

MONOPOLE FIELDS
FROM
VORTEX SHEETS
RECONCILING
ABELLIAN AND CENTER
DOMINANCE

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based on:

J. Fröhlich, P.M. Phys Rev D 64 (2001) 014505
hep-th 0011246

Nucl. Phys B511 (1999) 770

Aim of the talk:

propose a new order parameter for
the confinement - deconfinement transition
in $SU(2)$ Yang-Mills theory, based
on monopole field correlations constructed
in terms of center vortex sheets

Origin of confinement in QCD:
condensation of topological defects

center vortices ? monopoles ?

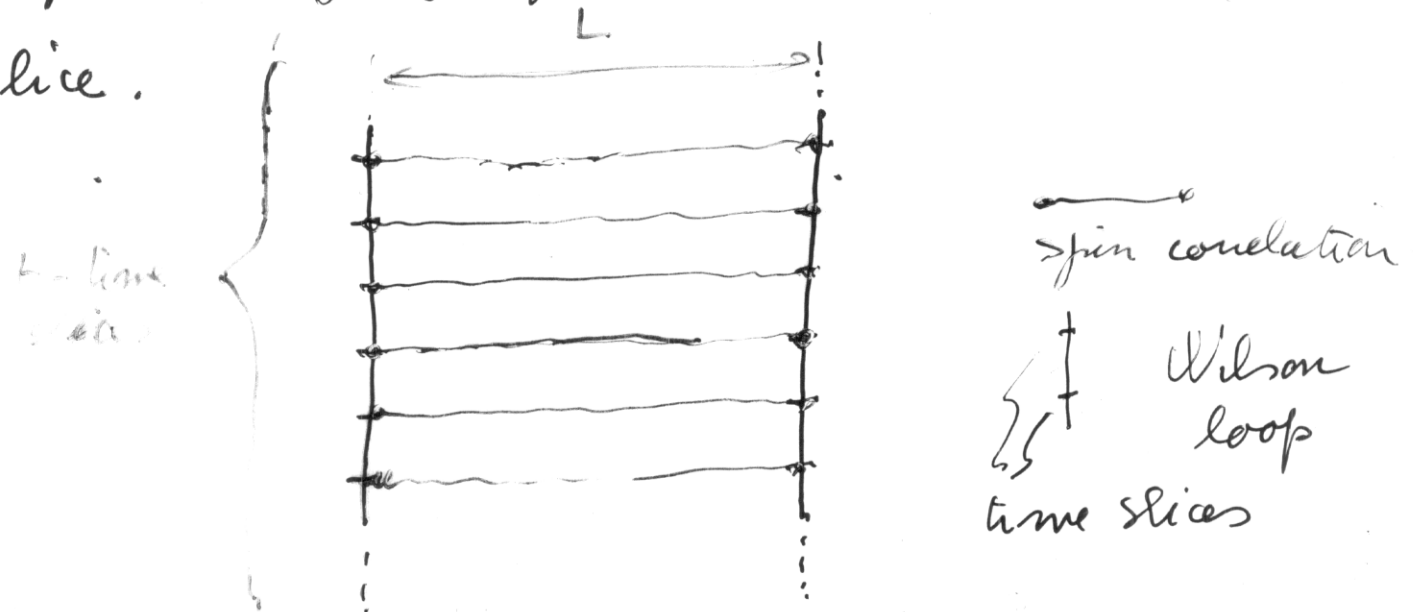
Consider for simplicity the simpler
SU(2) Yang-Mills model

1970
t Hooft
Polyakov
Mack
Nielsen-Olesen
Fradkin

Center vortices

1980
Del Debbio, Faber
Greenberg, Olexnik
Korvas, Tombelli
Engelhardt, Reinhardt...
D'Elia, de Forcellis...

basic idea: - The Wilson loop in time
direction in the temporal gauge can be
written as a product of two-point spin
correlation functions in 3-d NLS models,
coupled to gauge fields, one for each time
slice.



If the NL σ models are in the disordered
phase $\Rightarrow$ exponential decay of spin
correlation $\sim e^{-cL}$ $\Rightarrow$ area decay
for the Wilson loop $W \sim e^{-cL}$

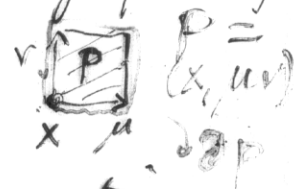
$\Rightarrow$ confinement by Wilson criterion

- Disorder in NL σ models due to
condensation of $\mathbb{Z}_2$ center vortex sheets

- Simplest (thin) vortex : location

plaquettes $p = (x, \mu\nu)$ where (U_μ, U_ν) SU(2) - gauge field)

$\text{sign Tr } U_{\mu\nu}(x) = \text{sign Tr } (U_{\partial p}) = -1$



L Wilson plaquette
 However close to the continuum these vortices

are dilute, hence we need to invoke

- "fat vortices" where $\text{sign Tr } (U_L) = -1$

for a loop L larger than a plaquette

or, presumably equivalent

- "P vortices" defined as follows

fix a projection gauge by maximizing

$$\sum_{x,\mu} \text{Tr} (U_\mu(x))^2$$

and in that gauge the location of a P-vortex is identified by the plaquettes p where

$$\prod_{(x,\mu) \in \partial p} \text{sign Tr}(U_\mu(x)) = -1$$

Numerical simulations show that P vortices condensate also near the continuum limit in the confining phase and (in finite temperature calculations) they are dilute in the deconfined phase.
 $\rightarrow$ center dominance

