High-Precision Tests of the MSSM with GigaZ

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We review the physics potential of the GigaZ option of the International Linear Collider (ILC) for probing the Minimal Supersymmetric Standard Model (MSSM) via the sensitivity of the electroweak precision observables measured at the ILC to quantum corrections [1]. A particular focus is put on the effective leptonic weak mixing angle, $\sin^2\theta_{\rm eff}$. The MSSM predictions take into account the complete one-loop results including the full complex phase dependence, all available MSSM two-loop corrections as well as the full Standard Model (SM) results. We find that the anticipated experimental accuracy at the ILC with GigaZ option may resolve the virtual effects of SUSY particles even in scenarios where the SUSY particles are so heavy that they escape direct detection at the LHC and the first phase of the ILC.

1 Introduction

Electroweak precision observables (EWPO) are very powerful for testing the Standard Model (SM) and extensions of it. A particularly attractive extension is the Minimal Supersymmetric Standard Model (MSSM), see Ref. [2] for a review of electroweak precision physics in the MSSM. In this context the Z-pole observables (and also the relation between the W- and Z-boson masses obtained from muon decay) play an important role. They comprise in particular the effective leptonic weak mixing angle, $\sin^2\theta_{\rm eff}$, the total Z-boson width, Γ_Z, the ratio of the hadronic to leptonic decay width of the Z, R_l, the ratio of the partial decay width for $Z \to b\bar{b}$ to the hadronic width, R_b, and the hadronic peak cross section, $\sigma^0_{\rm had}$. Performing fits in constrained SUSY models a certain preference for not too heavy SUSY particles has been found [3–7]. The prospective improvements in the experimental accuracies, in particular at the ILC with GigaZ option, will provide a high sensitivity to deviations both from the SM and the MSSM. In Tab. 1 we summarize the current experimental results [8–10] together with the anticipated improvements at the LHC and the ILC with GigaZ option, see Refs. [2, 11–13] for details.

In order to confront the predictions of supersymmetry (SUSY) with the electroweak precision data and to derive constraints on the supersymmetric parameters, it is desirable to achieve the same level of accuracy for the SUSY predictions as for the SM. In Refs. [14, 15] an new evaluation of M_W and the Z-pole observables in the MSSM has been presented. It includes the full one-loop result (for the first time with the full complex phase dependence), all available MSSM two-loop corrections (entering via the ρ parameter [16–18]), as well as the full SM results, see Refs. [14, 15] for details. The Higgs-boson sector has been implemented including higher-order corrections (as evaluated with `FeynHiggs` [19–21]). These corrections, being formally of higher-order, can give sizable contributions to the EWPO. The remaining theory uncertainties have been estimated to be $\delta M_W^{\rm theo} \lesssim 10$ MeV [14] and $\delta\sin^2\theta_{\rm eff}^{\rm theo} \lesssim 7 \times 10^{-5}$ [15]. It has furthermore been shown in Ref. [15] that M_W, $\sin^2\theta_{\rm eff}$ and Γ_Z show

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observable	central exp. value	$\sigma \equiv \sigma^{\rm today}$	$\sigma^{\rm LHC}$	$\sigma^{\rm ILC/GigaZ}$
M_W [GeV]	80.398	0.025	0.015	0.007
$\sin^2\theta_{\rm eff}$	0.23153	0.00016	0.00020–0.00014	0.000013
Γ_Z [GeV]	2.4952	0.0023	—	0.001
R_l	20.767	0.025	—	0.01
R_b	0.21629	0.00066	—	0.00014
$\sigma^0_{\rm had}$	41.540	0.037	—	0.025
m_t [GeV]	170.9	1.8	1.0	0.1

Table 1: Summary of the electroweak precision observables, including the top-quark mass, their current experimental central values and experimental errors, $\sigma \equiv \sigma^{\rm today}$ [8–10]. Also shown are the anticipated experimental accuracies at the LHC, $\sigma^{\rm LHC}$, and the ILC (including the GigaZ option), $\sigma^{\rm ILC}$. Each number represents the combined results of all detectors and channels at a given collider, taking into account correlated systematic uncertainties, see Refs. [2, 11–13] for details. Non-existing analyses are referred to as "—".

a pronounced sensitivity to the SUSY parameters, while the other EWPO exhibit only a small variation over the MSSM parameter space. In view of the extraordinary anticipated accuracy of $\delta\sin^2\theta_{\rm eff}^{\rm ILC/GigaZ} = 1.3 \times 10^{-5}$ [13], the effective leptonic weak mixing angle will be a highly sensitive probe of electroweak physics.

