{n*} \neq m_K$ at several values of the pion masses and momenta (and at different kaon masses) and extrapolate to the physical point by fitting the amplitude to its chiral expansion, including the chiral logarithms. Two extra operators needed with respect to Pallante and Golterman.

This is underway for $Q_{\mathbf{4}}$ and the electropenguins.

Summary of the main steps

for ‹ K^0 | $Q^{\Delta S=2}$ | K^0 › 1)-5)

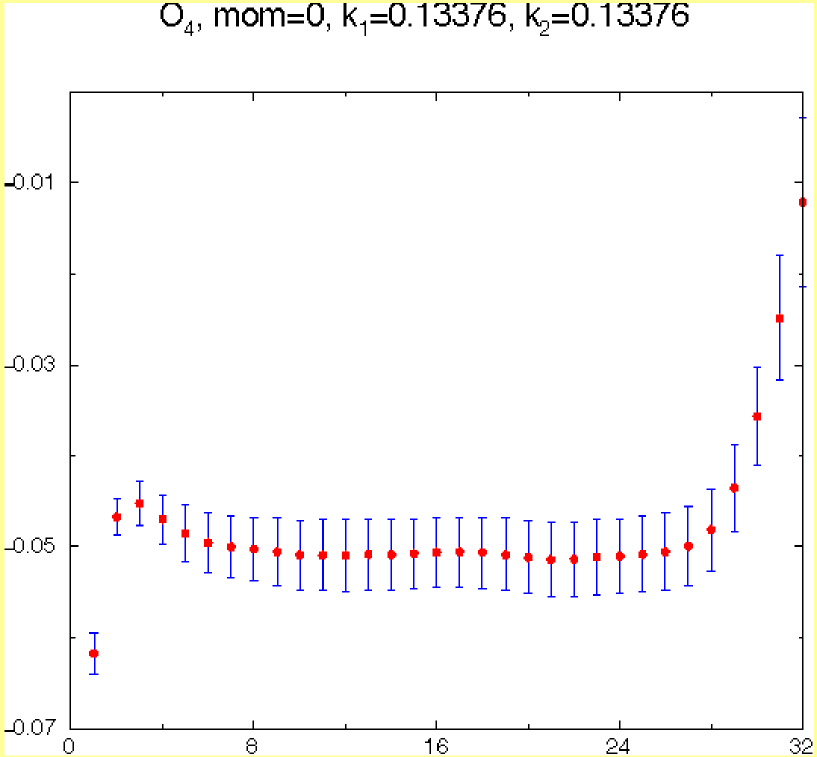
$K \to \pi\,\pi$

	$\Delta I=3/2$	$\Delta I=1/2$
1) Extraction of the signal	yes	yes (**NEW !!**)
2) Renormalization	non pert	pert !!
3) Chiral extrapolation to the physical point	yes	not yet (possible with more statistics)
4) Discretization errors	not yet (possible in the near future)	not yet (possible in the near future)
5) Quenching	possible in near future	possible

$\Delta I = 3/2$

THE SIGNAL: Improved action

350 Configurations $\beta = 6.0$ ($a^{-1} = 2$ GeV)

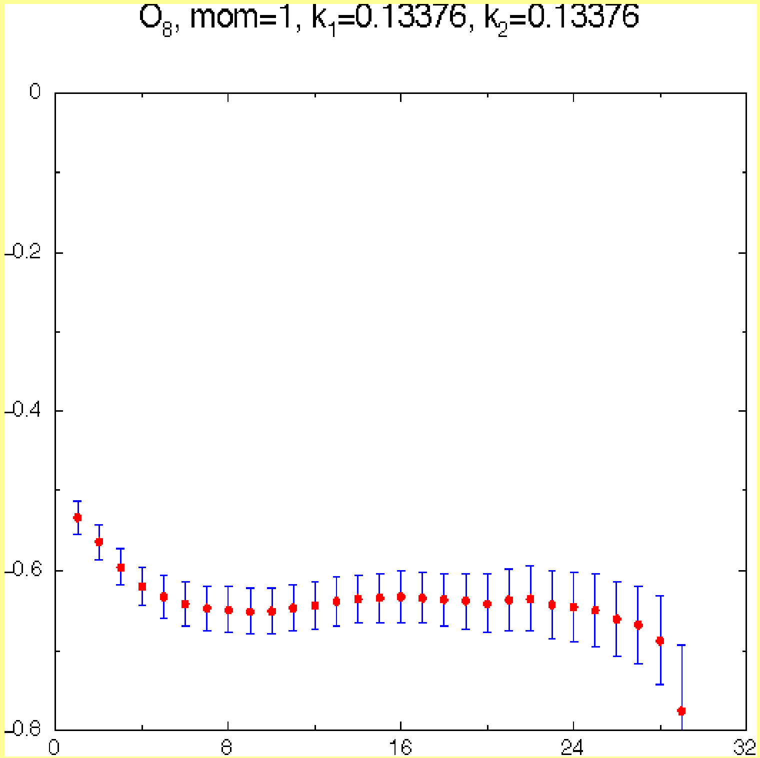


time

$\langle \pi\pi \ | Q_4 | \ K \rangle$

Present statistical error of O(10%)

