Light hadron spectrum and quark masses

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Introduction

study of light hadron spectrum:

- light hadron masses $\Rightarrow$ test of QCD
- determination of fundamental parameters of QCD; α_s, m_q

at lattice 2000:

in quenched QCD : matured

- ◦ low-lying hadron and quark masses are determined precisely ... besides quenching effects

in $N_f = 2$ QCD : in progress

- ◦ systematic studies of sea quark effects

in $N_f = 3$ QCD : exploratory

- ◦ developing algorithm
- ◦ few studies

contributions in this year

- B.Orth, “Full QCD beyond the rho decay threshold”
- C.Allton, “Scaling for Dynamical Fermion effects in UKQCD simulation”
- C.Gebert, “QCD spectroscopy’ with light quarks”
- L,Levkova, “Anisotoropic Lattice and Dynamical Fermions”
- D.Nelson, “Does $m_q = 0$?”
- S.Hashimoto, “Exploration of sea quark effects in two-flavor QCD with the $O(a)$ improved Wilson quark action”
- H.Neff, “Early plateau formulation for the η mass”
- K.Schilling, “A new approach to η' on the lattice”
- N.Ishizuka, “$I = 2$ Pion Scattering Length and Phase Shift with Wilson Fermions
- T.Yamazaki, “Maximum entropy analysis of hadron spectral functions and excited states in quenched lattice QCD”
- J.Hein, “Mass renormalization for improved staggerd quarks”
- J.Rolf, “Computation of the charm quark’s mass in quenched QCD”
- O.Philipenson, “Non-perturbative parton masses”
- F.Lee, “N^* Masses from an Anisotropic Lattice QCD Actions”
- H.B.Thacker, “Quenched Chiral Loop Effects in the Scalar Propagator”
- C.Rebbi, “First results from quenched QCD simulations with overlap fermions”
- Y.Aoki, “Hadron spectrum for quenched domain-wall fermions with DBW2 gauge action”
- C.McNeile, “Hadron spectroscopy of twisted mass lattice QCD at $\beta = 6.0$”
- A.Patel, “Transverse lattice QCD at strong coupling and large N”
- N.Ghaleh, “Transverse Momentum Distribution and Gluon Emission in Electron-Positron Annihilations at 60 GeV.”
- J.McIntosh, “Independent plaquette state study of compact $U(1)$ in 2+1 D utilizing an analytic Lanczos procedure”
- R.Frezzotti, “Quenched twisted mass QCD at small quark masses and in large volume”
- H.Moutrarde, “Preliminary calculation of $alpha_s$ from Green functions with dynamical quarks”
- N.Eicker, ”Light Quark Masses with $N_f = 2$ Wilson fermion: an Update from SESAM/TχL”
- A.Nakamura, “Study of sigma meson on the lattice”
- T.Draper, “Quenched Chiral Behavior of Hadron Masses with Overlap Fermions”
- D.Hepburn, “Light hadron spectrum using $O(a)$ improved action with $N_f = 2$”
- A.Hart, “Topology and the scalar glueball with $N_f = 2$ improved Wilson fermions”
- K.Jansen, “Scalar Condensate and light-quark masses from overlap fermions”
- S-J.Dong “Pion decay constant, Z_A, and quark masses from overlap fermions”

- S.Collins, A nonperturbative determination of c_A and the scaling of f_π and renormalised quark mass"
- S.Hawswirth, "First results from a parametrized fixed point action"
- U.Wolff, "First results on the running coupling in QCD with two massless flavors"
- H.Matsufuru, "Anisotropic $O(a)$ improved Wilson quark action and hadron spectroscopy"
- T.Umeda, "$O(a)$ improved Wilson quark action on anisotropic lattice"

- results with KS action $\Rightarrow$ review by Toussaint
- flavor singlet $\Rightarrow$ review by Edwards
- Ginsparg-Wilson fermion $\Rightarrow$ review by Kikukawa

  $\Rightarrow$ review by Hernandez

- my talk : studies with Wilson-type action

- outline

1. light hadron spectrum
   - in quenched QCD
   - in $N_f = 2$ full QCD

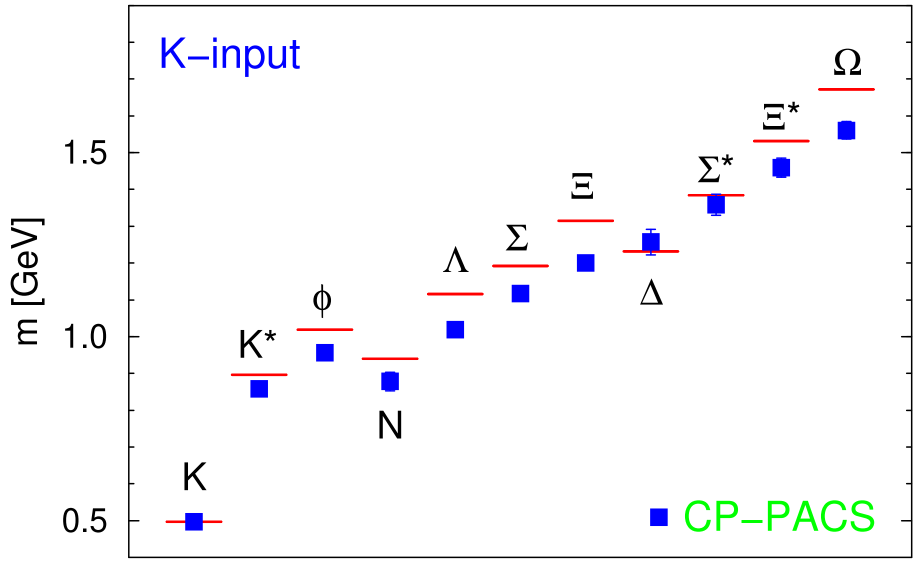
     search for sea quark effects
   - in $N_f = 3$ full QCD
2. test/application to chiral perturbation theory
3. determination of fundamental parameters of QCD
   - strong coupling
   - quark masses
4. summary

1. light hadron spectrum

1.1 quenched QCD

— before lattice2000 —

ground states : **already** studied well!!



decay constants, quark masses: non-perturbatively determined
(ALPHA, APE, UKQCD, QCDSF...)

– this year –

1. testing ground for new formulations

- anisotropic
- Ginsparg-Wilson fermions (domain-wall, overlap)
- twisted mass QCD

for improvement

- non-perturbative improvement

2. study of heavier/excited states

- negative parity baryon

1.1.1 tests of improved formulations

- calculation of meson spectrum

on anisotropic lattice

Matsufuru, *et al.*

plaq + clover (tadpole improved $c_{\rm SW}$),

$a_s^{-1} =$ 1.1-2.0 GeV, $La \sim$ 2 fm, 200-100conf, $\xi = 4$

domain-wall fermion

RBC(Y.Aoki):

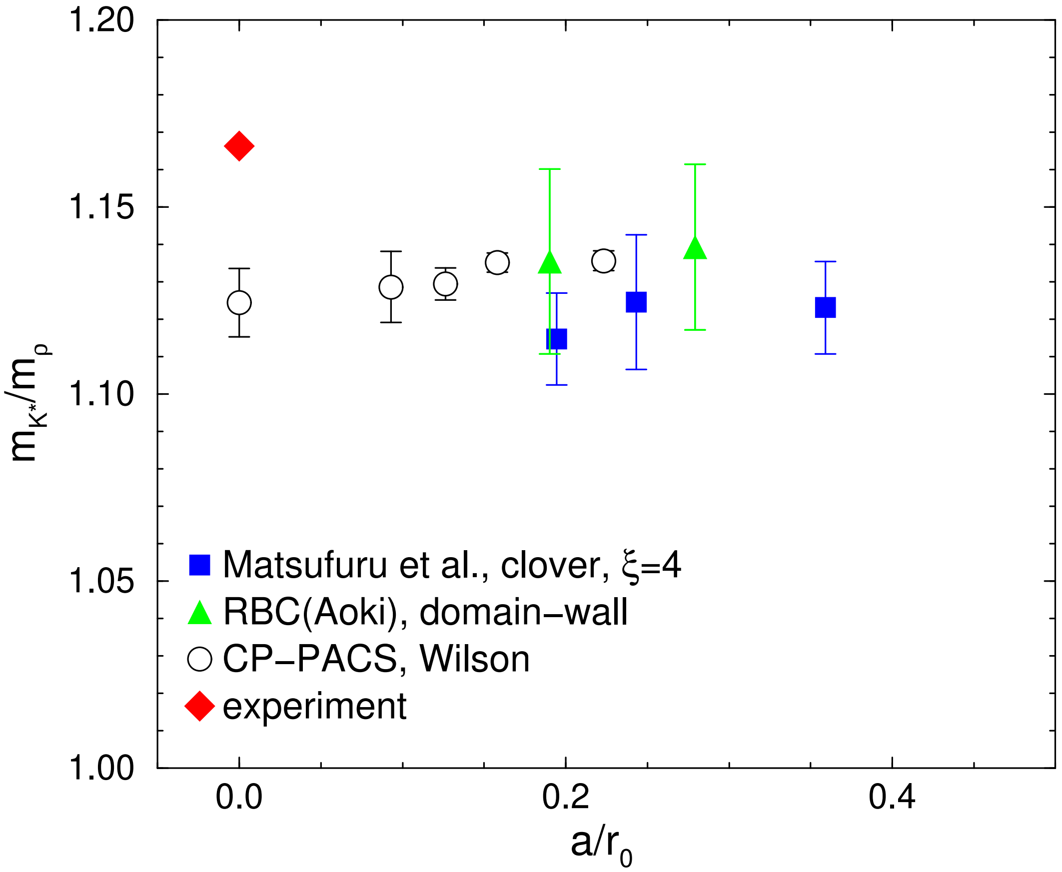
DBW2 gauge action(QCDTARO)

$(a^{-1}, L) \sim$ (1.3 GeV, 2.4 fm), (2.0 GeV, 1.6 fm),

$\sim$ 100 conf, $N_5 = 16$, $M_5 = 1.8$

$\Rightarrow$ consistency with previous calculations using Wilson/KS.

m_{K^*}/m_ρ



- consistent with previous calculation with 2-3% accuracy
- confirm small $m_{K^*} - m_K$ in quenched QCD

overlap fermions

- light quark masses with non-perturbative renomalization

  Giusti, *et al.*:

  $a^{-1} \sim 2$ GeV, $\quad L \sim 1.4$ fm, $\quad$ 54 conf

  Dong, *et al.*:

  $a^{-1} \sim 1.2$ GeV, $\quad L \sim 3.2$ fm, $\quad$ 63 conf

  $\Rightarrow$ (see below)

twisted mass lattice QCD

- suspress exceptional conf at small quark masses

$$D_{\mathrm{tmQCD}} = D_{\mathrm{Wilson}} + i\mu_q\gamma_5\tau_3$$

$\Rightarrow$ lower bound for eigenvalues of D_{tmQCD}

McNeile, *et al.*:

$a \sim$ 0.09 fm, $L =$1.4 fm

Della Morte, *et al.*:

$a \sim$ 0.07,0.09 fm, $L/a =$ 16, 24, $T/L =$ 2,3,
$m_{\mathrm{PS}}/m_{\mathrm{V}} \geq 0.47$

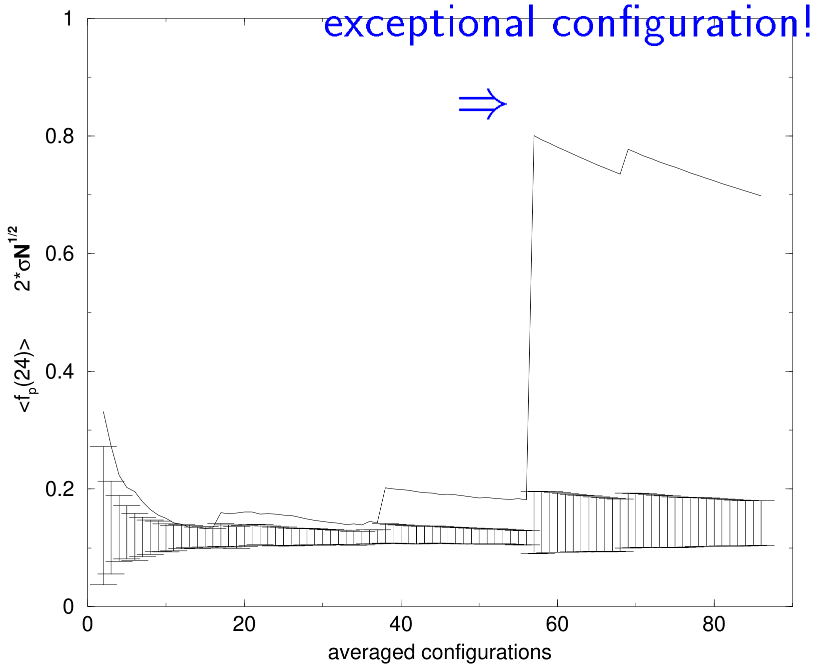
Della Morte, *et al.*

plot $\sigma\sqrt{N_{\rm conf}}$ vs $N_{\rm conf}$ $\quad\sigma$ = error of $\langle PP^\dagger\rangle$,

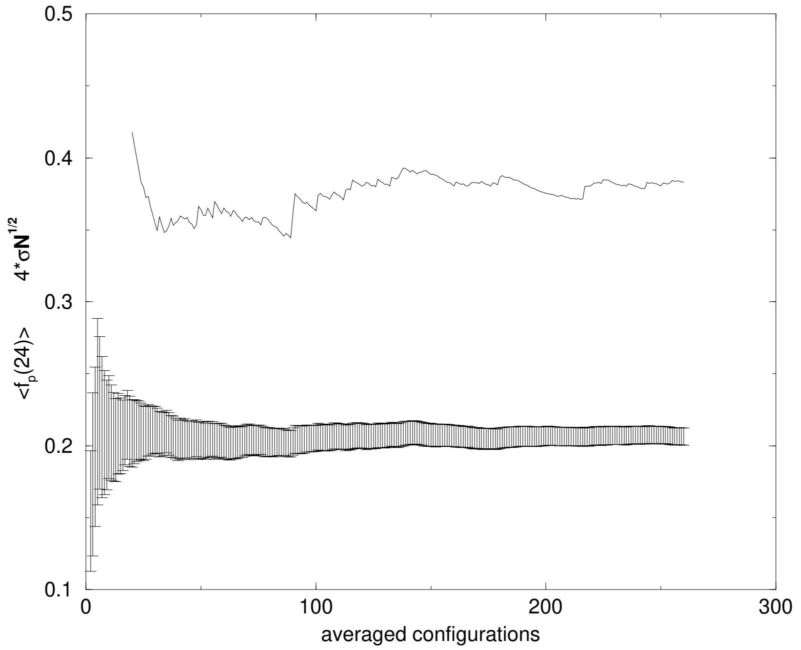
$\sigma \propto 1/\sqrt{N_{\rm conf}} \quad\Rightarrow\quad \sigma\sqrt{N_{\rm conf}} \sim$ const

$m_{\rm PS}/m_{\rm V} \sim 0.47$

2 $\sigma\sqrt{N}$ vs statistics in standard QCD



4 $\sigma\sqrt{N}$ vs statistics in twisted mass QCD



- exceptional configurations are suppressed in tm QCD.

