

# Light hadron spectrum and quark masses

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## study of light hadron spectrum:

- light hadron masses  $\Rightarrow$  test of QCD
- determination of fundamental parameters of QCD;  $\alpha_s, m_q$

## at lattice 2000:

### in quenched QCD : matured

- low-lying hadron and quark masses are determined precisely ... besides quenching effects

### in $N_f = 2$ QCD : in progress

- systematic studies of sea quark effects

### in $N_f = 3$ QCD : exploratory

- developing algorithm
- few studies

- B.Orth, "Full QCD beyond the rho decay threshold"
- C.Allton, "Scaling for Dynamical Fermion effects in UKQCD simulation"
- C.Gebert, "QCD spectroscopy' with light quarks"
- L.Levkova, "Anisotropic Lattice and Dynamical Fermions"
- D.Nelson, "Does  $m_q = 0$  ?"
- S.Hashimoto, "Exploration of sea quark effects in two-flavor QCD with the  $O(a)$  improved Wilson quark action"
- H.Neff, "Early plateau formulation for the  $\eta$  mass"
- K.Schilling, "A new approach to  $\eta'$  on the lattice"
- N.Ishizuka, " $I = 2$  Pion Scattering Length and Phase Shift with Wilson Fermions"
- T.Yamazaki, "Maximum entropy analysis of hadron spectral functions and excited states in quenched lattice QCD"
- J.Hein, "Mass renormalization for improved staggered quarks"
- J.Rolf, "Computation of the charm quark's mass in quenched QCD"
- O.Philipenson, "Non-perturbative parton masses"
- F.Lee, " $N^*$  Masses from an Anisotropic Lattice QCD Actions"
- H.B.Thacker, "Quenched Chiral Loop Effects in the Scalar Propagator"
- C.Rebbi, "First results from quenched QCD simulations with overlap fermions"
- Y.Aoki, "Hadron spectrum for quenched domain-wall fermions with DBW2 gauge action"
- C.McNeile, "Hadron spectroscopy of twisted mass lattice QCD at  $\beta = 6.0$ "
- A.Patel, "Transverse lattice QCD at strong coupling and large  $N$ "
- N.Ghaleh, "Transverse Momentum Distribution and Gluon Emission in Electron-Positron Annihilations at 60 GeV."
- J.McIntosh, "Independent plaquette state study of compact  $U(1)$  in 2+1 D utilizing an analytic Lanczos procedure"
- R.Frezzotti, "Quenched twisted mass QCD at small quark masses and in large volume"
- H.Moutrarde, "Preliminary calculation of  $\alpha_s$  from Green functions with dynamical quarks"
- N.Eicker, "Light Quark Masses with  $N_f = 2$  Wilson fermion: an Update from SESAM/T $\chi$ L"
- A.Nakamura, "Study of sigma meson on the lattice"
- T.Draper, "Quenched Chiral Behavior of Hadron Masses with Overlap Fermions"
- D.Hepburn, "Light hadron spectrum using  $O(a)$  improved action with  $N_f = 2$ "
- A.Hart, "Topology and the scalar glueball with  $N_f = 2$  improved Wilson fermions"
- K.Jansen, "Scalar Condensate and light-quark masses from overlap fermions"
- S-J.Dong "Pion decay constant,  $Z_A$ , and quark masses from overlap fermions"

quark mass"

- S.Hawswirth, "First results from a parametrized fixed point action"
- U.Wolff, "First results on the running coupling in QCD with two massless flavors"
- H.Matsufuru, "Anisotropic  $O(a)$  improved Wilson quark action and hadron spectroscopy"
- T.Umeda, " $O(a)$  improved Wilson quark action on anisotropic lattice"

- results with KS action  $\Rightarrow$  review by Toussaint
- flavor singlet  $\Rightarrow$  review by Edwards
- Ginsparg-Wilson fermion  $\Rightarrow$  review by Kikukawa  
 $\Rightarrow$  review by Hernandez
- my talk : studies with Wilson-type action

## 1. light hadron spectrum

- in quenched QCD
- in  $N_f = 2$  full QCD

search for sea quark effects

- in  $N_f = 3$  full QCD

## 2. test/application to chiral perturbation theory

## 3. determination of fundamental parameters of QCD

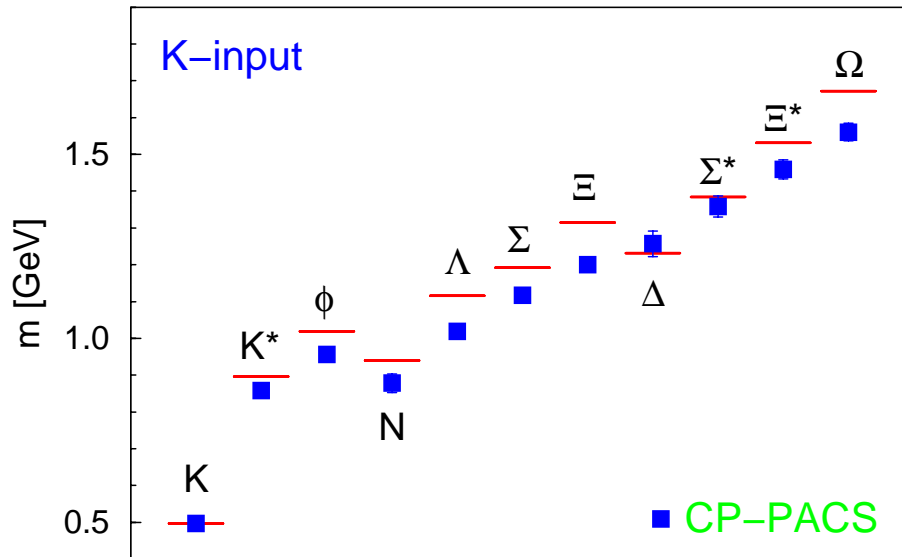
- strong coupling
- quark masses

## 4. summary

# 1. light hadron spectrum

— before lattice2000 —

ground states : **already** studied well!!



decay constants, quark masses: **non-perturbatively determined**  
(ALPHA, APE, UKQCD, QCDSF...)

— this year —

## 1. testing ground for new formulations

- anisotropic
- Ginsparg-Wilson fermions (domain-wall, overlap)
- twisted mass QCD

### for improvement

- non-perturbative improvement

## 2. study of heavier/excited states

- negative parity baryon

- calculation of meson spectrum

on anisotropic lattice

Matsufuru, *et al.*

plaq + clover (tadpole improved  $c_{\text{SW}}$ ),

$$a_s^{-1} = 1.1\text{-}2.0 \text{ GeV}, \quad La \sim 2 \text{ fm}, \quad 200\text{-}100\text{conf}, \quad \xi = 4$$

domain-wall fermion

RBC(Y.Aoki):

DBW2 gauge action(QCDTARO)

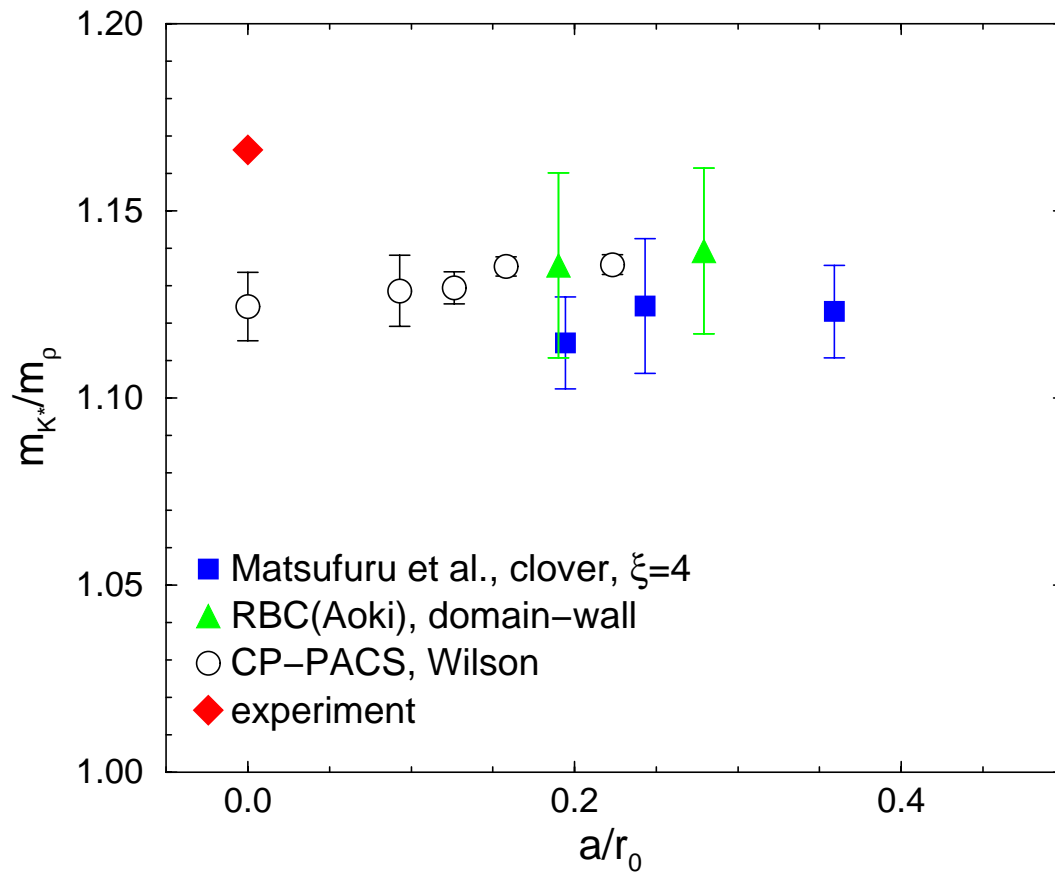
$$(a^{-1}, L) \sim (1.3 \text{ GeV}, 2.4 \text{ fm}), (2.0 \text{ GeV}, 1.6 \text{ fm}),$$

$$\sim 100 \text{ conf}, \quad N_5 = 16, \quad M_5 = 1.8$$

$\Rightarrow$  consistency with previous calculations using Wilson/KS.



$$\underline{m_{K^*}/m_\rho}$$



- consistent with previous calculation with 2-3% accuracy
- confirm small  $m_{K^*} - m_K$  in quenched QCD

- light quark masses with non-perturbative renormalization

Giusti, *et al.*:

$$a^{-1} \sim 2 \text{ GeV}, \quad L \sim 1.4 \text{ fm}, \quad 54 \text{ conf}$$

Dong, *et al.*:

$$a^{-1} \sim 1.2 \text{ GeV}, \quad L \sim 3.2 \text{ fm}, \quad 63 \text{ conf}$$

$\Rightarrow$  (see below)

- suppress exceptional conf at small quark masses

$$D_{\text{tmQCD}} = D_{\text{Wilson}} + i\mu_q \gamma_5 \tau_3$$

$\Rightarrow$  lower bound for eigenvalues of  $D_{\text{tmQCD}}$

McNeile, *et al.*:

$$a \sim 0.09 \text{ fm}, \quad L = 1.4 \text{ fm}$$

Della Morte, *et al.*:

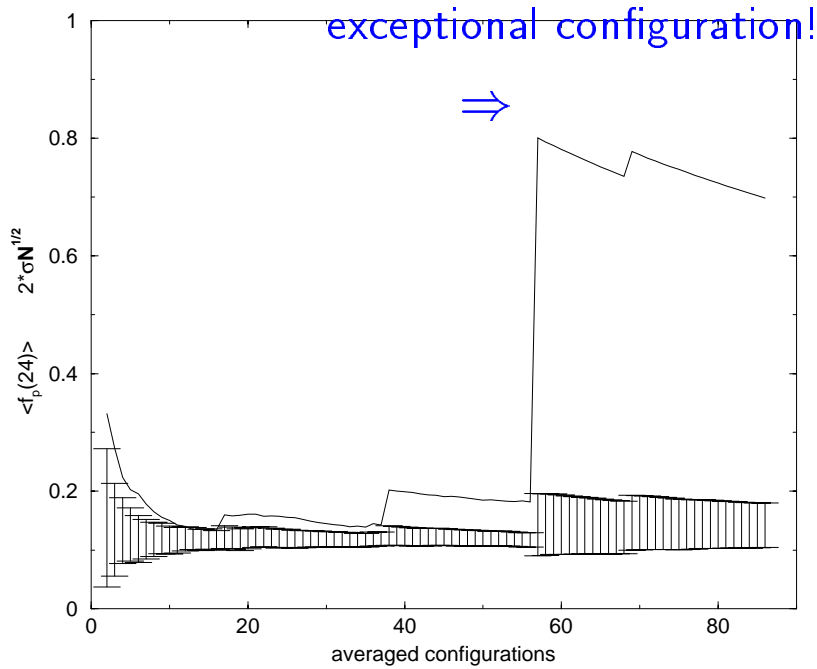
$$a \sim 0.07, 0.09 \text{ fm}, \quad L/a = 16, 24, \quad T/L = 2, 3, \\ m_{\text{PS}}/m_{\text{V}} \geq 0.47$$

plot  $\sigma\sqrt{N_{\text{conf}}}$  vs  $N_{\text{conf}}$      $\sigma$  = error of  $\langle PP^\dagger \rangle$ ,

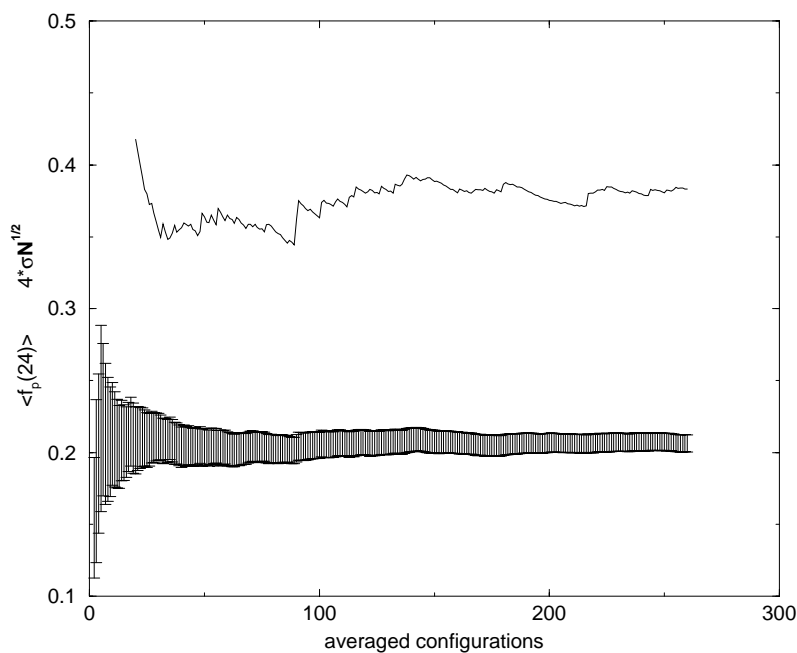
$$\sigma \propto 1/\sqrt{N_{\text{conf}}} \Rightarrow \sigma\sqrt{N_{\text{conf}}} \sim \text{const}$$

$$m_{\text{PS}}/m_{\text{V}} \sim 0.47$$

## 2 $\sigma\sqrt{N}$ vs statistics in standard QCD



## 4 $\sigma\sqrt{N}$ vs statistics in twisted mass QCD



- exceptional configurations are suppressed in tm QCD.