Di Giacomo, Paffuti, ... Polikarpov, Chernodub,	Monopoles	Di Giacomo, Paffuti, ... Polikarpov, Chernodub,
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Problem: how are defined the monopoles that condense, since there are no natural monopoles in an $SU(2)$ (continuum) theory

't Hooft: - Choose a scalar field $X(U)$
 $SU(2)$ -valued transforming in the adjoint repres.
 of the gauge group $SU(2)$

- gauge fixing : $X(U_m)$ diagonal
 $\rightarrow$ residual $U(1)$ gauge invariance

In this gauge

diag (U_m) = compact "photon field"

off-diag (U_m) = charged fields w.r.t.

the residual gauge group $U(1)$

The points in space-time where two
 eigenvalues of the matrix X coincide are
 the position of the monopole in this gauge.

Monopole condensation $\rightarrow$ dual.

Meissner effect $\rightarrow$ confinement by

squeezing of chromoelectric ($\approx U(1)$ residual)
 flux lines

The above scenario + assumption that off-
 diagonal d.o.f. qualitatively irrelevant =
 abelian dominance

Natural questions:

- Is there a "monopole field operator" $\hat{M}$ which plays the role of order parameter, i.e.

$$\langle \hat{M} \rangle = 0 \quad \text{deconfined phase}$$

$$\langle \hat{M} \rangle \neq 0 \quad \text{confined phase}$$

- does the choice of X influence $\langle \hat{M} \rangle$

Attempt for a positive answer (general idea)

- monopoles are dual to charged fields
- correlation functions for gauge invariant charged field via Dirac ansatz (dressing by Coulomb)
- non vanishing v.e.v. of charged field $\simeq$ Higgs mech.
- monopole correlation by duality and monopole non vanishing v.e.v. $\simeq$ dual Higgs mechanism
- for $SU(2)$ first obtain monopole correlations in abelian projection gauge $\xrightarrow{\text{via } X}$, then covariantize $\rightarrow \hat{M}$
 di Giacomo

Numerical simulations $\rightarrow$ this monopole field

operator is a good order parameter; transition
 and critical exponent
 is independent of the choice of X

However this monopole construction has a problem in the treatment of small scales, arising because dynamical charges and monopoles coexist in the model.

To understand where the problem appears:

Compare non-compact scalar QED (only charges)
with compact scalar QED (charges and monopoles)

non-compact scalar QED

basic fields A_μ real, θ U(1)-scalar
 $\downarrow$ gauge $\downarrow$
 "photon" "charges"

action $S(A_\mu, \theta) = \sum_{\gamma, \mu\nu} \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2(y) +$
 $+ \beta \sum_{\gamma, \mu} (e_\mu \theta - e A_\mu)^2(y)$
 $\hookrightarrow$ electric charge $\rightarrow$ exact. val $\langle \rangle$

dual model ($\beta \rightarrow \infty$): U(1) gauge theory in Villain form.

basic fields: dual U(1)-gauge $\tilde{A}_\mu$, integer valued
 "photon"

2-form $\eta_{\mu\nu}$ describing the "Dirac surfaces" swept
 by the Dirac strings attached to the worldlines of monopoles
 supported on plaquettes

action $\tilde{S}(\tilde{A}_\mu, u_{\mu\nu}) = \sum_{\gamma, \mu\nu} (\partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu + g u_{\mu\nu})^2(y)$

magnetic charge = $\frac{2\pi e}{g}$

$\hookrightarrow$ expect. $\langle \quad \rangle_{val}$

compact scalar QED

basic fields: A_μ $U(1)$ -gauge field, $u_{\mu\nu}$, θ

$\downarrow$ $\downarrow$ $\downarrow$
photons monopoles charges

action $S(A_\mu, u_{\mu\nu}, \theta) = \sum_{\gamma, \mu\nu} (\partial_\mu A_\nu - \partial_\nu A_\mu + g u_{\mu\nu})^2(y)$

$+ \beta \sum_{\gamma, \mu} \cos(\partial_\mu \theta + e A_\mu)(y)$

Dirac ansatz for a gauge-invariant charged field: Consider the

Coulomb field generated by a unit charge at a site $\vec{x}$ in the 3-d lattice at constant x^0

$$E_\mu^x = \begin{cases} \partial_i \Delta_3^{-1} \delta_x & \mu=i=1,2,3 \\ 0 & \mu=0 \end{cases}$$

$x = (x^0, \vec{x})$, $\Delta_3 = 3\text{-d. Laplacian}$, $\delta_x(y) = \delta_{xy}$

E_μ^x satisfies $\partial_\mu E_\mu^x = \delta_x$

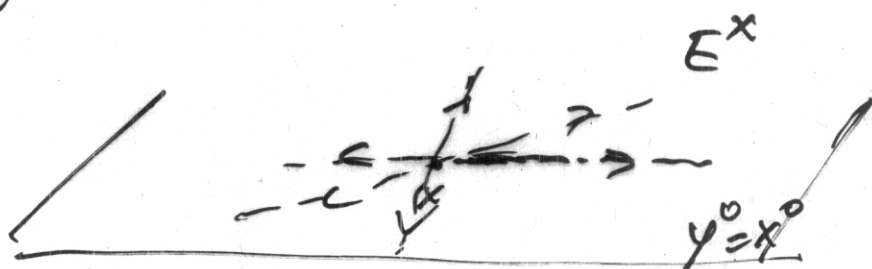
$\Rightarrow$ charged field defined by

$$e^{i\theta(x)} e^{ie \sum_{\gamma, \mu} E_{\mu}^x(\gamma) A_{\mu}(\gamma)} \equiv \phi(\bar{E}^x)$$

is gauge invariant

Picture

(1 space dimension omitted)



E_{μ} dresses the charged field θ by a cloud of soft "photons", in this way taking care of the Coulomb interaction tied to the charged particle even asymptotically.