1.1.2 non-perturbative(NP) improvement

- non-perturbative improvement

  $\Rightarrow$ precise determination of hadron and quark masses

- at Lattice 2000

  c_A from Schrödinger functional(SF)

  c_A from Ward identity(WI)

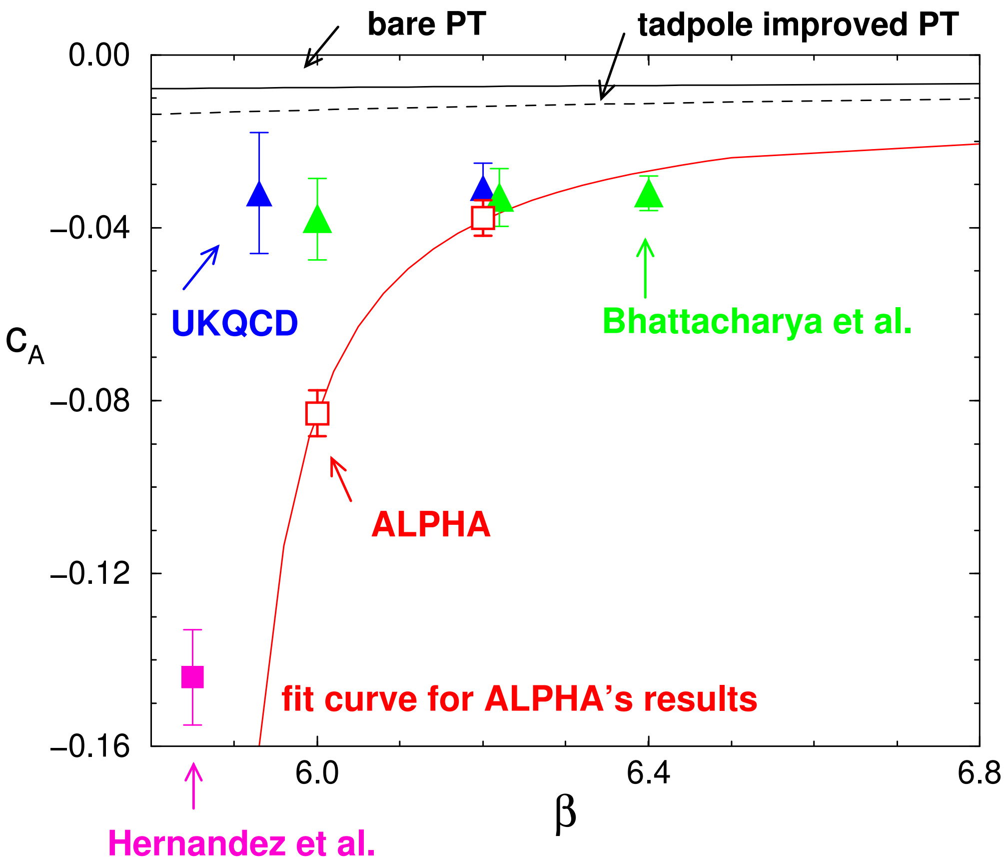
  $\Rightarrow$ different values

- new results for improvement coefficient c_A

Hernandez *et al.*: Schrödinger functional; at β=5.85

Bhattacharya *et al.*: Ward identity; at β=6.0, 6.2, 6.4

UKQCD(Collins): Ward identities; at β=5.93, 6.2

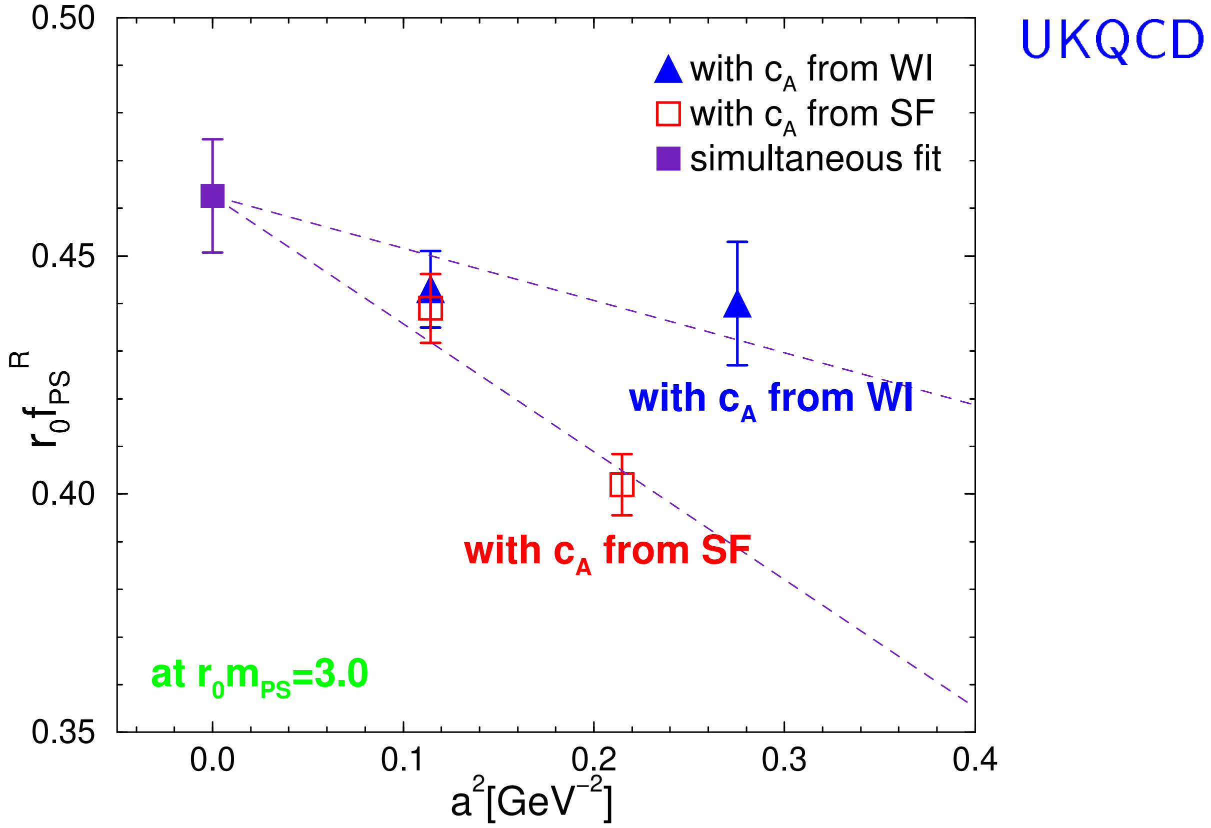


Hernandez, *et al.*

Bhattacharya *et al.*

UKQCD

decay constant at a reference quark mass



choice of improvement condition $\Rightarrow O(a)$ difference in c_A

1.1.3 negative parity baryon N^*

excited baryons $\Leftarrow$ experimental study in Jefferson Laboratory

negative parity baryon

$$N^*, \quad J^P = 1/2^-$$

$\Rightarrow$ parity mass splitting $N^* - N$

until Lattice 2000

Lee and Leinweber, LHPC, LHPC+UKQCD,...

- at large quark masses, or at fixed a
- finite size effects?

this year

- Sasaki *et al.*

  domain wall, $a^{-1} \sim$ 2 GeV, $L = 1.6$ fm

- LHPC+QCDSF+UKQCD

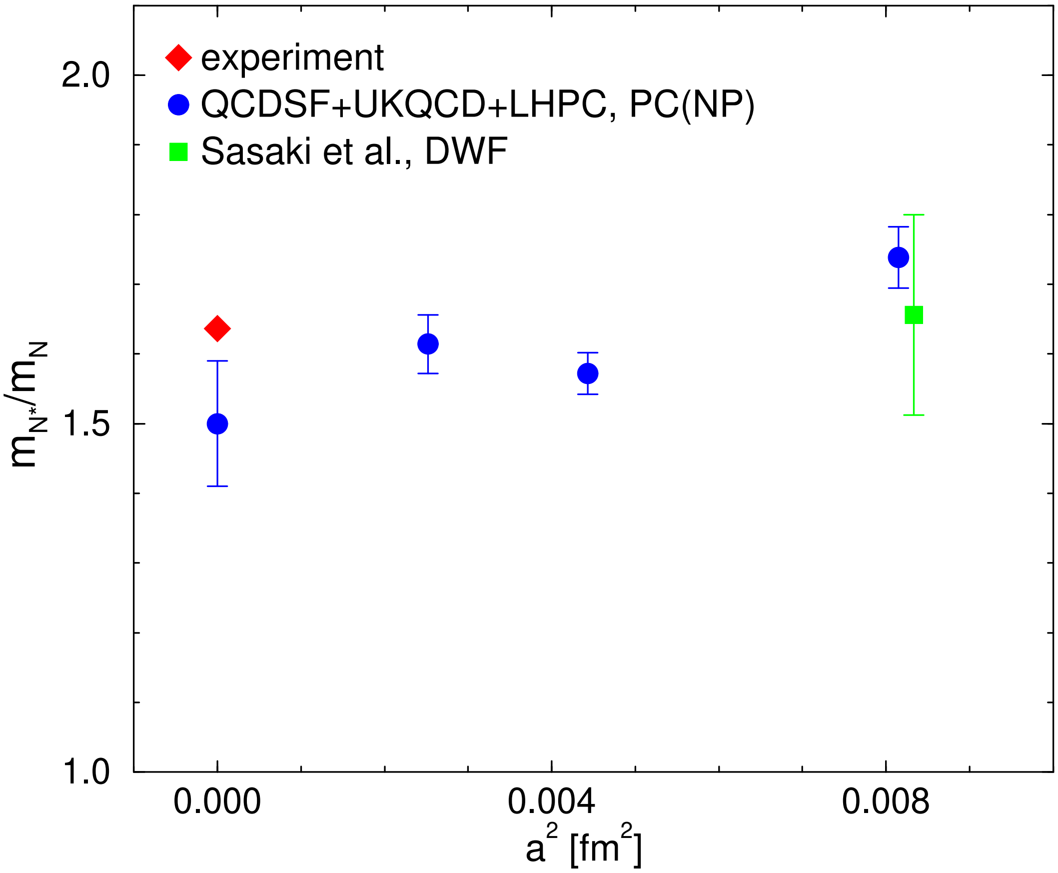
  systematic study of scaling violation, finite size effects

  clover(NP c_{SW}), $a^{-1} \sim$ 2-4 GeV, $L =$ 1.5-2.2 fm

- Lee et al.

  anisotropic ($\xi = 3$), D_{234}, $a_t^{-1} \sim$ 2.5 GeV, $L = 2.4$ fm

  chiral extrapolation has not been performed



- reproduce experimental N^* - N splitting within 10% accuracy.

- interesting to see other excitation
  - $N'(1440)$, $\Lambda(1405)$, ...

1.2 $N_f = 2$ QCD

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recent studies in $N_f = 2$ full QCD

group	action	a_t[fm]	La_s[fm]	$m_{\rm PS}/m_{\rm V}$
SESAM	P-W	0.07	1.2	0.69-0.83
TχL	P-W	0.07	1.8	0.55,0.7
SESAM/TχL	P-W	0.09	1.4	0.68-0.85
GRAL	P-W	0.12	1.9	0.4
UKQCD	P-C(1.76)	0.12	0.98-1.9	0.67-0.86
UKQCD	P-C(NP)	0.10	1.6	0.69-0.83
QCDSF	P-C(NP)	0.09	2.4,1.6	0.69,0.76
JLQCD	P-C(NP)	0.09	1.0-1.7	0.60-0.80
CP-PACS	R-C(TP)	0.11-0.22	2.5-2.6	0.55-0.81
Columbia	P-KS	0.09	1.5	0.57-0.70
MILC	P-KS	0.11-0.3	2.4-3.7	0.3-0.8
Columbia	PKS($\xi \neq 1$)	0.06-0.19	3.2-5.4	0.3-0.8
MILC	Sy-imp.KS	0.13	2.6	0.50

results with staggered fermion $\Rightarrow$ review by Toussaint.

results with Wilson fermion $\Rightarrow$ this talk.

Columbia(Levkova) : anisotropic lattices $\to$ phase transition.