- non-perturbative improvement

⇒ precise determination of hadron and quark masses

- at Lattice 2000

$c_A$  from Schrödinger functional(SF)

$c_A$  from Ward identity(WI)

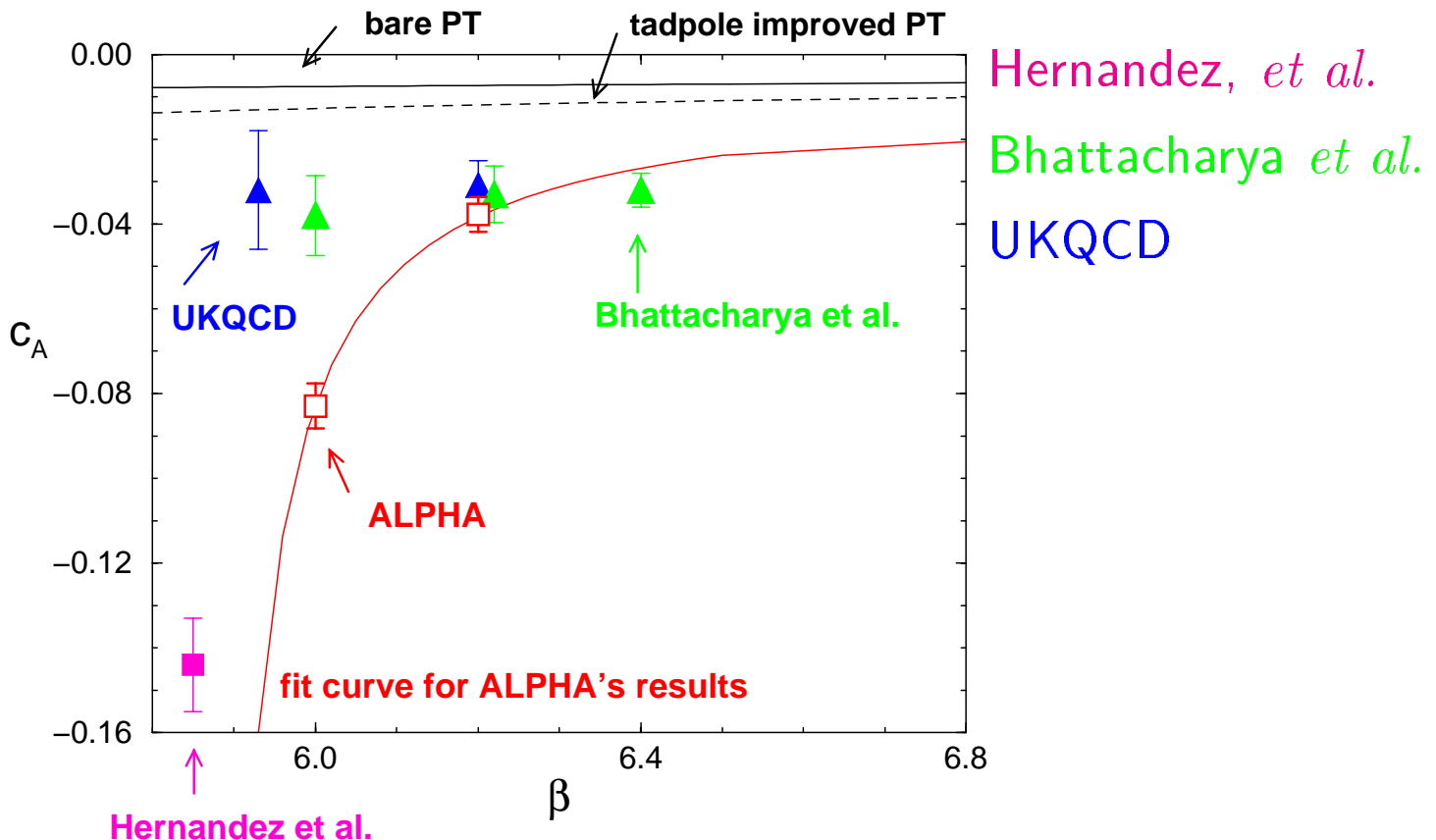
⇒ different values

- new results for improvement coefficient  $c_A$

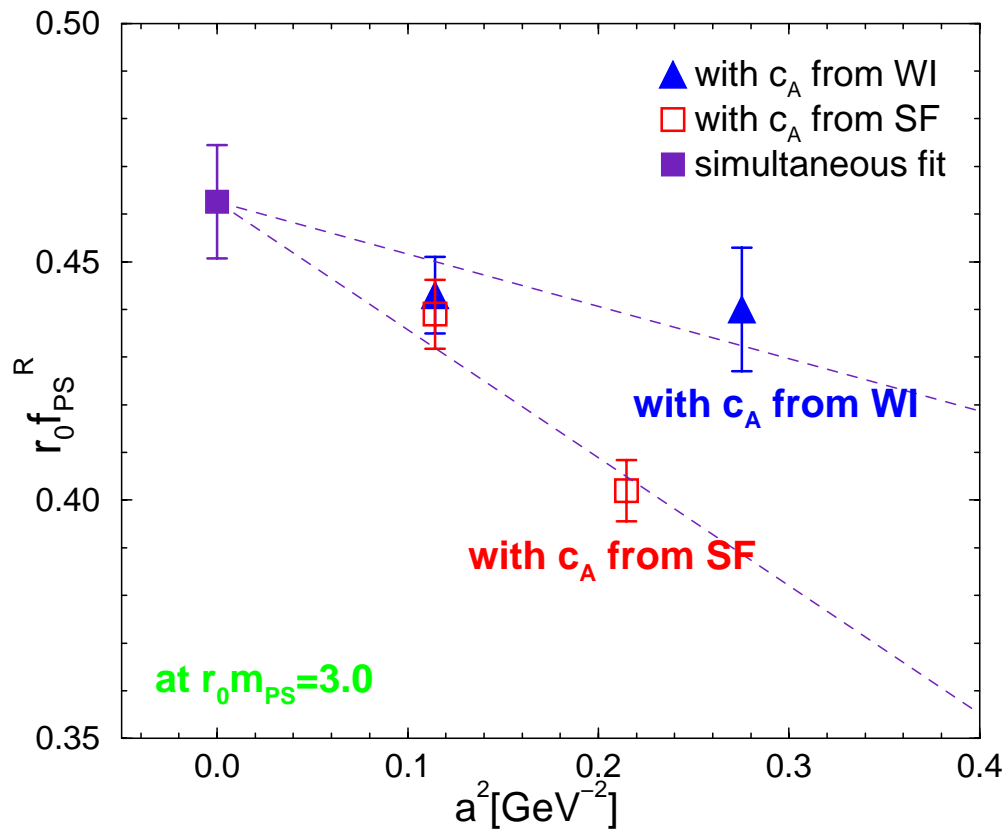
Hernandez *et al.*: Schrödinger functional; at  $\beta=5.85$

Bhattacharya *et al.*: Ward identity; at  $\beta=6.0, 6.2, 6.4$

UKQCD(Collins): Ward identities; at  $\beta=5.93, 6.2$



UKQCD



choice of improvement condition  $\Rightarrow O(a)$  difference in  $c_A$

excited baryons  $\Leftarrow$  experimental study in Jefferson Laboratory

negative parity baryon

$$N^*, \quad J^P = 1/2^-$$

$\Rightarrow$  parity mass splitting  $N^* - N$

until Lattice 2000

Lee and Leinweber, LHPC, LHPC+UKQCD,...

- at large quark masses, or at fixed  $a$
- finite size effects?

this year

- [Sasaki et al.](#)

domain wall,  $a^{-1} \sim 2 \text{ GeV}$ ,  $L = 1.6 \text{ fm}$

- [LHPC+QCDSF+UKQCD](#)

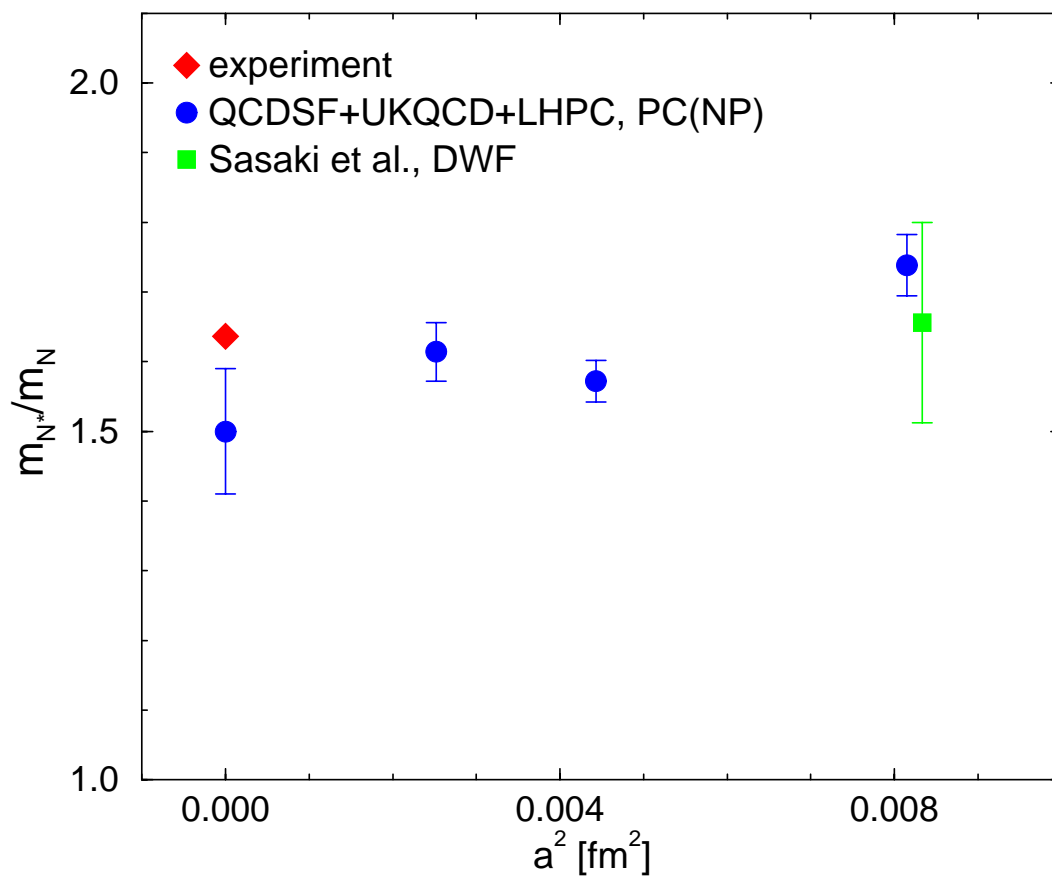
systematic study of scaling violation, finite size effects

clover(NP  $c_{\text{SW}}$ ),  $a^{-1} \sim 2\text{-}4 \text{ GeV}$ ,  $L = 1.5\text{-}2.2 \text{ fm}$

- [Lee et al.](#)

anisotropic ( $\xi = 3$ ),  $D_{234}$ ,  $a_t^{-1} \sim 2.5 \text{ GeV}$ ,  $L = 2.4 \text{ fm}$

chiral extrapolation has not been performed



- reproduce experimental  $N^*$  -  $N$  splitting within 10% accuracy.
- interesting to see other excitation
  - $N'(1440)$ ,  $\Lambda(1405)$ , ...



