Strings,
Non-commutative geometry
Quantum gravity
on the lattice

<u>Lattice 2000</u>:

Three theoretical talks by
Kawai, Loll and Sen.

Complementary view here:
<u>numerical</u> and <u>Lattice</u> aspects.

Reminder

lattice methods have been very succesful in the fields mentioned except for one point : Supersymmetry

Whenever the bosonic theory (strings, QG, NCYM) is known to exist the lattice formulation has

1. provided a rigorous definition of the theory
2. a scaling limit which could be analyzed by numerical/analytical methods
3. Often been superior to continuum methods even from an analytical point of view

<u>EX</u> 2d QG and non-critical strings for $c \leq 1$
(No problem with diff-inv.)

Non-commutative YM. Discovered in 83 from TEK

(NCYM → GOK, Gonzalez-Arroyo, Okawa and Korthals-Altes)

3d QG as a Turaev-Viro state-sum.

<u>Neg. EX</u> Bosonic strings in $d > 1$.

Theories which $\boxed{\text{may}}$ make sense

1. Superstring theories (and M-theory)

   Supersymmetry crucial and it has not (yet) been possible to implement it in local wordsheet approach ~ old matrix models, dynamical triangulations.

2. Four-dimensional quantum gravity

   We do not know if such a theory exists as a QFT an sich, but if it does the lattice seems a perfect tool for the search for a non-trivial fixed point, defining the theory

Rest of talk

(1) Explain how a new class of matrix models incorporate supersymmetry and might be a non-perturbative definition of superstring theory and discuss numerical studies of the theories

(2) Explain how they lead to NCYM and how this theory is defined in a natural way (in fact discovered) on the Lattice

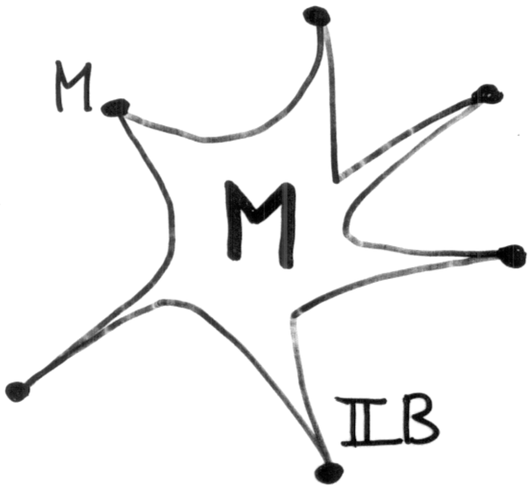
(3) Give a status report on Lattice quantum gravity research, in particular Lorentzian quantum gravity

String theory

$S =$ World sheet area + S.S.

$\Downarrow$

perturbative string theory



Second "string revolution"

(T-dualities)

(S-dualities)

In need of a <u>non-perturbative</u> <u>definition</u>

M(atrix) theory — IIB-matrix model

BFSS-model — IKKT-model

(Banks, Fishler, Shenker, Susskind) — (Ishibashi, Kawai, Kitazawa, Tsuchiya)

$\downarrow$

numerical work: Kabat, Lifschytz, Lowe; Janik, Wosiek (reported here)

Green-Schwarz superstring

in schild-gauge

$$S = \int d^2\xi \sqrt{g} \left(\frac{1}{4} \{X^\mu, X^\nu\}_{PB}^2 - \frac{i}{2} \bar{\Psi} \Gamma_\mu \{X^\mu, \Psi\} \right)$$

$$\{X, Y\}_{PB} = \frac{1}{\sqrt{g}} \varepsilon^{ab} \partial_a X \partial_b Y$$

$$\{X, Y\}_{PB} \to -i [X, Y]$$ $N \times N$ Hermitian

$$X^\mu(\xi_1, \xi_2) \longrightarrow X^\mu_{ij}$$ we need $N \to \infty$

$$\Psi^\alpha(\xi_1, \xi_2) \longrightarrow \Psi^\alpha_{ij}$$

$$\boxed{\begin{aligned} Z_N &= \int d\Psi \, d\bar{\Psi} \, dX_\mu \, e^{-S[\bar{\Psi}, \Psi, X]} \\ S &= -\mathrm{Tr}\left(\frac{1}{4} [X^\mu, X^\nu]^2 + \frac{1}{2} \bar{\Psi} \Gamma_\mu [X^\mu, \Psi] \right) \end{aligned}}$$

IKKT conjecture

If we consider the model for <u>finite</u> N we have an invariance which we might consider a natural kind of supersymmetry.