GRAL(Orth) : beyond $\rho \to \pi\pi$ threshold.

SESAM/TχL(Eicker) : light quark masses.

UKQCD+QCDSF(Schierholz) : strong coupling

- the main issue

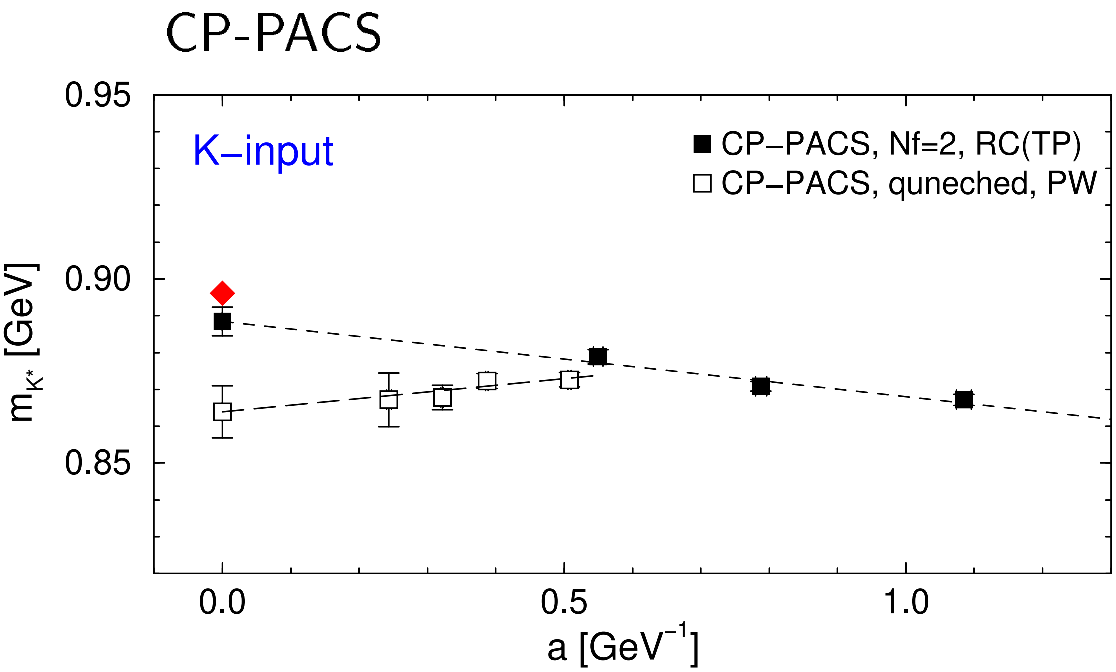
search for sea quark effects

small hyperfine splitting in quenched QCD

$\Rightarrow$ closer to experiment in $N_f = 2$ QCD?

SESAM/TχL, UKQCD, CP-PACS, QCDSF, JLQCD, MILC, ...

at Lattice 2000 (review by S.Aoki)



$\Rightarrow$ Vector meson masses are closer to experiment

$\Rightarrow$ But after continuum extrapolation

This year

UKQCD

- clover, non-perturbative $c_{\rm SW}$
- fixed a/r_0 (a ~0.1 fm)
- $L \sim$ 1.6 fm
- $m_{\rm PS}/m_{\rm V} \geq$ 0.7

JLQCD

- clover, non-perturbative $c_{\rm SW}$
- fixed β ($a \sim$ 0.1 fm at physical $m_{q,{\rm sea}}$)
- $L \sim$ 1.1, 1.4, 1.8 fm ($\Rightarrow$ finite size effect)
- $m_{\rm PS}/m_{\rm V} \geq$ 0.6

- UKQCD, JLQCD : leading scaling violation $O(a^2)$

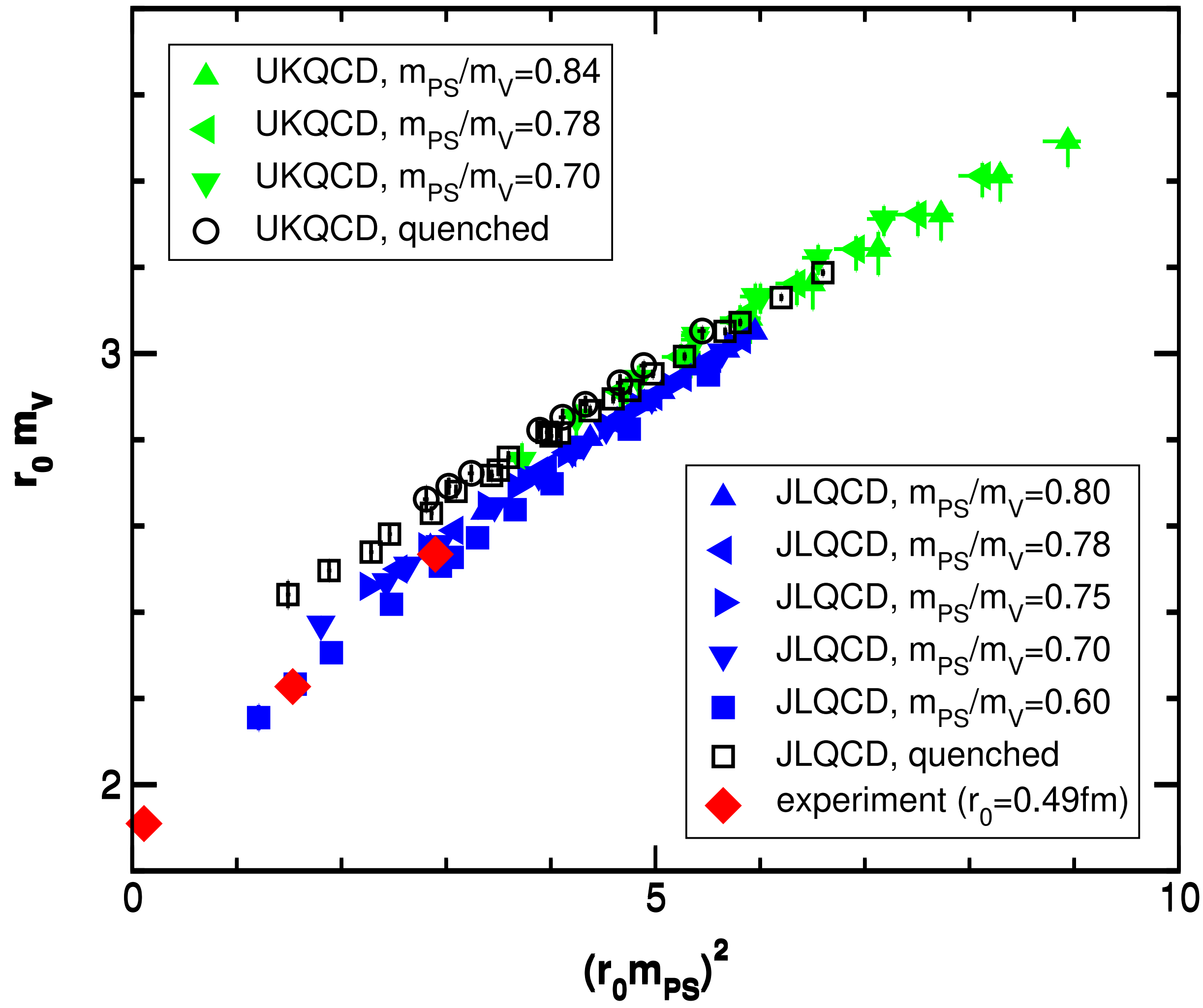
  $\Rightarrow$ smaller scaling violation compared to CP-PACS

  CP-PACS : RG gauge + tadpole improved clover

- should provide an interesting check of sea quark effects.

1.2.1 meson masses

meson masses at simulation points



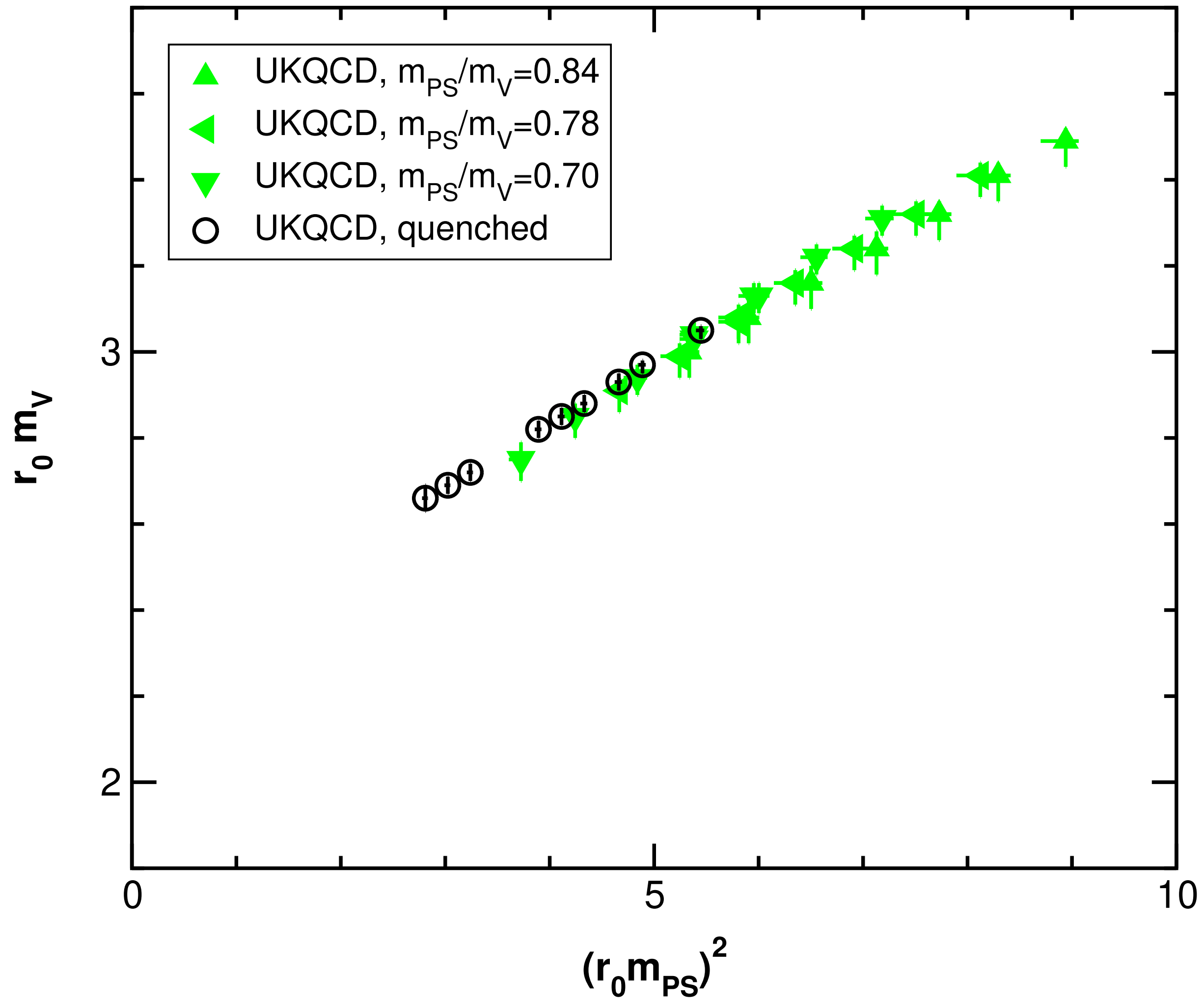
- UKQCD
  - $m_{\rm PS}/m_{\rm V} \geq 0.7$
  - sea quark effect is not clear from this plot.
- JLQCD

  clear sea quark effects

  larger slope $\Rightarrow$ larger hyperfine splitting

1.2.1 meson masses

meson masses at simulation points



- UKQCD
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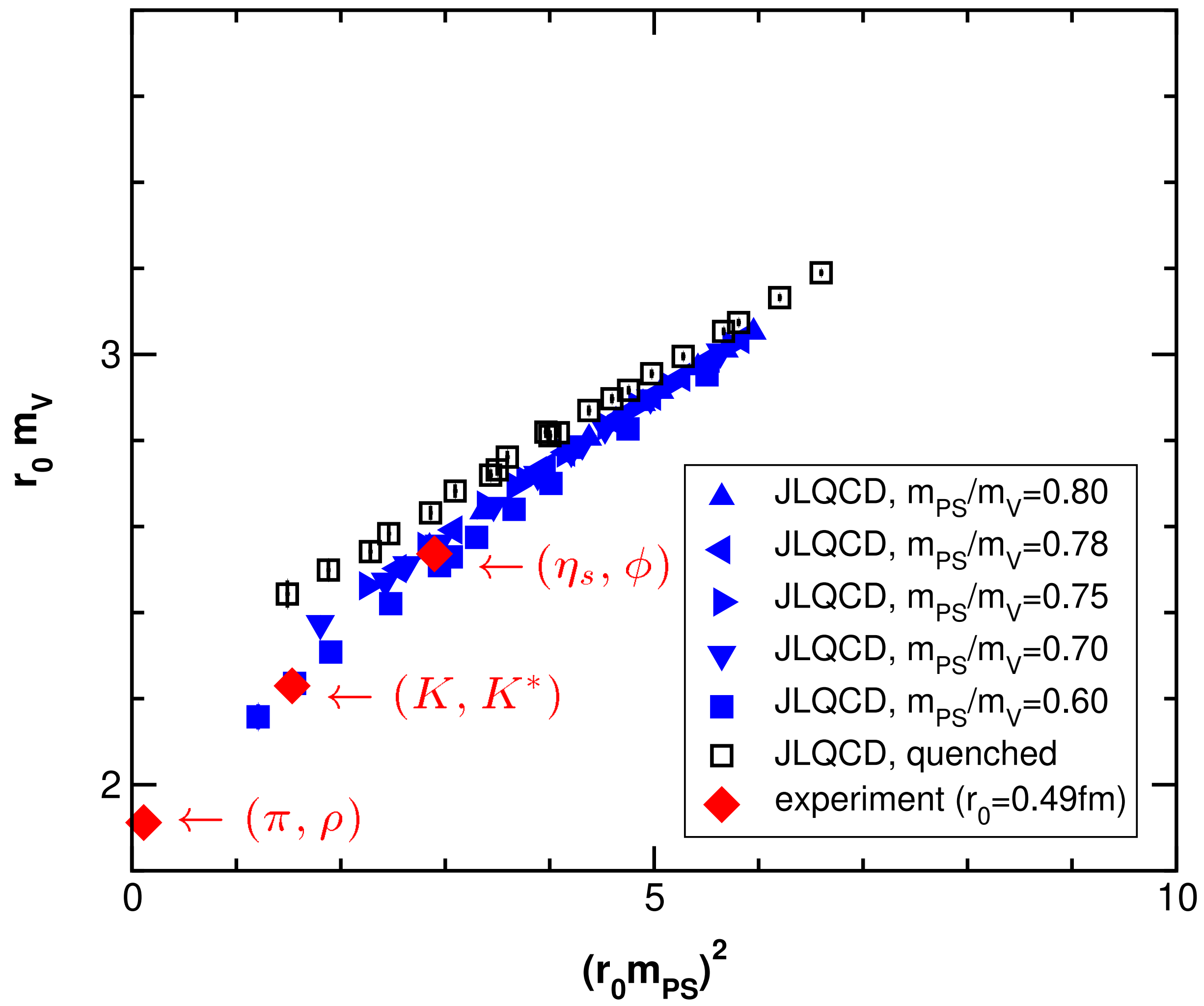