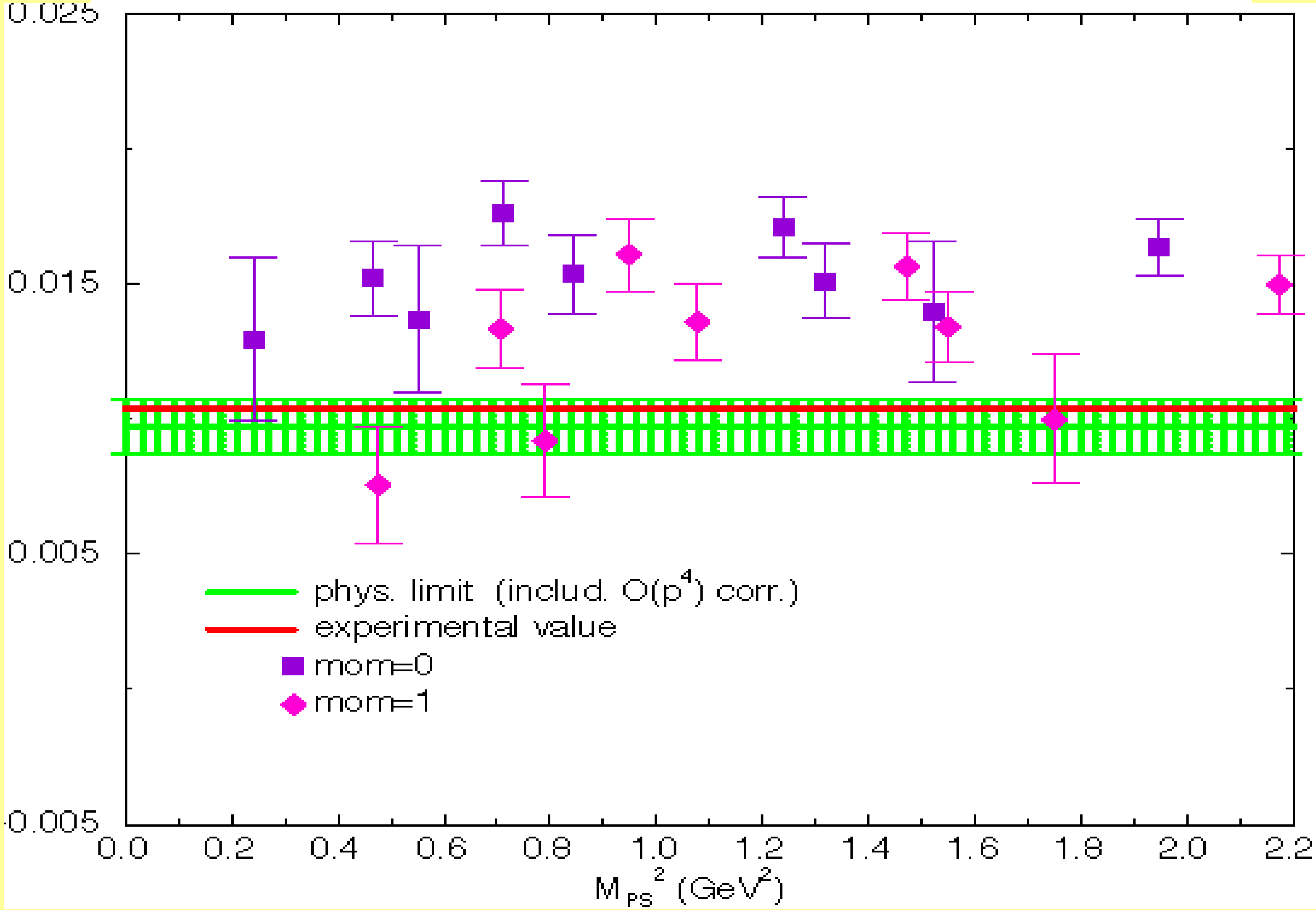
$\langle \pi\pi \ | Q_8 | \ K \rangle$



time

Future statistical error < 3%

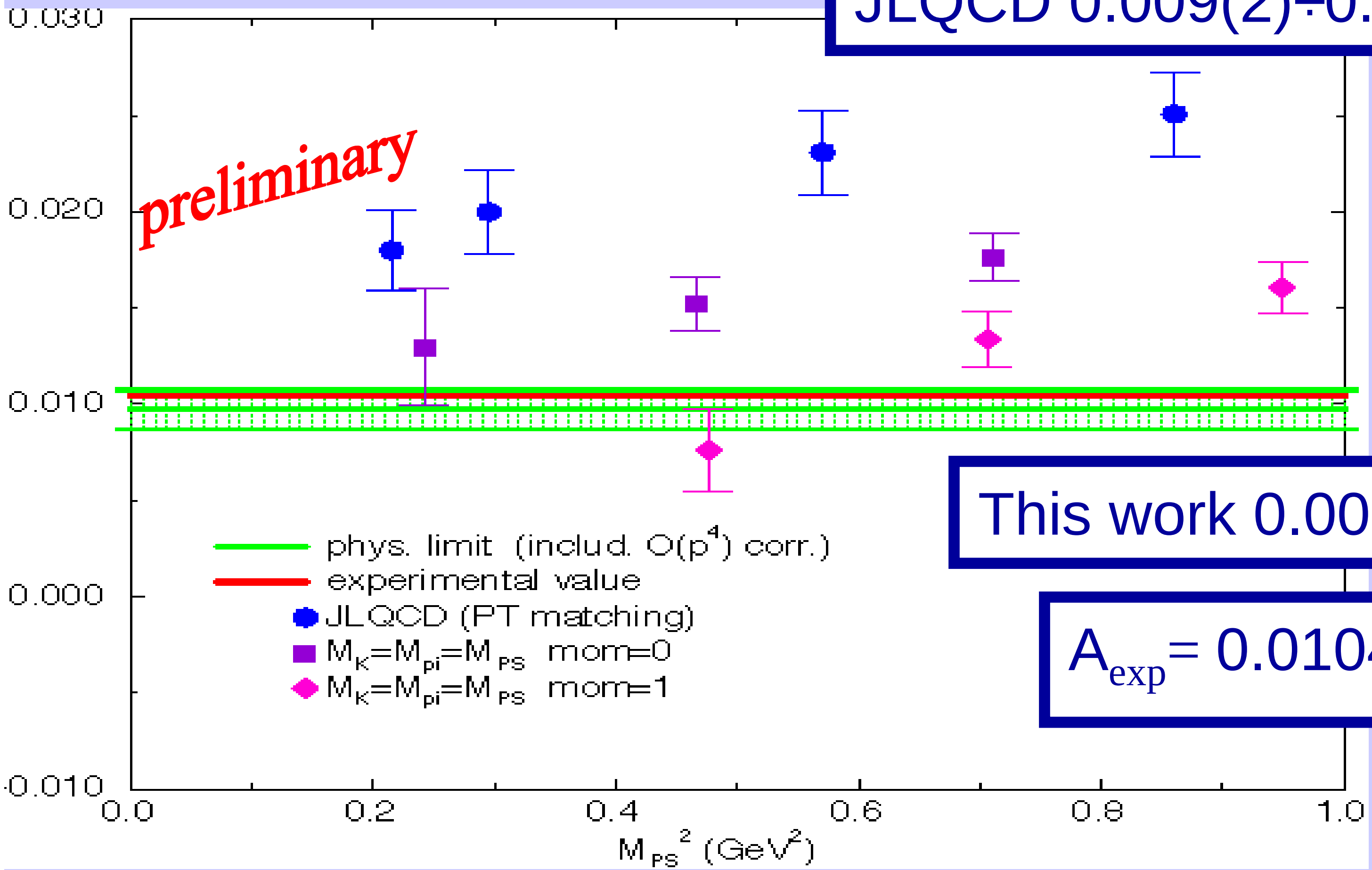
THE CHIRAL BEHAVIOUR FOR $\langle\pi | Q_4 | K\rangle$



for the chiral behaviour of $\langle Q_4\rangle$ see for example Pallante and Golterman and Lin;
chiral logs and extra operators not yet included; $\cos\delta(E) \approx 1$

NEW !! THE CHIRAL BEHAVIOUR FOR $\langle \pi\, \pi\, | H_W | \, K \rangle_{I=2}$ and a comparison with JLQCD **Phys. Rev. D58 (1998) 054503** (non improved perturbative renormalization) & experiments

