Lattice 2001 : Reflections

all my apologies are in this title

P. Hasenfratz
Bern

1. $K \to \pi\pi$ from $K \to \pi$ & $K \to 0$ with domain-wall fermions

CP-PACS
RBC

plenary: Martinelli

It is exciting and frightening to see how brains, software and hardware come together in a highly relevant project and push it to the end, which does not necessarily mean full success.

2. Chiral symmetric fermions

plenaries: Kikukawa
Hernandez

went through a rapid development during the last years

3. Selected topics

topological susceptibility

an algorithm

cut-off effects

1. $K\to\pi\pi$ from $K\to\pi$ & $K\to 0$ from DWF

CP-PACS (Noaki)
RBC (Mawhinney, Cristian, Bloom)

relevant physics
active field of current research
complex problem with relations to exciting theoretical methods
central problem in this community since the early days of lattice computations

* * *

$K\to\pi\pi$ exhibits two significant phenomena

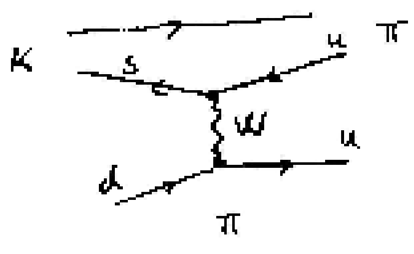
$$\frac{\Gamma(K_s\to\pi^+\pi^-)}{\Gamma(K^+\to\pi^+\pi^0)} \sim 450 \qquad \begin{array}{l}\Delta I = 1/2\ (\&\ 3/2)\\ \Delta I = 3/2\end{array}$$

and (direct) CP violation

non-trivial interplay of strong & electroweak forces

several energy scales $\quad m_t, \ldots, m_u, m_d$

ⓐ OPE



dressed with QCD interactions

the 4-fermion interaction is almost pointlike

factorization $\qquad A = C(\mu/M_W, \alpha_s)\cdot\langle\mathcal{O}\rangle + O(\mu^2/M$

if $\quad \mu \gg \Lambda_{QCD} \quad$ expect: $C(\mu/M_W, \alpha_s)$ can be calculated in PT

if the charm quark is not simulated : $\mu \lesssim 1.3\,\mathrm{GeV}$

given μ/Λ_{QCD} ; is PT applicable?

an important technical issue

one can check consistency within PT itself,
or preferably
compare with the full NP result

a) physical quantities with high energy scales

short distance part of the potential
small volume, high temperature physics, ...

b) finite and divergent renormalization constants
Z

c) cut-off effects

a) very few tests

Necco, Sommer (2001)

$q-\bar{q}$ potential/force

Bali, Boyle, Davies

$0.05\ \mathrm{fm} \leq r \leq 0.8\ \mathrm{fm}$

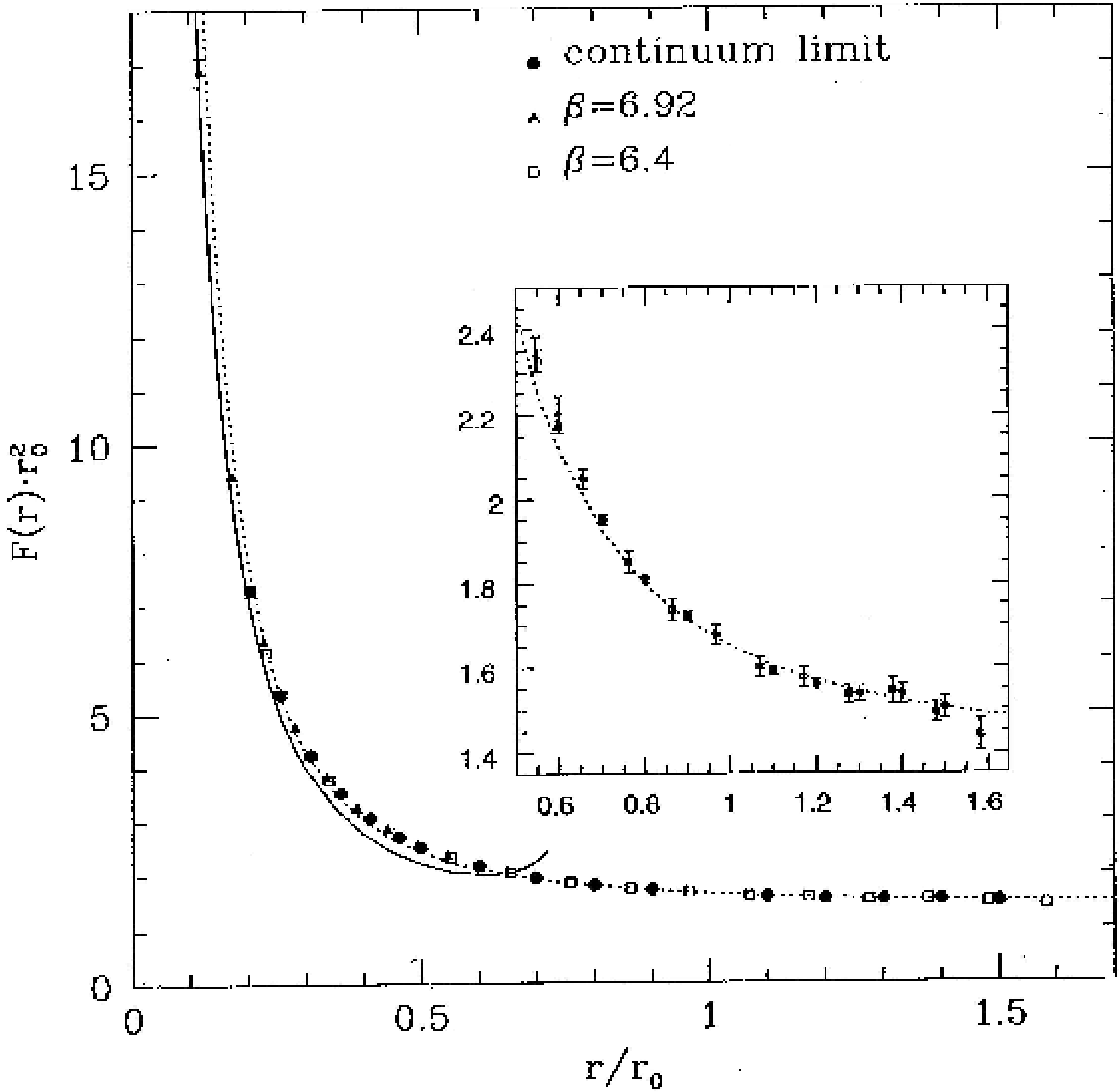
Λ_{QCD} is known (quenched)

Alpha
Capitani, Lüscher, Sommer, Wittig (98)

→ PT for the force is a parameter free prediction
(3 loop)

Fischler (77)
Billoire (80)
Peter (97)
Schröder (98)

Necco, Sommer
(2001)