## 1.2 $N_f = 2$ QCD

## recent studies in $N_f=2$ full QCD

group	action	$a_t$ [fm]	$La_s$ [fm]	$m_{PS}/m_V$
SESAM	P-W	0.07	1.2	0.69-0.83
$T_\chi L$	P-W	0.07	1.8	0.55, 0.7
SESAM/ $T_\chi L$	P-W	0.09	1.4	0.68-0.85
GRAL	P-W	0.12	1.9	0.4
UKQCD	P-C(1.76)	0.12	0.98-1.9	0.67-0.86
UKQCD	P-C(NP)	0.10	1.6	0.69-0.83
QCDSF	P-C(NP)	0.09	2.4, 1.6	0.69, 0.76
JLQCD	P-C(NP)	0.09	1.0-1.7	0.60-0.80
CP-PACS	R-C(TP)	0.11-0.22	2.5-2.6	0.55-0.81
Columbia	P-KS	0.09	1.5	0.57-0.70
MILC	P-KS	0.11-0.3	2.4-3.7	0.3-0.8
Columbia	PKS( $\xi \neq 1$ )	0.06-0.19	3.2-5.4	0.3-0.8
MILC	Sy-imp.KS	0.13	2.6	0.50

results with staggered fermion  $\Rightarrow$  review by Toussaint.

results with Wilson fermion  $\Rightarrow$  this talk.

Columbia(Levkova) : anisotropic lattices  $\rightarrow$  phase transition.

GRAL(Orth) : beyond  $\rho \rightarrow \pi\pi$  threshold.

SESAM/ $T_\chi L$ (Eicker) : light quark masses.

UKQCD+QCDSF(Schierholz) : strong coupling

- the main issue

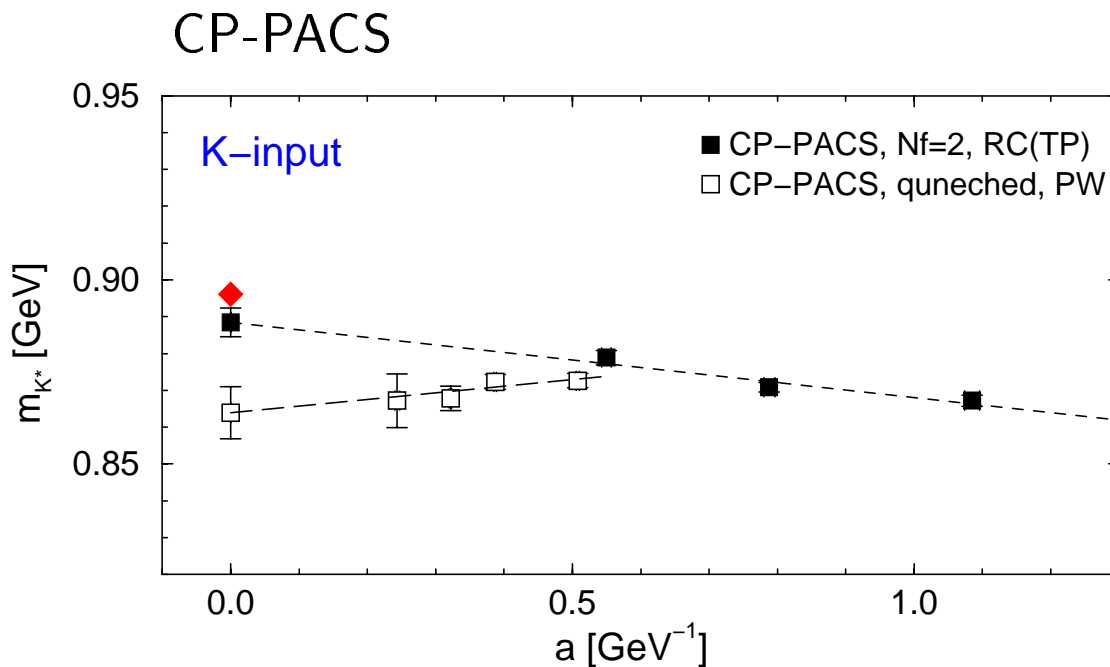
## search for sea quark effects

small hyperfine splitting in quenched QCD

⇒ closer to experiment in  $N_f=2$  QCD?

SESAM/ $T_\chi$ L, UKQCD, CP-PACS, QCDSF, JLQCD, MILC, ...

at Lattice 2000 (review by S.Aoki)



⇒ Vector meson masses are closer to experiment

⇒ But after continuum extrapolation

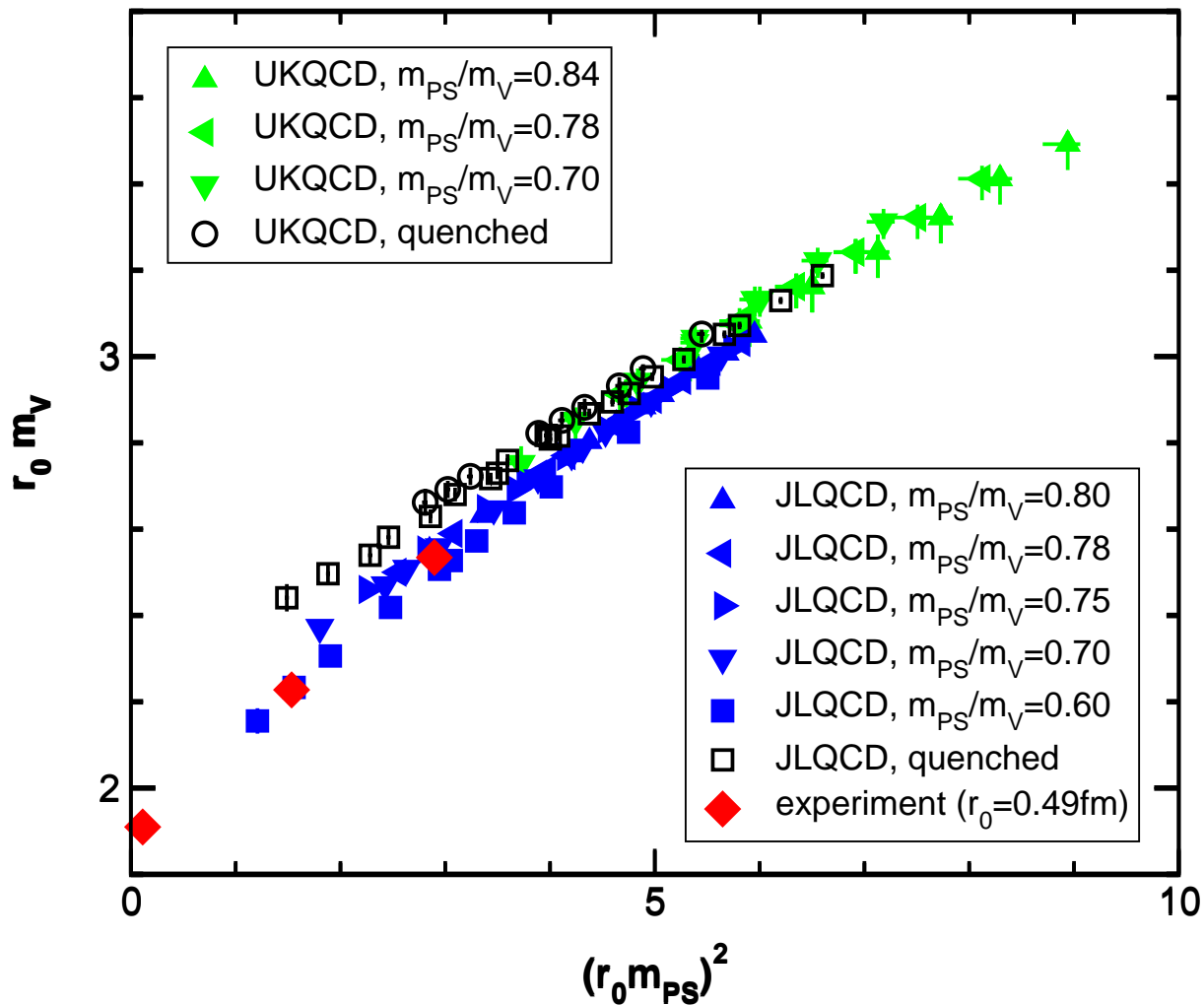
## UKQCD

- clover, non-perturbative  $c_{\text{SW}}$
- fixed  $a/r_0$  ( $a \sim 0.1$  fm)
- $L \sim 1.6$  fm
- $m_{\text{PS}}/m_{\text{V}} \geq 0.7$

## JLQCD

- clover, non-perturbative  $c_{\text{SW}}$
  - fixed  $\beta$  ( $a \sim 0.1$  fm at physical  $m_{q,\text{sea}}$ )
  - $L \sim 1.1, 1.4, 1.8$  fm ( $\Rightarrow$  finite size effect)
  - $m_{\text{PS}}/m_{\text{V}} \geq 0.6$
- 
- UKQCD, JLQCD : leading scaling violation  $O(a^2)$ 
    - $\Rightarrow$  smaller scaling violation compared to CP-PACS
    - CP-PACS : RG gauge + tadpole improved clover
  - should provide an interesting check of sea quark effects.

## meson masses at simulation points



- UKQCD

- $m_{PS}/m_V \geq 0.7$

- sea quark effect is not clear from this plot.