JLQCD 0.009(2)÷0.011(2) GeV^3

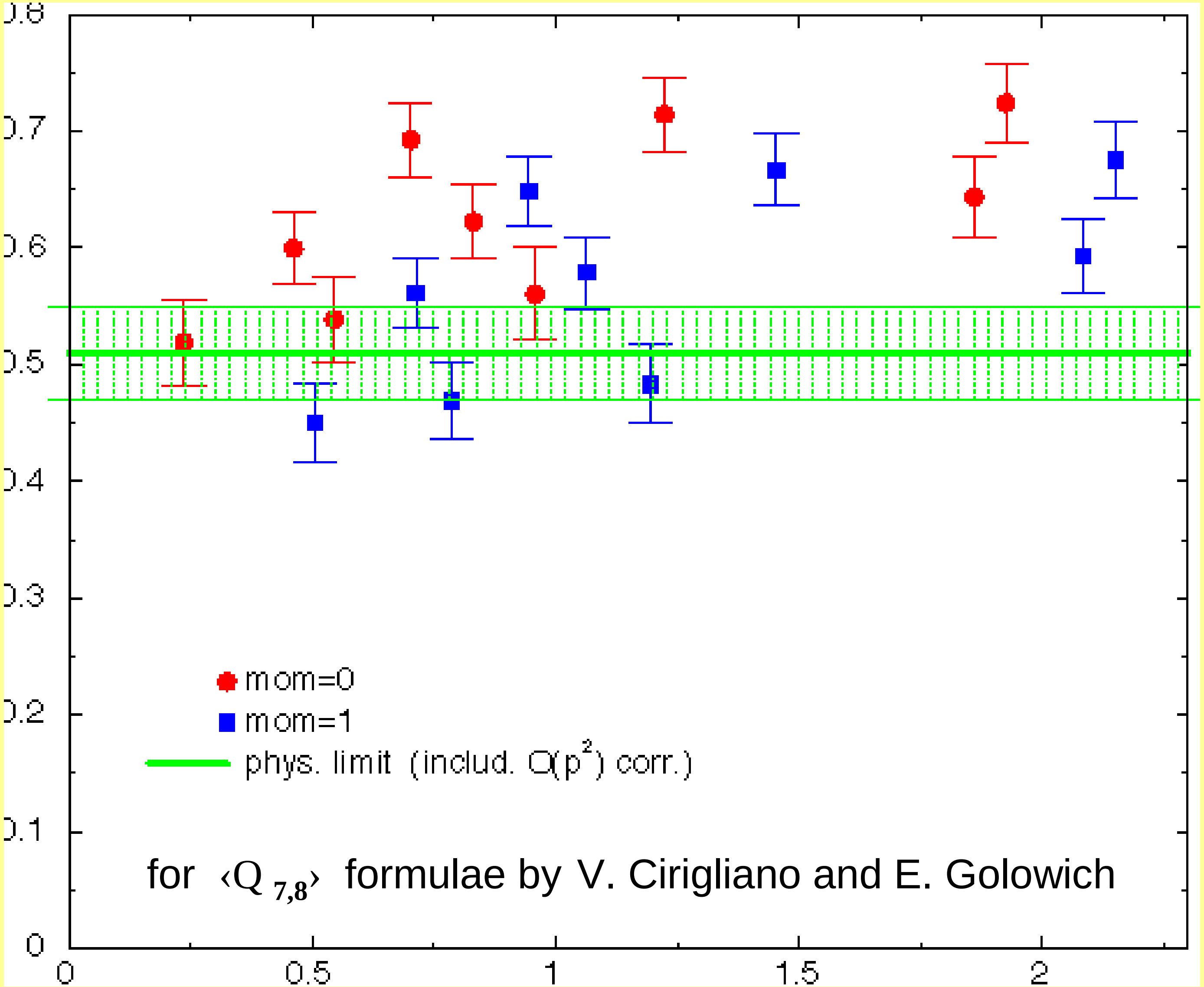


This work 0.0097(10) GeV^3

A_{exp}= 0.0104098 GeV^3

Lattice QCD finds B_K = 0.86 and a value of $\langle \pi\, \pi\, | H_W | \, K \rangle_{I=2}$ compatible with exps

THE CHIRAL BEHAVIOUR FOR $\langle \pi\, \pi\, |Q_8|\, K \rangle_{I=2}$



RI-MOM renormalization scheme

Results for Q $_{7,8}$ and comparison with other determinations ($\overline{\text{MS}}$)

RBC and CPPACS for comparison

~ 2.2

GeV^3

	$\langle Q_8 \rangle$	$\langle Q_7 \rangle$
K ’ š š (SPQ$_{cd}$R) NEW!!	0.53 ± 0.06	0.02 ± 0.01
J. Donoghue and E. Golowich	1.3 ± 0.3	0.22 ± 0.05
M. Knecht, S. Peris and E. De Rafael	3.5 ± 1.1	0.11 ± 0.03
Donini et al. (Rome)	0.5 ± 0.1	0.11 ± 0.04
D. Becirevic et al. (SPQ$_{cd}$R) NEW!!	0.49 ± 0.06	0.10(2)(1)

preliminary

preliminary

from K ⌫ π

$\Delta I = 1/2$ and ε'/ε

The subtractions of the power divergencies, necessary to obtain finite matrix elements, are the major obstacle in lattice calculations

1) these subtractions are present for both the methods which have been proposed

K ⌫ π π from K ⌫ π and K ⌫ 0 — Direct K ⌫ π π calculation

2) the subtractions are not needed for $\langle \pi\, \pi\, | Q_{4,7,8} | K \rangle_{I=2}$

Example $\Delta I = 1/2$ from K ⌫ π π

$$Q^{\pm} = [\, s\, \gamma_\mu (1-\gamma_5)\, d \;\; u\, \gamma_\mu (1-\gamma_5)\, u \pm s\, \gamma_\mu (1-\gamma_5)\, u \;\; u\, \gamma_\mu (1-\gamma_5) d \,]$$

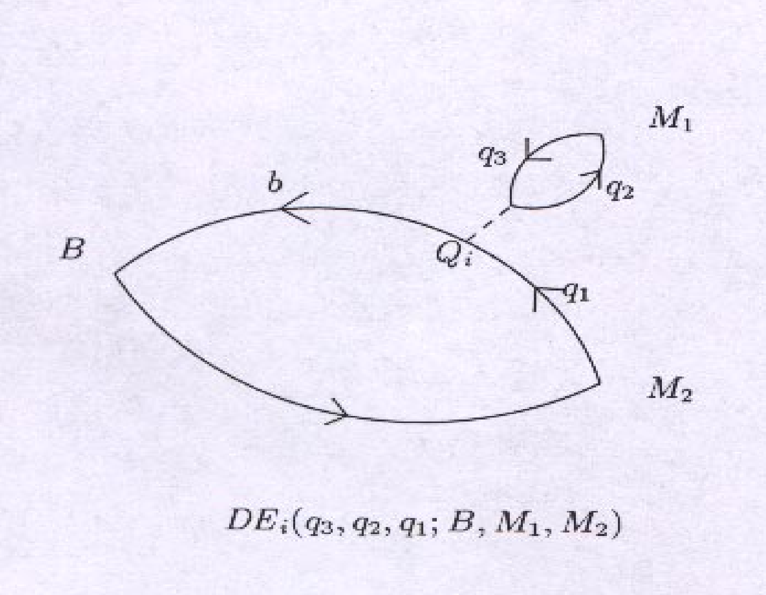
$$-\, [\, c \leftrightarrow u \,]$$

this is the subtraction !!