	$\beta = 0.0$			$\beta = 0.2$			$\beta = 0.4$		
	LANL	ALPHA	P. Th.	LANL	ALPHA	P. Th.	LANL	ALPHA	P. Th.
c_{SW}	1.769	1.769	1.521	1.614	1.614	1.481	1.526	1.526	1.449
Z_V^0	+0.770(1)	+0.7809(6)	+0.810	+0.7874(4)	+0.7922(4)(9)	+0.821	+0.802(1)	+0.8032(6)(12)	+0.830
Z_A^0	+0.807(2)(8)	+0.7906(94)	+0.829	+0.818(2)(5)	+0.807(8)(2)	+0.839	+0.827(1)(4)	+0.827(8)(1)	+0.847
Z_P^0/Z_S^0	0.802 +0.842(5)(1)	N.A.	+0.956	0.815 +0.884(3)(1)	N.A.	+0.959	0.822 +0.901(2)(5)	N.A.	+0.962
c_A	−0.037(5) −0.037(4)(8)	−0.083(5)	−0.013	−0.033(5) −0.032(3)(6)	−0.038(4)	−0.012	−0.032(5) −0.029(2)(4)	−0.025(2)	−0.011
c_V	−0.107(17)(4)	−0.32(7)	−0.028	−0.09(2)(1)	−0.21(7)	−0.026	−0.08(1)(2)	−0.13(5)	−0.024
c_T	+0.06(1)(3) 0.076(10)	N.A.	+0.020	+0.051(7)(17) 0.059	N.A.	+0.019	+0.041(3)(23) 0.051	N.A.	+0.018
$\tilde{b}_V$	+1.43(1)(4)	N.A.	+1.106	+1.30(1)(1)	N.A.	+1.099	+1.24(1)(1)	N.A.	+1.093
b_V	+1.52(1)	+1.54(2)	+1.274	+1.42(1)	+1.41(2)	+1.255	+1.39(1)	+1.36(3)	+1.239
$\tilde{b}_A - \tilde{b}_V$	−0.26(3)(4)	N.A.	−0.002	−0.11(3)(4)	N.A.	−0.002	−0.09(1)(1)	N.A.	−0.002
$b_A - b_V$	−0.24(3)(4)	N.A.	−0.002	−0.11(3)(4)	N.A.	−0.002	−0.08(1)(1)	N.A.	−0.002
$\tilde{b}_P - \tilde{b}_S$	−0.06(4)(3)	N.A.	−0.066	−0.09(2)(1)	N.A.	−0.062	−0.090(10)(1)	N.A.	−0.059
$\tilde{b}_P - \tilde{b}_A$	−0.07(4)(5)	N.A.	+0.002	−0.09(3)(3)	N.A.	+0.001	−0.12(2)(5)	N.A.	+0.001
$\tilde{b}_A$	+1.17(4)(6)	N.A.	+1.104	+1.19(3)(5)	N.A.	+1.097	+1.16(2)(3)	N.A.	+1.092
b_A	+1.28(3)(4)	N.A.	+1.271	+1.32(3)(4)	N.A.	+1.252	+1.31(2)(1)	N.A.	+1.237
$\tilde{b}_P$	+1.10(5)(13)	N.A.	+1.106	+1.11(4)(7)	N.A.	+1.099	+1.04(3)(7)	N.A.	+1.093
$\tilde{b}_S$	+1.16(6)(11)	N.A.	+1.172	+1.19(4)(6)	N.A.	+1.161	+1.13(3)(8)	N.A.	+1.151

TABLE VIII: Final results for improvement and renormalization constants. The first error is statistical, and the second, where present, corresponds to the difference between using 2-point and 3-point discretization of the derivative used in the extraction of c_A.

Bhattacharya, Gupta, Lee, Sharpe

bosonic string model $F(r) = \sigma + \frac{\pi}{12 r^2}$

PT works – within the errors – for $r \lesssim 0.15$ fm

~ 1.3 GeV

?!

Kuti (parallel talk)

Kuti, Juge, Morningstar

b) Z factors

PT is poor

quenched QCD, $\mu = 2$ GeV

Z_P:

fermion type	1-loop	NP
Wilson Gimenez et al.	0.62	0.45
clover (NP) Becirevic et al.	0.59	0.39
staggered Aoki et al.	0.54	0.34

c) improvement coeff.s

scale $\sim \frac{1}{a} \sim 2\text{-}3$ GeV

PT is not reliable

analogies between

Bhattacharya et al.

* * * Symanzik pr. & OPE

however: a, b, c, form 3 different types for PT!

The "tree graph" above generates 2 operators

$$Q_1 = (\bar s_a u_b)_L (\bar u_b d_a)_L \qquad (\bar s u)_L = \bar s \gamma^\mu (1-\gamma_5) u$$

$$Q_2 = \quad a \quad a \qquad b \quad b \quad \leftarrow \text{color} \qquad \Big\} (8_L, 1_R) \oplus (27_L, 1_R)$$

$$\langle \pi\pi^I | Q_i | K \rangle \qquad i = 1,2 \qquad I = 0,2$$

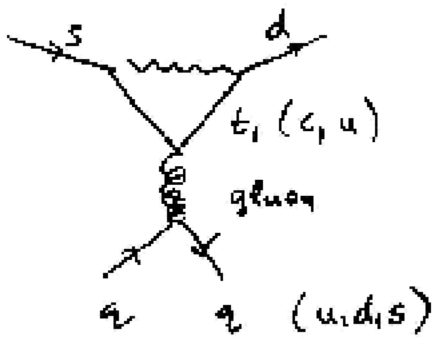
the sum over i, weighted with the Wilson coeff's $z_i(\mu)$ should $\sim$ explain $\omega = \frac{\mathrm{Re}\, A_0}{\mathrm{Re}\, A_2} \sim 22$ $\qquad K \to \pi\pi^I = A_I e^{i\delta}$

* * *

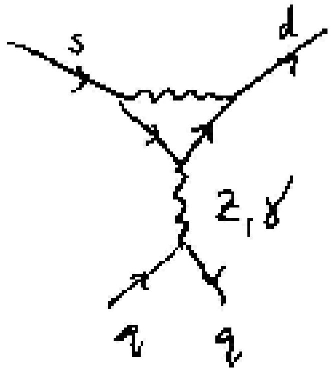
"tree graph" does not know about the 3rd generation

CP conserving

we have to go beyond the leading appr.