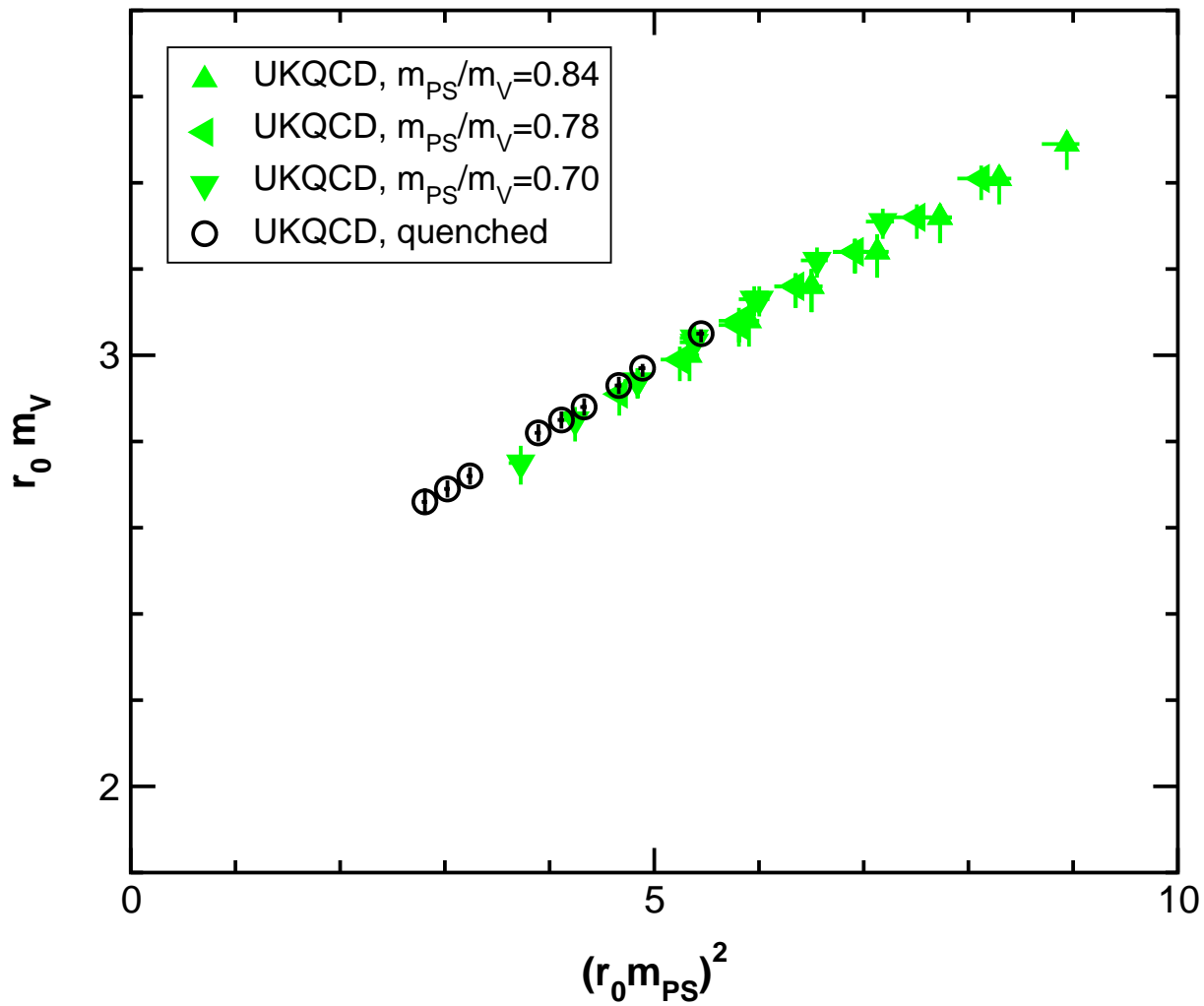
- JLQCD

- clear sea quark effects

- larger slope  $\Rightarrow$  larger hyperfine splitting

## meson masses at simulation points

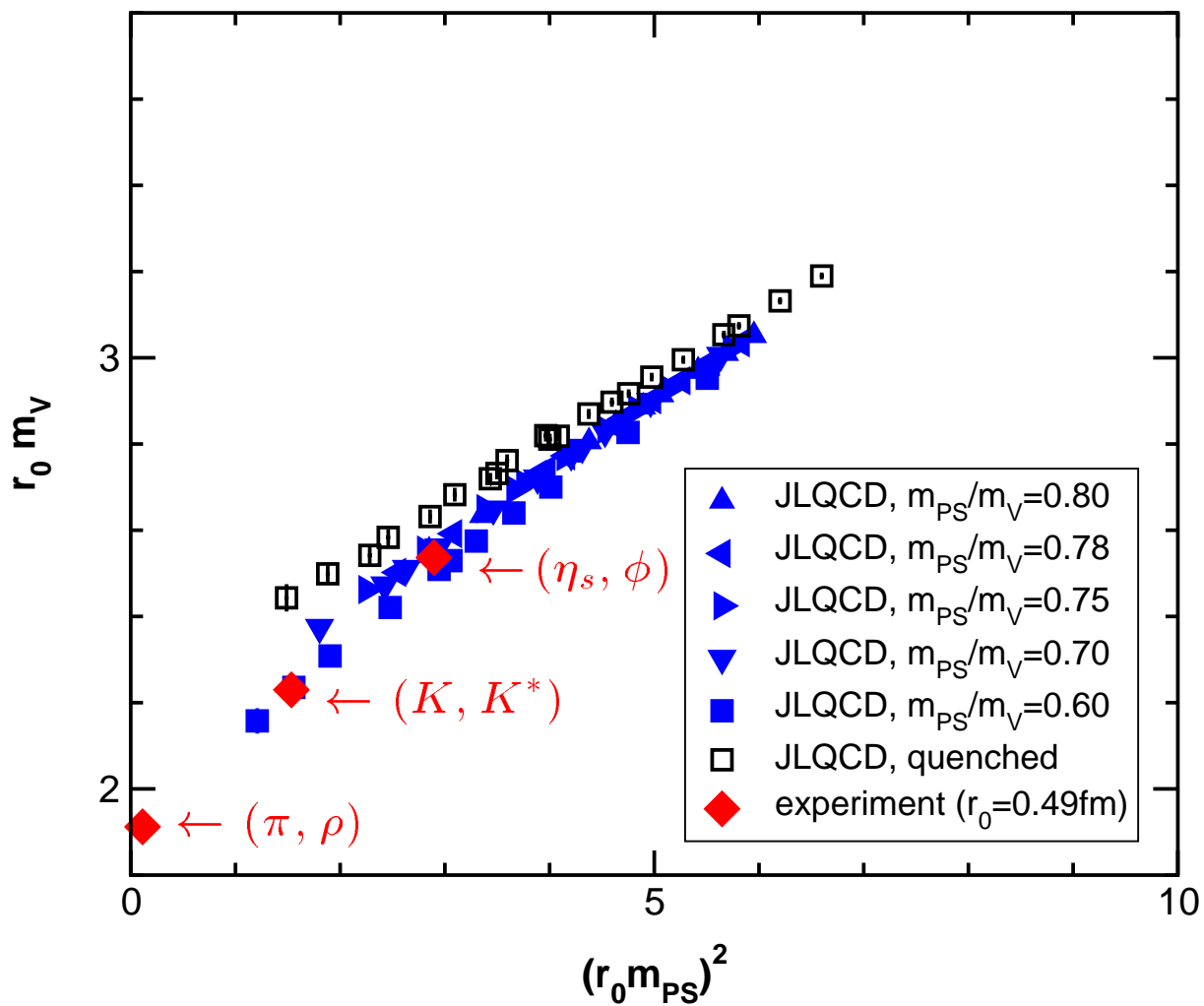
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- UKQCD

- $m_{PS}/m_V \geq 0.7$

- sea quark effect is not clear from this plot.



- JLQCD

- $m_{PS}/m_V \geq 0.6$

- clear sea quark effect

larger slope  $\Rightarrow$  larger hyperfine splitting

$$m_V \frac{dm_V}{dm_{PS}^2} \quad @ \quad \frac{m_V}{m_{PS}} = \frac{m_{K^*}}{m_K} \quad (\text{Lacock and Michael})$$

- monitor the slope of “ $m_V$  vs  $m_{PS}^2$ ” plot

in full QCD

1) calculate  $J$  at fixed sea quark masses, and take  $m_{\text{sea}} \rightarrow m_{ud}$

$$J \equiv \lim_{m_{\text{sea}} \rightarrow m_{ud}} J(m_{\text{sea}}), \quad J(m_{\text{sea}}) \equiv m_V \frac{dm_V}{dm_{PS}^2} \Big|_{m_{\text{sea}}}$$

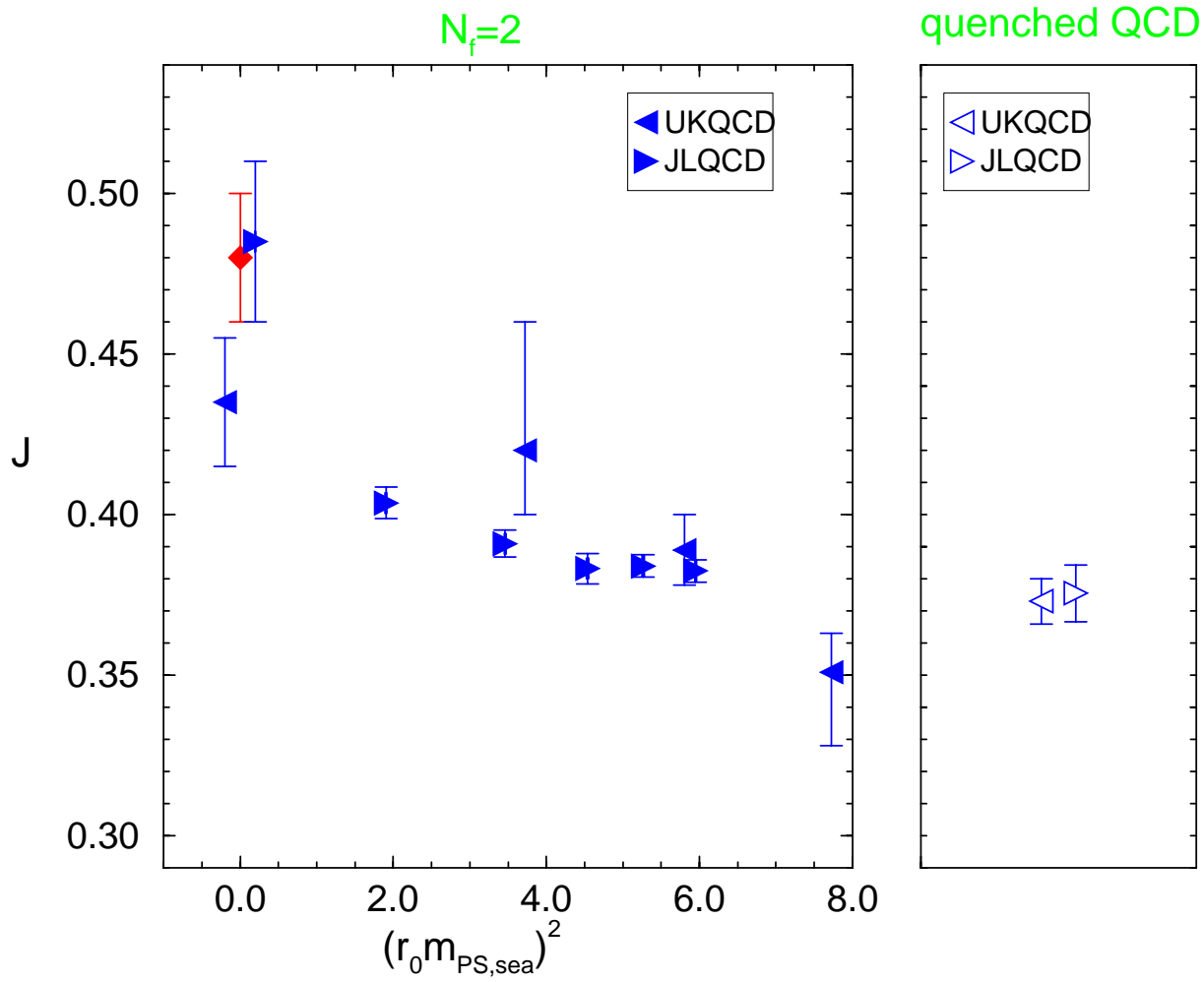
2) calculate  $J$  from meson masses at physical quark mass

$$J \equiv m_{K^*} \frac{m_{K^*} - m_\rho}{m_K^2 - m_\pi^2}$$

3) calculate  $J$  as mass ratio at simulation point(MILC)

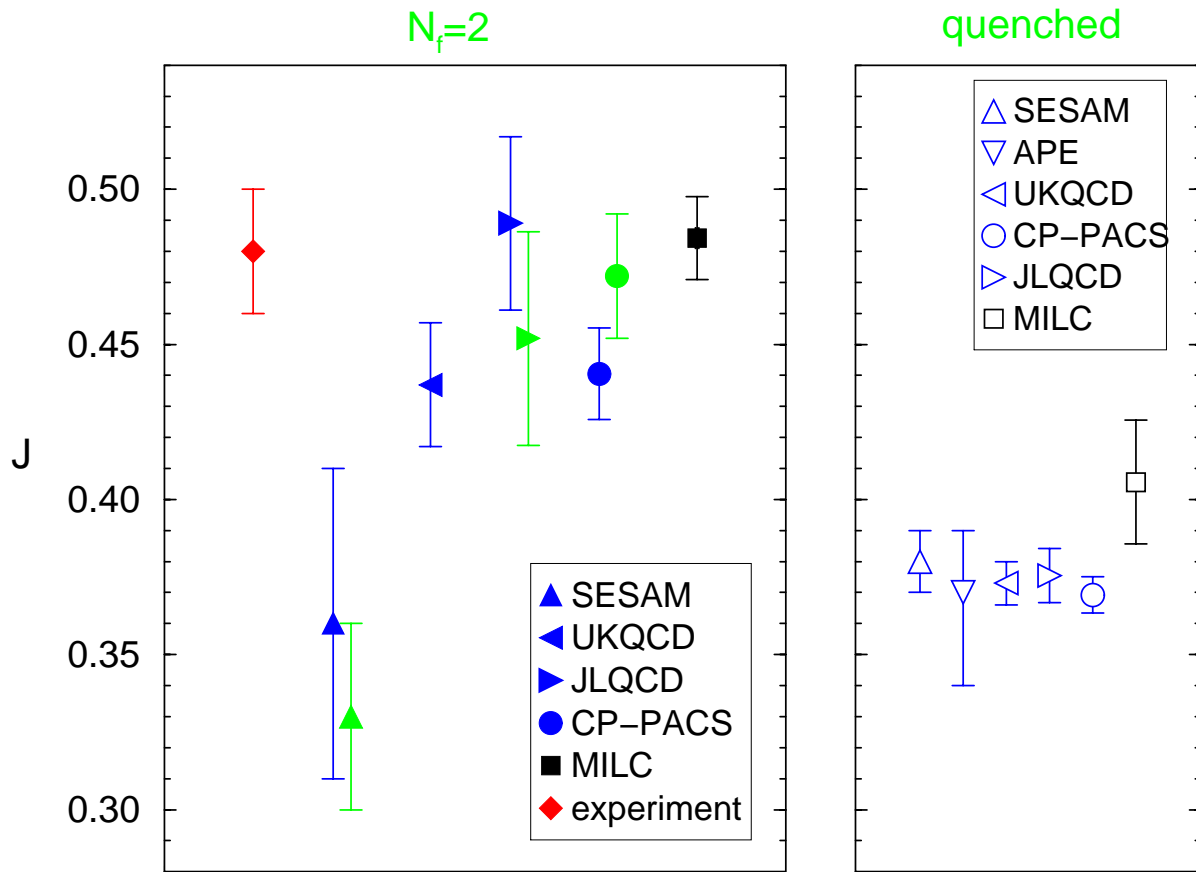
$$J \equiv m_V(m_1, m_2) \frac{m_V(m_2, m_2) - m_V(m_1, m_1)}{2(m_{PS}(m_1, m_2)^2 - m_{PS}(m_1, m_1)^2)}$$





1)  $J$  at fixed sea quark masses, and take  $m_{\text{sea}} \rightarrow m_{ud}$