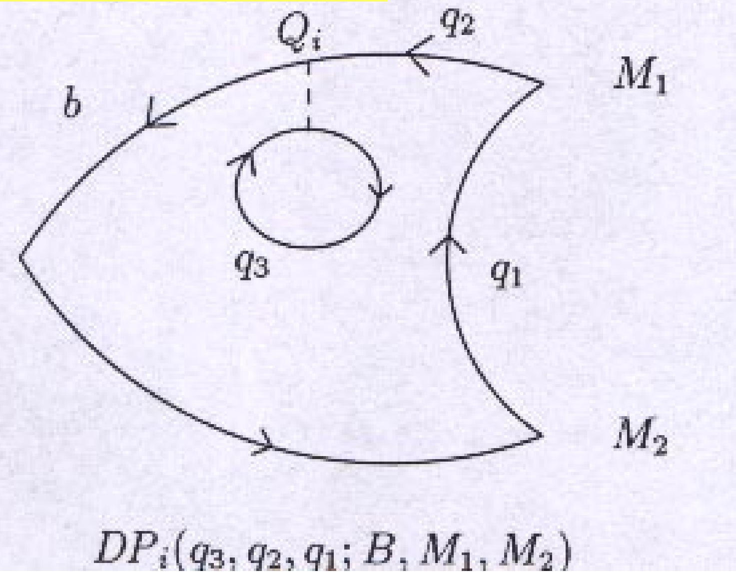
ΔI = 1/2 and ε'/ε

The most important contributions are expected to come from penguin diagrams

Disconnected emission



Disconnected Penguin



$B_{1,2} \approx 4,5$

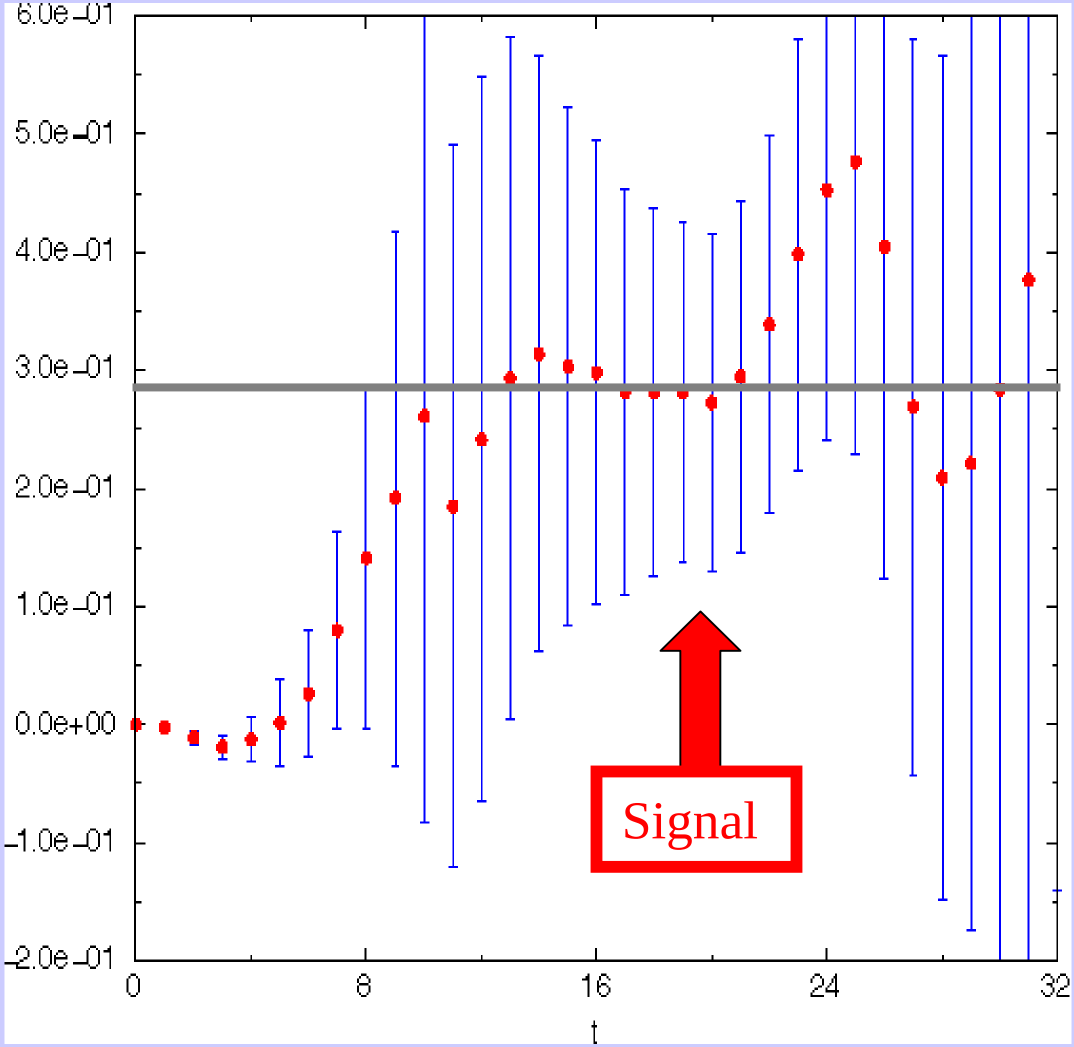
$B_6 \approx 3$

Lattice calculations have shown that it is non possible to explain the octet enhancement with emission diagrams only; penguins are at the origin of the Power divergences. They are absent in ΔI=3/2 amplitudes.

Matrix element of $\langle \pi\, \pi\, | Q^- |\, K \rangle_{I=0}$ without $Z(\mu a)$

only penguin contractions with GIM subtractions

$m_K = 2\, m_\pi$



preliminary

1) Data with 340 confs
2) Statistical error 50÷70%
3) Needed about 5000 confs for an error of 20 % (quenched)
4) Actually about 20 confs/day (9 months)
6) With further improment of the programmes and the 3rd machine 45/day (4 months)