QCD penguin



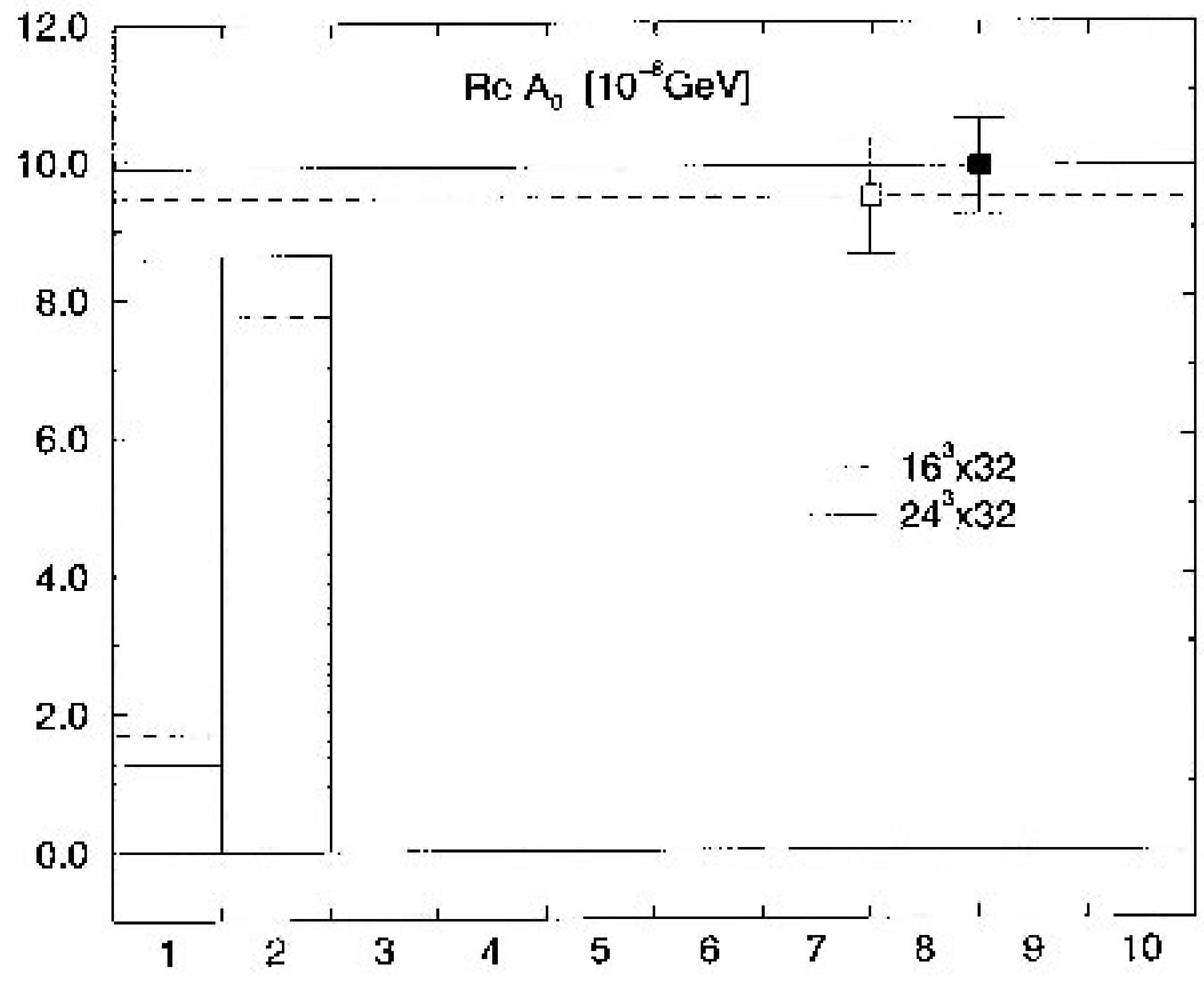
electroweak penguin

z_i, y_i : Munich / Rome group

$$\mathrm{Im}\, A_I = - \mathrm{Im}\left(V_{ts}^* V_{td}\right) \frac{G_F}{\sqrt{2}} \sum_{i=3}^{10} y_i(\mu) \langle Q_i \rangle_I$$

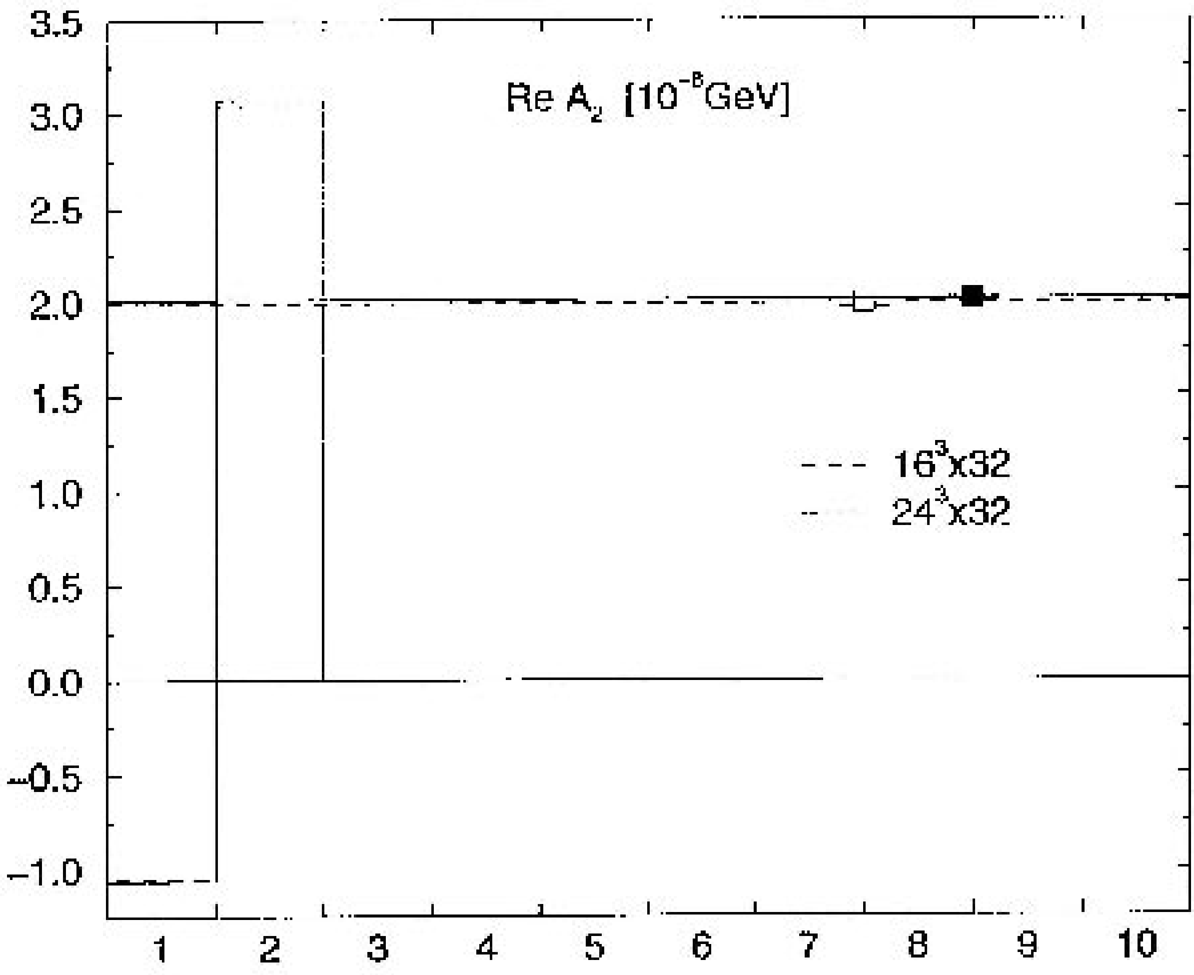
Re A_0 is dominated by Q_1, Q_2

CP-PACS



Re A_2 is dominated by Q_1, Q_2

CP-PACS



it is expected (factorization arguments) that

$\langle Q_6 \rangle_0$ & $\langle Q_8 \rangle_2$ dominate (confirmed by the results)

$$\varepsilon'/\varepsilon \simeq \omega \frac{G_F}{2|\varepsilon| \, \mathrm{Re} A_0} \, \mathrm{Im}(V_{ts}^* V_{td}) \left(y_6 \langle Q_6 \rangle_0 - \omega \, y_8 \langle Q_8 \rangle \right)$$

potential cancellation

$(8_L, 1_R)$ $\quad Q_6 = (\bar{s}d)_L \, (\bar{q}q)_R$ $\quad$ QCD

$(8_L, 8_R)$ $\quad Q_8 = \frac{1}{2} (\bar{s}d)_L \left(2(\bar{u}u)_R - (\bar{d}d)_R - (\bar{s}s)_R \right)$ $\quad$ electroweak penguin

(b) determination of the $\langle \pi\pi | Q_i | K^0 \rangle$ bare matrixelements

without reduction

Becirevic et al.