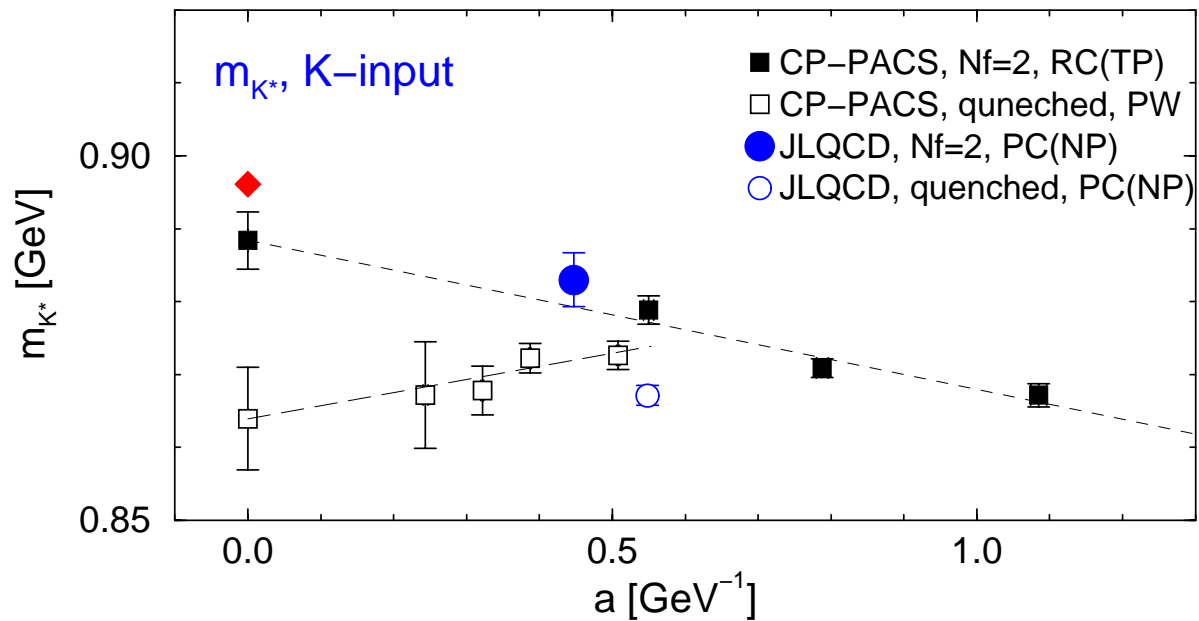
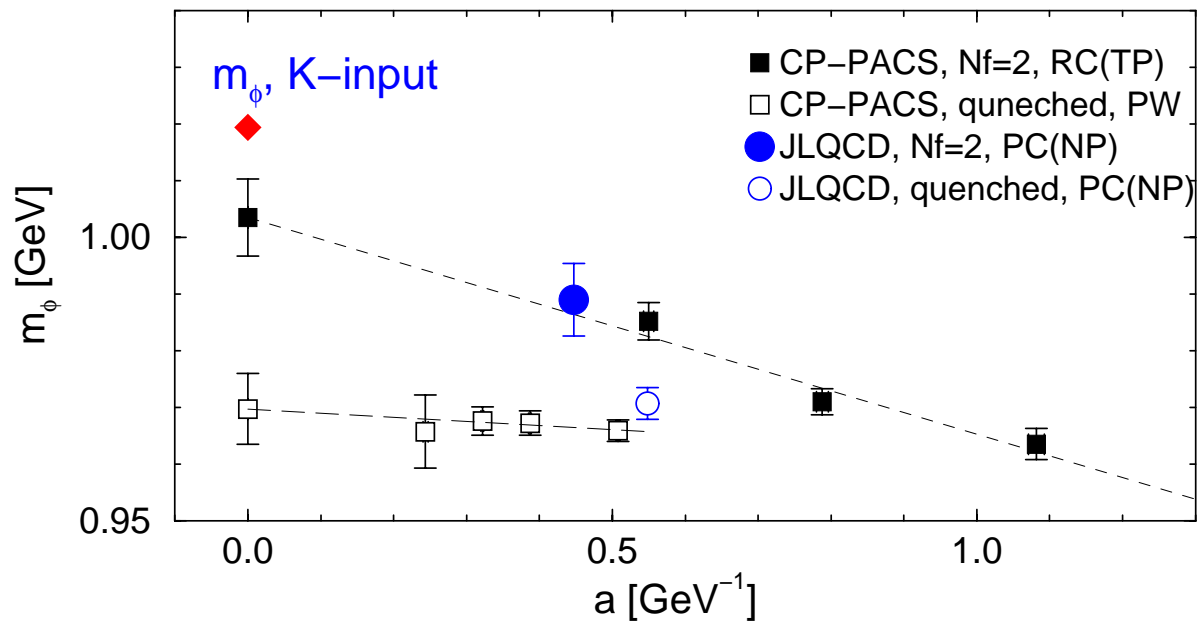
- $J$  increases as sea quark mass decreases



- 1)  $J$  at fixed sea quark masses, and take  $m_{\text{sea}} \rightarrow m_{ud}$
- 2)  $J$  from meson masses at physical quark mass
- 3)  $J$  as mass ratio at simulation point(MILC)

• sea quark effects  $\Rightarrow$  **closer to experiment**

except SESAM  $\Leftarrow$  heavy sea quark mass ( $m_{\text{PS}}/m_{\text{V}} \geq 0.69$ ) ?



- sea quark effect is confirmed using different action

JLQCD: plaq + clover(non-perturbative  $c_{SW}$ )

CP-PACS: RG + clover(tadpole improved  $c_{SW}$ )

- recent studies of baryon spectrum

CP-PACS :  $V \sim 2.5$  fm.

UKQCD :  $V \sim 1.6$  fm.

JLQCD :  $V \sim 1.1 - 1.8$  fm.

These volumes are sufficiently large?

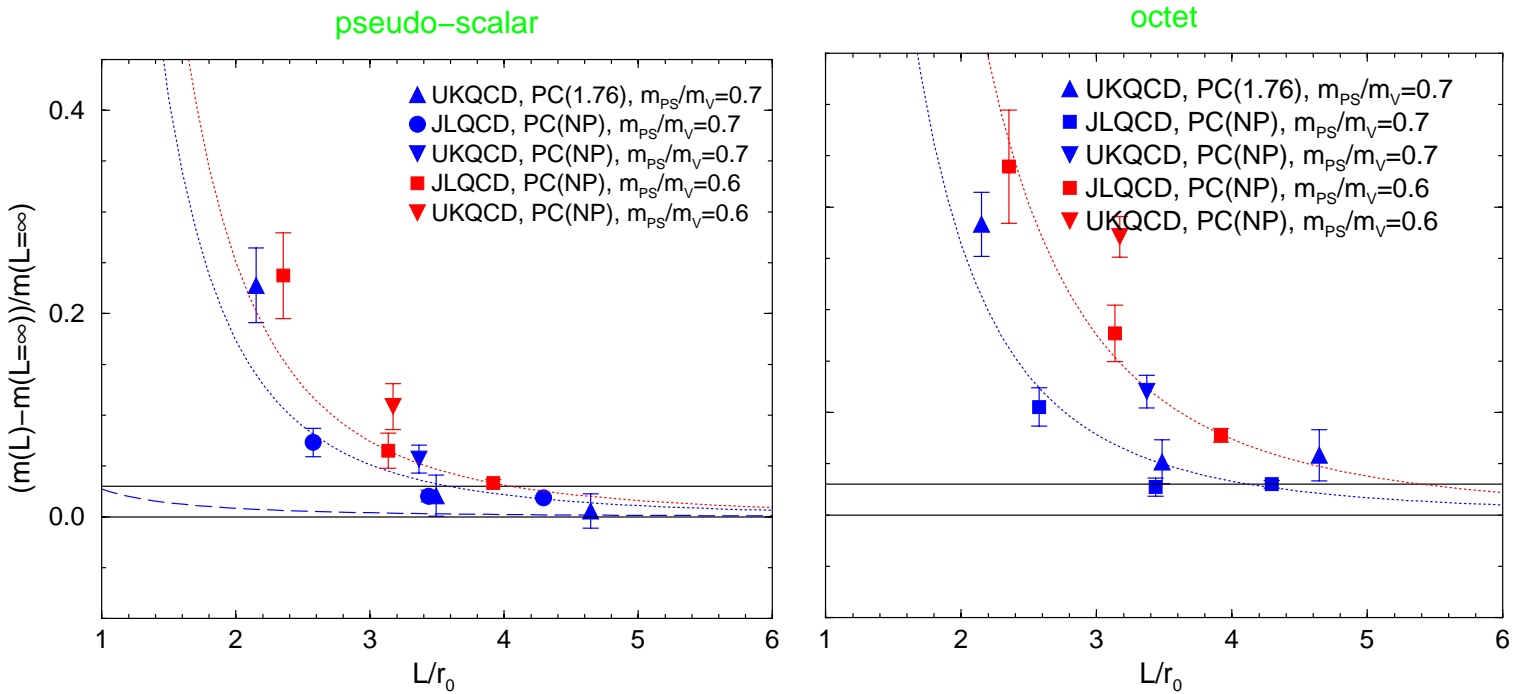
⇒ check finite size effects in JLQCD and UKQCD data

- mass shift at finite volume

$$(m(L) - m(\infty))/m(\infty) \quad \text{vs} \quad L/r_0$$

blue symbols : at  $m_{PS}/m_V=0.7$

red symbols : at  $m_{PS}/m_V=0.6$

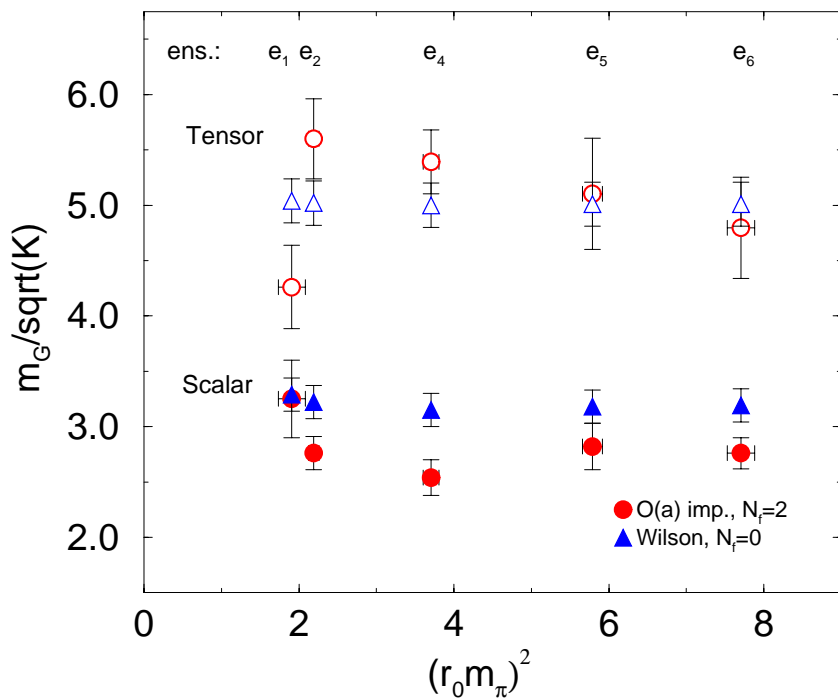


- volume for  $FSE \leq 3\%$ , at  $a=0.1$  fm

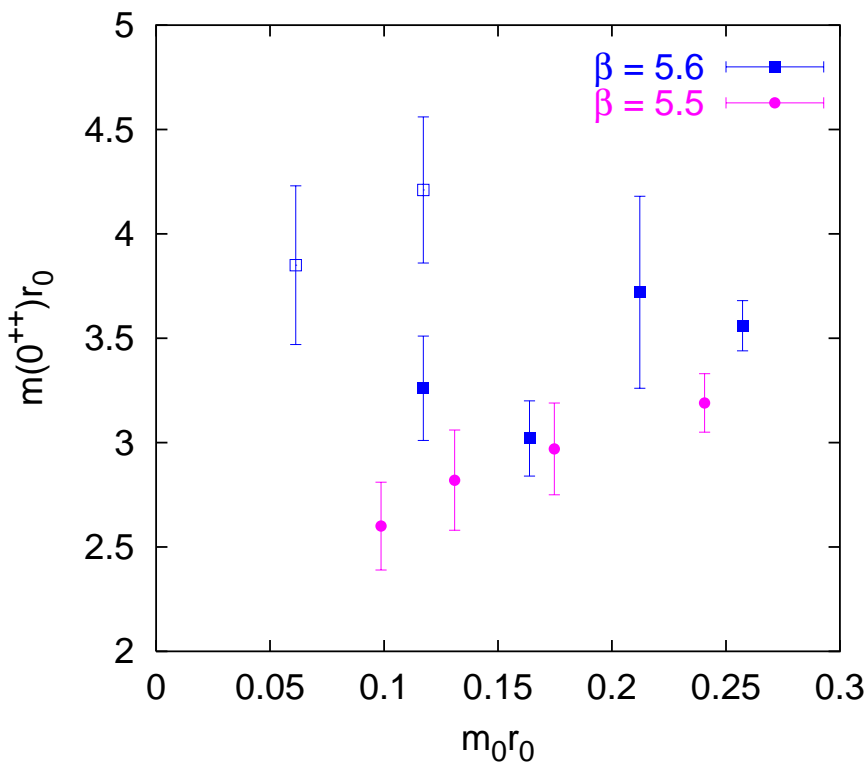
	$m_{PS}/m_V \sim 0.7$	$m_{PS}/m_V \sim 0.6$
meson	$16^3$	$20^3$
baryon	$24^3$	$32^3$

- much larger volumes are required for baryons!
- still, we haven't obtained definite conclusions

UKQCD :  $\sim 15\%$  decrease in  $0^{++}$  at  $a^{-1} \sim 2$  GeV



SESAM :  $\sim 25\%$  decrease in  $0^{++}$  at  $a^{-1} \sim 2.7$  GeV



- decreases in  $0^{++} \Rightarrow$  indication of sea quark effects?
- need more precise data and study of scaling violation.

## SCALAR collaboration (Nakamura)

- calculation of scalar meson in  $N_f = 2$  full QCD

Wilson fermion,  $a = 0.2$  fm,  $m_{PS}/m_V = 0.9$

$\Rightarrow$  poster by Nakamura

## 1.3 $N_f = 3$ QCD



this year

- with KS fermion

## MILC

- improved KS
- standard hybrid-R

- with Wilson-type action

algorithm for odd number of flavors  $\Rightarrow$  review by Peardon

exact algorithms have been developed.