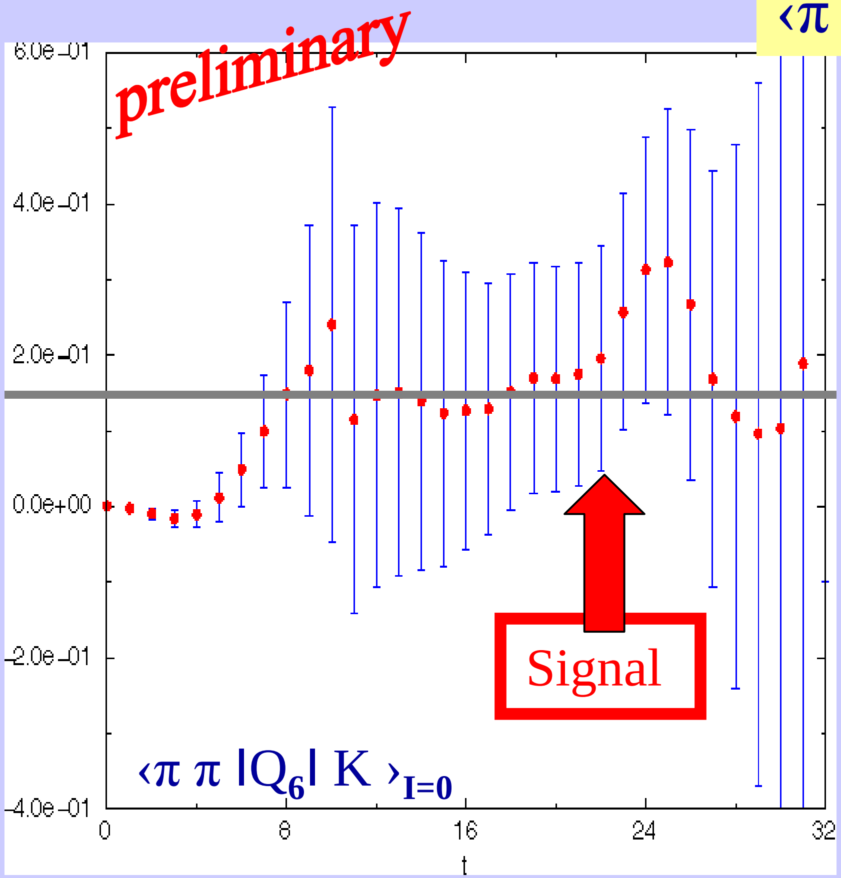
we are looking for better 2 pion sources

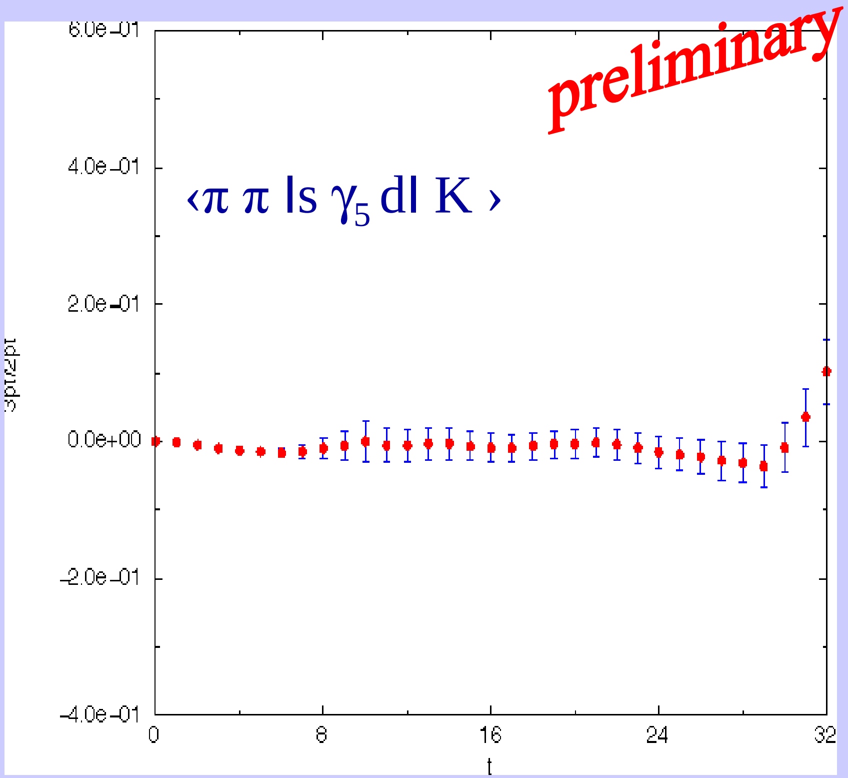
For bare ops $\langle \pi\, \pi\, | Q^- |\, K \rangle_{I=0} / \langle \pi\, \pi\, | Q_+ |\, K \rangle_{I=2} \approx$ **9 !!**

Matrix element of $\langle\pi\,\pi\,|Q_6|\,K\,\rangle_{I=0}$ without $Z(\mu a)$
only penguin contractions

$m_K = 2\,m_\pi$

No subtraction needed $\langle\pi\,\pi\,|s\,\gamma_5\,d|\,K\,\rangle =$
$\langle\pi\,\pi\,|\partial_\mu\,s\,\gamma_\mu\gamma_5\,d|\,K\,\rangle/(m_s+m_d) = 0$





See C. Dawson et al. Nucl. Phys. B514 (1998) 313

K ⌫ $\pi\pi$ from K ⌫ π and K ⌫ 0 and Domain Wall Fermions

- Impossible (in practice) with Wilson (Improved Fermions) because of (power) subtractions;
- With Domain Wall Fermions (DWF) only one subtraction is required (see below); Also true with Overlap Fermions
- Computer much more demanding than Wilson or Improved Fermions thus only K ⌫ π so far;
- A very good control of residual chiral symmetry breaking is required. The error decreases as exp(- const. L_5) but remember that we have power divergences;
- Problems with the extrapolation to the physical point (Pich Pallante see also talk by Colangelo) ??

A subtraction is needed:

$\langle\pi \,|\, Q_i^{sub} \,|\, K\rangle = \langle\pi \,|\, Q_i^{sub} \,|\, K\rangle - c_i\,(m_s + m_d)\,\langle\pi \,|\, s\,d \,|\, K\rangle$;

c_i obtained from the condition

$\langle 0 |\, Q_i \,|\, K\rangle - c_i\,(m_s - m_d)\,\langle 0 |\, s\,\gamma_5\, d \,|\, K\rangle = 0$;

c_i is obtained either using non degenerate quarks (RBC) or from the derivative of the 2-point correlation function (CP-PACS)