Lin (parallel)

Martinelli (plenary)

Maiani, Testa

Lellouch, Lüscher

Lin, Martinelli, Sachrajda, Testa

CP-PACS, RBC: $K \to \pi\pi$ from $K \to \pi$ & $K \to 0$

$\underbrace{K \to \pi \;\&\; K \to 0}_{\text{calculated at } m_K = m_\pi = M}$ (unphysical)

Bernard et al.

connected using tree level χPT

$\Delta I = 1/2$

$$\langle \pi^+ \pi^- | Q_i^{1/2} | K^0 \rangle = i \, \frac{m_K^2 - m_\pi^2}{\sqrt{2} f_\pi M^2} \langle \pi^+ | Q_i^{1/2} - \alpha_i Q_{sub} | K^+ \rangle$$

where α_i is obtained from

$$\langle 0 | Q_i^{1/2} - \alpha_i Q_{sub} | K^+ \rangle = 0$$

$$Q_{sub} = (m_s + m_d) \, \bar{s}d - (m_s - m_d) \, \bar{s}\gamma_5 d$$

$(8_L, 8_R)$

$\Delta I = 3/2$ is similar, but without subtraction

ⓒ renormalization

CP-PACS PT

RBC NP

standard

$$Q^{latt}(1/a) \to Q^{\overline{MS}}(1/a) \qquad 1/a \sim 2\,GeV$$

$$\downarrow \text{RG}$$

$$Q^{\overline{MS}}(\mu = 1.3\,GeV)$$

RBC NP renormalization

Landau gauge

"we see no effects" of Gribov ambiguity

PT does not know about Gribov copies ...

question: if we worry about Gribov copies then how can we assume the validity of perturbation theory?

Noaki

Physical results

Mawhinney, Blum, Calin

Soni

CP-PACS $\Delta I = 3/2$ amplitude ($\mathrm{Re}A_2$) : $\sim$ exp.

$\Delta I = 1/2$: a factor of 2 smaller

ϵ'/ϵ : is negative, small $\sim 10^{-4}$

RBC $\mathrm{Re}\,A_0$, $\mathrm{Re}\,A_2$, $\mathrm{Re}A_0/\mathrm{Re}A_2$: $\sim$ exp

ϵ'/ϵ : is negative, small

... tour de force ...

large chiral corrections?

quenching ?

missing final state int.s?

Golterman, Pallante ?

- the whole reduction is on the <u>tree level</u>

  on the p^4 level the low energy constants in $K \to \pi\pi$ can not be determined from $K \to \pi$, $K \to 0$

- $O(p^4)$ (P)(Q) χPT can be used to fit $K \to \pi$, $K \to 0$

  but $K \to \pi\pi$ remains on tree level

- the $O(p^4)$ corrections are estimated to be significant

  $(m_K \sim 500\,MeV)$ Golterman, Pallante

- with the "reduction" the $\pi - \pi$ final state inf. is lost

  an uncontrolled approximation? plenary: Colangelo

- the "reduction" relies heavily on chiral symm.

  with Wilson fermion $\Delta I = 1/2$ sector beyond reach

- can the power div. subtraction be numerically controlled? (repeatedly discussed at this conference)

  it seems so

- chiral behaviour?

  after subtraction all the matrixelements should go to zero in the chiral limit

$$\langle \quad \rangle = a_0 + a_1 M^2 + \cdots$$

should be 0 ↑ this is what we need

2. Chiral symmetric fermions

$\left\{ \begin{array}{l} \text{Kikukawa} \\ \text{Hernandez} \end{array} \right.$

plenary

$\left\{ \begin{array}{l} \text{Damgaard} \\ \text{Edwards} \end{array} \right.$

Ginsparg - Wilson (GW) relation (1982)

"mildest way" to break chiral symmetry

$$\gamma_5 D + D\gamma_5 = D\gamma_5 D \qquad * \qquad D = D(U)$$

$$D^{ab}_{\alpha\beta}(x,x')$$

or equivalently

$$\gamma_5 D^{-1}(x,x') + D^{-1}(x,x')\gamma_5 = \gamma_5 \delta_{xx'} \qquad **$$

the GW relation was not derived

it was guessed from what was found in a free field theory

$**$ indicates that D will respect physical chiral symmetry

$*$ is a non-linear equation

$\nexists$ ultralocal solution — Horvath

D has a tail — Bietenholz

the solution D should be $\boxed{\text{local}}$, however

$$D(x,x') \underset{|x-x'|\gg 1}{\sim} e^{-M(a|x-x'|)}, \quad M \sim \Lambda^{\text{cut}}$$

on "locality of the action in a QFT"

↓

universality

↓

predictive power

based on intuition

collective motion of a large # of degrees of freedom

the microscopic details forgotten

if, however, the direct couplings in the action (hand made!) compete with this collective motion, the model is good for nothing.

We are ready to give up – for a period – unitarity (quenching), but we do not give up predictive power.

For this reason the careful checks concerning the locality of GW Dirac operators are very important.

Hernandez, Jansen, Lüscher

Kikukawa (plenary)

Ishibashi — parallel
Holland — poster
Berici — parallel

skeleton in the closet: $\det(\mathcal{D}(U)_{\text{staggered}})^{1/2}$?

folklor is not very convincing; precise tests (or theory) would be needed

can not be related to locality → quenched Aoki (2000)

problems with the Edinburgh plot

? → unquenched Toussaint plenary

first solution to GW: fixed-point action (1997)

"classically perfect action"

defined by classical equation

handling these classical eqs within a MC is far too expensive → parametrization

Jörg, Holland, Hasenfratz, Niedermayer, H.

all its nice properties become approximate

first tests: $a \sim 0.17\,\mathrm{fm}$, $9^3 \times 24$

$m_\pi / m_\rho \sim 0.35$ can be reached without exceptional confs.

$m_{res} \approx 0$ $\langle\bar\psi\psi\rangle$, χ_{top}, ...

↑ chiral symm is needed to very high prec.

D^{FP}_{par} as input to overlap proj.

the other two known solutions: overlap and domain-wall

↑ catching up

Kaplan, Shamir

Narayanan, Neuberger

Kikukawa, Noguchi

Bonici

⋮

↑ used in large scale simulations

RIKEN-BNL-Columbia Y. Aoki spectroscopy

interesting results with overlap D at this conference (and before)

Berruto, Giusti, Hoelbling, Rebbi

Giusti, Hoelbling, Rebbi: light m_q, NP renorm.

Hernandez, Jansen, Lellouch, Wittig

Dong, Draper, Horvath, Lee, Liu, Zhang

Remark: although "twisted mass QCD" has Wilson like chiral properties, allows to simulate light pions

$+ i\mu \gamma_5 \tau_3$ in the action

- 1-loop improvement — Frezotti, Sint, Weisz
- large scale simulations — Della Morte, Frezotti, Heitger, Mc Neile
- induced new theoretical ideas in matrix element renorm. — Becirevic, Gimenez, Lubicz, Martinelli, Papinutto

with exact, or approx. GW actions:

Low lying eigenvalues, eigenmodes — DeGrand, Anna Hasenfratz

local chirality — Gattringer, Göckeler, Rakow, S. Schaefer, A. Schäfer

topology vs. temperature — Gattringer et al

different $\mathcal{D}$'s on smooth instanton backgrounds — Gattringer et al.