- JLQCD
  - ◦ $m_{\rm PS}/m_{\rm V} \geq 0.6$
  - ◦ clear sea quark effect

  larger slope ⇒ larger hyperfine splitting

J parameter

$$m_{\rm V}\frac{dm_{\rm V}}{dm_{\rm PS}^2} \quad @ \quad \frac{m_{\rm V}}{m_{\rm PS}}=\frac{m_{K^*}}{m_K} \qquad \text{(Lacock and Michael)}$$

- monitor the slope of "$m_{\rm V}$ vs $m_{\rm PS}^2$" plot

in full QCD

1) calculate J at fixed sea quark masses, and take $m_{\rm sea} \to m_{ud}$

$$J \equiv \lim_{m_{\rm sea}\to m_{ud}} J(m_{\rm sea}), \quad J(m_{\rm sea}) \equiv m_{\rm V}\frac{dm_{\rm V}}{dm_{\rm PS}^2}\bigg|_{m_{\rm sea}}$$

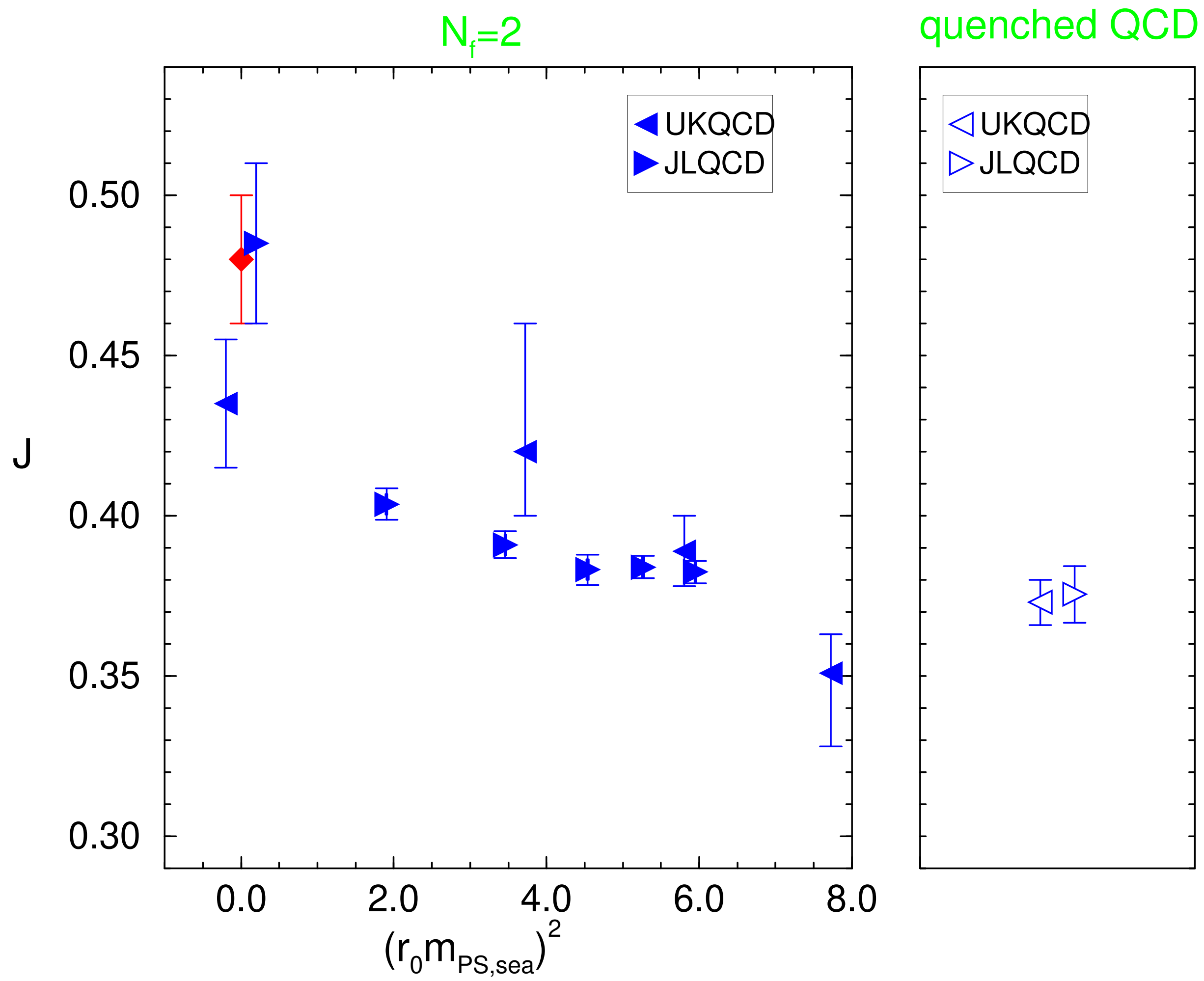
2) calculate J from meson masses at physical quark mass

$$J \equiv m_{K^*}\frac{m_{K^*}-m_\rho}{m_K^2-m_\pi^2}$$

3) calculate J as mass ratio at simulation point(MILC)

$$J \equiv m_{\rm V}(m_1,m_2)\frac{m_{\rm V}(m_2,m_2)-m_{\rm V}(m_1,m_1)}{2\,(m_{\rm PS}(m_1,m_2)^2-m_{\rm PS}(m_1,m_1)^2)}$$

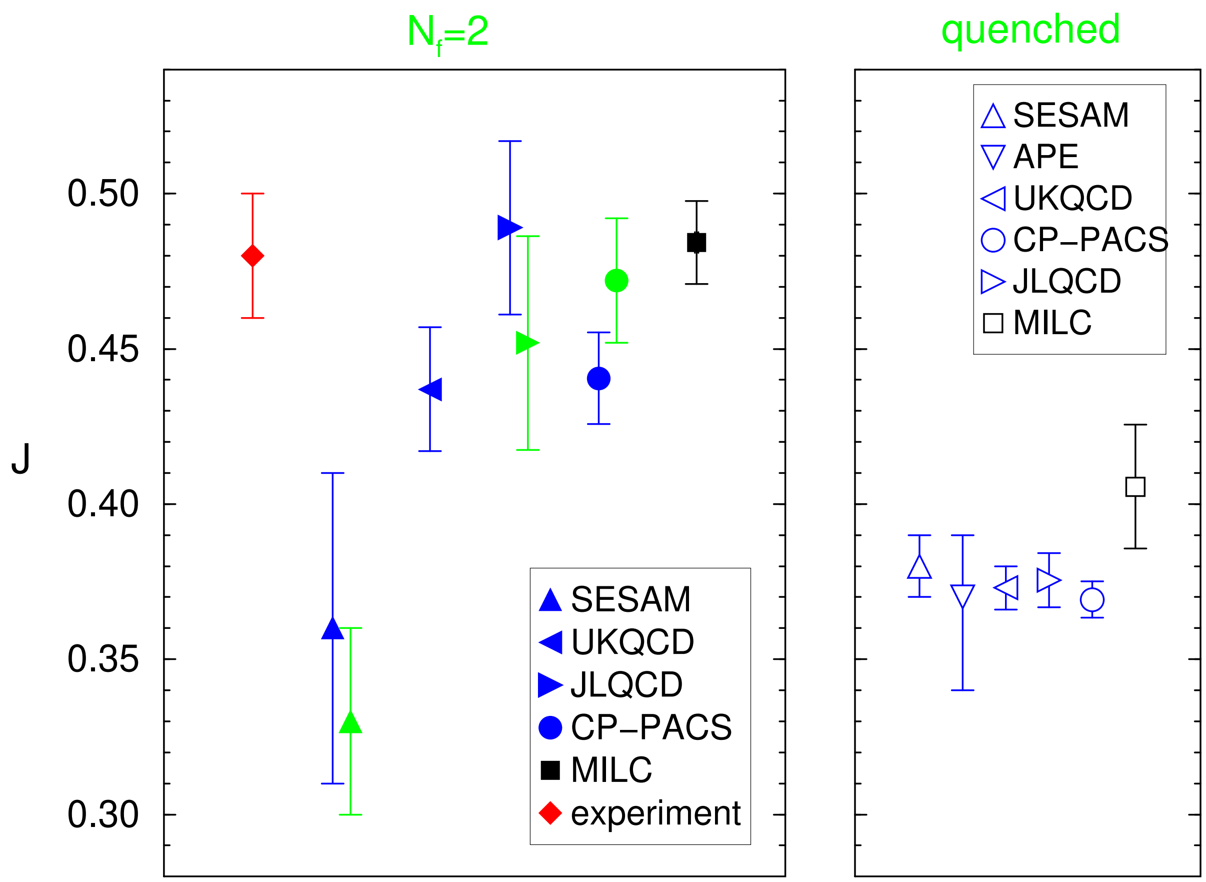
sea quark mass dependence



1) J at fixed sea quark masses, and take $m_{\mathrm{sea}} \to m_{ud}$

- J increases as sea quark mass decreases

summary of recent results



1) J at fixed sea quark masses, and take $m_{\rm sea} \to m_{ud}$

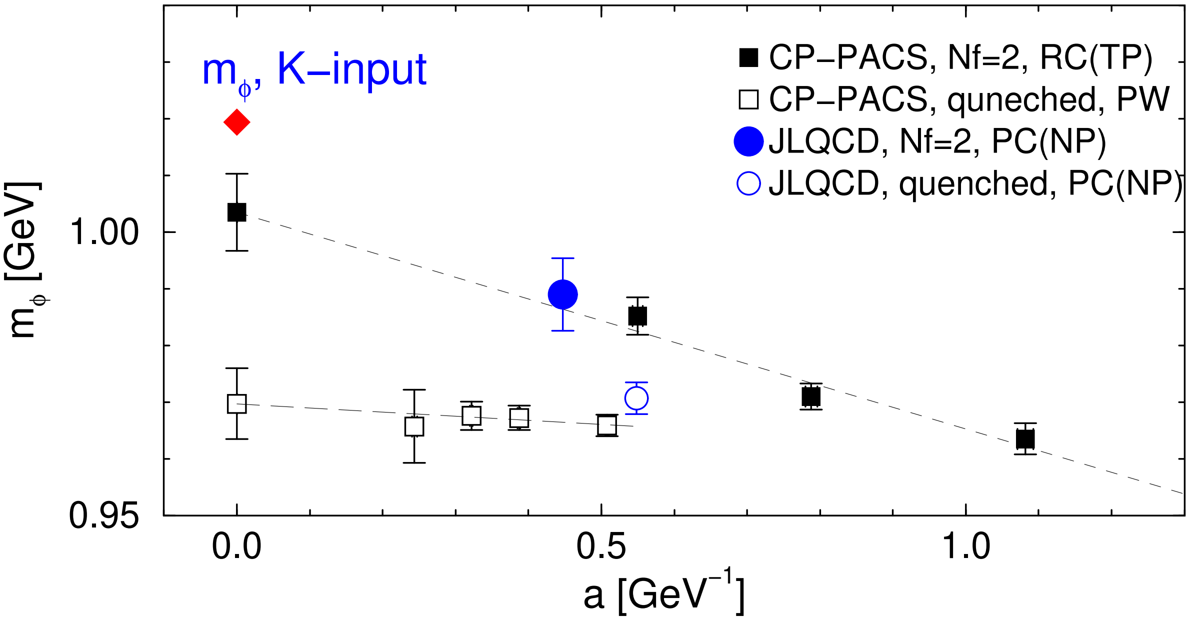
2) J from meson masses at physical quark mass