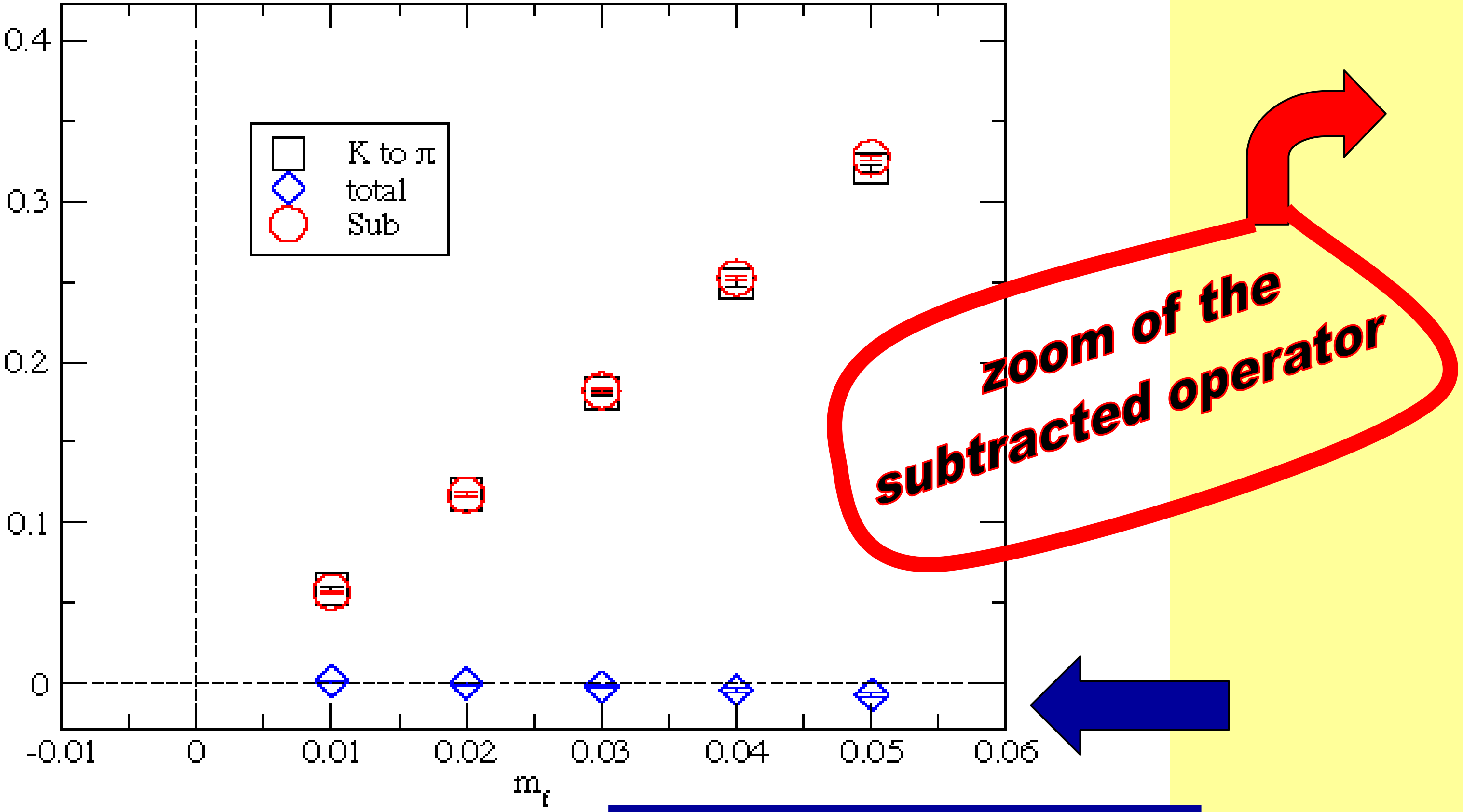
C. Bernard et al. Phys. Rev. D32 (1985) 2343.

Preliminary numerical results for all the operators were presented by CP-PACS and RBC) at Lattice 2000;

Physics Results at Lattice 2001.

QCD Penguin $\langle \pi^- | Q_{6,\mathrm{lat}} | K^- \rangle$ and $2\, m_f\, \eta_6\, \langle \pi^- | \bar{s} d_{\mathrm{lat}} | K^- \rangle$

Divergent subtraction is almost complete. Physical slope is roughly 50 x smaller than the unsubtracted one



Data are highly correlated!

RBC Standard Gauge Field Action β=6.0 DWF

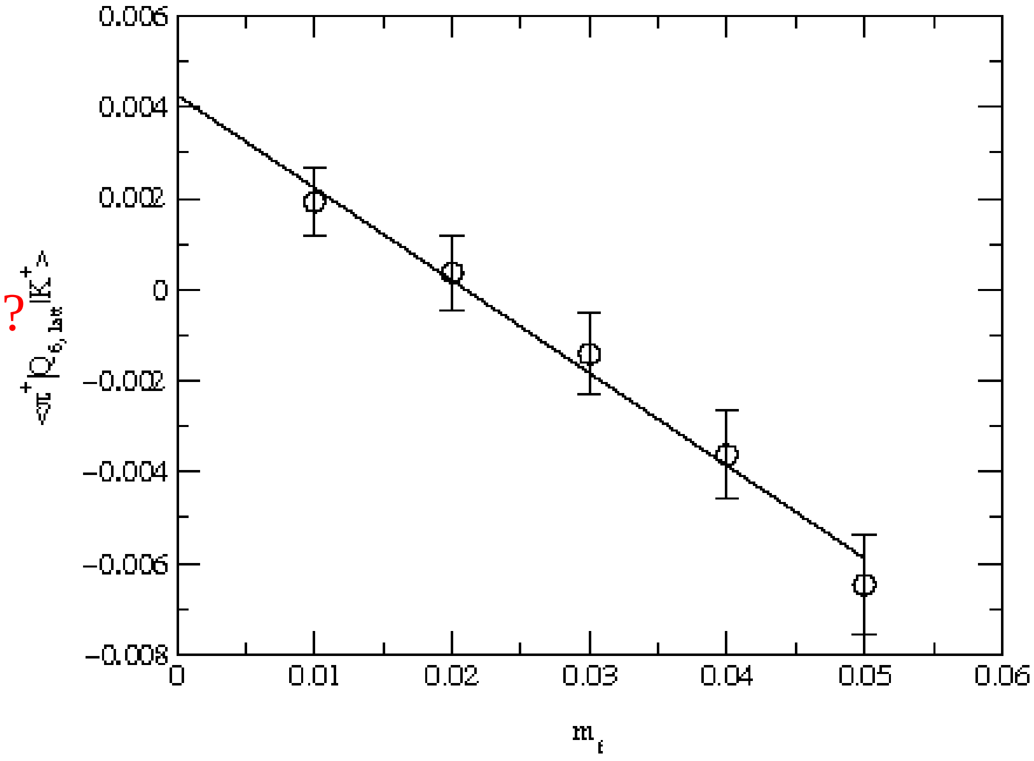
Subtracted QCD Penguin $\langle \pi^+ | Q_{6,\mathrm{lat}} | K^+ \rangle$ (important to ϵ')

Does not vanish as $m_f \to -m_{res}$ because valence quark loop is sensitive to **high energy** chiral symmetry breaking effects (mixing between domain walls). Effect is an additive shift in the quark mass, which is eliminated by taking the slope.