Duality: The 2-point monopole correlation $G(x, x')$ function in the dual model is the dual of the

2-point correlation function for the field $\phi(\bar{E}^x)$

To construct it we need to choose a path

$\int^{xx'}$ joining x to x' (1-form) and defining

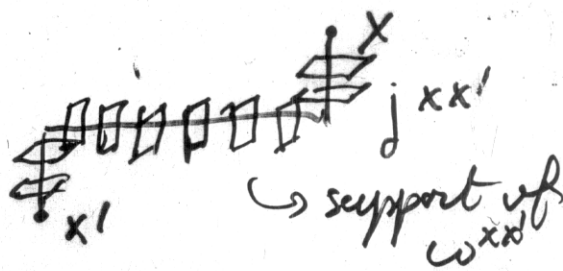
the 3-forms

supported on cubes

$$\omega^{xx'} = \star \int^{xx'} \quad B^x = \star E^x$$

Picture

(1 space dim suppressed
cubes $\rightarrow$ plaquettes)

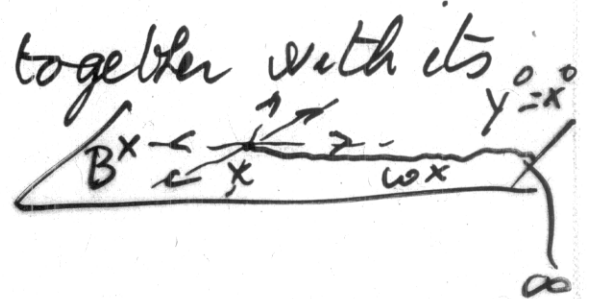


$$G(x, x') = \langle e^{-[\tilde{S}(\tilde{A}_\mu, n_{\mu\nu} + \partial_\rho \tilde{\Delta}'(B^x - B^{x'} + \omega^{xx'})) - \tilde{S}(\tilde{A}_\mu, n_{\mu\nu})]} \rangle$$

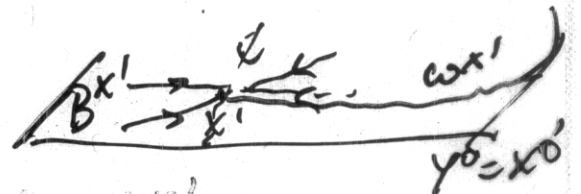
$$\equiv \langle D(g(B^x - B^{x'} + \omega^{xx'})) \rangle^{\sim} = \langle \phi(E^x) \bar{\phi}(E^{x'}) \rangle$$

($\Delta = 4$ -d laplacian)

Choosing $\omega^{xx'}$ as a sum $\omega^x - \omega^{x'}$ of the dual of two paths each at constant time, from x to ∞ and from ∞ to x' (+ monopole b.c.) we can interpret $B^x + \omega^x$ as the magnetic field of a monopole of charge 1 at x together with its Dirac string ω^x and



describing a cloud of soft "photons" dressing the monopole.



In the $U(1)$ gauge theory $G(x, x')$ is independent of the choice of $\omega^{xx'}$ (Dirac string independence)

i) duality exchanges Wilson and 't Hooft loops:

L loop, Σ surface $\partial \Sigma = L$

$\downarrow L_\mu$ current $\downarrow \Sigma_{\mu\nu}$ current Wilson loop

$$\langle e^{ie \sum_{\gamma, \mu} A_\mu(\gamma) L_\mu(\gamma)} \rangle = \langle e^{-[\tilde{S}(\tilde{A}_\mu, n_{\mu\nu} + * \Sigma_{\mu\nu}) - \tilde{S}(\tilde{A}_\mu, n_{\mu\nu})]} \rangle^{\sim} \equiv \langle D(\Sigma) \rangle^{\sim}$$

ii) From (obvious generalizations of) the above charged and monopole field correlation reconstruction theorem $\rightarrow$

$\phi(x)$ charged field operator
 $\hat{M}(x)$ monopole field operator

non local

vacuum $|\Omega\rangle$ and its dual $|\tilde{\Omega}\rangle$ such that

$$G(x, x') = \langle \tilde{\Omega} | \hat{M}(x) \hat{M}(x')^\dagger | \tilde{\Omega} \rangle = \langle \Omega | \hat{\phi}(x) \hat{\phi}(x')^\dagger | \Omega \rangle$$

and $\hat{M}$ is a good order parameter for the confinement - deconfinement transition in the $U(1)$ gauge theory, $\hat{\phi}$ a good order parameter for the Higgs - Coulomb transition in $\downarrow$ non-compact scalar QED (Theorem Fröhlich-M.)

