Monte Carlo based studies of polarized positrons source for the International Linear Collider (ILC)

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Simulation of polarization dependent processes for the development of a polarized positron source for the ILC.

I. Production:
- Helical undulator: circ. pol. photons
- Conversion Target: via pair production longitudinally polarized $e^-$ and $e^+$
- Capture of the polarized $e^+$

II. Polarimetry:
- Transmission polarimetry via Compton scattering (E166)
- other possibilities: Møller / Bhabha scattering ?
GEANT4 Status

Particle:
- Particle definition
- Momentum
- Polarization

Material/Volume:
- Atomic composition
- Density
- Fields (electric/magnetic)

Processes:
- at rest: particle decay, annihilation
- along step: ionization, multiple scattering
- post step: Compton scattering, Bremsstrahlung, Møller-, Bhabha scattering, Pair production

- only low energy Compton scattering (linear pol. optical photons on unpol. e−)
- no polarization of the medium
- placeholder for pol. vector of particles (3-vector) exists
What is needed in GEANT4 for polarization studies

TARGET

Gammas:
- Gamma Conversion
- Compton Scattering
- PhotoElectric Effect

Electrons and Positrons:
- Multiple Scattering
- Ionization
- Bremsstrahlung

Diagnostics (Polarimetry)
Cross sections polarization dependent

- Compton Scattering
- Bhabha Scattering
- Moller Scattering
- Positron annihilation in Flight
- ........

Polarization transfer to e-/e+

Polarization tracking (depolarization effects ?)

MAGNETIC FIELD:

Compton Scattering
Bhabha Scattering
Moller Scattering
Positron annihilation in Flight

Polarized beam
- From the Helical Undulator

0.5 X0 W (Tungsten) -> E166
X0 W (Tungsten) = 3.5 mm
Proposal for the implementation

• Use the 3-vector (particlePolarization) for bookkeeping of particle polarization
  but: how to define the polarization?
• introduce a polarization manager to handle the medium-polarization
• implement polarization dependent cross sections for the desired processes
  in a universal way (for future extension)

Proposal: use Spin density matrix and Stokes-Parameters

Advantages:
• can handle all polarization states
• provides a unique definition of the polarization
**Stokes parameters**

G. Stokes, Trans. Cambridge Phil. Soc. 9 (1852) 399

Wave function:

\[ \Psi(x, t) = a_1 \Psi_1 + a_2 \Psi_2 \]

Jones vector:

\[ \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad |a_1|^2 + |a_2|^2 = 1 \]

Spin density matrix:

\[ \rho = \mathbf{a} \otimes \mathbf{a}^* = \begin{pmatrix} a_1 a_1^* & a_1 a_2^* \\ a_2 a_1^* & a_2 a_2^* \end{pmatrix} = \frac{1}{2} (1 + \xi \sigma) \]

Stokes parameter:

\[ \xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} = \mathbf{a}^\dagger \mathbf{\sigma} \mathbf{a} \]

\[ \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

\[ \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]

\[ \sigma_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \]
## Stokes parameters

<table>
<thead>
<tr>
<th>Stokes parameter</th>
<th>Photon observation</th>
<th>Fermion observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_1$</td>
<td>Plane polarization</td>
<td>Spin in $z$ - direction</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>Plane polarization at an angle of $\pi/4$ to the right</td>
<td>Spin in $x$ - direction</td>
</tr>
<tr>
<td>$\xi_3$</td>
<td>Left/Right circular polarization</td>
<td>Spin in $y$ - direction</td>
</tr>
</tbody>
</table>

Example linear polarized photon:

$$E = \cos \phi E_1 + \sin \phi E_2$$

$$\xi = \begin{pmatrix} \cos^2 \phi - \sin^2 \phi \\ 2 \sin \phi \cos \phi \\ 0 \end{pmatrix}$$
Matrix formalism

\[
\begin{pmatrix}
  I \\
  \xi
\end{pmatrix} = T \begin{pmatrix}
  I_0 \\
  \xi_0
\end{pmatrix}
\]

Transformation matrix

\[
T = \begin{pmatrix}
  S & A_1 & A_2 & A_3 \\
  P_1 & M_{11} & M_{21} & M_{31} \\
  P_2 & M_{12} & M_{22} & M_{32} \\
  P_3 & M_{13} & M_{23} & M_{33}
\end{pmatrix}
\]

- Differential cross section
- Asymmetry
- Polarization
- Depolarization and polarization transfer
Pair production in field of nucleus

\[
T = \begin{pmatrix}
I & -D & 0 & 0 \\
0 & 0 & 0 & -L \\
0 & 0 & 0 & -T \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
I = [p^2 + (p - k)^2](3 + F(p, k; Z)) - 2p(p - k)(1 + G(p, k; Z))
\]

\[
D = 8p(p - k) G(p, k; Z)
\]

\[
L = k\{(2p - k)[3 + F(p, k; Z)] + 2(p - k)[1 + G(p, k; Z)]\}
\]

\[
T = 4k(p - k) H(p, k; Z)
\]
Compton scattering

\[
T = \begin{pmatrix}
I & A & 0 & E \\
A & B & 0 & H_1 \\
0 & 0 & C & H_2 \\
F & G_1 & G_2 & D
\end{pmatrix}
\]

Independent of electron spin $S$:
(I, A, B, C, D)

- $I = 1 + \cos^2 \theta + (k_0 - k)(1 - \cos \theta)$
- $A = \sin^2 \theta$
- $D = 2 \cos \theta + (k_0 - k)(1 - \cos \theta) \cos \theta$

Dependent on electron spin $S$:
(E, F, G_i, H_i)

- $E = -(1 - \cos \theta)(k_0 \cos \theta + k) \cdot S$
- $F = -(1 - \cos \theta)(k \cos \theta + k_0) \cdot S$
Proposal for the implementation

**Input:**
Stokes parameters of the particle (beam) $\rightarrow$ G4ThreeVector (bookkeeping already included in GEANT4)
Polarization of the Volume $\rightarrow$ G4ThreeVector (PolarizationManager)

**Calculate cross sections** (total, differential) $\rightarrow$ interaction length
Derive polarization transfer from initial to final state

**Output:**
Stokes parameters of the final states
Results Compton scattering

E_γ = 10 MeV

Polarized:
- \( P_e = +1 \)
- \( P_γ = +1 \)
- \( P_e = -1 \)
- \( P_γ = +1 \)

Unpolarized:
- \( P_e = 0 \)
- \( P_γ = 0 \)
Polarization transfer in the pair production process

Conversion target

Polarized Photon Beam

Stokes Vector

Positron Polarization profile from polarized Undulator photons (creation point)

Positron Polarization at the creation state

$\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$

Polarization transfer in the pair production process
Polarization transfer in the pair production process

Stokes Vector

Unpolarized Photon Beam

Conversion target

Positron Polarization profile from polarized Undulator photons (creation point)
Target studies - results

Polarization profile of the first harmonic undulator photons

Positron Polarization profile created by the undulator photons (creation point)

The target may filter positrons with high degree of polarization

$0.5 \times X_0 W$
Outlook

- **Continue the implementation of Polarization in G4 processes**
  - Focus on stokes formalism *(describes all polarization states for $\gamma, e^+, e^-$)*
  - First priority to processes needed for the for polarized positron source
  - Other processes will not be neglected

- **Cross check with other existing simulation packages (EGS4...)**

- **Possibility to simulate and cross check with experimental results (E166)**

- **Contact and collaborate with other groups (developing polarized Geant4)**
  - Coordinate the work and the approach on the implementation.

- **Propose the polarized processes to the G4 collaboration (official release)**