Schwinger model (an ideal testing ground) — Lang, Giusti, Hoelbling, Rebbi

– breakthrough on chiral gauge theories

Lüscher

$U(1)$ NP

Suzuki

general compact PT (all orders)

Kikukawa, Nakayama

electroweak $SU(2) \times U(1)$ NP

– admissibility condition

$\rightarrow$ $\# \cdot Z_\nu$ ν : topological charge

$\uparrow$?

absolute size of baryon number violation?

Suzuki

real repr. Weyl fermions

$\downarrow$ equivalent to

Majorana fermions (L–R symm.)

on the lattice

based on this observation suggested a

"natural choice" for # for Weyl fermions in complex repr.

this paper influences an interesting problem

$$\psi_L = \hat{P}_L \psi \qquad \hat{P}_L = \frac{1}{2}(1-\hat{\gamma}_5)$$

$$\bar{\psi}_L = \bar{\psi} P_R \qquad P_R = \frac{1}{2}(1+\gamma_5)$$

asymmetric

$$\bar{\psi}_L D \psi_L \xrightarrow[CP]{} \bar{\psi}_L D \psi_L - \frac{1}{2} \bar{\psi} D \gamma_5 D \psi \Big|_{\{U\} \to \{U^{CP}\}}$$

on that level CP is violated, unlike in the "cont."

one can introduce a cont. parameter in $\hat{P}_L$, P_R

but CP violation remains, or locality is violated

as a consequence

$$\left| \det D_L(U^{CP})_{\not{0}} \right| = \left(\frac{2}{a}\right)^{-Q(U)} \left| \det D_L(U)_{\not{0}} \right|$$

zero modes are taken out

violates CP ?

Suzuki's suggested factor would just eliminate this.

Is this suggestion consistent with clustering?

3. Selected topics

χ_{top} topological susceptibility

N_f deg. flavours

$$* \quad \chi^{(unquenched)} \doteq \frac{\langle \nu^2 \rangle}{V} = \Sigma\, m_q \frac{1}{N_f}$$

$$= f_\pi^2\, m_\pi^2 \frac{1}{2N_f}\Big|_{m_q \to 0}$$

$-\Sigma = \langle \bar{u} u \rangle = \dots$

$f_\pi = 93\,\mathrm{MeV}$

Crewther (77)

Di Vecchia, Veneziano (1980)

Leutwyler, Smilga ('92)

on the lattice (GW)

Chandrasekharan

Witten - Veneziano relation

$$* \quad \chi^q = f_\pi^2\, m_\eta^2 \frac{1}{2N_f}\Big|_{N_c \to \infty}$$

the same relation as above but

$$\chi^{unq} \longrightarrow \chi^q \equiv \chi^{YM(SU(N_c))}\Big|_{N_c \to \infty}$$

$$m_\pi^2 \longrightarrow m_\eta^2$$

Giusti, Rossi, Testa, Veneziano (2001)

rederived this relation on the lattice (GW)

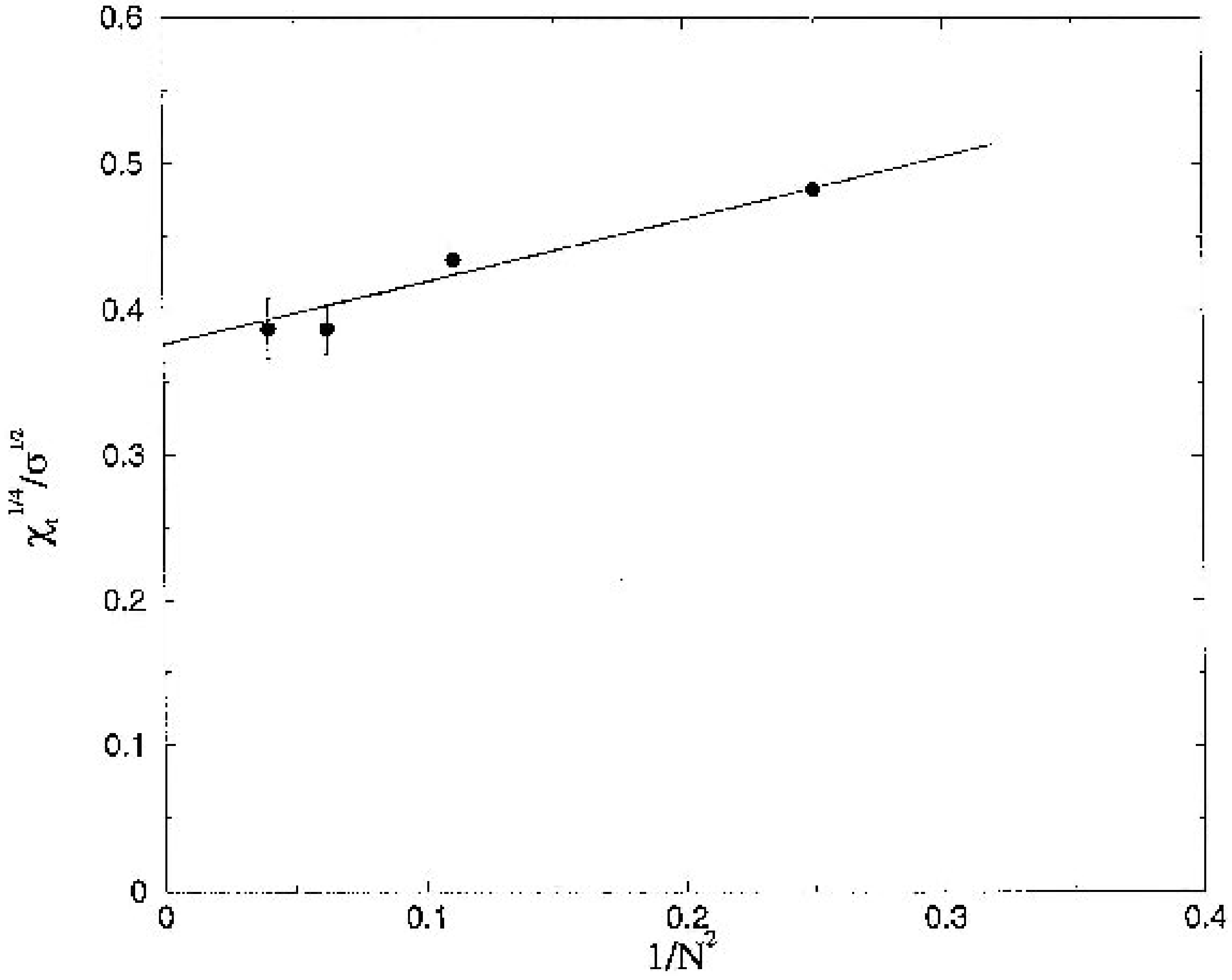
their result confirms the WV relation if

$q(x) = \frac{1}{2}\, \mathrm{tr}^{c,D}\, \gamma_5 D(x,x)$ is used as the topological charge density