$S = 0$ (classical eq. of motion)

$[X_\mu, [X_\mu, X_\nu]] = 0$

A solution $[X^\alpha_\mu, X^\alpha_\nu] = 0$

Simultaneously diagonalizable.

The spray of eigenvalues i R^{10} is viewed as defining classical space-time.

Valleys $[X_\mu, X_\nu] = 0$ potential danger for the existence of

$$\int d\bar{\Psi}\, d\Psi\, dX_\mu\, e^{-S}$$

$$\int dX_\mu\, e^{\mathrm{Tr}[X_\mu, X_\nu]^2}$$

$\boxed{\text{Clarified}}$ (Austing, Wheater)

SS exists for all N in $D = 4, 6, 10$

B exists for $D \geq 3$ and $N \geq 4$.

In a non-perturbative string theory the dimensionality of $\boxed{\text{REAL}}$ space-time should come out automatecally.

Can we see it ?

Space-time uncertainty:

$$\Delta^2 = \frac{1}{N}\left(\mathrm{Tr}\, X_\mu^2 - \max_{U\in SU(N)} \sum_i \left\{(U X_\mu U^\dagger)_{ii}\right\}^2\right)$$

$\Delta^2 = 0$ if X_μ's simultaneously diag.

$$(X_\mu)_i \equiv \left(U_{max} X_\mu U^\dagger_{max}\right)_{ii}$$

Extend of spacetime R

$$R^2 \equiv \frac{1}{N}\langle \mathrm{Tr}\, X_\mu^2 \rangle = \frac{2}{N(N-1)} \sum_{i<j}^{N} \langle (x_i^\mu - x_j^\mu)^2 \rangle$$

$$\rho(r) = \frac{2}{N(N-1)} \left\langle \sum_{i<j}^{N} \delta\left(r - \sqrt{(x_i^\mu - x_j^\mu)^2}\right)\right\rangle$$

For SS-SU(2) (Staudacher, Krauth)

$$\rho(r) \sim r^{-2D+5}$$

$$\boxed{\frac{1}{N}\langle \mathrm{Tr}\, X^{2n} \rangle = \infty \quad , n \text{ large}}$$

Source of ambiguity!

$$T_{\mu\nu} \equiv \frac{2}{N(N-1)} \left\langle \sum_{i<j} (X_i^\mu - X_j^\mu)(X_i^\nu - X_j^\nu) \right\rangle$$

10×10 matrix. Eigenvalues $\lambda_1 > \cdots > \lambda_{10} > 0$

<u>principal moments of inertia for space-time</u>

We would like to see:

1. 4d flat pancake for $N \to \infty$
2. $\Delta/R \to 0$ for $N \to \infty$

$$Z = \int dX_\mu \, d\Psi \, d\bar{\Psi} \; e^{-S[X, \Psi, \bar{\Psi}]}$$

$$= \int dX_\mu \, e^{+\frac{1}{4} \mathrm{Tr}[X^\mu, X^\nu]^2} \, \mathrm{Det}\, M(X)$$

Unfortunately Det M is complex in $D=10$

Anagnostopoulos and Nishimura (reported here) have developed a method for dealing with complex determinants of the kind encountered. It seems to work for the RMM for QCD at finite density.