- multi-boson (Lüscher, Borici and de Forcrand, Alexandrou *et al.*)
- polynomial HMC (Takaishi and Forcrand)

## Gebert *et al.*

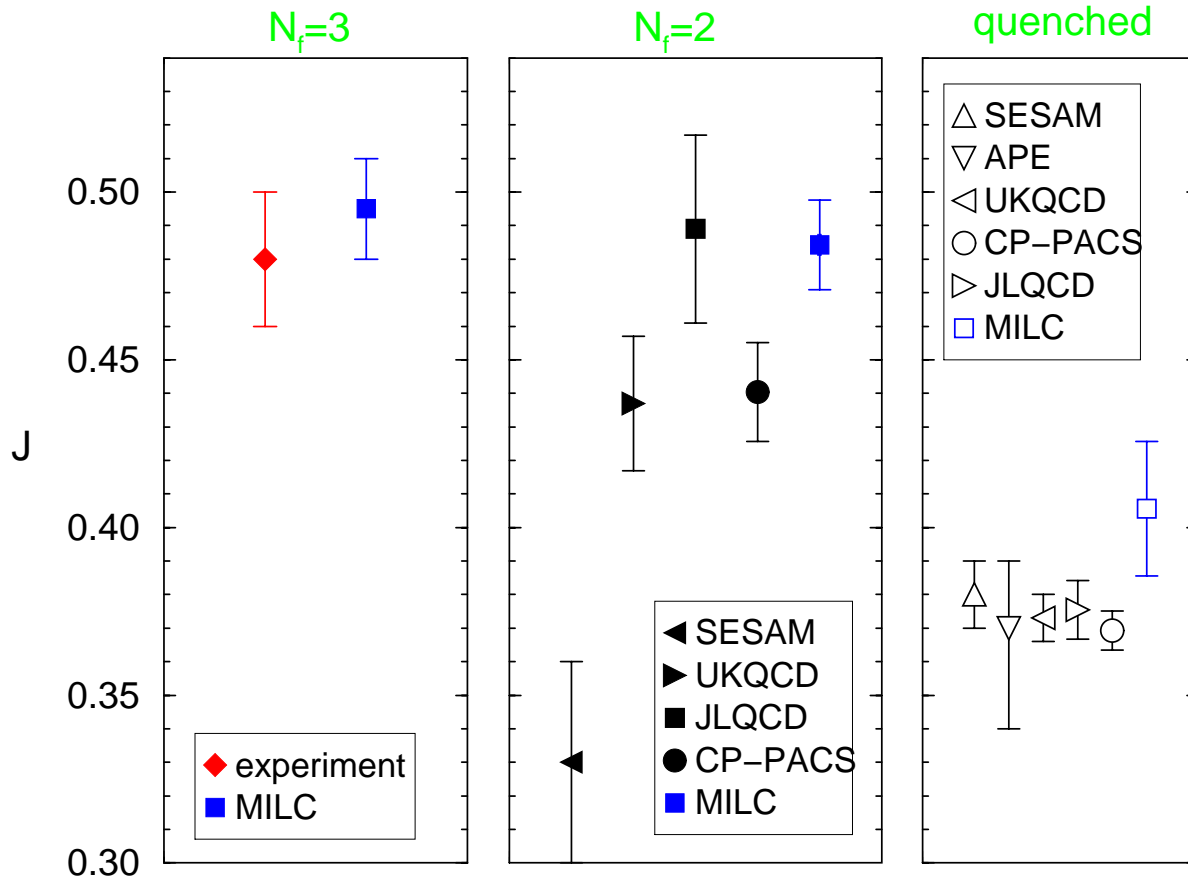
- plaquette gauge + Wilson
- two-step multi-boson (Montvay)

## JLQCD

- plaquette gauge + clover
- polynomial HMC (Ishikawa(JLQCD))

## MILC ( $\Rightarrow$ review by Toussaint )

$J$  parameter:



- MILC's results

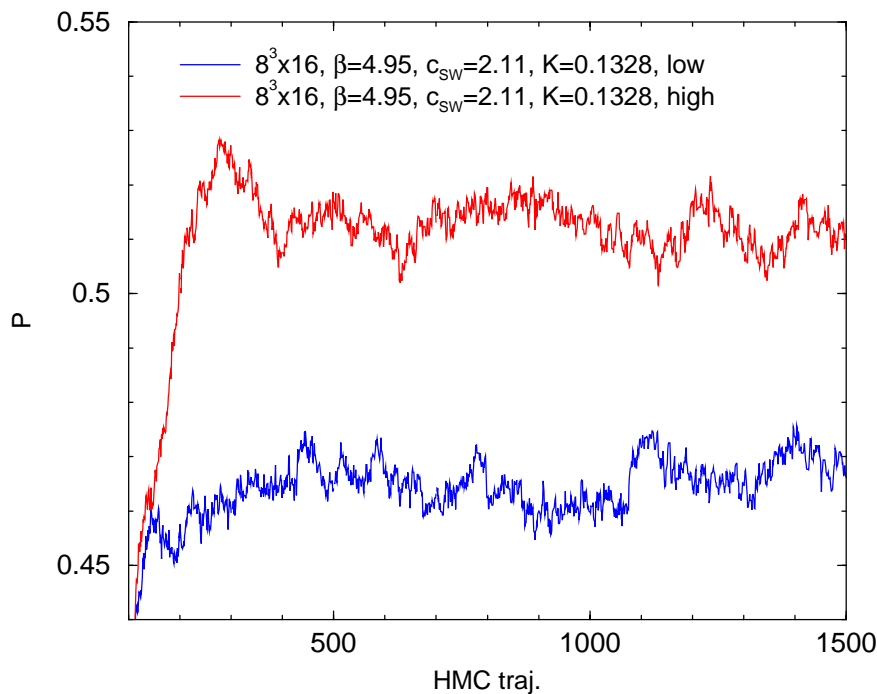
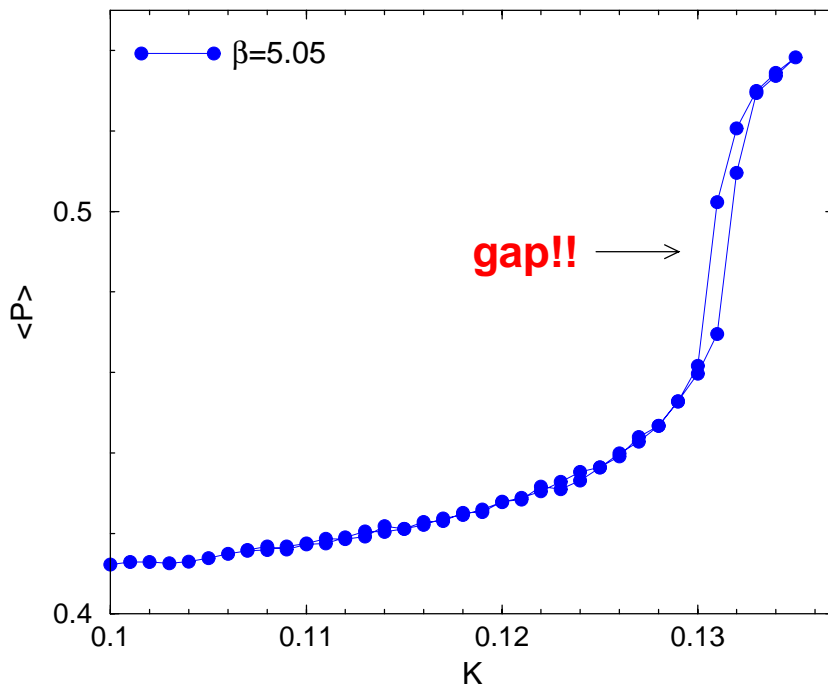
- clear sea quark effects

- consistent with experiment in full QCD

- no systematic deviation between KS and Wilson/clover

## JLQCD(Okawa)

- first systematic simulations with  $O(a)$  improved Wilson action  
degenerated 3 flavors, tadpole improved  $c_{SW}$   
study of phase structure  $\Rightarrow$  study of light hadron spectrum
- unexpected phase transition is found!!



$$8^3 \times 16, \beta=4.95$$

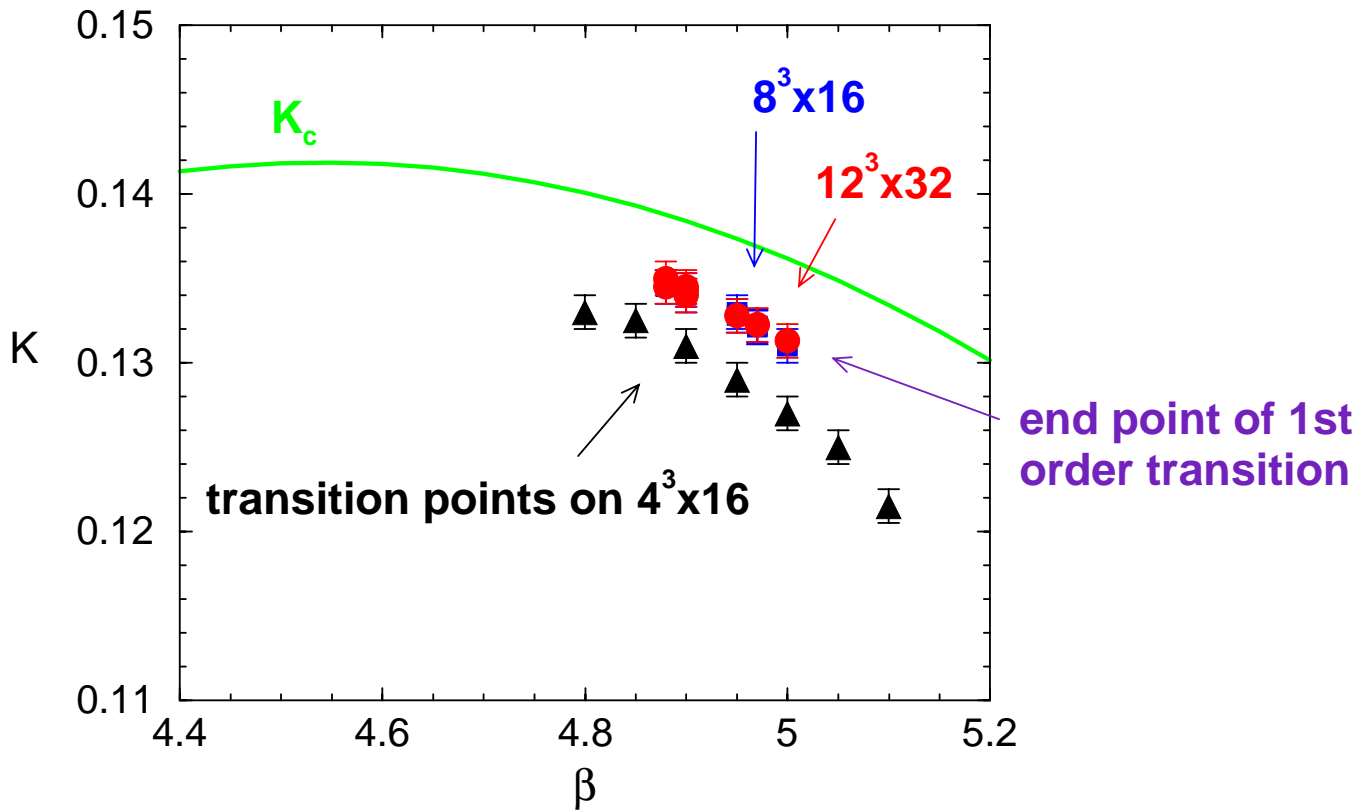
$$K_{ud} = K_s = 0.1328$$

$$a^{-1} = 1.393(13) \text{ GeV}$$

$$m_{PS}/m_V = 0.87$$

$$a^{-1} = 0.827(4) \text{ GeV}$$

$$m_{PS}/m_V = 0.86$$



- bulk transition at  $T = 0$ .  $\Rightarrow$  disappears at  $\beta \sim 5.0$   
 $a^{-1} \sim 2.6$  GeV at the end point
- this phase transition = lattice artifact
  - plaquette gauge + clover quarks  $\Rightarrow a^{-1} > 2.6$  GeV.
- phase transition disappears for
  - improved gauge + clover quark
  - plaquette/improved gauge + Wilson quark
- with improved gauge and/or Wilson quark  
 $\Rightarrow$  coarse lattices are OK!

## 2. test/application of ChPT

- test of ChPT using lattice data

- simulated quark masses  $\gg m_{ud}$

- $\Rightarrow$  ChPT is used as a guide for extrapolation to  $m_{ud}$

- tests of ChPT at simulated quark mass: important!

- $\Leftarrow$  fit lattice data ( $m_{PS}$  and  $f_{PS}$ ) to ChPT formulae.

- determination of low-energy constants  $\alpha_i$  ( $i=1-10$ )

(recent work by Sharpe and Shores)

- fit lattice data to ChPT formulae

- $\Rightarrow$  low-energy constants

- $\Rightarrow$  information on physics in real world from ChPT

- $m_u = 0$ ? (Cohen, Kaplan and Nelson)

phenomenological value : “standard”  $\alpha_8 \sim 0.8$

“ $m_u = 0$ ”  $\alpha_8 \sim -0.9$

(Gasser, Leutwyler, Bijnens, Ecker, ..., 1984-1992)

studies in quenched QCD

- Bardeen *et al.*, 2000

- ALPHA, 2000

## UKQCD : first estimate of $\alpha_8$ in $N_f=2$ full QCD

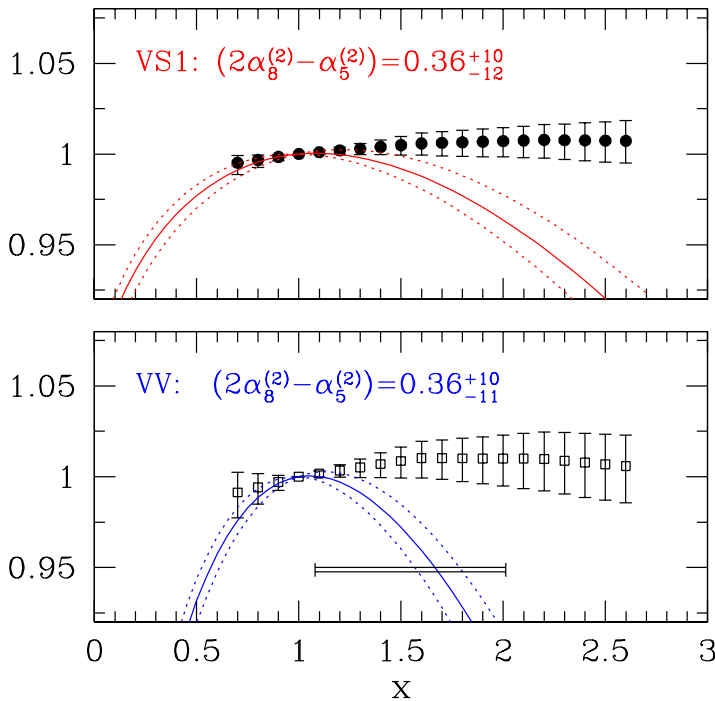
- “ratio” method of ALPHA, PQChPT formulae by Sharpe,

$$R_M^{VV} \equiv \frac{2y/m_{\text{PS}}^2(y)}{2y_{\text{ref}}/m_{\text{PS}}^2(y_{\text{ref}})} = (x-1) y_{\text{ref}} (\underline{2\alpha_8 - \alpha_5}) + \text{log terms}$$