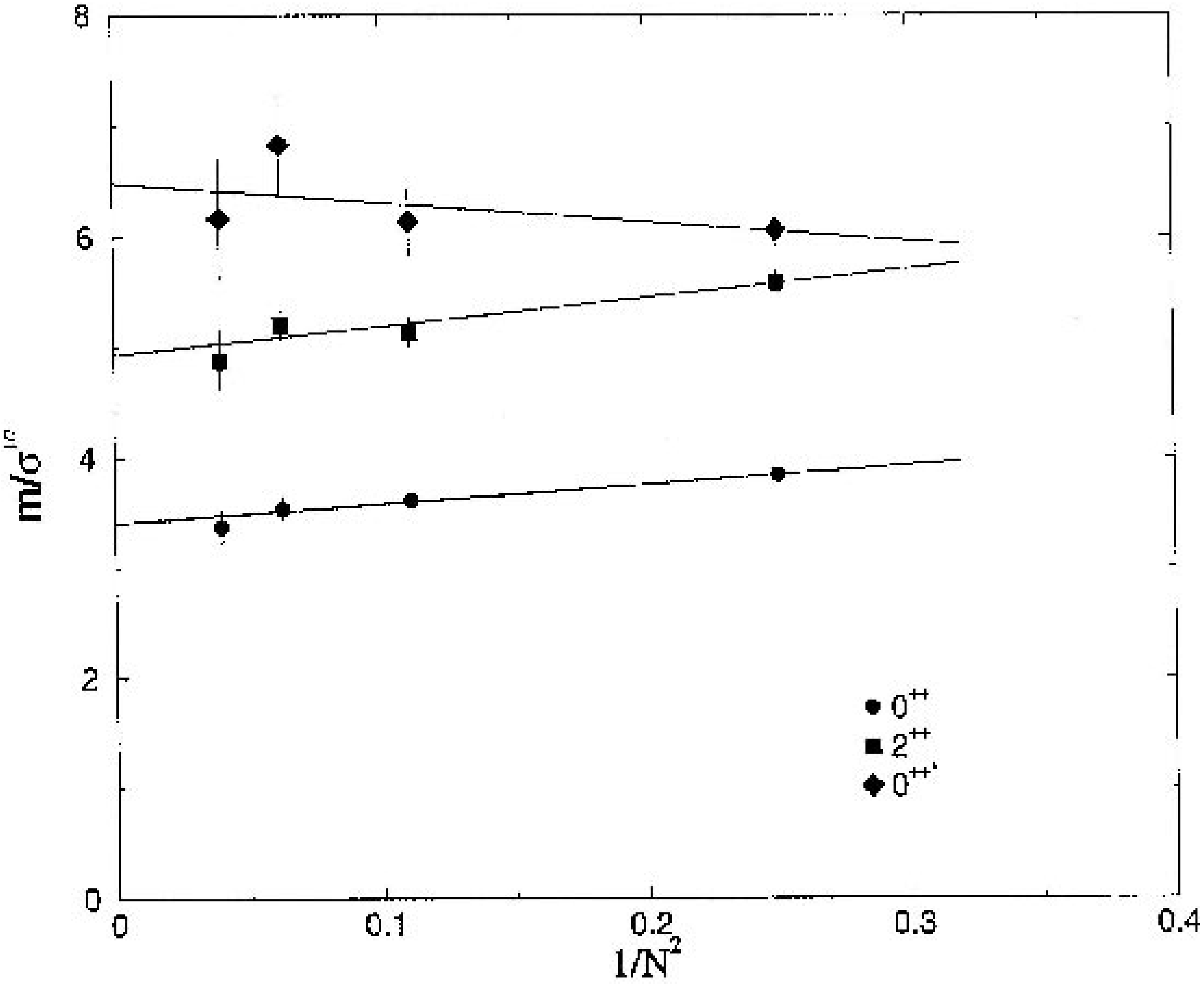
the derivation of χ and WV

shows close analogies

how physics changes with N_c ? Lucini, Teper



Lucini, Teper



how physics changes with N_c ? Lucini, Teper $SU(N_c)$

numerical results

CP-PACS 2001

quenched, naive Q $L = 1.5\,fm$

cooling $a \in (0.09 - 0.27)\,fm$

no renormalization

$r_0 \chi^{\frac{1}{4}} = 0.0570(43)$

$\chi^{\frac{1}{4}} = (197 \pm ^{3}_{0} \pm 12 \text{ MeV})^4$

dynamical, naive Q $a \in (0.11 - 0.22)\,fm$

cooling mean field clover

no renormalization

Hart, Teper $N_f = 2$ UKQCD conf.s $16^3 \times 32$

$a \sim 0.1\,fm$

Anna Hasenfratz

reanalysed the staggered, $N_f = 2$ MILC confs.