ii) Proposed adaptation of $U(1)$ monopole construction to $SU(2)$

U_μ - $SU(2)$ gauge field

action $S_{YM}(U_\mu) = \beta \sum_{y, \mu\nu} (1 - \text{Tr } U_{\mu\nu}(y))$

Wilson plaquette $U_{\mu\nu}(y) = U_\mu(y) U_\nu(y+\hat{\mu}) U_\mu^\dagger(y+\hat{\mu}+\hat{\nu}) U_\nu^\dagger(y)$

expectation value $\langle \dots \rangle_{YM}$

Fix an abelian projection gauge choosing

an $su(2)$ -valued scalar field X^a , $a=1,2,3$

and denote U_μ in this gauge by $\bar{U}_\mu$

expect. value $\langle \rangle_X$

Decomposition
$$\bar{U}_\mu = \begin{pmatrix} (1-|c_\mu|^2)^{\frac{1}{2}} & -\bar{c}_\mu \\ c_\mu & (1-|c_\mu|^2)^{\frac{1}{2}} \end{pmatrix} e^{i \frac{\tilde{A}_\mu}{2} \sigma_3}$$

$c_\mu(x) \equiv c_\mu^1(x) + i c_\mu^2(x) \in \mathbb{C}$, $\tilde{A}_\mu$ $U(1)$ -gauge field with range $(-2\pi, 2\pi)$

Integrating out $c_\mu \sim$ compact QED in $\tilde{A}_\mu$

Analogy with compact QED $\rightarrow$ monopole 2pt funt. in this abelian projection gauge:

$$\begin{aligned} & \langle D(g(B^x - B^{x'} + \omega^{xx'})) \rangle_X = \\ & = \langle e^{-S_{YM}(\bar{U}_{\mu\nu})} e^{i \frac{\sigma_3}{2} \int d^3x \, 2\pi g \, \partial^\mu \Delta^{-1}(B^x - B^{x'} + \omega^{xx'})_{\mu\nu}} \rangle_X \\ & e^{S_{YM}(\bar{U}_{\mu\nu})} \end{aligned}$$

where

$$\bar{U}_{\mu\nu} = \left(e^{i \frac{\sigma_3}{2} \cdot \vec{c}} \right)_{\mu\nu} e^{i \frac{\sigma_3}{2} (\partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu)}$$

Wilson plaquette in the X -projection gauge

Covariantization: replace σ_3 by $X \xleftarrow{\text{transform indices.}}$

gauge invariant correlation, replace $\langle \rangle_X$ by $\langle \rangle_{YM}$
 $\Rightarrow$ M. Di Giacomo et al

Naively one would think that the same construction of charged and dual monopole correlations works also for compact scalar QED (and hence the SVCE case would be O.K. too), but it does not: If dynamical monopoles and charges coexist, the above construction starts to depend unspllycally on Dirac strings!

Why?

Consider compact scalar QED; the matter partition function Z_m can be expanded in terms of integer currents j_μ :

$$Z_m \equiv \int \prod_x d\theta_x e^{\beta \sum_{\gamma, \mu} \cos(Q_\mu \theta + e A_\mu)(\gamma)} =$$

$$= \int \mathcal{D}V(j_\mu) e^{ie \sum_{\gamma, \mu} j_\mu(\gamma) A_\mu(\gamma)} = \int \mathcal{D}V(j_\mu)$$

for a suitable measure $\mathcal{D}V(j^\mu)$ on currents $j^\mu: \partial^\mu j_\mu = 0$ representing virtual charged particle-antiparticle worldlines. The coupling of Dirac surfaces $u_{\mu\nu}$ to j_μ

is given by i.e.g. $\sum_{\gamma, \gamma', \mu, \nu} j_\mu(\gamma) \Delta^{-1}(\gamma - \gamma') \partial_\nu u_{\mu\nu}(\gamma')$

¹³ Change of Dirac surface



for fixed
monopole
weldlines

$$S \rightarrow S' = S + \partial V$$

$\downarrow$ n_{pm} $\downarrow$ $n'_{pm} = n_{pm} + \partial_p V_m$

$\downarrow$ $\partial_\mu V_p$ volume element

$\Rightarrow$ change in the interaction $j_\mu - n_{\mu\nu}$:

i.e.g. $\sum_{y,\mu} j_\mu(y) V_\mu(y)$

exponentiated is irrelevant if

e.g. $= 2\pi l$

$l \in \mathbb{Z}$

Dirac
quantisation
condition
for charges

since j_μ, V_μ are integer valued

Consistency: Dirac quantisation condition for fluxes

e.g. $\sum_{y,\mu} j_\mu(y) V_\mu(y) \in 2\pi \mathbb{Z}$

Consistency condition satisfied with Wilson

loop insertion: $j_\mu \rightarrow j_\mu + L_\mu$ still integer

but it does not hold in the correlation
function of charged field because this involves

the non-integer current E_μ^x , so that a change
of Dirac surface $\Rightarrow$ i.e.g. $\sum_{y,\mu} E_\mu^x(y) V_\mu(y) \notin i2\pi \mathbb{Z}$

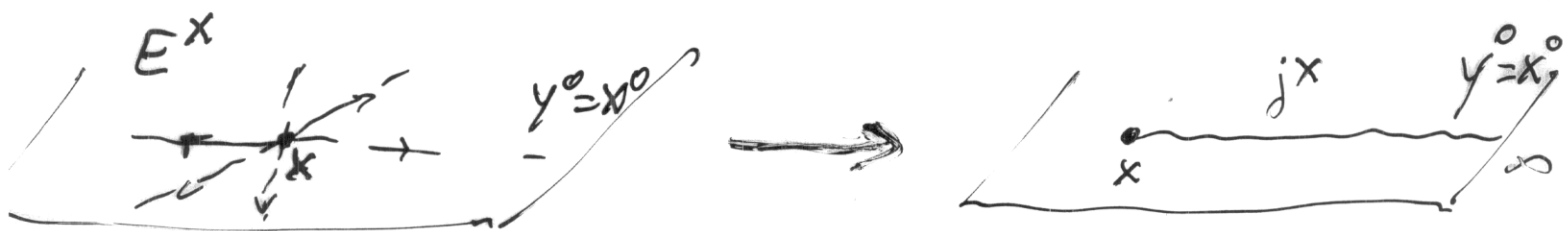
even if e.g. $\in 2\pi \mathbb{Z} \Rightarrow$ inconsistency