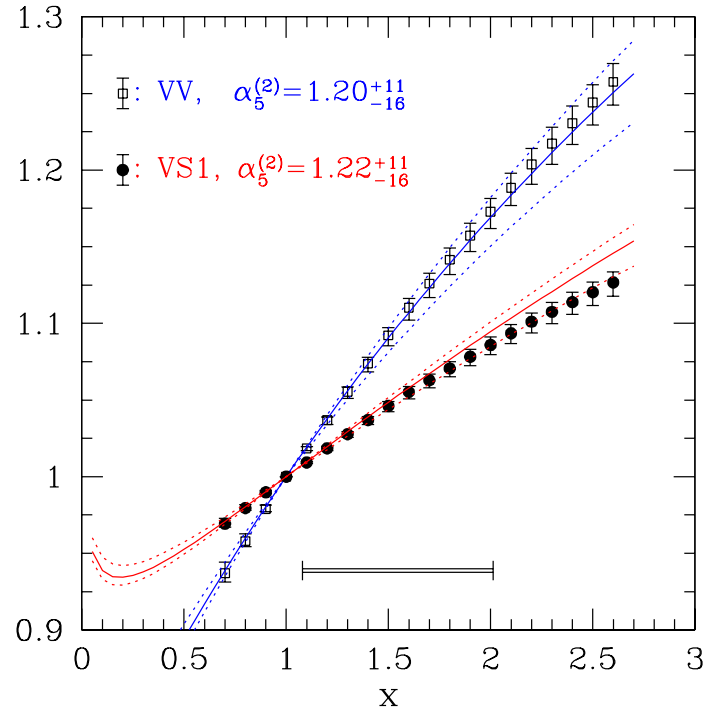
$$R_F^{VV} \equiv \frac{f_{\text{PS}}(y)}{f_{\text{PS}}(y_{\text{ref}})} = (x-1) y_{\text{ref}} \underline{\alpha_5}/2 + \text{log terms}$$

$$y \equiv B_0 m_q / (4\pi F)^2, \quad x = y/y_{\text{ref}}$$

(b):  $R_M$



(a):  $R_F$



$$\bullet \alpha_5^{N_f=2} = 0.99(6)(20), \quad \alpha_8^{N_f=2} = 0.67(4)(20) \Rightarrow m_u \neq 0$$

but...

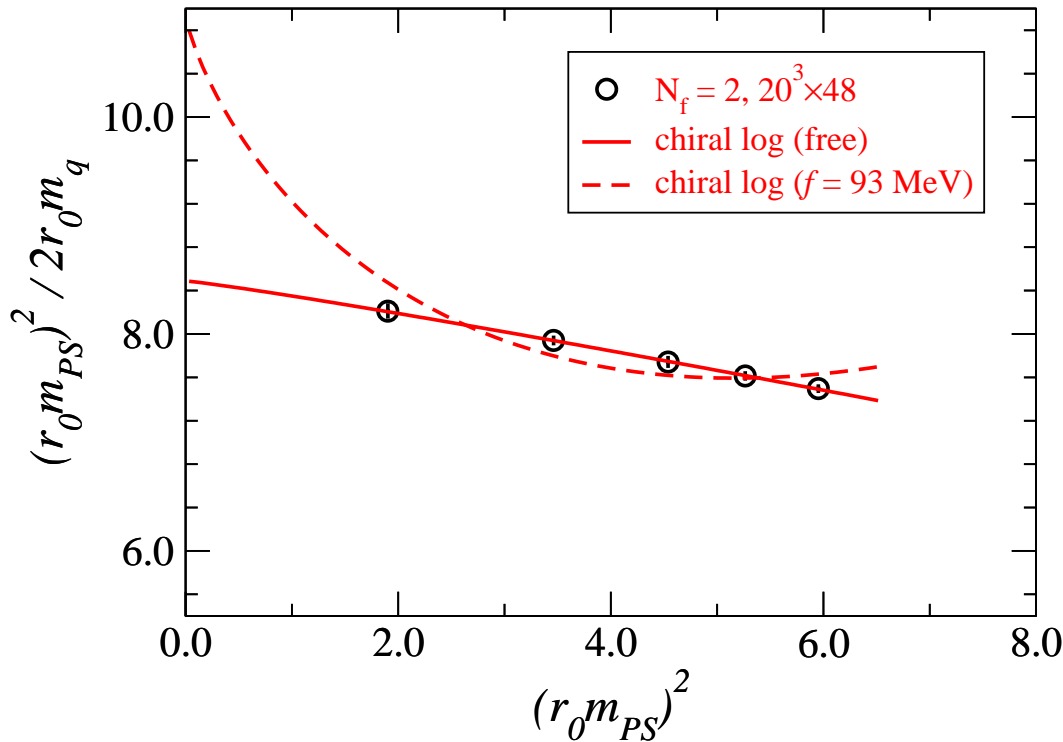
- fit for  $R_M$  does not seem good...

$\Rightarrow$  Are lattice data really consistent with PQChPT?

## examination of ChPT log behavior in $N_f=2$ QCD

$$m_q \equiv m_{q,\text{val}} = m_{q,\text{sea}}$$

$$\frac{M_{PS}^2}{2B_0 m_q} = 1 + \frac{y}{N_f} \ln[y] + y \{(\alpha_8 - \alpha_5) + N_f (\alpha_8 - \alpha_5)\}$$



- no clear evidence of chiral log

- no evidence also for “ $f_{PS}$  vs  $m_{PS}^2$ ”

$\Rightarrow$  not consistent with ChPT ( $m_{PS}/m_V \sim 0.6-0.8$ )



- finite size effects
- lattice artifacts
- higher order of ChPT
- sea quark mass ( $m_{\text{PS}}/m_{\text{V}} \sim 0.6-0.8$ ) isn't small enough

D.Nelson *et al.*

KS,  $N_f = 3$ ,  $m_q/m_s \geq 0.1$ ,

$R_M$  : better consistency with ChPT

### important works in future

- explore (much) smaller sea quark masses also with Wilson type action.  
      $\Rightarrow$  attempt by GRAL collaboration
- extend UKQCD's and JLQCD's attempts toward small sea quark masses.

### 3. determination of fundamental parameters of QCD; $\alpha_s$ and $m_q$

- famous calculation by NRQCD (Davies *et al.*, 1997)

$$\alpha_{\overline{\text{MS}}}^{(N_f=5)}(M_Z) = 0.1174(24)$$

KS action for light quarks

$N_f = 0, 2$  (extrapolated to  $N_f = 3$ )

$\alpha_{\overline{\text{MS}}}$  from  $\alpha_P$ , scale from  $\Upsilon$

---

important to confirm this results using Wilson-type action

- SESAM, 1999

similar calculation with Wilson action

$$\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.1118(17)$$

⇒ significantly smaller

scaling violation?

- Wilson action

Ph.Boucaud, *et al.*

$$N_f = 0, 2$$

$\alpha_{\overline{\text{MS}}}$  from three gluon vertex

scale from  $\rho$

$$\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.113(2)$$

- $O(a)$  improved Wilson action

UKQCD at Lattice00

$$N_f = 0, 2, \quad \alpha_{\overline{\text{MS}}} \text{ from } \alpha_P,$$

scale from  $\Upsilon$

QCDSF+UKQCD

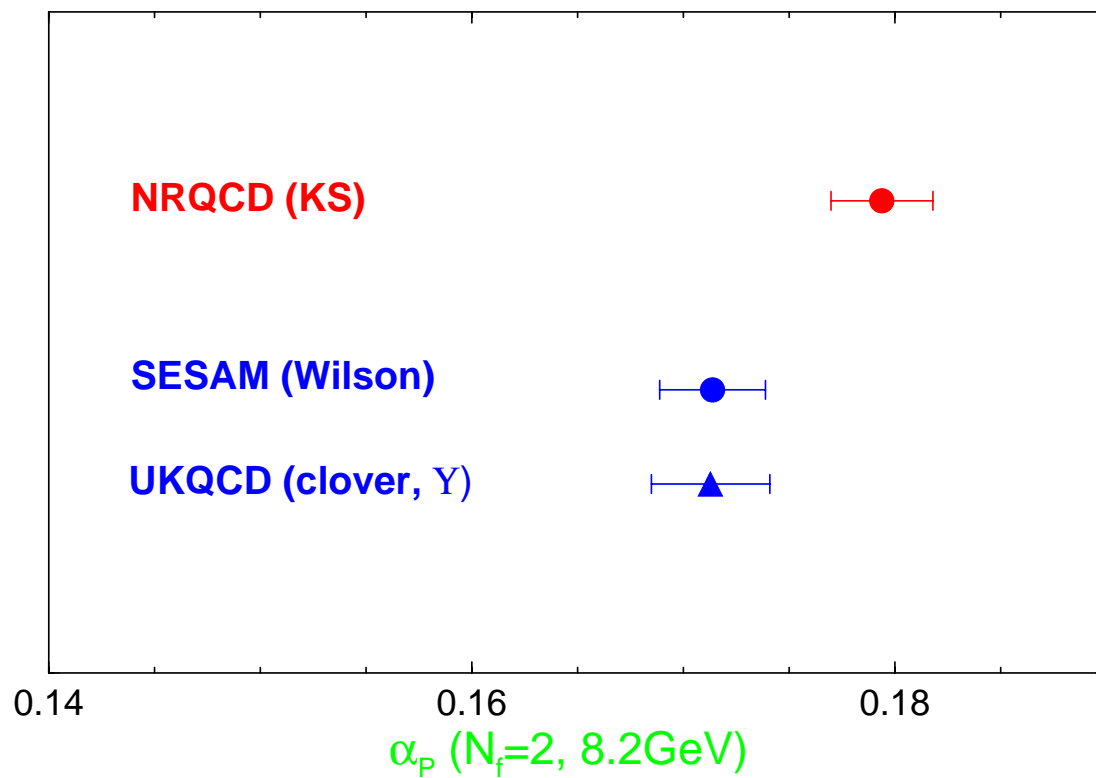
$$N_f = 0, 2, \quad \alpha_{\overline{\text{MS}}} \text{ from } \alpha_P$$

scale from  $r_0$

$$\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.1076(20)(18)$$

ALPHA

- non-perturbative evolution of  $\alpha_{\text{SF}}$  in  $N_f = 2$
- determination of scale : in progress



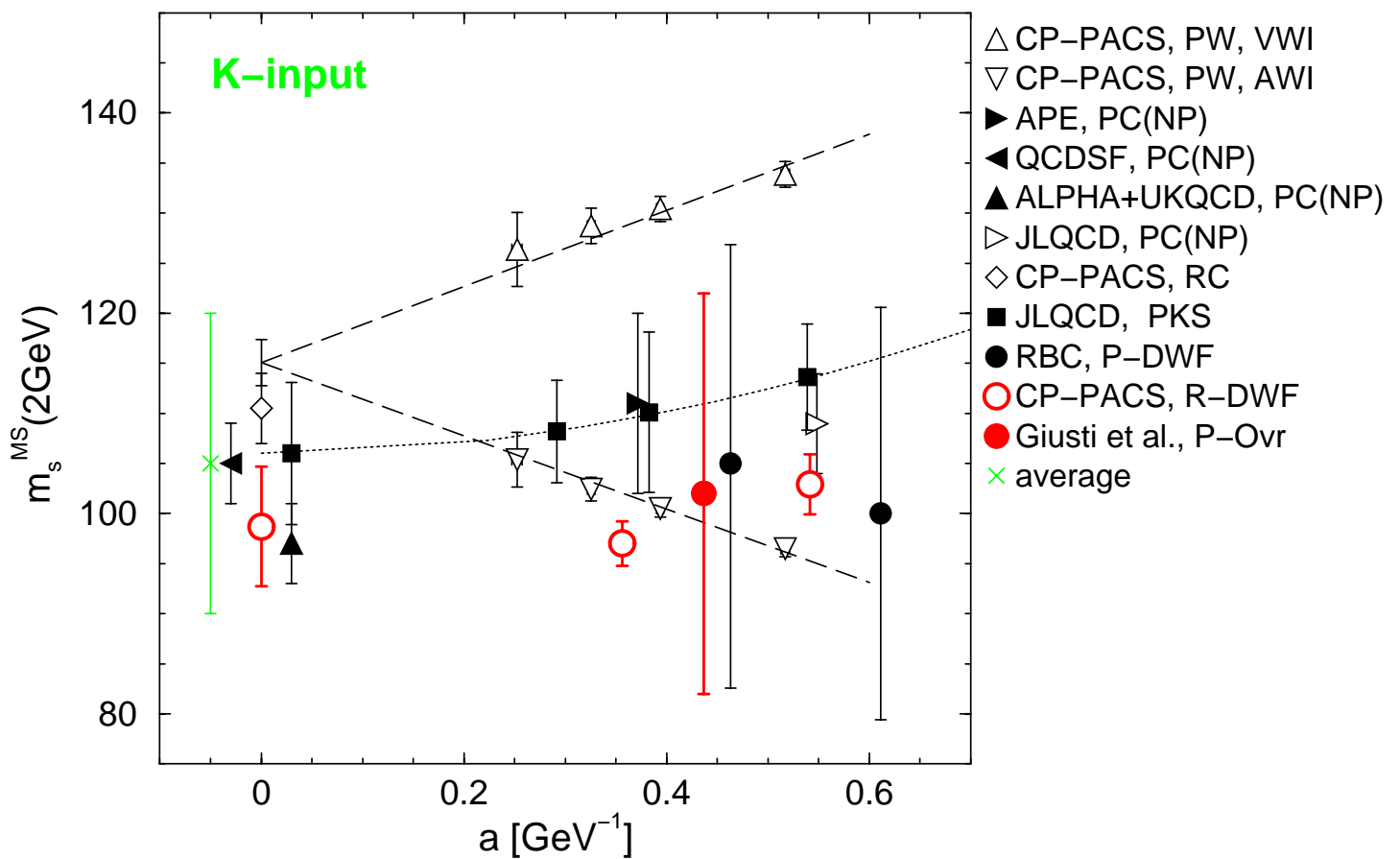
- all results using scale from  $\Upsilon$ .
- smaller values for Wilson-type action.
- discrepancy is present before conversion to  $\overline{\text{MS}}$  scheme.
- reason is still unclear.
  - scaling violation?
  - higher order correction?