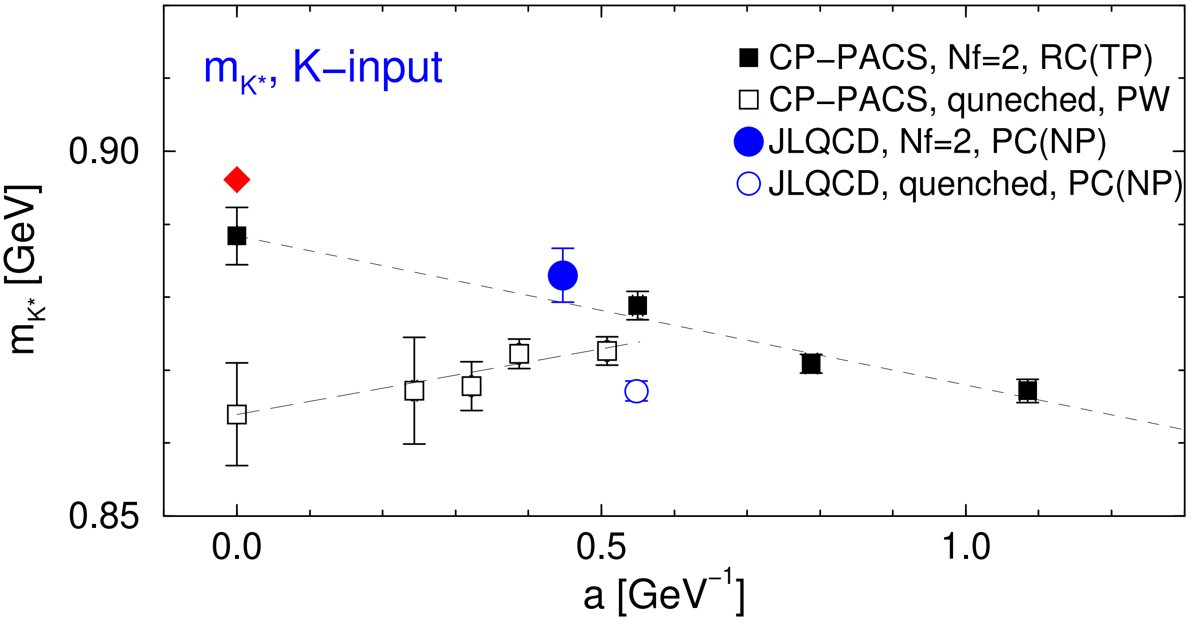
3) J as mass ratio at simulation point(MILC)

- sea quark effects $\Rightarrow$ **closer to experiment**

  except SESAM $\Leftarrow$ heavy sea quark mass $(m_{\rm PS}/m_{\rm V} \geq 0.69)$?

meson masses





- sea quark effect is confirmed using different action

JLQCD: plaq + clover(non-perturbative c_{SW})

CP-PACS: RG + clover(tadpole improved c_{SW})

1.2.2 baryon masses

- recent studies of baryon spectrum

  CP-PACS : $V \sim 2.5$ fm.

  UKQCD : $V \sim 1.6$ fm.

  JLQCD : $V \sim 1.1$ - 1.8 fm.

These volumes are sufficiently large?

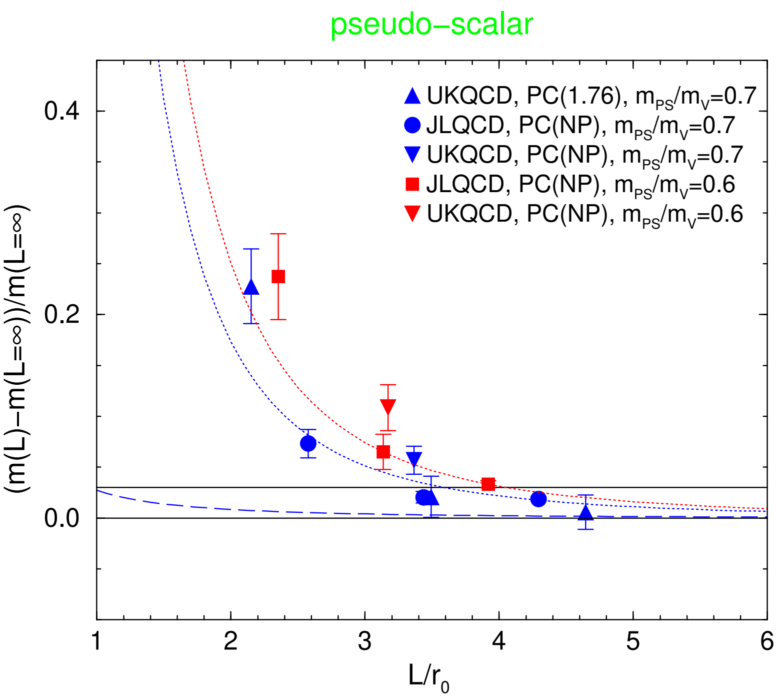
$\Rightarrow$ check finite size effects in JLQCD and UKQCD data

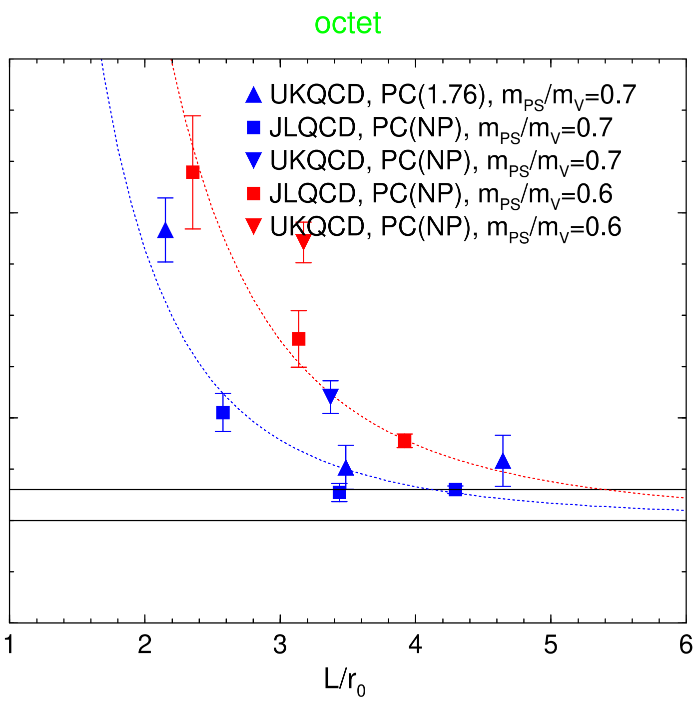
$$m(L\!=\!\infty) \quad \Leftarrow \quad m(L) = m(\infty) + c/V$$

- mass shift at finite volume

$$(m(L) - m(\infty))/m(\infty) \quad \text{vs} \quad L/r_0$$

blue symbols : at $m_{\rm PS}/m_{\rm V}$=0.7
red symbols : at $m_{\rm PS}/m_{\rm V}$=0.6





- volume for FSE $\leq 3\%$, at a=0.1 fm

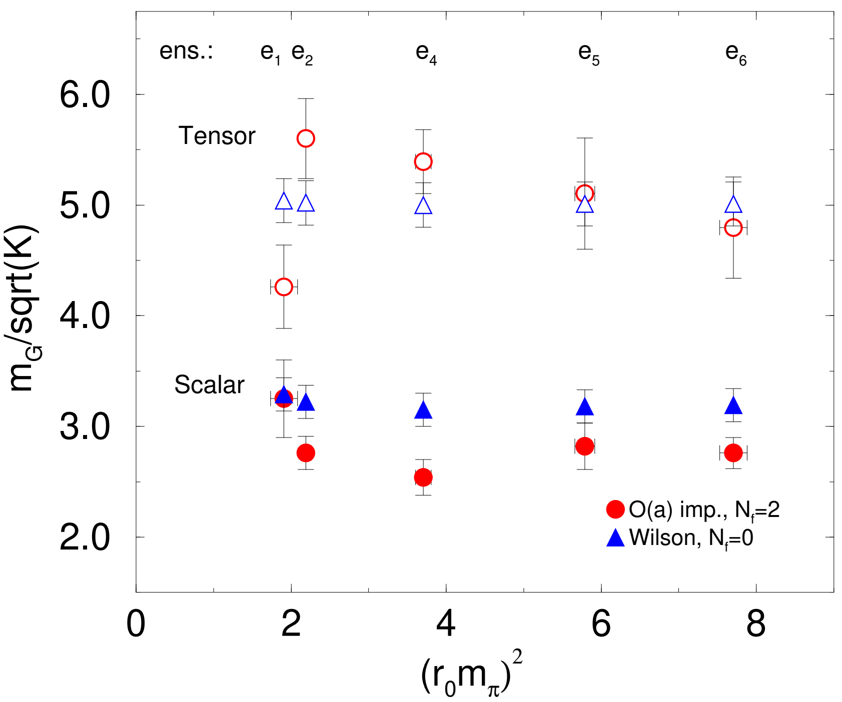
	$m_{\rm PS}/m_{\rm V}\sim$0.7	$m_{\rm PS}/m_{\rm V}\sim$0.6
meson	16^3	20^3
baryon	24^3	32^3

∘ much larger volumes are required for baryons!

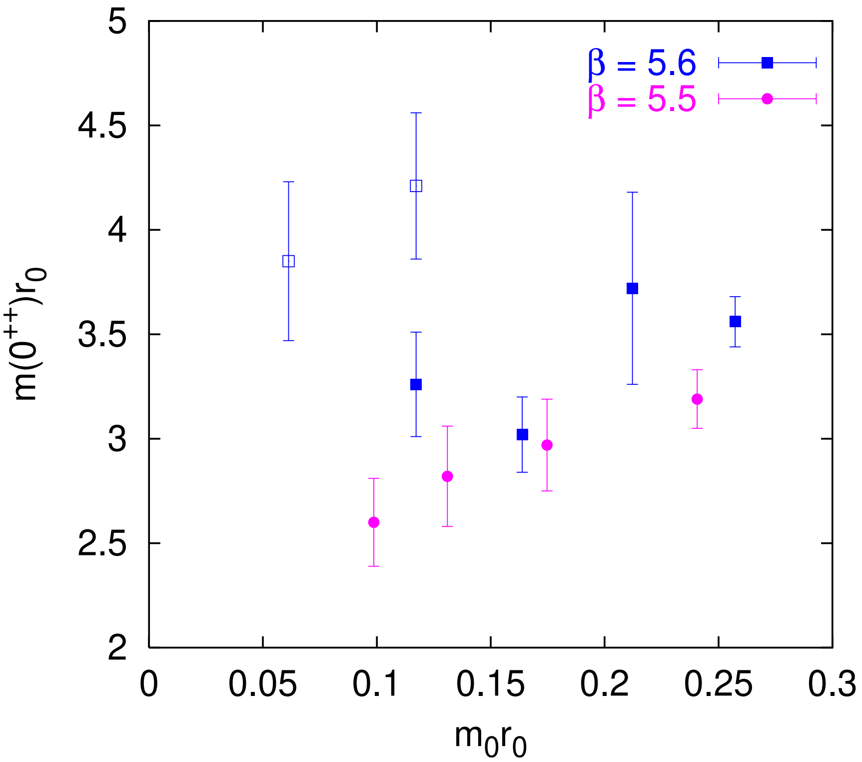
∘ still, we haven't obtained definite conclusions

1.2.3 glueball

UKQCD : $\sim$ 15% decrease in 0^{++} at $a^{-1} \sim 2$ GeV



SESAM : $\sim$ 25% decrease in 0^{++} at $a^{-1} \sim 2.7$ GeV



- decreases in 0^{++} $\Rightarrow$ indication of sea quark effects?
- need more precise data and study of scaling violation.

1.2.4 scalar meson

SCALAR collaboration (Nakamura)

- calculation of scalar meson in $N_f = 2$ full QCD

  Wilson fermion, a =0.2 fm, $m_{\rm PS}/m_{\rm V}$ =0.9

  $\Rightarrow$ poster by Nakamura

1.3 $N_f = 3$ QCD

1.3 $N_f = 3$ QCD

this year

- with KS fermion

  MILC

  - improved KS
  - standard hybrid-R

- with Wilson-type action

  algorithm for odd number of flavors $\Rightarrow$ review by Peardon

  exact algorithms have been developed.