HYP smeared topological charge

renormalization calculated

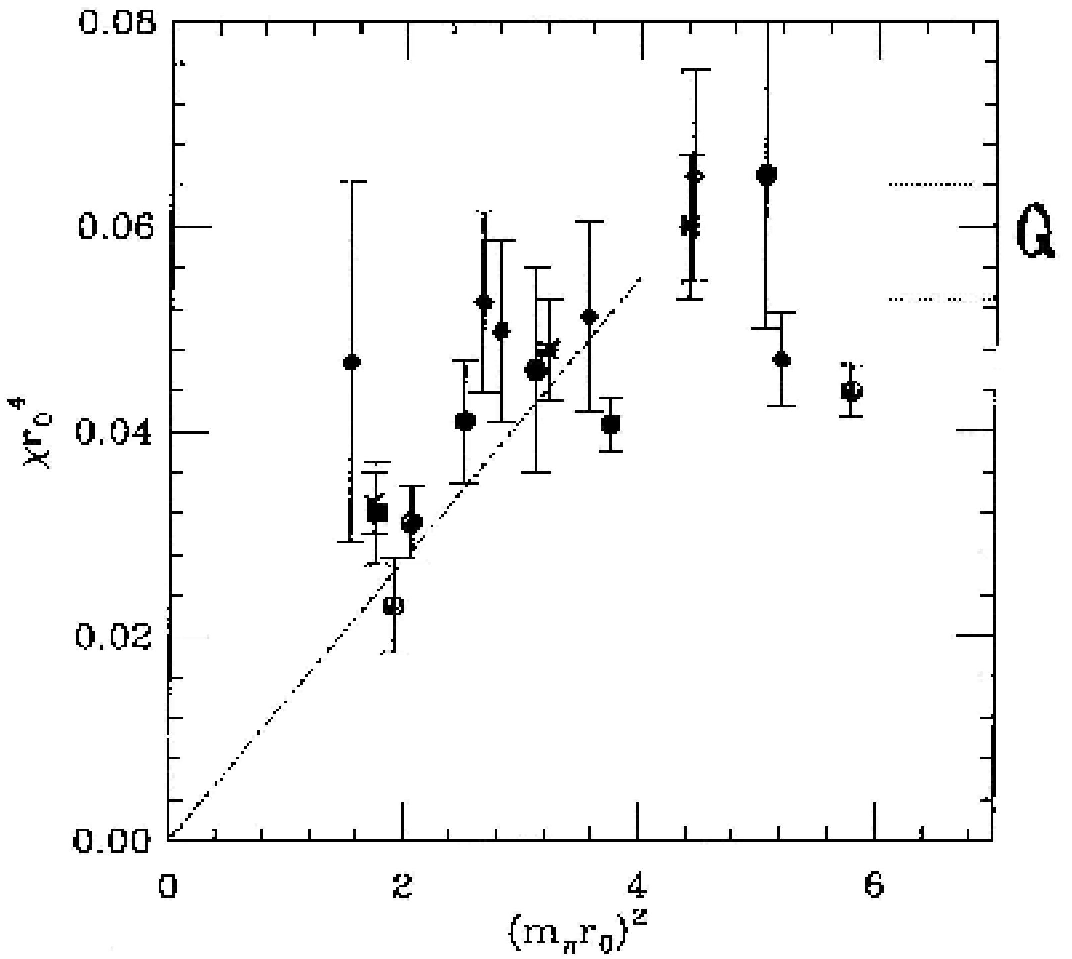
Dürr

from Anna Hasenfratz

The topological susceptibility provides a sensitive test of chiral behavior

$$\chi = \frac{m_q}{n_f}\Sigma + O(m_q^2) = \frac{f_\pi^2 m_\pi^2}{2n_f} + O(m_\pi^4)$$

Thin link actions with $a \sim 0.1$fm



Filled symbols:staggered; diamond:Wilson; octagon,burst:clover;

An algorithm

plenary S. Hands

"sign problems" and cluster algorithms Chandrasekharan (Bangalore 2000)

in QCD?

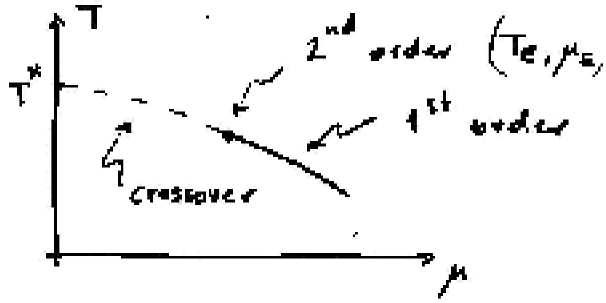
* * *

Fodor, Katz — a new algorithm to study QCD at finite T and μ

promising results

a physics motivated variation of the Ferrenberg - Swendsen reweighting technique

$N_f = 3$, physical $m_{u,d,s}$, expected:



suggestion: produce confs. at $\mu = 0$, $T = T^*$ and reweight to transition (crossover) confs.

works much better than the traditional reweighting Barbour et al.

the crossover identified by looking for Lee-Yang zeros

$\beta_{zero} = \beta_{zero}(L, \mu)$ — L: spatial size, μ: real

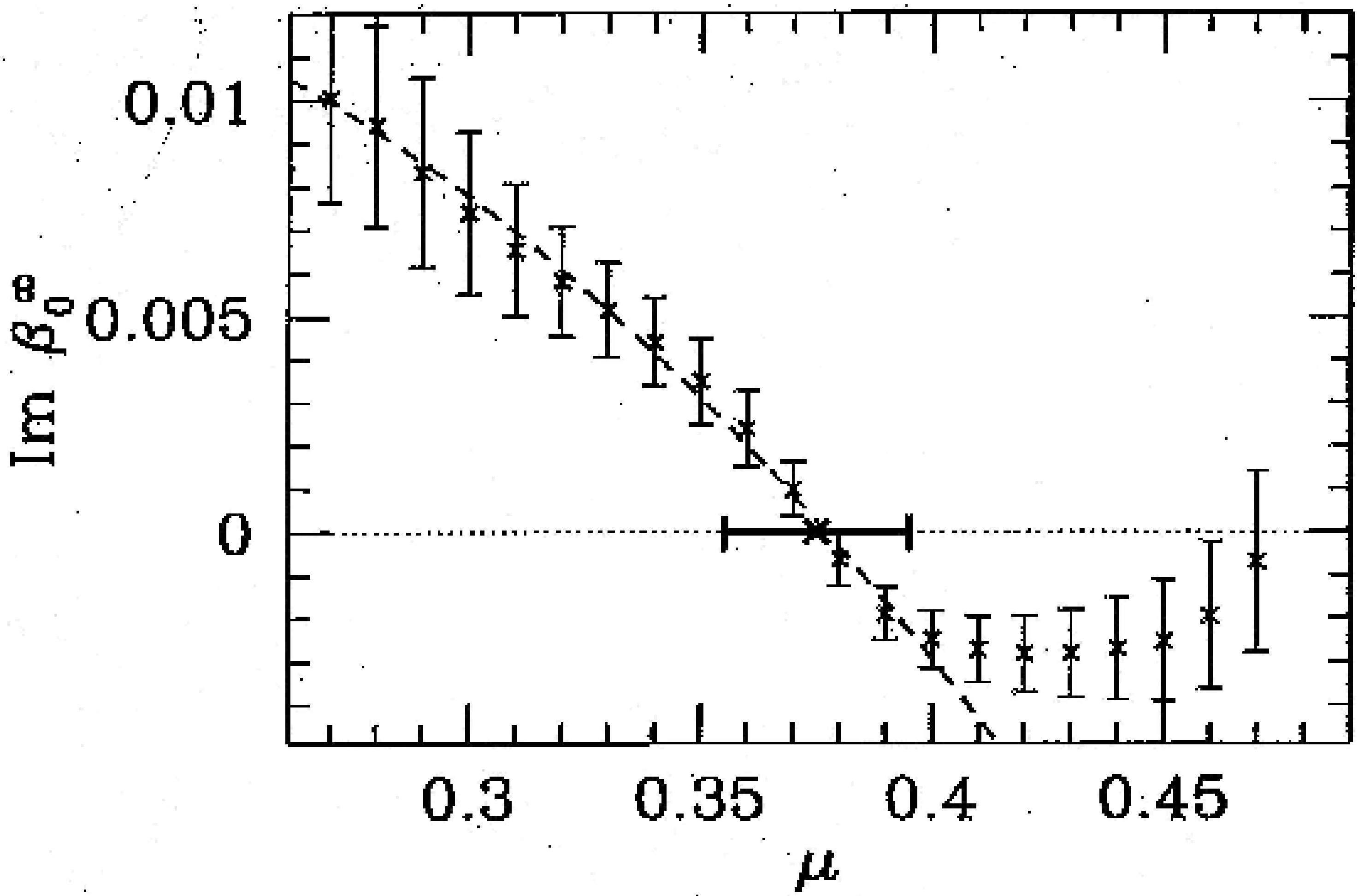
$\mathrm{Im}\left(\beta_{zero}(L \to \infty, \mu)\right) \neq 0$ crossover

$= 0$ real phase tr.