The same inconsistency via duality propagates to the monopole correlation function in the dual model (a $\mathbb{Z}$ -scalar QED with $\tilde{e} = g$, $\tilde{g} = e$) and it is responsible of the dependence on the Dirac string $\omega^{xx'}$ in $SU(2)$ -Yang Mills theory where both 't Hooft magnetic monopoles and electrically charged (off-diag) degrees of freedom are present $\hookrightarrow$ at order ε^2

How to avoid the inconsistency ($eg = 2\pi$)

Naive recipe: replace E_m^x (real-valued) by an integer current j_m^x , a "Mandelstam string" supported on a path at constant time from x to ∞ .

Picture (1 space dim. suppressed)



However such choice produces infrared divergences due to the infinite self-energy and interaction energy associated to j_μ^x

(whereas E_μ^x is infrared safe) because:

$$\sum_{y, y', \mu} (j_\mu^x - j_\mu^{x'})(y) \Delta^-(y - y') (j_\mu^x - j_\mu^{x'})(y') = \infty$$

but

$$\sum_{y, y', \mu} (E_\mu^x - E_\mu^{x'})(y) \Delta^-(y - y') (E_\mu^x - E_\mu^{x'})(y') < \infty$$

Proper recipe: integrate over a family of integer "Mandelstam strings" j_μ^x weighted by a (complex) measure $\mathcal{D}\mu(j^x)$ such that at large scales

$$\int \mathcal{D}\mu(j^x) e^{ie \sum_{y, \mu} A_\mu(y) j_\mu^x(y)} \sim e^{ie \sum_{y, \mu} A_\mu(y) E_\mu^x}$$

↑
"rough" support on average

Basic idea of the construction of $\mathcal{D}\mu(j^x)$

Let ξ be a real field in the 3-space at fixed time x^0 , minimally coupled to a real gauge field A_μ with action

$$S(\xi | A_\mu) = \frac{\beta}{2} \sum_{y \in \mathbb{Z}^3, y^0 = x^0, \mu=1,2,3} (\partial_\mu \xi - e A_\mu)^2(y)$$

Corresponding expectation value $\langle \dots \rangle^{x^0}(A)$

$$\langle e^{i\xi(x)} \rangle^{x^0}(A) \sim \text{const } e^{ie \sum_{\gamma, \mu} A_\mu(y) E_\mu^\gamma(y)}$$

$$E_\mu^x = \partial_i \Delta_{\mu 3}^{-1} \delta_x$$

Fourier transforming in A

$$\langle e^{i\xi(x)} \rangle^{x^0}(A) = \int \mathcal{D}\mu_0(j^x) e^{ie \sum_{\gamma, \mu} A_\mu(y) j_\mu^x(y)} \quad (*)$$

for a suitable measure $\mathcal{D}\mu_0(j^x)$ on real-valued currents j_μ^x (from x to ∞)

$\Rightarrow$ Dirac's dressing transformation for non-compact QED can be realized as integral over real Mandelstam strings

For compact QED with monopoles, A_μ should be taken $U(1)$ -valued and ξ replaced by a $U(1)$ scalar χ with action

$$S(\chi | A_\mu) = \beta_\chi \sum_{\substack{\gamma, \mu = 1, 2, 3 \\ \gamma^0 = x^0}} \cos(\partial_\mu \chi - e A_\mu)$$

$\Rightarrow$ representation analogous to $(*)$ but with j^x integer: $\langle e^{i\chi(x)} \rangle^{x^0}(A_\mu) \sim \int \mathcal{D}\mu(j^x) e^{ie \sum_{\gamma, \mu} A_\mu(y) j_\mu^x(y)}$
 where $\mathcal{D}\mu$ is a measure on integer currents from x to ∞

$S(X|A_\mu)$ describes a spin field X in the presence of an external gauge field A_μ in 3-d. For large β_X and small in average gauge fluctuations, the spin system is in a phase where the symmetry

$$\underline{X \rightarrow X + \text{const}}$$

is spontaneously broken. The physics at large scales is dominated by the Goldstone boson, a real field ξ ^{X spin waves}, which is the decompactification of X . Corrections are due to vortices, "irrelevant" in the scaling limit; the compactness of A_μ also becomes irrelevant in the scaling limit

$$\Rightarrow \langle e^{iX(x)} \rangle^{x^0}(A_\mu) \underset{\text{large scales}}{\sim} \langle e^{i\xi(x)} \rangle^{x^0}(A_\mu) \\ \simeq e^{ie \sum_{\nu,\mu} A_\mu(y) E_\mu^x(y)}$$

hence we recover the same infrared behaviour of Dirac's ansatz

$$\langle j_\mu^x \rangle^{x^0}(A_\mu) \sim E_\mu^x \Rightarrow \mathcal{P}_\mu(j^x) \text{ peaked at large scales around its mean } E_\mu^x$$

non compact scalar QED $\rightarrow$ compact scalar QED

dressing
for
charged
field

$$e^{ie \sum_{\gamma \in \mu} E_{\mu}^x(\gamma) A_{\mu}(\gamma)} \rightarrow \int \mathcal{D}\mu(j^x) e^{ie \sum_{\gamma \in \mu} j_{\mu}^x(\gamma) A_{\mu}(\gamma)}$$