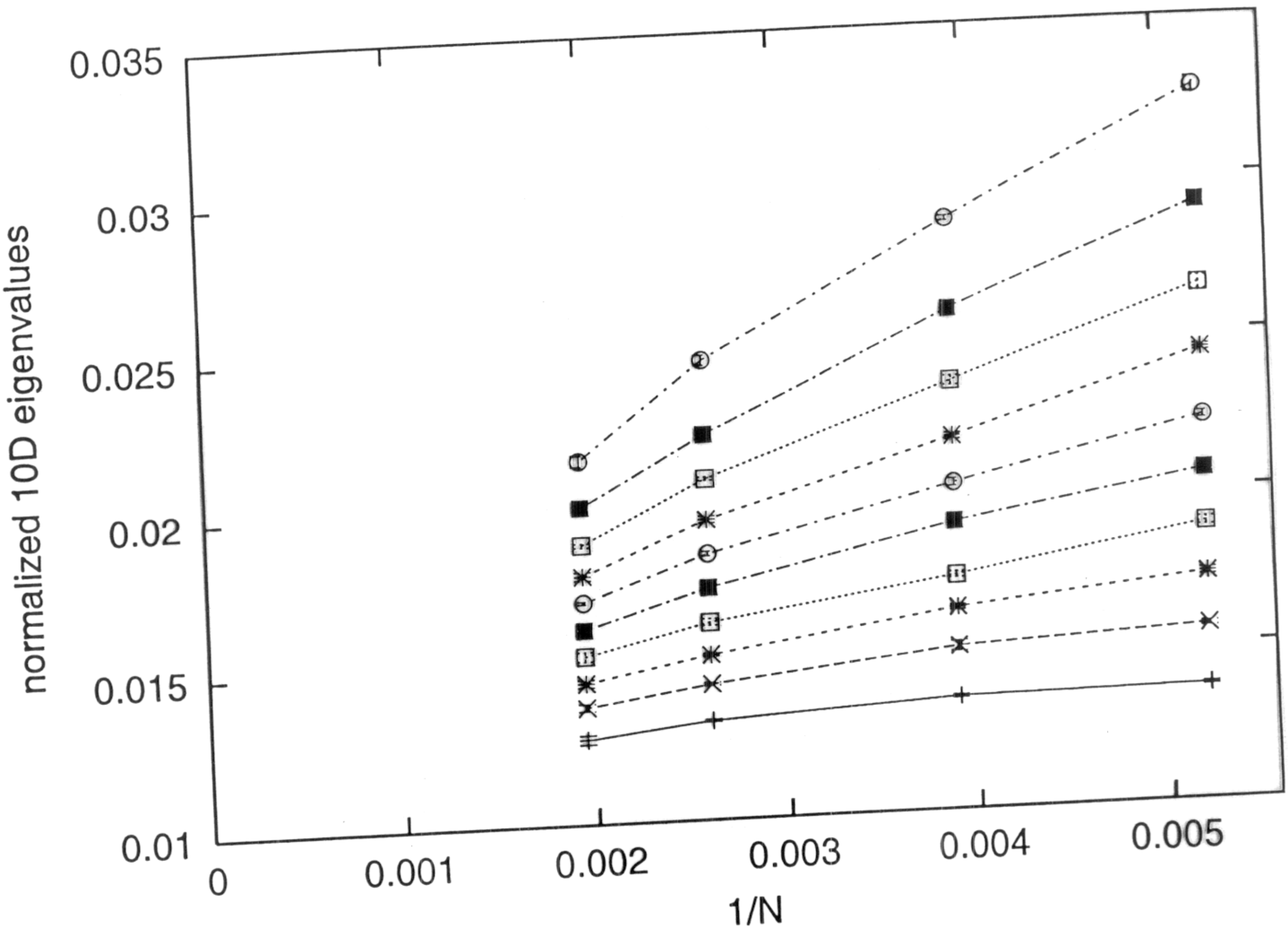
$D=10$ has been simulated with

$$\mathrm{Det}\, M \longrightarrow |\mathrm{Det}\, M|$$

using a one-loop approximation to the full theory

No spontaneous symmetry breaking to 4d observed. No reduction $\Delta/R \to 0$

(13)



0.035
0.03
0.025
0.02
0.015
0.01
normalized 10D eigenvalues
0
0.001
0.002
0.003
0.004
0.005
1/N

SS-Theory in $D=4$:

$\operatorname{Det} M(X)$ real, positive

One can investigate the full theory but unfortunately $R^2 = \infty$ and

$T_{\mu\nu} = \infty \qquad \left(\langle \operatorname{Tr} X_\mu^2 \rangle = \infty \right)$

Two groups have performed simulations

Bialas, Burda, Petersson, Tabaczek

Anagnostopoulos, Bietenholz, Nishimura, Hofheinz, Hotta, J.A

Using $T_{\mu\nu}$ one finds SSB to $d=1$!