2 $\sin^2\theta_{\rm eff}$ in a global MSSM scan

We first analyse the sensitivity of $\sin^2\theta_{\rm eff}$ to higher-order effects in the MSSM by scanning over a broad range of the SUSY parameter space. The following SUSY parameters are varied independently of each other in a random parameter scan within the given range:

$$\begin{aligned}
\text{sleptons} &: \; M_{\tilde F,\tilde F'} = 100\ldots 2000~\text{GeV}, \\
\text{light squarks} &: \; M_{\tilde F,\tilde F'_{\rm up/down}} = 100\ldots 2000~\text{GeV}, \\
\tilde t/\tilde b~\text{doublet} &: \; M_{\tilde F,\tilde F'_{\rm up/down}} = 100\ldots 2000~\text{GeV}, \quad A_{\tau,t,b} = -2000\ldots 2000~\text{GeV}, \\
\text{gauginos} &: \; M_{1,2} = 100\ldots 2000~\text{GeV}, \quad m_{\tilde g} = 195\ldots 1500~\text{GeV}, \\
&\; \mu = -2000\ldots 2000~\text{GeV}, \\
\text{Higgs} &: \; M_A = 90\ldots 1000~\text{GeV}, \quad \tan\beta = 1.1\ldots 60. \qquad (1)
\end{aligned}$$

Here $M_{\tilde F,\tilde F'}$ are the diagonal soft SUSY-breaking parameters in the sfermion sector, A_f denote the trilinear couplings, $M_{1,2}$ are the soft SUSY-breaking parameters in the chargino and neutralino sectors, $m_{\tilde g}$ is the gluino mass, μ the Higgs mixing parameter, M_A the $\mathcal{CP}$-odd Higgs boson mass, and $\tan\beta$ is the ratio of the two vacuum expectation values. Only the constraints on the MSSM parameter space from the LEP Higgs searches [22, 23] and the lower bounds on the SUSY particle masses from direct searches as given in Ref. [24] were taken into account. Apart from these constraints no other restrictions on the MSSM parameter space were made.

In Fig. 1 we compare the SM and the MSSM predictions for $\sin^2\theta_{\rm eff}$ as a function of m_t as obtained from the scatter data. The predictions within the two models give rise to

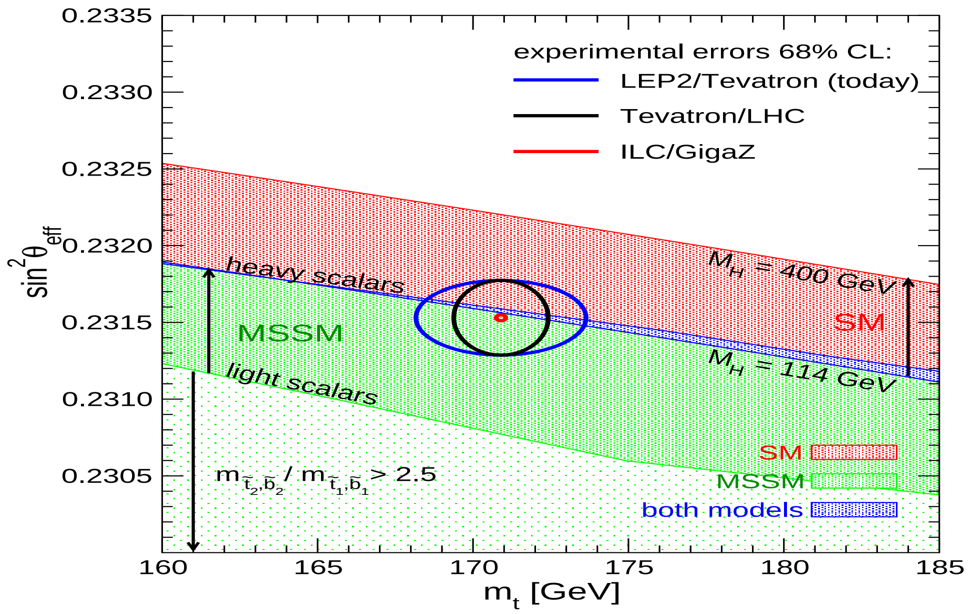

Figure 1: MSSM parameter scan for $\sin^2\theta_{\mathrm{eff}}$ as a function of m_t over the ranges given in eq. (1). Todays 68% C.L. ellipses as well as future precisions, drawn around todays central value, are indicated in the plot.