  - multi-boson (Lüscher, Borici and de Forcrand, Alexandrou *et al.*)
  - polynomial HMC (Takaishi and Forcrand)

  Gebert *et al.*

  - plaquette gauge + Wilson
  - two-step multi-boson (Montvay)

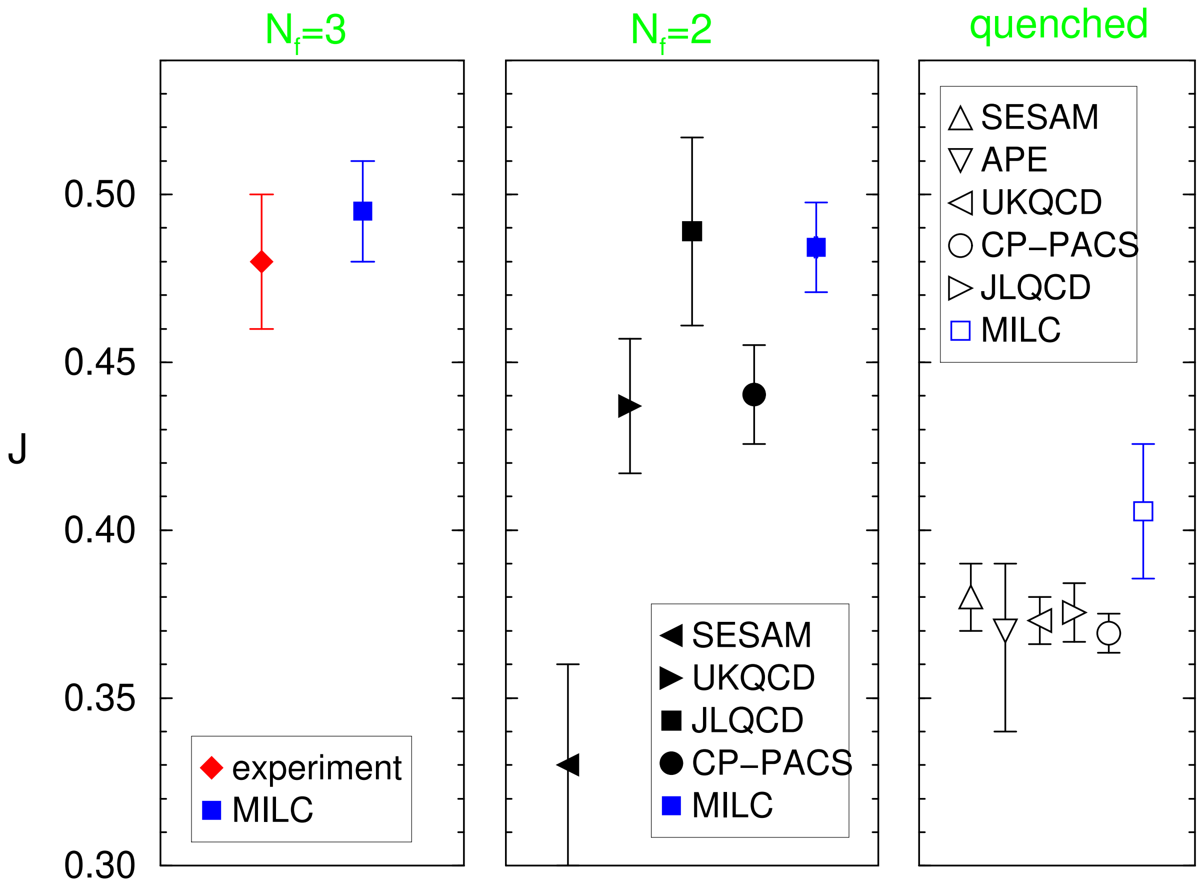
  JLQCD

  - plaquette gauge + clover
  - polynomial HMC (Ishikawa(JLQCD))

1.3.1 with KS action

MILC ($\Rightarrow$ review by Toussaint)

J parameter:



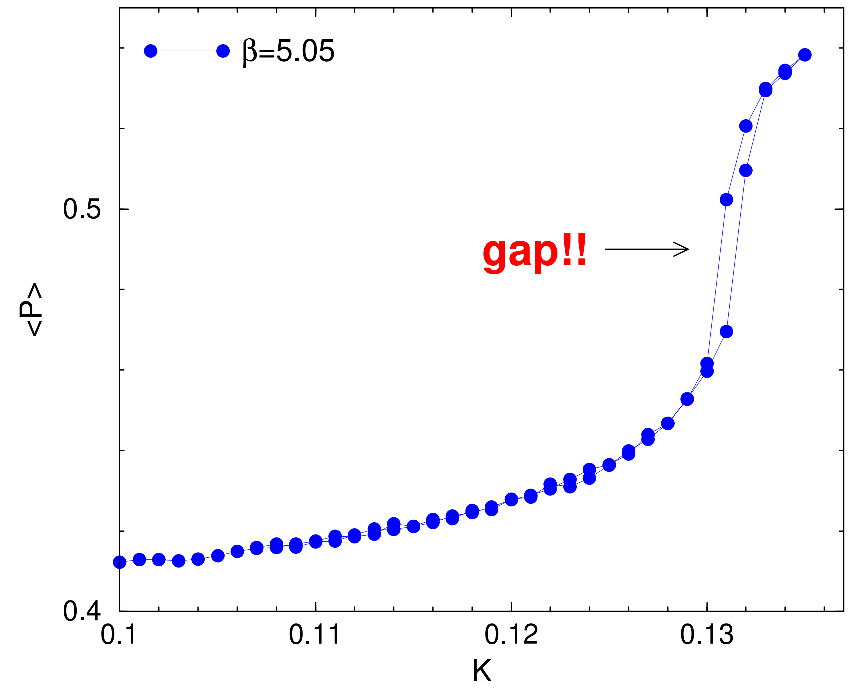
- MILC's results
  - clear sea quark effects
  - consistent with experiment in full QCD
- no systematic deviation between KS and Wilson/clover

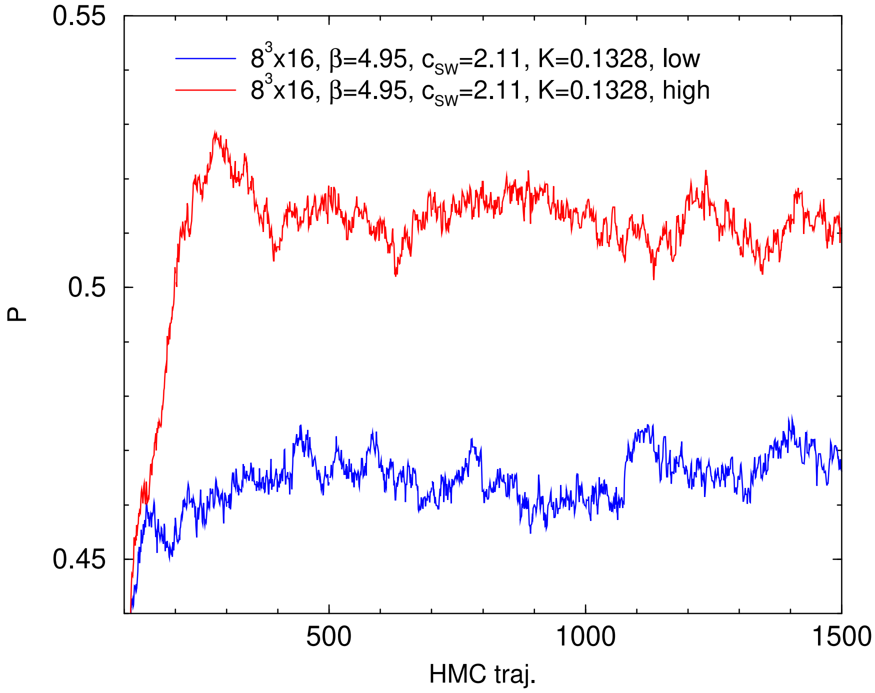
1.3.2 with Wilson-type action

JLQCD(Okawa)

- first systematic simulations with $O(a)$ improved Wilson action
  degenerated 3 flavors, tadpole improved $c_{\rm SW}$
  study of phase structure $\Rightarrow$ study of light hadron spectrum

- unexpected phase transition is found!!





$8^3\times 16,\ \beta=4.95$

$K_{ud}=K_s=0.1328$

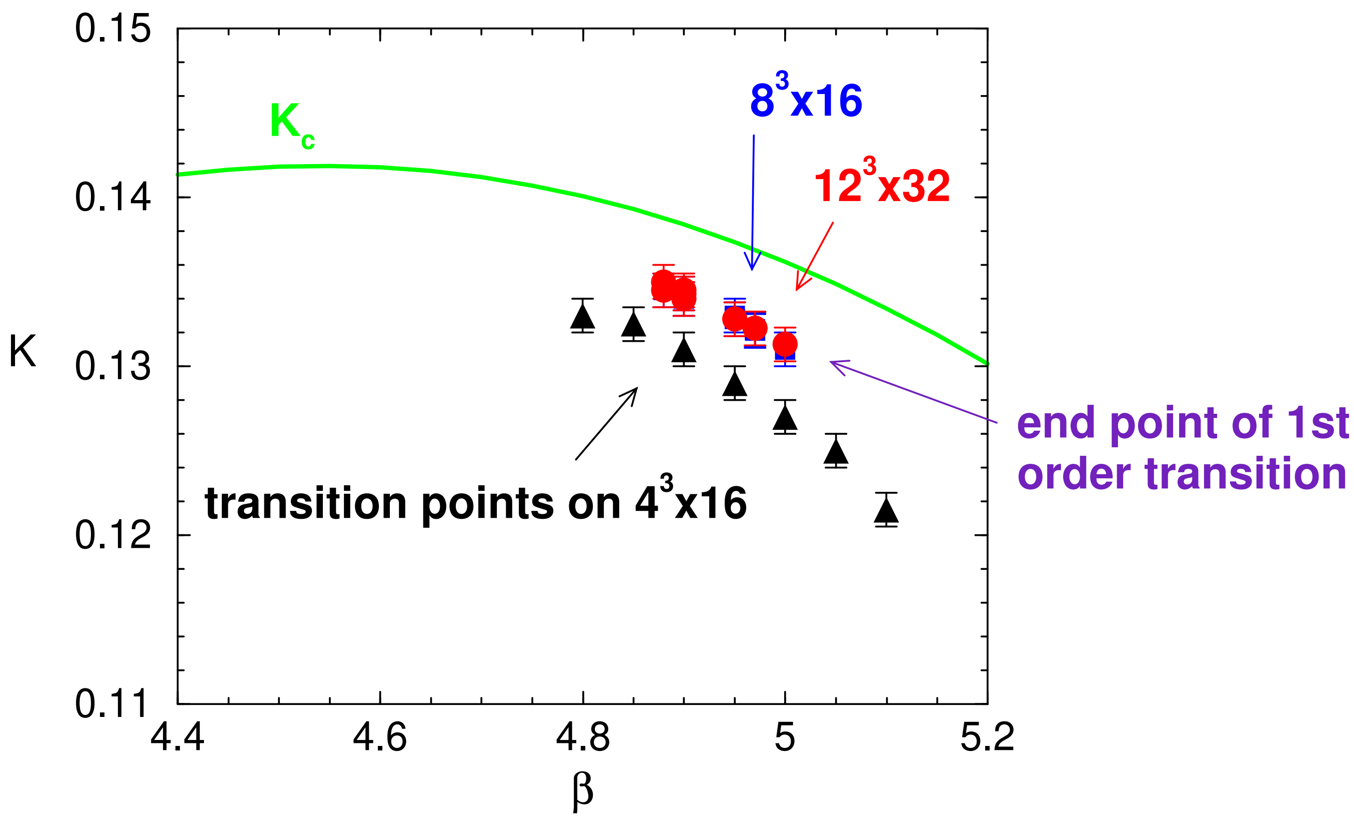
$a^{-1}=1.393(13)$ GeV

$m_{\rm PS}/m_{\rm V}=0.87$

$a^{-1}=0.827(4)$ GeV

$m_{\rm PS}/m_{\rm V}=0.86$

phase diagram



- bulk transition at $T = 0.$ $\Rightarrow$ disappears at $\beta \sim$ 5.0

  $a^{-1} \sim$ 2.6 GeV at the end point

- this phase transition $=$ lattice artifact
  - plaquette gauge $+$ clover quarks $\Rightarrow$ a^{-1} $>$2.6 GeV.

- phase transition disappears for
  - improved gauge $+$ clover quark
  - plaquette/improved gauge $+$ Wilson quark

- with improved gauge and/or Wilson quark

  $\Rightarrow$ coarse lattices are OK!

2. test/application of ChPT

2. *test/application of ChPT*

- test of ChPT using lattice data

  - simulated quark masses $\gg m_{ud}$

    $\Rightarrow$ ChPT is used as a guide for extrapolation to m_{ud}

  - tests of ChPT at simulated quark mass: important!

    $\Leftarrow$ fit lattice data ($m_{\rm PS}$ and $f_{\rm PS}$) to ChPT formulae.

- determination of low-energy constants α_i (i=1-10)

  (recent work by Sharpe and Shoresh)

  - fit lattice data to ChPT formulae

    $\Rightarrow$ low-energy constants

    $\Rightarrow$ information on physics in real world from ChPT

  - m_u=0? (Cohen, Kaplan and Nelson)

  phenomenological value : "standard" $\alpha_8 \sim 0.8$

  "$m_u = 0$" $\alpha_8 \sim -0.9$

  (Gasser,Leutwyler,Bijnens,Ecker,...,1984-1992)

  studies in quenched QCD

  - Bardeen *et al.*, 2000
  - ALPHA, 2000

this year

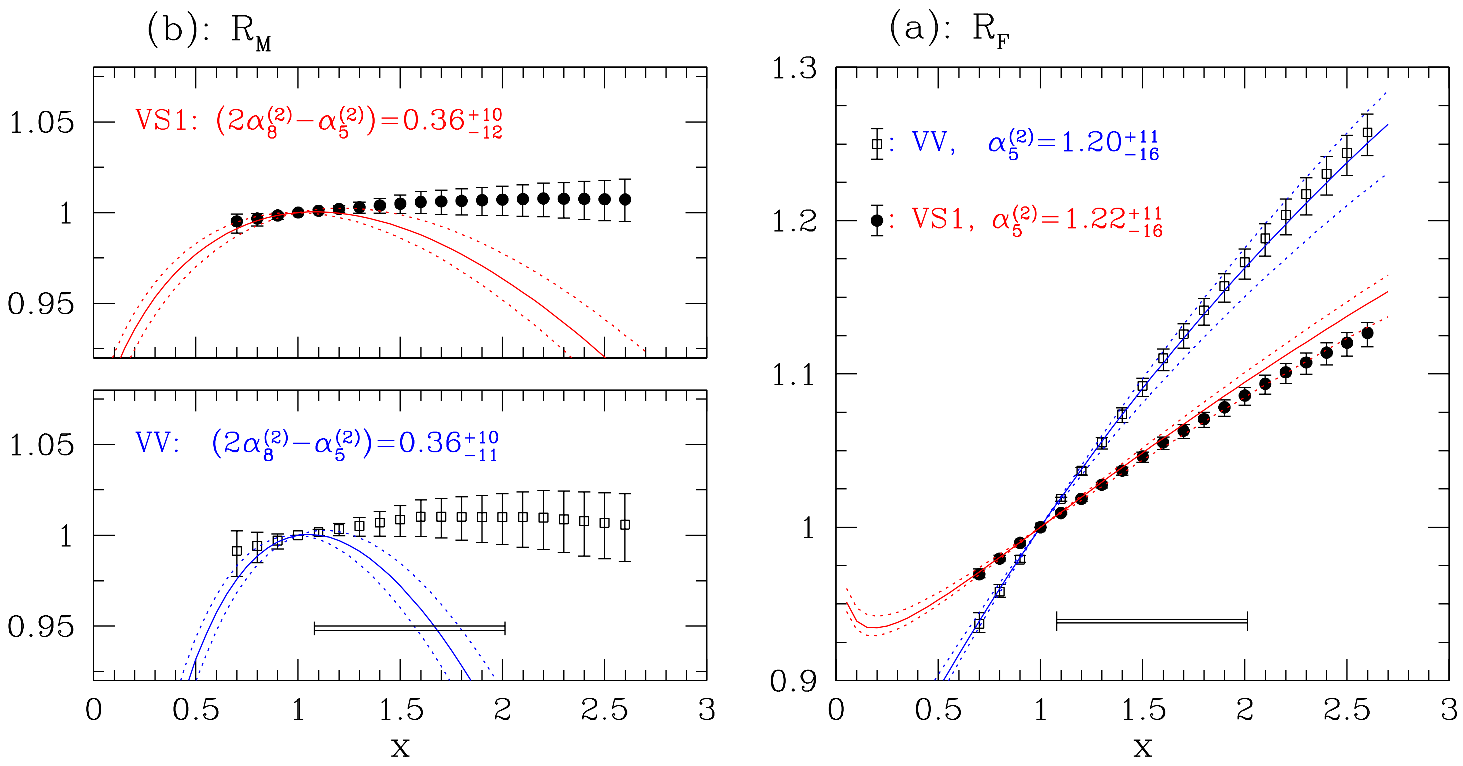
UKQCD : first estimate of α_8 in $N_f=2$ full QCD