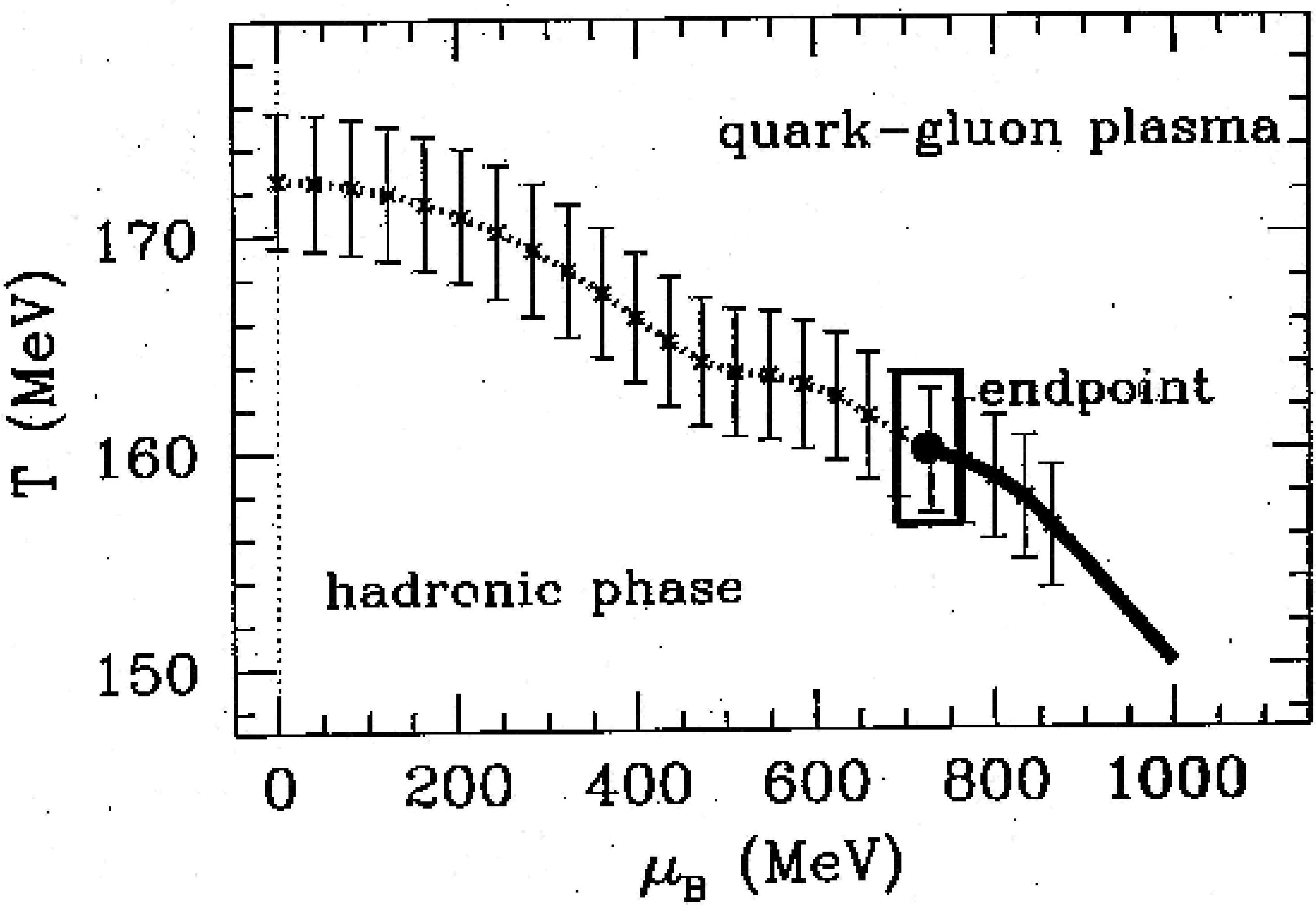
$T_E = (160 \pm 3.5 \pm ?)$ MeV $\qquad \mu_E = (725 \pm 35 \pm ?)$ MeV

Lee-Yang zeros

Fodor, Katz



Fodor, Katz


quark-gluon plasma
endpoint
hadronic phase
170
160
150
T (MeV)
0
200
400
600
800
1000
μ_B (MeV)

An unexpected cut-off effect

$d = 2$ $O(3)$ non-linear σ-model

AF theory of bosons

expected cut-off effect (leading) a^2 times logs

Hasenbusch, Niedermayer, Seefeld, Wolff, H.

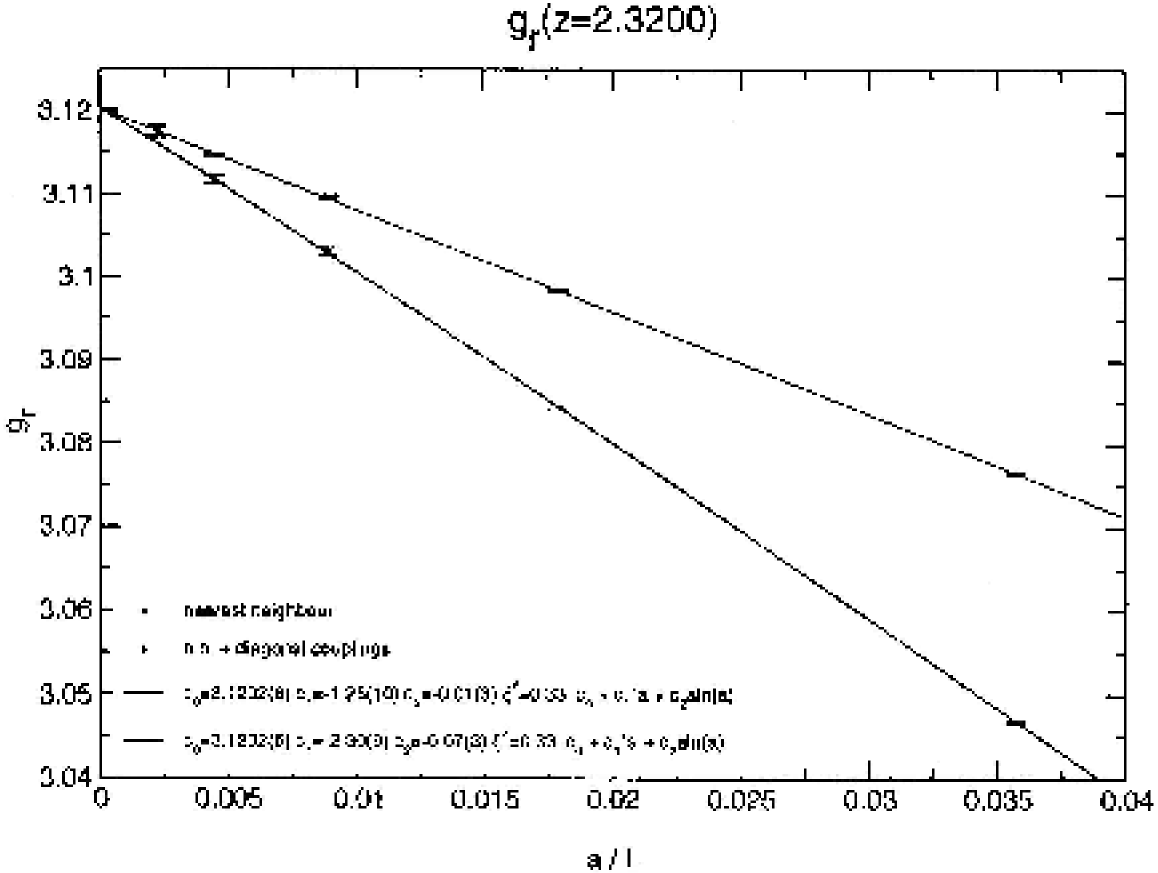
two different actions; two different physical quantities:

does not look like $\sim a^2$ at all

renormalized zero momentum
4-point coupling in finite
physical volume

$z = L/\xi$

Hasenbusch et al.



LWW finite size scaling function

Hasenbusch et al.