### 3.2.1 quenched QCD

#### strange quark mass

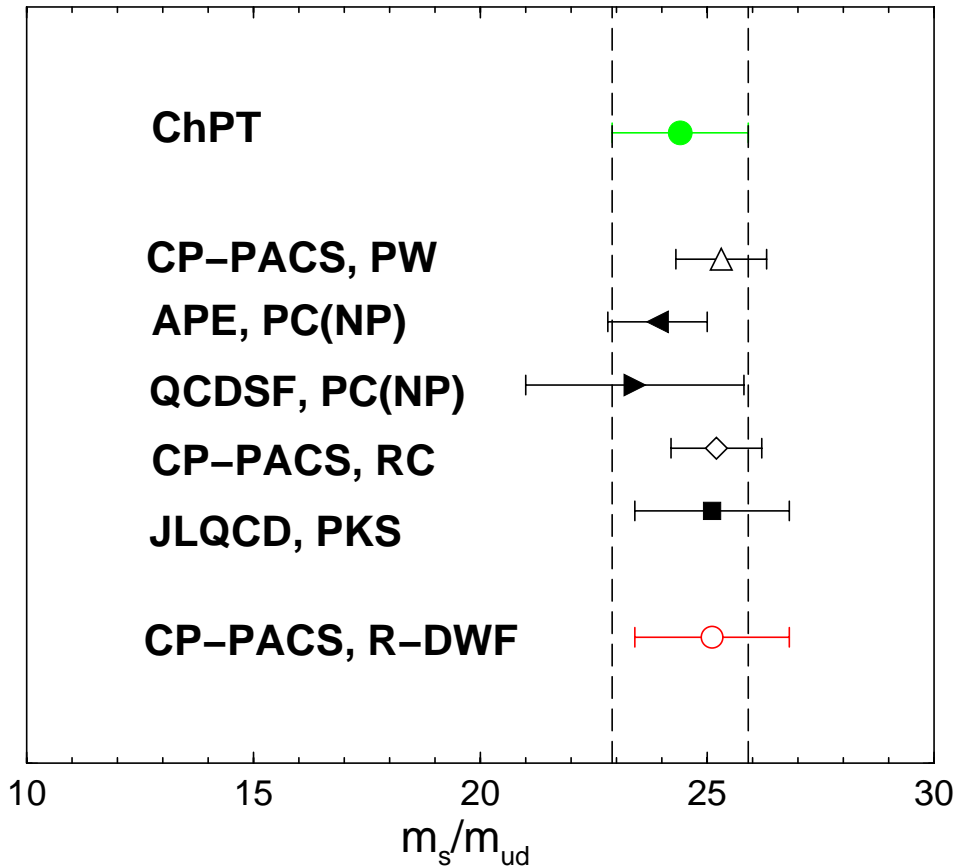
CP-PACS : domain-wall,  $a_\rho^{-1} = 1.8, 2.8 \text{ GeV}$ ,  $La \sim 2.6, 2.3 \text{ fm}$

Giusti *et al.* : overlap,  $a_{f_K}^{-1} = 2.3 \text{ GeV}$ ,  $La \sim 1.4 \text{ fm}$



$$m_s^{\text{MS}}(2 \text{ GeV}) = 105 \pm 15 \text{ MeV}$$

$m_s/m_{ud}$



- new result from CP-PACS :  
consistent with previous results and ChPT

- use ChPT ratio, average of  $m_s$   
 $\Rightarrow m_{ud}^{\overline{\text{MS}}}(2 \text{ GeV}) = 4.5(7) \text{ MeV}$

- results with overlap fermion

Giusti *et al.*,  $m_{ud}^{\overline{\text{MS}}}(2 \text{ GeV}) = 4.2(0.2)(0.7) \text{ MeV}$

Dong *et al.*,  $m_{ud}^{\overline{\text{MS}}}(2 \text{ GeV}) = 4.6(1.2)(0.2) \text{ MeV}$

- this year

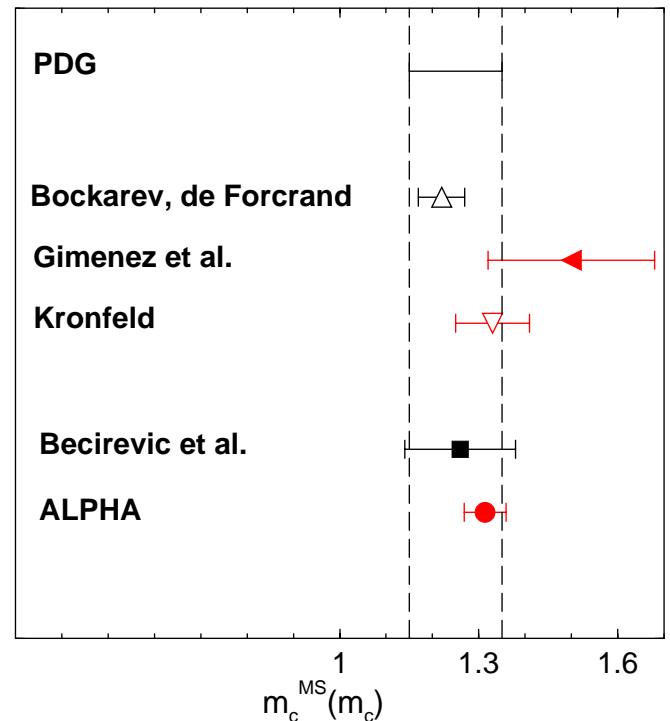
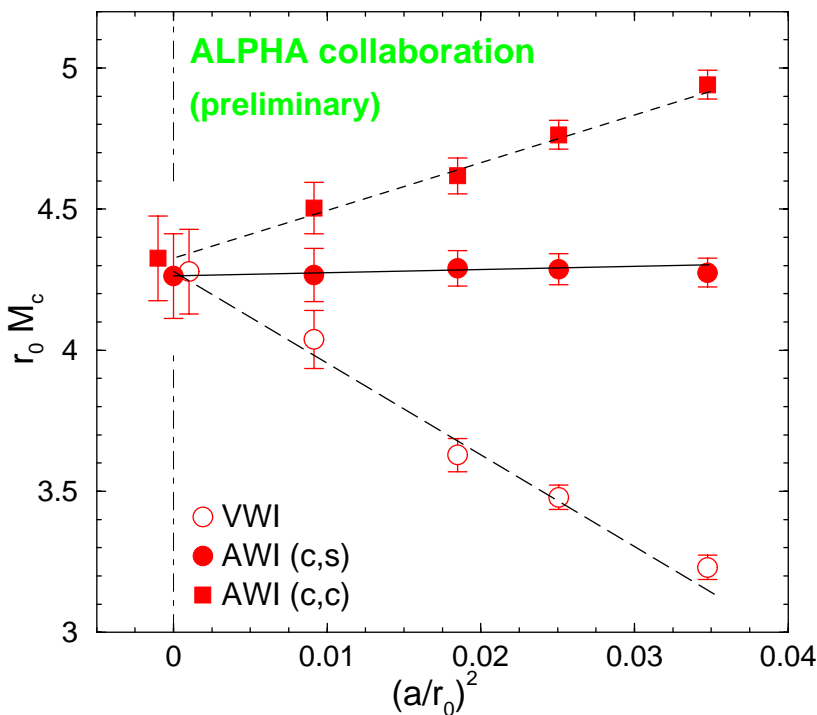
Becirevic *et al.* : NP clover, NP  $Z$ ,  $a \sim 0.07$  fm

ALPHA(Rolf) : NP clover, NP  $Z$ ,  $a \rightarrow 0$

Juge *et al.* : clover, 2loop matching,  $a \rightarrow 0$

- in relativistic lattice formulations

$O((am_c)^n)$  error  $\Rightarrow$  should take continuum limit



- Juge, *et al.* : very preliminary,  $m_c^{\overline{\text{MS}}}(m_c) = 1.28(4)(??)$

- new results are  $m_c^{\overline{\text{MS}}}(m_c) \sim 1.3$  GeV

- largest error in ALPHA's result  $\Leftarrow$  uncertainty in scale

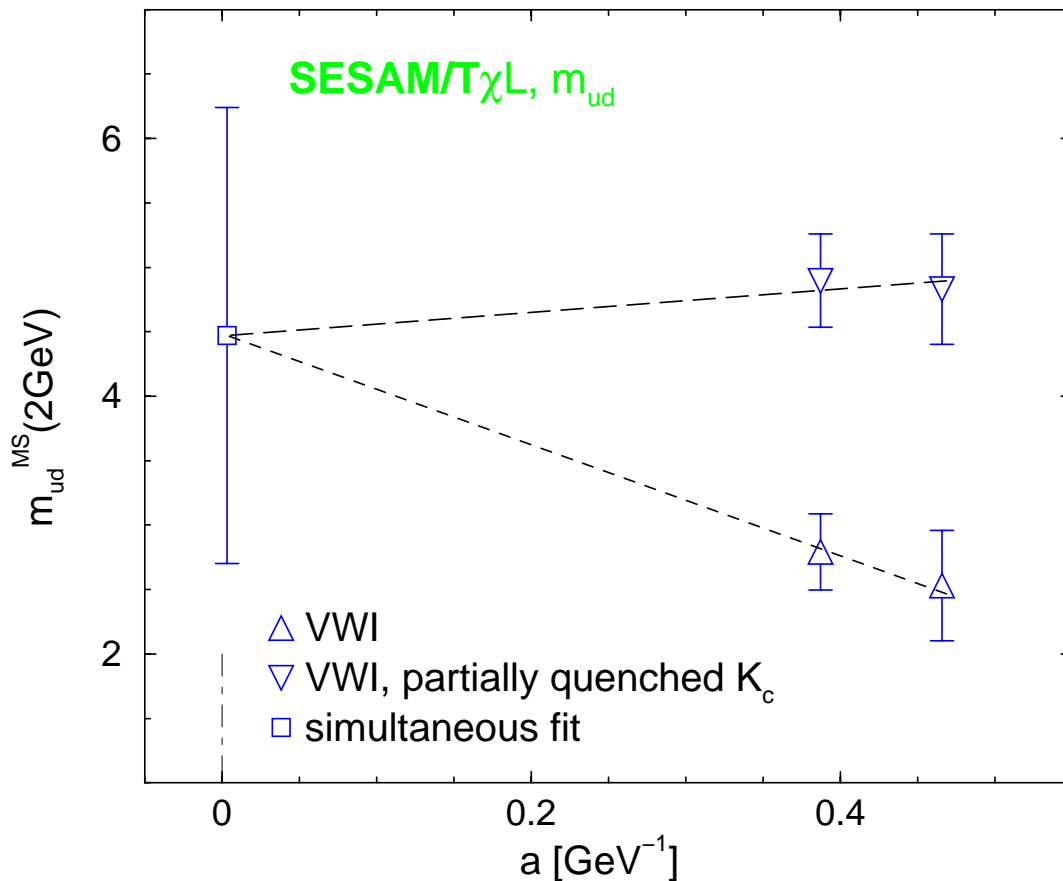
$\Rightarrow$  determination in full QCD

bottom quark mass  $\Rightarrow$  review by Ryan



- this year

SESAM/T $\chi$ L (Eicker) : 1 loop  $Z$ ,  $a^{-1} \sim 2.2, 2.6$  GeV



◦  $m_{ud} = 4.5(1.7)$  MeV

◦  $m_s = 92(83)$  MeV

◦  $m_s/m_{ud} = 20(16)$  MeV

◦ large errors come from continuum extrapolation

⇒ calculation at small lattice spacing is needed.

## quenched QCD : matured

- various methods have been tested and ready for applications.
  - non-perturbative improvement/renormalization
  - improved formulations  
(anisotropic lattices, domain-wall, overlap, twisted mass QCD)
- ⇒ implementation in full QCD.

## $N_f=2$ full QCD : systematic investigations by many groups

- sea quark effects in meson spectrum
  - good! ... but, at finite lattice spacing (except CP-PACS)
- extensive simulations are still needed for
  - scaling violation : fine lattices
  - baryon : larger volumes
  - test of ChPT : smaller sea quark masses
  - high performance computer / “improved” formulations
- few progress for quark masses
  - NP improvement/renormalization are required

$N_f=3$  full QCD : real world!

- ready for systematic studies.

  - ⇐ developement of algorithm/study of phase structure

- several groups started simulations with KS/Wilson action

⇒ We hope see interesting results at Lattice2002!!