- ∘ "ratio" method of ALPHA, PQChPT formulae by Sharpe,

$$R_M^{VV} \equiv \frac{2y/m_{\rm PS}^2(y)}{2y_{\rm ref}/m_{\rm PS}^2(y_{\rm ref})} = (x-1)\,y_{\rm ref}\,\left(\underline{2\alpha_8-\alpha_5}\right) + \text{log terms}$$

$$R_F^{VV} \equiv \frac{f_{\rm PS}(y)}{f_{\rm PS}(y_{\rm ref})} = (x-1)\,y_{\rm ref}\,\underline{\alpha_5}/2 + \text{log terms}$$

$$y \equiv B_0 m_q/(4\pi F)^2, \quad x = y/y_{\rm ref}$$



- $\alpha_5^{N_f=2} = 0.99(6)(20), \quad \alpha_8^{N_f=2} = 0.67(4)(20) \quad \Rightarrow \quad m_u \neq 0$

but...

- ∘ fit for R_M does not seem good...

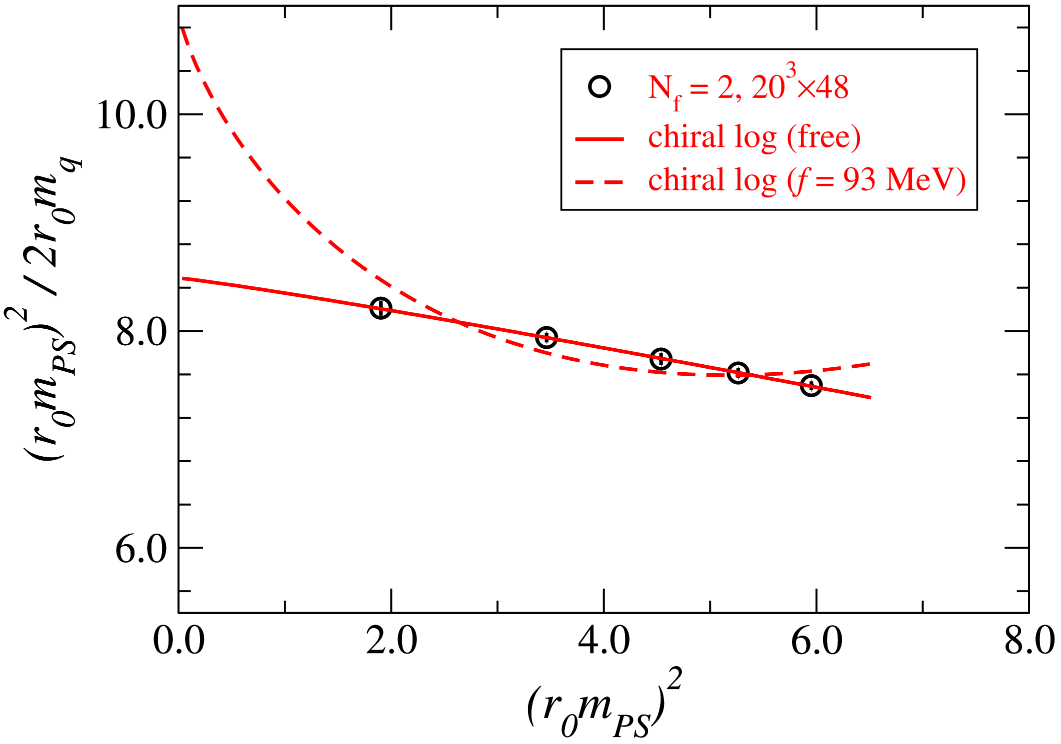
$\Rightarrow$ Are lattice data really consistent with PQChPT?

JLQCD (Hashimoto)

examination of ChPT log behavior in $N_f = 2$ QCD

$$m_q \equiv m_{q,\mathrm{val}} = m_{q,\mathrm{sea}}$$

$$\frac{M_{PS}^2}{2B_0 m_q} = 1 + \underline{\frac{y}{N_f}\ln[y]} + y\{(\alpha_8 - \alpha_5) + N_f(\alpha_8 - \alpha_5)\}$$



- no clear evidence of chiral log
- no evidence also for "f_{PS} vs m_{PS}^2"

$\Rightarrow$ not consistent with ChPT ($m_{\mathrm{PS}}/m_{\mathrm{V}} \sim$ 0.6-0.8) possible reasons

- finite size effects
- lattice artifacts
- higher order of ChPT
- sea quark mass ($m_{\rm PS}/m_{\rm V} \sim 0.6$-$0.8$) isn't small enough

  D.Nelson *et al.*

  KS, $N_f = 3$, $m_q/m_s \geq 0.1$,

  R_M : better consistency with ChPT

important works in future

- explore (much) smaller sea quark masses also with Wilson type action.

  $\Rightarrow$ attempt by GRAL collaboration

- extend UKQCD's and JLQCD's attempts toward small sea quark masses.

3. determination of fundamental parameters of QCD; α_s and m_q

3.1. strong coupling

- famous calculation by NRQCD (Davies *et al.*, 1997)

$$\alpha_{\overline{\mathrm{MS}}}^{(N_f=5)}(M_Z) = 0.1174(24)$$

KS action for light quarks

$N_f = 0,\ 2$ (extrapolated to $N_f = 3$)

$\alpha_{\overline{\mathrm{MS}}}$ from α_P, scale from Υ

---

important to confirm this results using Wilson-type action

- SESAM, 1999

similar calculation with Wilson action

$$\alpha_{\overline{\mathrm{MS}}}^{(5)}(M_Z) = 0.1118(17)$$

$\Rightarrow$ significantly smaller

scaling violation?

- recent calculations ($N_f = 2$)

  - Wilson action

    Ph.Boucaud, *et al.* — $N_f = 0$, 2

    $\alpha_{\overline{\mathrm{MS}}}$ from three gluon vertex

    scale from ρ

    $\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.113(2)$

  - $O(a)$ improved Wilson action

    UKQCD at Lattice00 — $N_f = 0$, 2, $\alpha_{\overline{\mathrm{MS}}}$ from α_P,

    scale from Υ

    QCDSF+UKQCD — $N_f = 0$, 2, $\alpha_{\overline{\mathrm{MS}}}$ from α_P

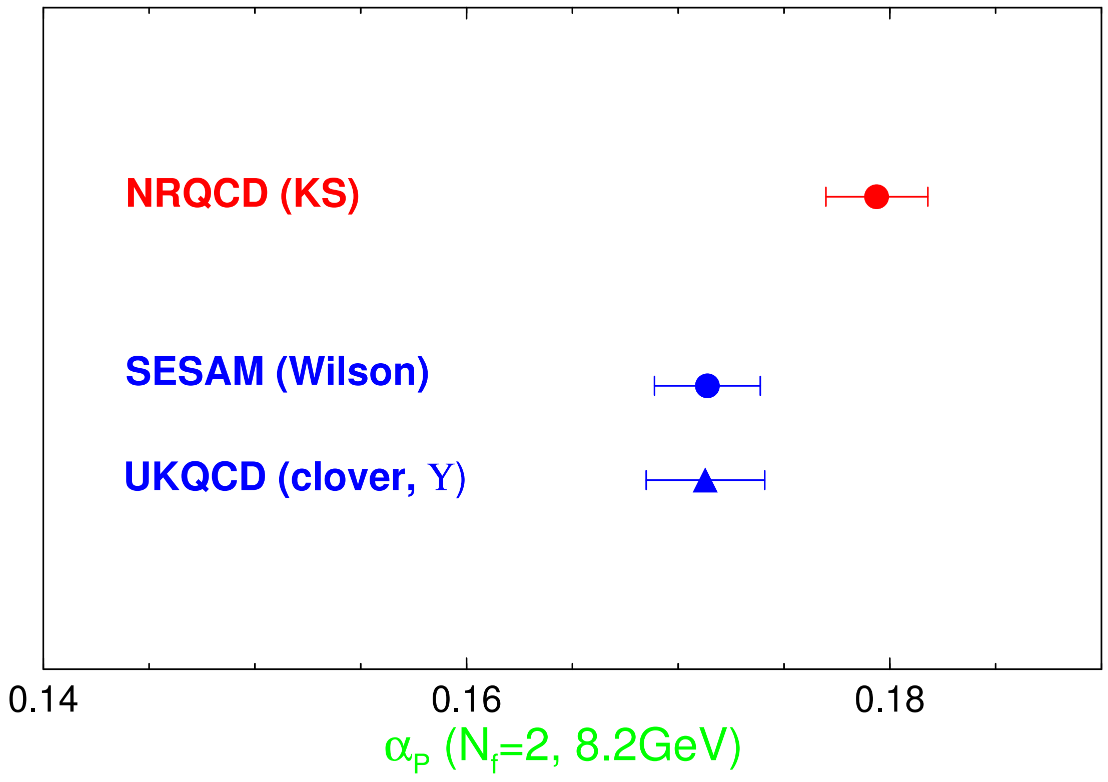
    scale from r_0

    $\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.1076(20)(18)$

ALPHA

- non-perturbative evolution of α_{SF} in $N_f = 2$
- determination of scale : in progress

- α_P in $N_f = 2$ at $q = 8.2$ GeV



- all results using scale from Υ.
- smaller values for Wilson-type action.
- discrepancy is present before conversion to $\overline{\mathrm{MS}}$ scheme.
- reason is still unclear.
  - scaling violation?
  - higher order correction?