Picture

$$\text{Picture} \quad \begin{array}{c} E^x \\ \nearrow \\ x \\ \searrow \end{array} \xrightarrow{\gamma^0 = x^0} \int \mathcal{D}\mu(j^x) \quad \begin{array}{c} \gamma^0 = x^0 \\ \nearrow \\ x \\ \searrow \end{array}$$

Dual monopole correlation function $G(x, x')$

Let $\Sigma(j^x - j^{x'} + j^{xx'})$ be a surface bounded
by the path (closed at ∞) $j^x - j^{x'} + j^{xx'}$, $j^{xx'} = \partial \omega^{xx'}$

Picture

$$\text{Picture} \quad \begin{array}{c} x' \\ \nearrow \\ j^{xx'} \\ \searrow \\ x \end{array} \xrightarrow{\gamma^0 = x^0} \infty \quad \Sigma(j^x - j^{x'} + j^{xx'})$$

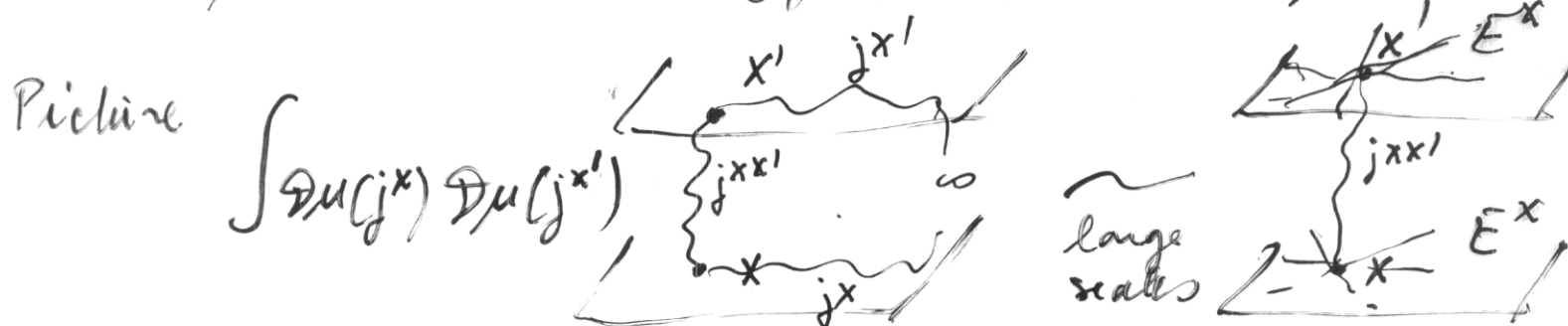
't Hooft loop

$$\begin{aligned} G(x, x') &= \int \mathcal{D}\mu(j^x) \mathcal{D}\mu(j^{x'}) \langle \mathcal{D}(\Sigma(j^x - j^{x'} + j^{xx'})) \rangle \\ &= \int \mathcal{D}\mu(j^x) \mathcal{D}\mu(j^{x'}) \langle e^{i\theta(x)} e^{ie \sum_{\gamma \in \mu} (j_{\mu}^x - j_{\mu}^{x'}) A_{\mu}(\gamma)} e^{i\theta(x')} \rangle \end{aligned}$$

If $e g \in 2\pi \mathbb{Z}$ Dirac quantisation condition for fluxes
is satisfied and the correlation function is independent
of the choice of Dirac strings,

but at large scales, since $\mathcal{D}\mu(j^x)$ is peaked around E^x , 19

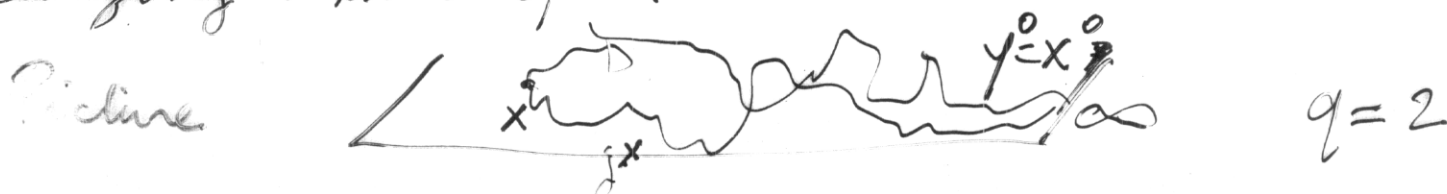
$$G(x, x') \sim \langle D(g(B^x - B^{x'} + \omega^{xx'})) \rangle$$



i.e. it behaves like the dual Dirac ansatz, with $\tilde{g} = e$ $\tilde{e} = g$ $B^x = \star E^x$, hence it has no infrared divergences. (mathematically verified in gaussian approx.)

Remark If the charged field has charge $e = q$, $q \in \mathbb{Z}$, the monopole charge $g = 2\pi$, $eq = 2\pi q$

the measure in the modified ansatz, $\mathcal{D}\mu_q(j^x)$, is constructed in terms of a 3-d U(1) scalar field χ of charge q and it involves currents j^x generically supported on q paths each one carrying a unit flux



The corresponding monopole 2-point function $G(x, x')$ is constructed by means of q -sheet surfaces $\Sigma(j^x * j^{x'} + j^{xx'})$, the sheets being joined at the support of $j^{xx'}$, current of charge q
 $\Rightarrow \hat{M}$ order parameter

(Preliminary numerical results positive: Polakow et al.)