two bands in the m_t–$\sin^2\theta_{\mathrm{eff}}$ plane with only a relatively small overlap region (indicated by a dark-shaded (blue) area). The allowed parameter region in the SM (the medium-shaded (red) and dark-shaded (blue) bands) arises from varying the only free parameter of the model, the mass of the SM Higgs boson, from $M_H^{\mathrm{SM}} = 114$ GeV, the LEP exclusion bound [23] (lower edge of the dark-shaded (blue) area), to 400 GeV (upper edge of the medium-shaded (red) area). The very light-shaded (green), the light shaded (green) and the dark-shaded (blue) areas indicate allowed regions for the unconstrained MSSM. In the very light-shaded region at least one of the ratios $m_{\tilde{t}_2}/m_{\tilde{t}_1}$ or $m_{\tilde{b}_2}/m_{\tilde{b}_1}$ exceeds 2.5 (with the convention that $m_{\tilde{f}_1} \leq m_{\tilde{f}_2}$), while the decoupling limit with SUSY masses of $\mathcal{O}(2\text{ TeV})$ yields the upper edge of the dark-shaded (blue) area. Thus, the overlap region between the predictions of the two models corresponds in the SM to the region where the Higgs boson is light, i.e., in the MSSM allowed region ($M_h \lesssim 130$ GeV [19, 20]). In the MSSM it corresponds to the case where all superpartners are heavy, i.e., the decoupling region of the MSSM. The 68% C.L. experimental results for m_t and $\sin^2\theta_{\mathrm{eff}}$ are indicated in the plot. As can be seen from Fig. 1, the current experimental 68% C.L. region for m_t and $\sin^2\theta_{\mathrm{eff}}$ is in good agreement with both models and does not indicate a preference for one of the two models. The prospective accuracies for the Tevatron/LHC and the ILC with GigaZ option, see Tab. 1, are also shown in the plot (using the current central values). Especially the ILC/GigaZ precision indicates the strong potential for a significant improvement of the sensitivity of the electroweak precision tests [12]. A comparison of the MSSM parameter space preferred by $\sin^2\theta_{\mathrm{eff}}$ and the directly measured values will constitute a highly sensitive test of the model.

3 Scenario where no SUSY particles are observed at the LHC

It is interesting to investigate whether the high accuracy achievable at the GigaZ option of the ILC would provide sensitivity to indirect effects of SUSY particles even in a scenario where the (strongly interacting) superpartners are so heavy that they escape detection at the LHC.