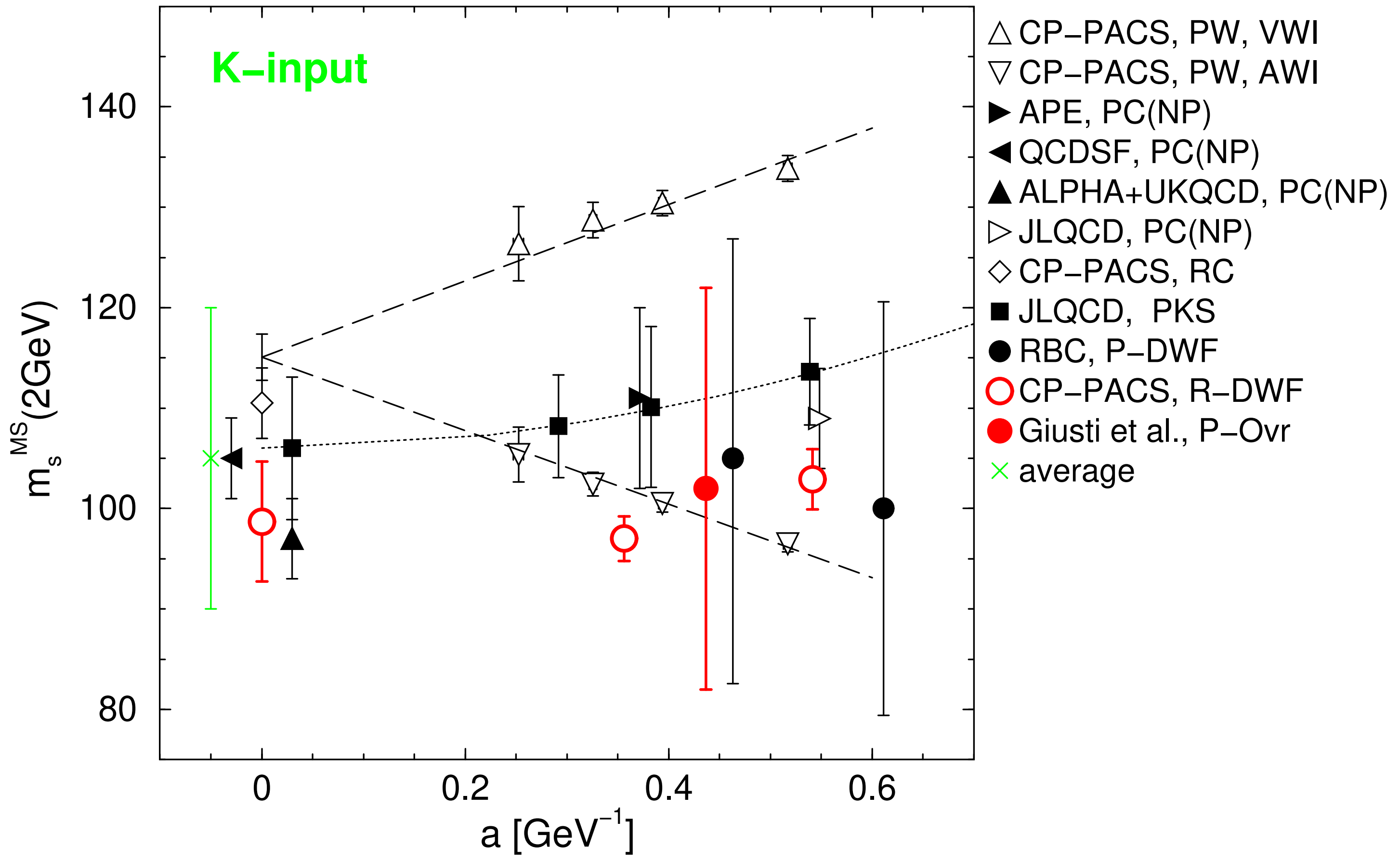
3.2 quark masses

3.2.1 quenched QCD

strange quark mass

CP-PACS : domain-wall, $a_\rho^{-1} = 1.8,\ 2.8$ GeV, $La \sim 2.6,\ 2.3$ fm

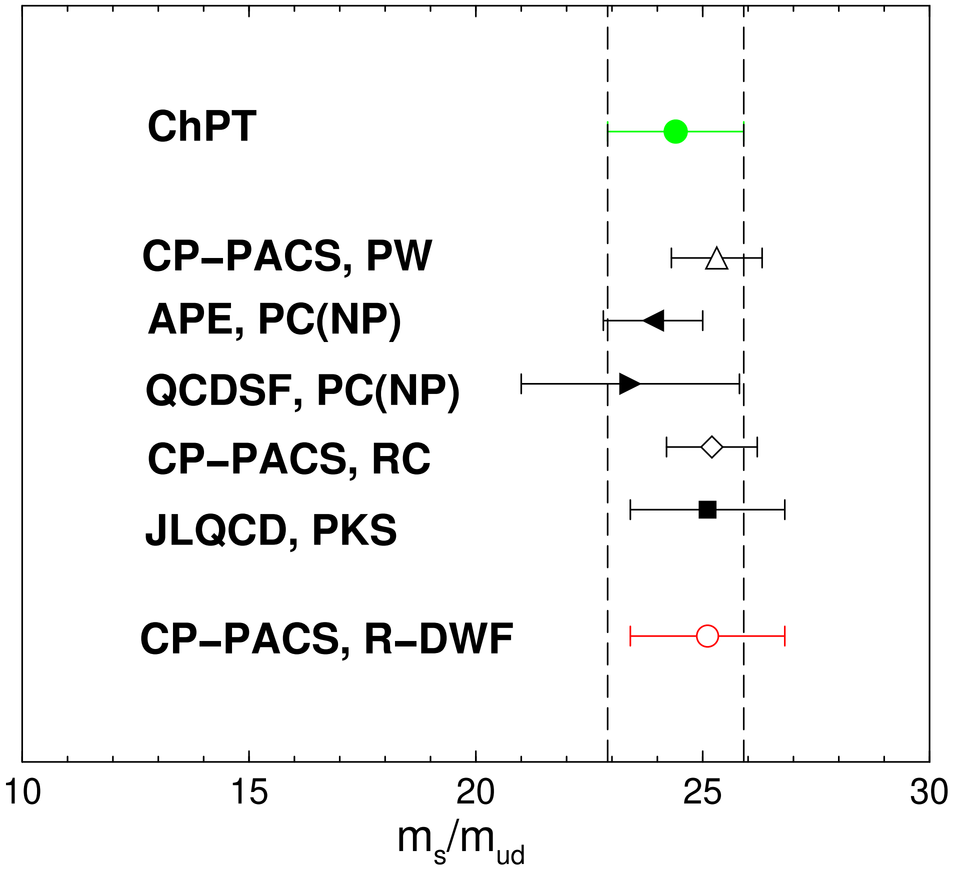
Giusti *et al.* : overlap, $a_{f_K}^{-1} = 2.3$ GeV, $La \sim 1.4$ fm



$$m_s^{\overline{\rm MS}}(2\ \text{GeV}) = 105 \pm 15\ \text{MeV}$$

ud quark mass

m_s/m_{ud}



- new result from CP-PACS :

  consistent with previous results and ChPT

- use ChPT ratio, average of m_s

  $\Rightarrow m_{ud}^{\overline{\rm MS}}(2\ \text{GeV}) = 4.5(7)$ MeV

- results with overlap fermion

  Giusti *et al.*, $m_{ud}^{\overline{\rm MS}}(2\ \text{GeV}) = 4.2(0.2)(0.7)$ MeV

  Dong *et al.*, $m_{ud}^{\overline{\rm MS}}(2\ \text{GeV}) = 4.6(1.2)(0.2)$ MeV

charm quark mass

- this year

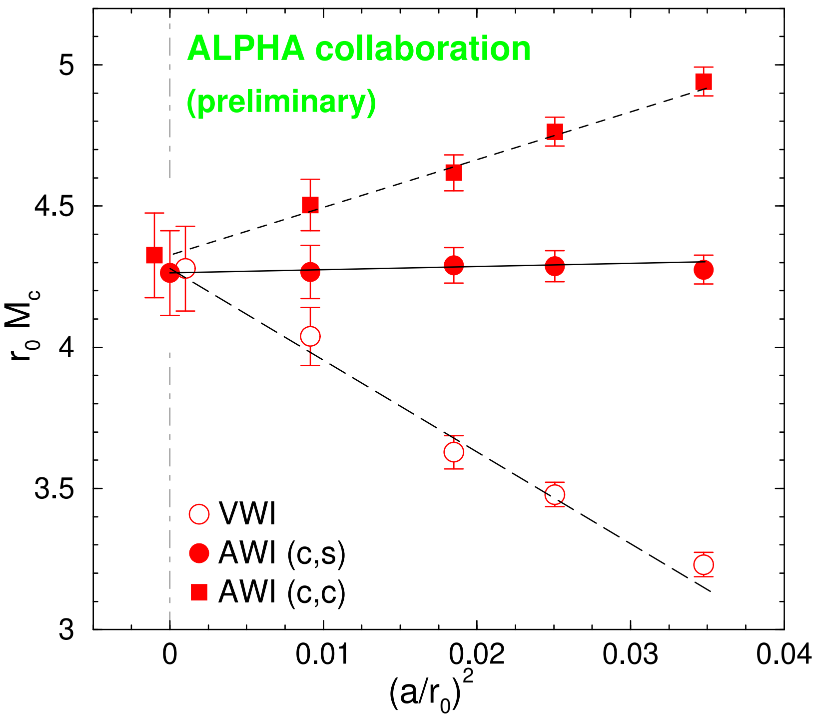
  Becirevic *et al.* : NP clover, NP Z, $a \sim$ 0.07 fm

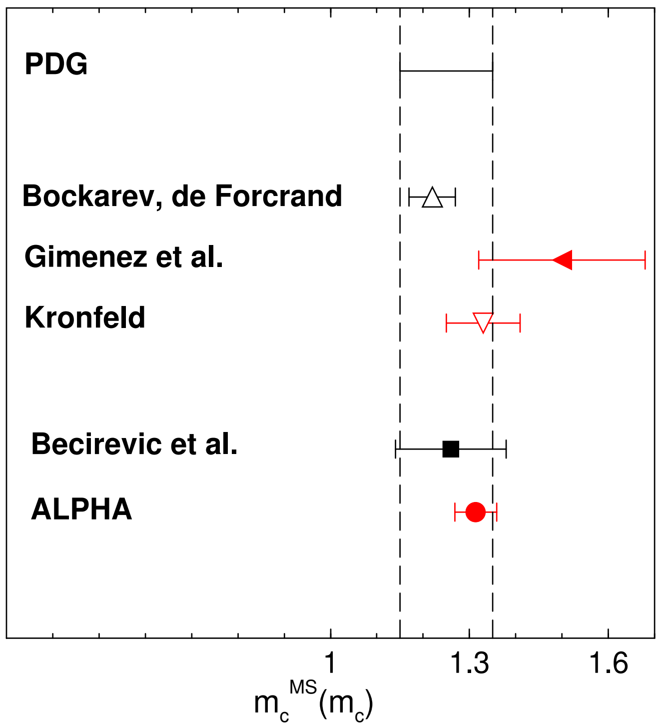
  ALPHA(Rolf) : NP clover, NP Z, $a \to 0$

  Juge *et al.* : clover, 2loop matching, $a \to 0$

- in relativistic lattice formulations

  $O((am_c)^n)$ error $\Rightarrow$ should take continuum limit





- Juge, *et al.* : very pleliminary, $m_c^{\overline{\rm MS}}(m_c) = 1.28(4)(??)$

- new results are $m_c^{\overline{\rm MS}}(m_c) \sim$ 1.3 GeV

- largest error in ALPHA's result $\Leftarrow$ uncertainty in scale

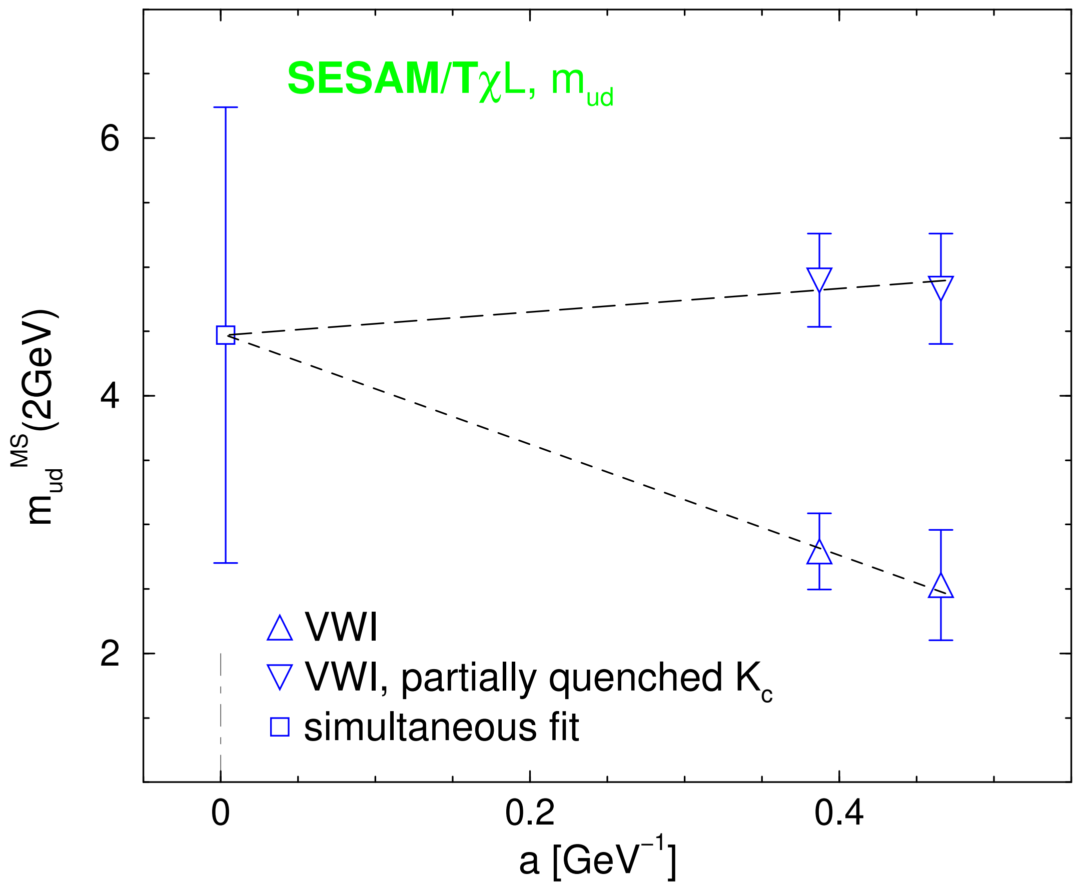
  $\Rightarrow$ determination in full QCD

bottom quark mass $\Rightarrow$ review by Ryan

3.2.2 $N_f = 2$ full QCD

- this year

SESAM/TχL (Eicker) : 1 loop Z, $a^{-1} \sim$ 2.2, 2.6 GeV



- ◦ $m_{ud} =$ 4.5(1.7) MeV
- ◦ $m_s =$ 92(83) MeV
- ◦ $m_s/m_{ud} =$ 20(16) MeV

- ◦ large errors come from continuum extrpoaltion

  $\Rightarrow$ calculation at small lattice spacing is needed.

4. Summary

qunched QCD : matured

- various methods have been tested and ready for applications.
  - non-perturbative improvement/renomalization
  - improved formulations

    (anisotropic lattices, domain-wall, overlap, twisted mass QCD)

  $\Rightarrow$ implementation in full QCD.

$N_f = 2$ full QCD : systematic invetigations by many groups

- sea quark effects in meson spectrum

  good! ... but, at finite lattice spacing (except CP-PACS)

- extensive simulations are still needed for

  scaling violation : fine lattices

  baryon : larger volumes

  test of ChPT : smaller sea quark masses

    high performance computer / "improved" formulations

- few progress for quark masses

  NP improvement/renormalization are required

$N_f = 3$ full QCD : real world!

- ready for systematic studies.

  $\Leftarrow$ developement of algorithm/study of phase structure

- several groups started simulations with KS/Wilson action

$\Rightarrow$ We hope see interesting results at Lattice2002!!