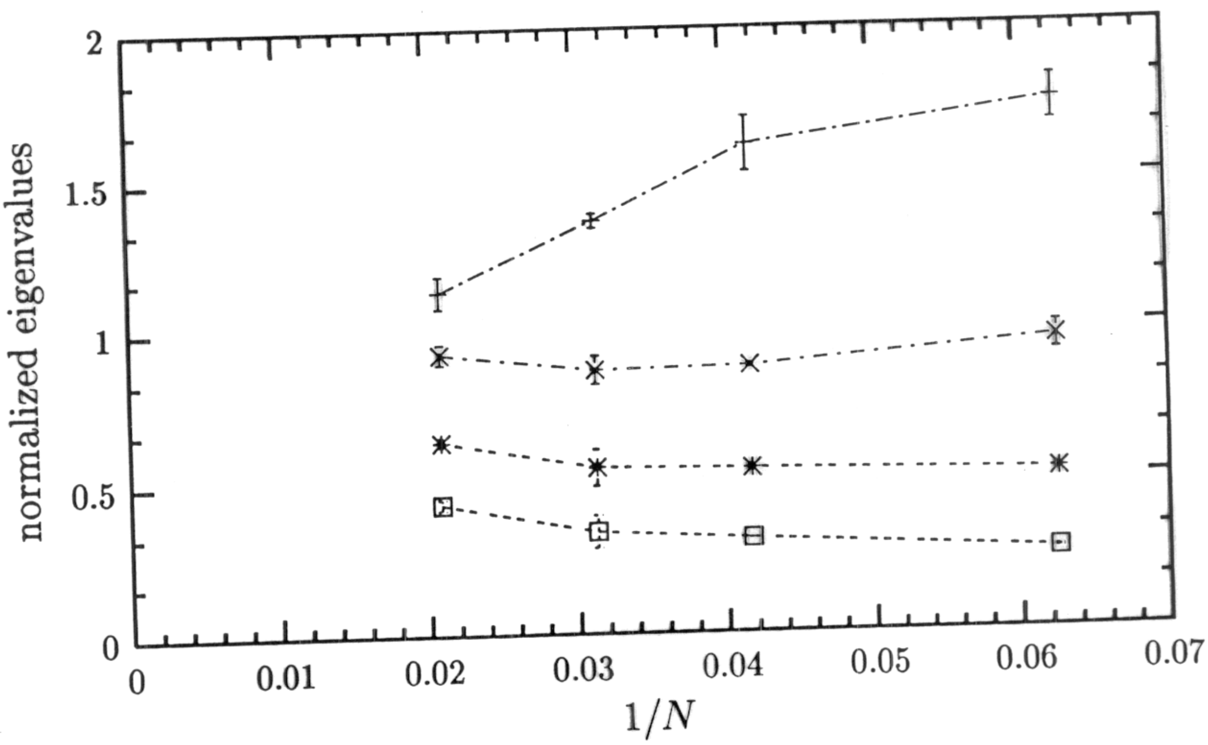
One eigenvalue diverges

However, using a modified definition which should be finite one sees no SSB ?

✦ We define

$$T_{\mu\nu}^{(\text{new})} = \frac{2}{N(N-1)} \left\langle \sum_{i<j} \frac{(x_{i\mu} - x_{j\mu})(x_{i\nu} - x_{j\nu})}{\sqrt{(x_i - x_j)^2}} \right\rangle$$

which has eigenvalues $\infty > r_1 > r_2 > \ldots > r_D > 0$.



NCYM on the lattice

Made famous by Seiberg and Witten.

From IIB-matrix point of view the can be obtain by expanding around "classical" BPS-saturated solutions:

$$[X_\mu,[X_\mu,X_\nu]]=0 \qquad [X_\mu^{(0)},X_\nu^{(0)}]=iC_{\mu\nu}$$

(Note that $X_\mu^{(0)}$ has to be infinite dimensional. Will we ever capture them in numerical simulations)

Studied intensively as toy models of string theories, but have a definition by themselves.

A natural regularization provided by the Lattice via TEK

(1) It respects non-commutative gauge sym. in the same way as ordinary gauge sym. is respected by the Lattice

(2) It can be formulated for finite N

(3) It preserves Morita equivalence

$$Z_{TEK} = \int \prod_\mu dU_\mu \; e^{\frac{1}{2g^2}\sum_{\mu\neq\nu}\left(Z_{\mu\nu}^* \operatorname{tr} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c.\right)}$$

Wilson lattice U(N) gauge theory on a hypercube with twisted boundary conditions:

$$Z_{\mu\nu} = e^{\frac{4\pi i}{N} n_{\mu\nu}} \in \mathbb{Z}_N \qquad (n_{\mu\nu} = -n_{\nu\mu} \text{ int.})$$

The classical vacuum is non-trivial

$$U_\mu^{cl} = \Gamma_\mu \quad , \quad \Gamma_\mu \Gamma_\nu = Z_{\mu\nu} \Gamma_\nu \Gamma_\mu$$

These twist-eaters known for any $Z_{\mu\nu}$ (Gonzales-Arroyo, Van Baal, ...)