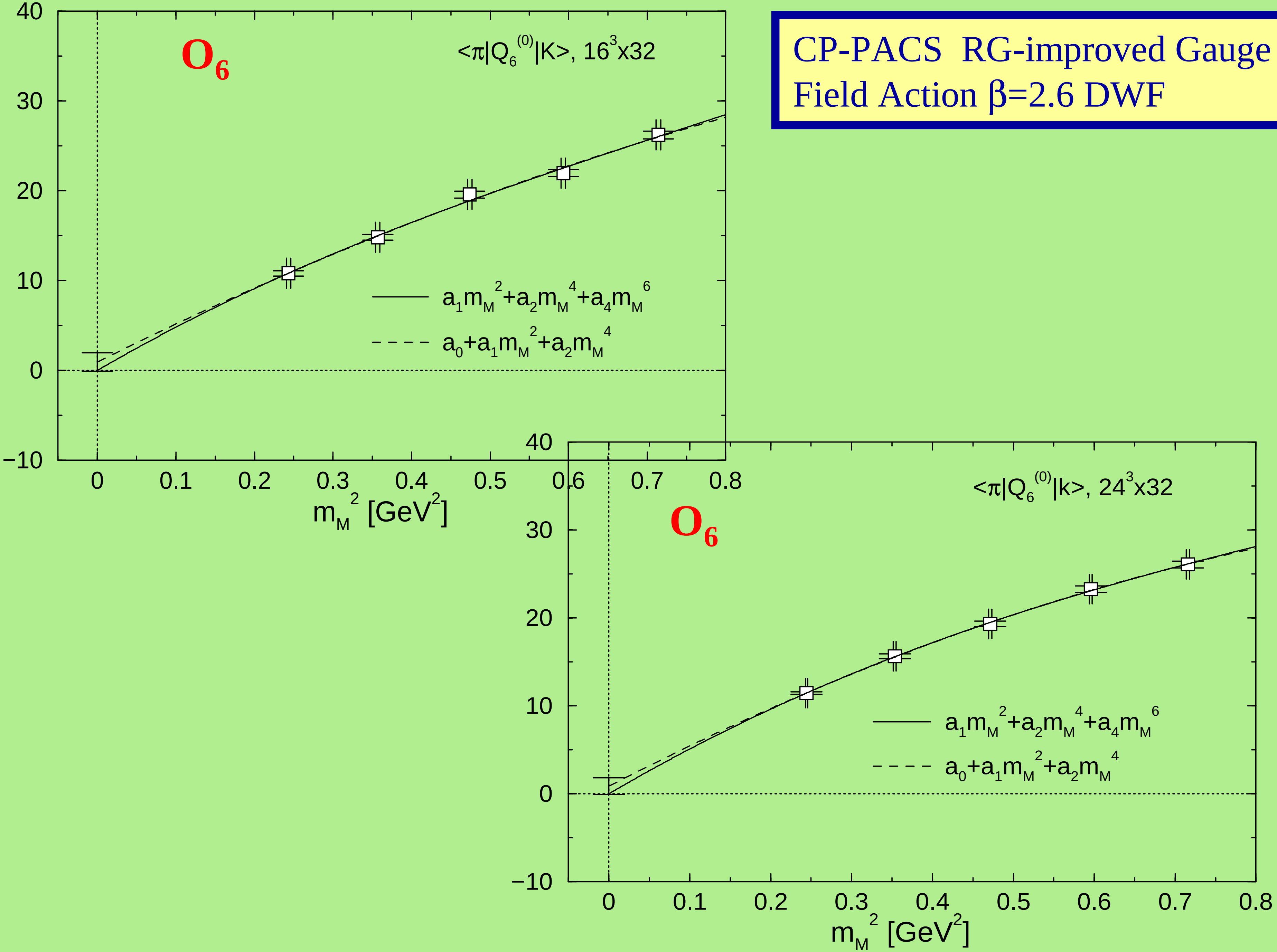
$$\alpha_6 = \frac{f^2}{4} \times \text{slope} = (-8.12 \pm 0.98) \times 10^{-5}$$

RBC Standard Gauge Field Action β=6.0 DWF

Linear fit ??

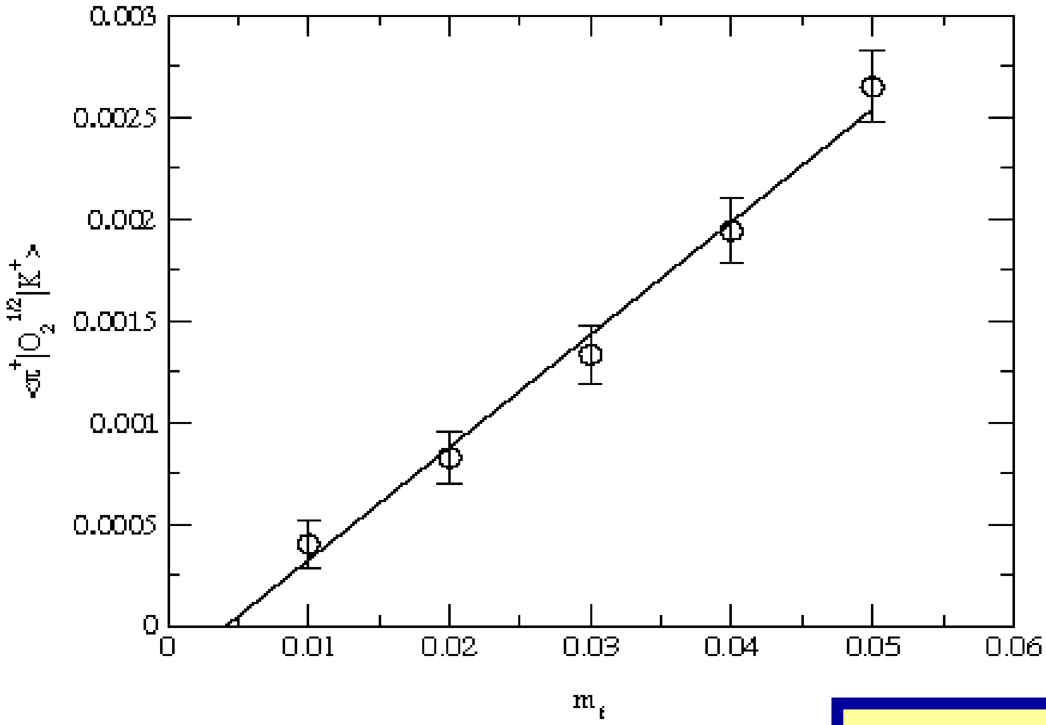


CP-PACS RG-improved Gauge Field Action β=2.6 DWF



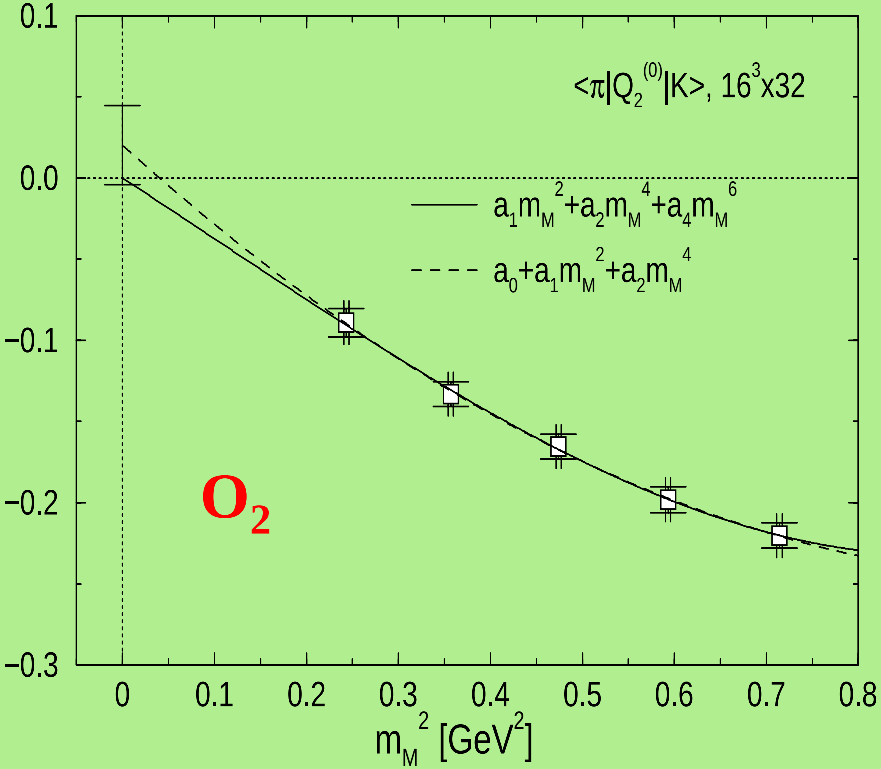
Subtracted $\langle \pi \; | Q_{2,\mathrm{lat}}^{1/2} | K \; \rangle$ (important to $\mathrm{Re} A_0$)