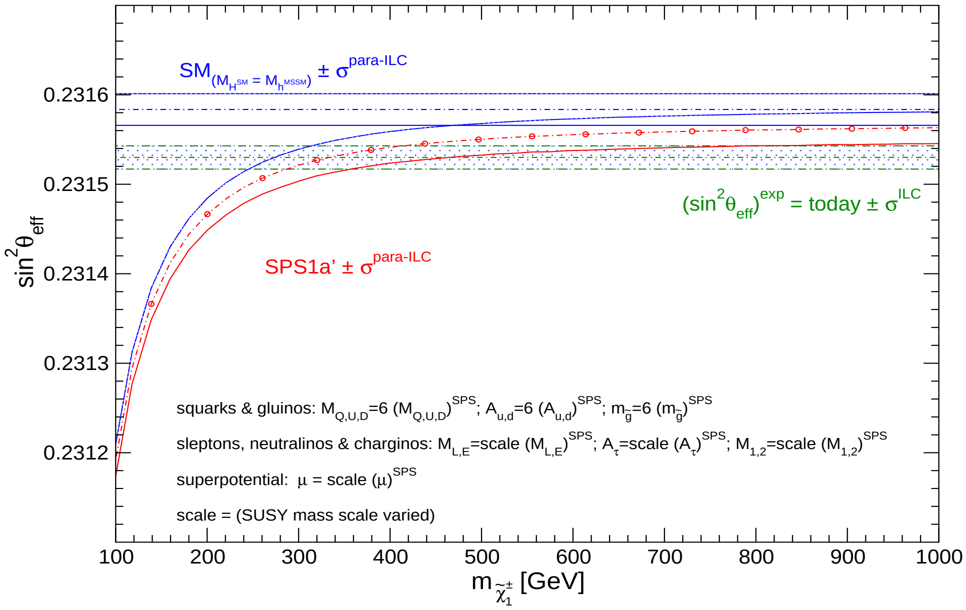

Figure 2: Theoretical prediction for $\sin^2\theta_{\text{eff}}$ in the SM and the MSSM (including prospective parametric theoretical uncertainties) compared to the experimental precision at the ILC with GigaZ option. An SPS1a′ inspired scenario is used, where the squark and gluino mass parameters are fixed to 6 times their SPS 1a′ values. The other mass parameters are varied with a common scalefactor.

We consider in this context a scenario with very heavy squarks and a very heavy gluino. It is based on the values of the SPS 1a′ benchmark scenario [25], but the squark and gluino mass parameters are fixed to 6 times their SPS 1a′ values. The other masses are scaled with a common scale factor except M_A which we keep fixed at its SPS 1a′ value. In this scenario the strongly interacting particles are too heavy to be detected at the LHC, while, depending on the scale-factor, some colour-neutral particles may be in the ILC reach. In Fig. 2 we show the prediction for $\sin^2\theta_{\text{eff}}$ in this SPS 1a′ inspired scenario as a function of the lighter chargino mass, $m_{\tilde{\chi}_1^\pm}$. The prediction includes the parametric uncertainty, $\sigma^{\text{para-ILC}}$, induced by the ILC measurement of m_t, $\delta m_t = 100$ MeV [26], and the numerically more relevant prospective future uncertainty on $\Delta\alpha^{(5)}_{\text{had}}$, $\delta(\Delta\alpha^{(5)}_{\text{had}}) = 5 \times 10^{-5}$ [27]. The MSSM prediction for $\sin^2\theta_{\text{eff}}$ is compared with the experimental resolution with GigaZ precision, $\sigma^{\text{ILC}} = 0.000013$, using for simplicity the current experimental central value. The SM prediction (with $M_H^{\text{SM}} = M_h^{\text{MSSM}}$) is also shown, applying again the parametric uncertainty $\sigma^{\text{para-ILC}}$.

Despite the fact that no coloured SUSY particles would be observed at the LHC in this scenario, the ILC with its high-precision measurement of $\sin^2\theta_{\rm eff}$ in the GigaZ mode could resolve indirect effects of SUSY up to $m_{\tilde{\chi}_1^\pm} \lesssim 500$ GeV. This means that the high-precision measurements at the ILC with GigaZ option could be sensitive to indirect effects of SUSY even in a scenario where SUSY particles have *neither* been directly detected at the LHC nor the first phase of the ILC with a centre of mass energy of up to 500 GeV.

4 Conclusions

EWPO provide a very powerful test of the SM and the MSSM. We have reviewed results for M_W and Z boson observables such as $\sin^2\theta_{\rm eff}$, Γ_Z, $R_{\rm l}$, $R_{\rm b}$, $\sigma^0_{\rm had}$. Within the MSSM new results for the EWPO containing the complete one-loop results with complex parameters and all available higher-order corrections in the SM and the MSSM have recently become available. The sensitivity to higher-order effects will drastically improve with the ILC precision (including the GigaZ option) on the EWPO and m_t. This has been illustrated in two examples. A general scan over the MSSM parameter space for $\sin^2\theta_{\rm eff}$ and m_t currently does not prefer the SM or the MSSM over the other. However, the anticipated GigaZ precision indicates the high potential for a significant improvement of the sensitivity of the electroweak precision tests. In a second example we have assumed a scenario with very heavy SUSY particles, outside the reach of the LHC and the first stage of the ILC with $\sqrt{s} = 500$ GeV. It has been shown that even in such a scenario the GigaZ precision on $\sin^2\theta_{\rm eff}$ may resolve virtual effects of SUSY particles, providing a possible hint to the existence of new physics.

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