Monopole correlation functions for $SU(2)$ YM

Preliminaries: charges of fields

- 't Hooft regular monopoles in an abelian projection gauge have magnetic charge $q = 4\pi$
 - singular gauge-invariant π_2 -monopoles have magnetic charge $2\pi \cong \pi_2$ charge - 1 but near the continuum are dilute (Mack, Petkova)
 - off diagonal c_μ have electric charge $e = 1$
- $\Rightarrow$ For regular monopoles Dirac quantization condition for charges is satisfied with

$$eq = 2\pi \cdot (2) \Rightarrow q = 2$$

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Our proposal for monopole 2-pt, in $SU(2)$ YM

$$G(x, x') = \int \mathcal{D}\mu_2(j^x) \mathcal{D}\mu_2(j^{x'}) \langle D(\Sigma(j^x - j^{x'} + j^{xx'})) \rangle_{YM}$$

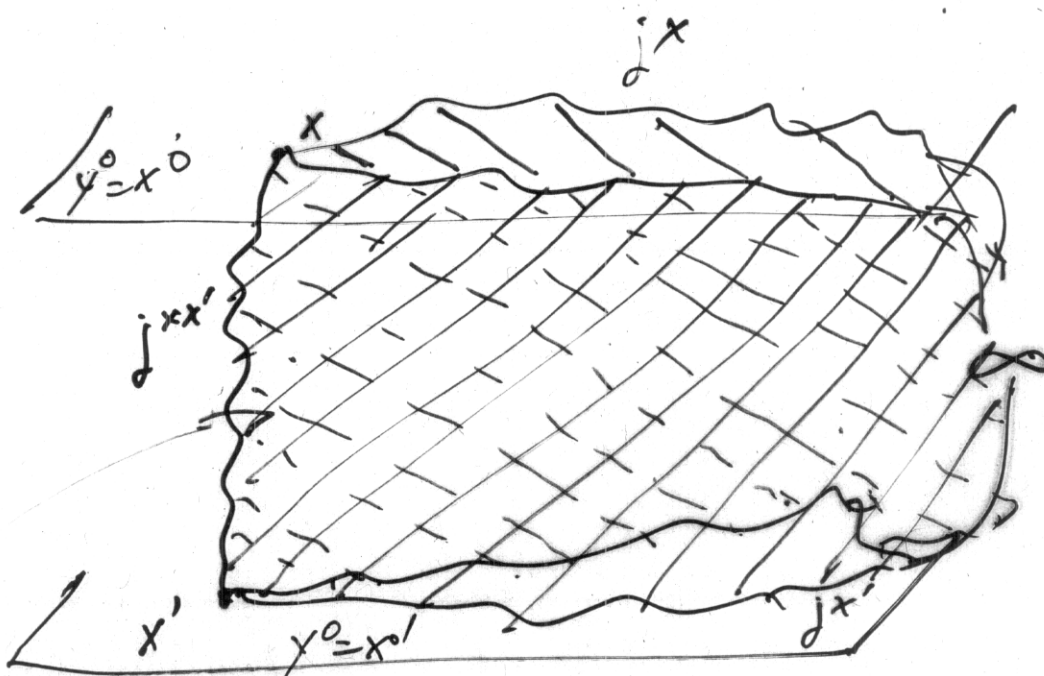
t Hooft disorder loop

$\Sigma(j^x - j^{x'} + j^{xx'})$ ^{2-sheet} surface with boundary given by the support of $j^x, j^{x'}$ (currents generically supported on 2 paths each of flux 1) and $j^{xx'}$ of flux 2 joining x to x' .

D is the corresponding 't Hooft disorder loop constructed modifying the plaquette term in the Yang-Mills action as

$$Tr U_{\mu\nu}(x) e^{i \frac{1}{3} \pi * \Sigma(j^x - j^{x'} + j^{xx'})(x)_{\mu\nu}}$$

Picture
(1 space dimension suppressed)



$\mathbb{Z}_2$ -vortex sheets $\Sigma(j^x - j^{x'} + j^{xx'})$

This definition

- does not involve the choice of an abelian projection
- it is gauge invariant
- it is consistent at small scales since it respects Dirac quantisation condition for fluxes (in particular it is independent of the choice of the Dirac string $\omega^{xx'} = +j^{xx'}$)

Σ takes integer values, hence $e^{i\pi \tilde{G}_3^* \Sigma} \in \mathbb{Z}_2 \Rightarrow$

one can substitute $\tilde{G}_3$ with a Lie-algebra valued

field X defining an abelian projection

$$(e^{i\pi X \Sigma} \in \mathbb{Z}_2)$$

In this projection gauge, $SU(2)$ YM is a $U(1)$ theory and since $\mathcal{D}_{\mu_2}(j^x)$ is peaked at large scales around $2B^x$

$$\int \mathcal{D}_{\mu_2}(j^x) \mathcal{D}_{\mu_2}(j^{x'}) \langle D(\Sigma(j^x - j^{x'} + j^{xx'})) \rangle_{YM} \sim \text{large scale}$$
$$\sim \langle D(2(B^x - B^{x'} + \omega^{xx'})) \rangle_{YM}^x$$

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i.e. we recover the $U(1)$ monopole correlation proposed by Di Giacomo et al with charge $q=2$.