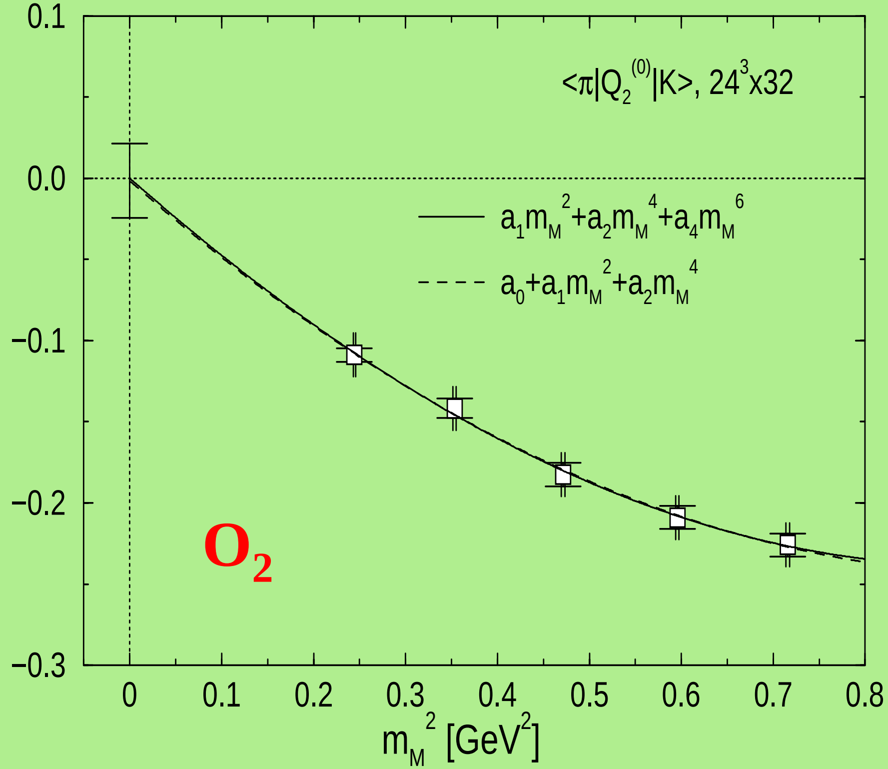
Explict chiral symmetry breaking effects smaller than for Q_6.



$$\alpha_2 = \frac{f^2}{4} \times \textbf{slope} = (2.22 \pm 0.16) \times 10^{-5}$$

RBC Standard Gauge
Field Action β=6.0 DWF





CP-PACS RG-improved Gauge Field Action β=2.6 DWF

Physics Results from RBC and CP-PACS

talks by Mawhinney,Calin,Blum and Soni (RBC)

Noaki (CP-PACS)

	$Re(A_0)$	$Re(A_2)$	$Re(A_0)/Re(A_2)$	ε'/ε
RBC	$29 \div 31\ 10^{-8}$	$1.1 \div 1.2\ 10^{-8}$	$24 \div 27$	$-4 \div -8\ 10^{-4}$
CP PACS	$16 \div 21\ 10^{-8}$	$1.3 \div 1.5\ 10^{-8}$	$9 \div 12$	$-2 \div -7\ 10^{-4}$
EXP	$33.3\ 10^{-8}$	$1.5\ 10^{-8}$	22.2	$17.2 \pm 1.8\ 10^{-4}$

- **Chirality**
- **Subtraction**
- **Low Ren.Scale**
- **Quenching**
- **FSI**
- **New Physics**
- **A combination ?**

Even by doubling O_6 one cannot agree with the data

K ⌫ π π and Staggered Fermions (Poster by W.Lee) will certainly help to clarify the situation I am not allowed to quote any number

Unphysical quenched contributions $\Delta I = 1/2$ and ε'/ε Golterman and Pallante presented by Pallante

Q_6 is an (8,1) operator. In the quenched case it may mix with an (8,8) operator , as it can be explicit checked in one loop **quenched** chiral perturbation theory

This correction is potentially more important for Q_6 since

$\langle \pi \mid Q_6 \mid K \rangle \sim m^2$

$\langle \pi \mid (8,8) \mid K \rangle \sim 1$ **thus, the one loop correction**

$\langle \pi \mid \Delta Q^{(8,8)}_6 \mid K \rangle \sim m^2$

GP suggested to remove $\bar{q}\, q$ contractions in Eye/Ann Diagrams to get rid of the unphysical (8,8) contributions

THE COMPARISON GIVES US AN INDICATION OF THE SYST. ERROR

Figure

A TESTING GROUND FOR K ⌫ $\pi\ \pi$ CALCULATIONS

J. DONOGHUE AT KAON 2001

study K ⌫ $\pi\ \pi\ l\ \nu_l$ namely

$$\langle\ \pi\pi\ S=0,\ I=0\ \ |\ \bar{u}\ \gamma_\mu\ (1-\gamma_5\ )s\ |K\ \rangle$$

Simple case for Maiani-Testa theorem

Renormalization trivial (no mixing no power div.)

Chiral expansion known at 2 loops

Conclusions and Outlook

MANY PROGRESSES

1) The possibility of computing the physical K ⌫ π π amplitude has been demonstrated by LL (see also LMST);

2) For the first time there is a signal for K ⌫ π π penguin-like contractions of $\mathbf{Q_{1,2,6}}$. More work is needed to reduce the uncertainties (statistical and systematic);

3) The new results with Domain Wall Fermions for K ⌫ π amplitudes are really puzzling;

4) The chiral extrapolation to the physical point (quenched, unquenched, infinite and finite volumes) is <u>critical;</u>

4) The extension of LL/LMST to non-leptonic B-decays (e.g. B ⌫ K π), for which the two light mesons are above the inelastic threshold, remains an open problem worth being investigated.

David and Golia by Caravaggio

… follow Martin Lüscher suggestion, small and smart is often better than big and ….