Let $\boxed{N = L^{D/2}}$ (D even). The twist eaters can be used to construct a map (Weyl-map)

$$U_\mu^{ij} \longrightarrow \mathcal{U}_\mu(x) \qquad x \in \mathbb{Z}_L^D$$

(recall $X_\mu(\xi_1, \xi_2) \to X_\mu^{ij}$)

$$A \to A(x) \quad , \quad AB \to A(x) * B(x)$$

$$A(x) * B(x) = \sum_{y,z} e^{2i\theta^{-1}_{\mu\nu} y_\mu z_\nu} A(x+y) B(x+z)$$

$\theta_{\mu\nu} \sim n_{\mu\nu}$

$$\boxed{Z_{TEK}(U(N)) = Z^{NC}_{L^D}(U(1))}$$

$$U^{ij}_\mu \to \mathcal{U}_\mu(x)$$

$$U_\mu U^\dagger_\mu = 1 \equiv \mathcal{U}_\mu(x) * \mathcal{U}^*_\mu(x) = 1$$

$$Z^{NC}_{L^D}(U(1)) = \int \prod d\mathcal{U}_\mu(x)\, e^{-S^{NC}(\mathcal{U})}$$

$$S^{NC}(\mathcal{U}) = \frac{1}{2e^2} \sum_{\mu \neq \nu,\, x} \mathcal{U}_\mu(x) * \mathcal{U}_\nu(x+\hat{\mu}) * \mathcal{U}^\dagger_\mu(x+\hat{\nu}) * \mathcal{U}^\dagger_\nu(x)$$

$$e^2 = g^2 N$$

The identity is itself the simplest example of Morita equivalence:

<u>a $U(N)$ commutative gauge theory on a 1^D lattice $\equiv$ non-commutative $U(1)$ theory on a L^D lattice, $L^D = N$</u>

$$\text{DOF}: \underset{U(N)}{D N^2} = \underset{U(1)}{D L^D}$$

QG on the lattice

Can 4dQG be described as a non-trivial fixed point in a regularized Euclidean Lattice theory?

$2+\varepsilon$ expansion gives support to this idea.

But recent results in Regge calculus and using dynamical triangulations in the negative

(Exception: Horata, Yukawa this conference)

(Claim: $U(1)$ field can lead to second order phase transition)

$\boxed{\text{Main problem}}$: very degenerate geometries are entropically dominant. Somehow they are not tamed in the right way by the action.

The action has its own problem: In the continuum it is unbounded from below: The conformal factor

$$g_{\mu\nu}(x) = e^{\phi(x)}\hat{g}_{\mu\nu}(x) \Rightarrow \quad -(\partial_\mu\phi)^2$$

On computer one sees a transition from one class of pathological configurations (degenerate) to another class (branched polymers)

But first order

This is an indication that the measure used is wrong:

<u>classically</u> the conformal factor is not a d.o.f and it should not appear so in the quantum theory either.

Motivated by these considerations it was suggested that only a restricted class of geometries should enter in the path integral:

<u>The ones which could be associated with non-degenerate Lorentzian geometries</u> (R. Loll):

$$\boxed{\text{Lorentzian Dynamical triangulations}}$$

The approach is closely related to a proper-time description of gravity and in this gauge it has been proven (moduli a few technical assumptions) that <u>the conformal factor is cancelled by a measure factor</u>, (Loll, Das Gupta this conference)

Good indications that the same is true in 3d LDT.
Computer simulations suggest a continuum limit and the model admits a matrix model formulation
So again one might be in the situation where a lattice model formulation is convenient even from an analytical point of view.

3d QG is interesting as a test for the LDT philosophy :

1. EDT failed
2. It has a conformal factor problem (contrary to 2d QG)
3. It is formally non-renormalizable in the gravitational coupling constant.
4. STILL it should make sense since there is no field theoretical d.o.f.

$$\boxed{\text{After that the unchartered area of 4d LDT } \ldots\ldots}$$

Already in use : Study of BH

(Dittrich, Kappel, Loll)