- From our monopole correlation functions constructed in terms of double-sheet surfaces of center vortices one can reconstruct a monopole field operator $\hat{M}$. Since at large scales we recover Di Giacomo et al order parameter, sensitive to large scale properties, we expect that $\hat{M}$ is a well defined order parameter for the confinement-deconfinement transition, with

$$\langle R | \hat{M} | R \rangle \begin{cases} = 0 & \text{deconfinement} \\ \neq 0 & \text{confinement} \end{cases}$$

Our construction, independent of projections, might explain why the choice of the abelian projection is numerically irrelevant in the determination of the critical exponents of the transition.

(Di Giacomo, Passutti)

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- In an abelian projection gauge $j^{xx'}$ could be interpreted as the worldline of a regular monopole, whereas the average effect of j^x and $j^{x'}$ describes the charge-2 Coulomb field attached to it in a (partially explicit) dual Higgs model.

But j^x and $j^{x'}$ carry a magnetic charge $2\pi \in \mathbb{Z}_2$, hence gauge independent. Therefore the location of creation and annihilation of regular monopoles are also gauge independent, hence intrinsic to $SU(2)$

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Relation with center dominance

(due to the construction in terms of $\mathbb{Z}_2$ -vortex sheets,

- Replace U_μ $SU(2)$ -valued with $\tilde{U}_\mu$, $SO(3) \simeq SU(2)/\mathbb{Z}_2$ -valued and a 2-form $P_{\mu\nu}$, $\mathbb{Z}_2 \in \{0,1\}$ valued.

Mack-Peterson constraint: $\forall$ cube c

$$e^{i\pi \sum_{(y,\mu\nu) \in \partial c} P_{\mu\nu}(y)} = e^{i \sum_{(y,\mu\nu) \in \partial c} \arg \text{Tr}(U_{\mu\nu}(y))} = e^{i \sum_{(y,\mu\nu) \in \partial c} \arg \text{Tr}(\tilde{U}_{\mu\nu}(y))}$$

$D(\Sigma)$ rewritten in terms of $\tilde{U}_\mu, g_{\mu\nu}$:

$$\begin{aligned} \text{Tr } U_{\mu\nu}(y) e^{i\pi \sigma_3 \otimes \Sigma_{\mu\nu}(y)} &= \\ &= \text{Tr } \tilde{U}_{\mu\nu}(y) e^{i\pi [g_{\mu\nu}(y) + \otimes \Sigma_{\mu\nu}(y)]} \end{aligned}$$

$\Rightarrow$ the introduction of the 't Hooft loop is obtained by shifting $g_{\mu\nu}$ by $\otimes \Sigma_{\mu\nu}$

The constraint between $\tilde{U}_\mu$ and $g_{\mu\nu}$ can be solved by:

$$\begin{aligned} e^{i\pi g_{\mu\nu}(y)} &= \text{sign Tr } U_{\mu\nu}(y) \left(\prod_{y', \mu' \in \partial(y, \mu\nu)} \text{sign Tr } U_{\mu'\nu'}(y') \right) \\ &= \text{sign Tr } \tilde{U}_{\mu\nu}(y) \prod_{y', \mu' \in \partial(y, \mu\nu)} \text{sign Tr } \tilde{U}_{\mu'\nu'}(y') \end{aligned}$$

used in this vertex used in P-vertex

gauge-dependent but abelian projection independent

In center projection gauge $p = (y, \mu\nu)$ is the location of P-vortex if

$$\prod_{(y', \mu') \in \partial(y, \mu\nu)} \text{sign Tr } U_{\mu'\nu'}(y') = -1$$

Numerics $\Rightarrow$ P-vortex sheets percolate in the confinement phase and are dilute in the deconfinement phase.

Close to the continuum the plaquettes where $\text{sign Tr } U_{\mu\nu}(y) = -1$ (gauge-invariant) are dilute. $\Rightarrow$

To discuss the transition we can replace P-vortex sheets with sheets where $g_{\mu\nu} \neq 0$.

$\Rightarrow$ One can argue that if such sheets percolate the introduction even of an infinite sheet like $\sum (j^x - j^{x'} + j^{xx'})$ is a small perturbation and it does not destroy clustering in (x, x') , whereas if such sheets are dilute the introduction of $\sum (j^x - j^{x'} + j^{xx'})$ does lead to clustering $\Rightarrow$

One can interpret a non-vanishing expectation value of the monopole operator $\hat{M}$ in the center gauge as due to a ^{center dominant} condensation of vortex sheets!

- The two-sheet structure of monopole correlation appears as the counterpart of the monopole loop-2-sheet closed vortex surfaces found in the center projection (Del Debbio et al '97, Engelhardt et al, de Forcrand et al 2000)

SUMMARIZING

- We propose a new order parameter for the confinement-deconfinement transition in $SU(2)$ YM, defined as vacuum expectation value of a monopole field operator $\hat{m}$ whose correlation functions _{↳ abelian dominance} are constructed in terms of center vortex sheets
- Such correlation functions are intrinsic to $SU(2)$, but they can be identified with Green functions of regular monopoles in any abelian projection
- $\langle \hat{m} \rangle \neq 0$ can also be interpreted as due to condensation of center vortices _{↳ center dominance}
- Our construction avoids some small-scale inconsistencies of previous monopole approaches by rejecting Dirac quantization for fluxes