About the Running and Decoupling in the MSSM

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The evolution of the strong coupling constant α_s from M_Z to the GUT scale is presented, involving three-loop running and two-loop decoupling. Accordingly, the two-loop transition from the $\overline{\text{MS}}$- to the $\overline{\text{DR}}$-scheme is properly taken into account. We find that the three-loop effects are comparable to the experimental uncertainty for α_s.

1 Introduction

The observation that the gauge couplings of the strong, electromagnetic and weak interaction tend to unify in the Minimal Supersymmetric Standard Model (MSSM) at a high energy scale $\mu_{\text{GUT}} \simeq 10^{16}$ GeV and the consistent predictions made for SM parameters, such as the top quark mass and the ratio of the bottom quark to the tau lepton masses, using constraints on the Yukawa sector of SUSY-GUT models, brought SUSY in the center of the phenomenological studies.

Nevertheless, SUSY is only an approximate symmetry in nature and several scenarios for the mechanism of SUSY breaking have been proposed. A possibility to constrain the type and scale of SUSY breaking is to study, with very high precision, the relations between the MSSM parameters evaluated at the electroweak and the GUT scales. The extrapolation over many orders of magnitude requires high-precision experimental data at the low energy scale. A first set of precision measurements is expected from the CERN Large Hadron Collider (LHC) with an accuracy at the percent level. A comprehensive high-precision analysis can be performed at the International Linear Collider (ILC), for which the estimated experimental accuracy is at the per mill level. In this respect, it is necessary that the same precision is reached also on the theory side in order to match with the data [2]. Running analyses based on full two-loop renormalization group equations (RGEs) [3, 4] for all parameters and one-loop threshold corrections [5] are currently implemented in the public programs ISAJET [6], SOFTSUSY [7], SPHENO [8], SuSpect [9]. The agreement between the different codes is in general within one percent [10]. A first three-loop running analysis, based, however, only on one-loop threshold effects, was carried out in Ref. [11].

In this talk, we report on the evaluation of the strong coupling α_s in MSSM, based on three-loop RGEs [12] and two-loop threshold corrections [13]. On the one hand, the three-loop corrections reduce significantly the dependence on the scale at which heavy particles are integrated out [14] and on the other hand, they are essential for phenomenological studies, because they are as large as, or greater than, the effects induced by the current experimental accuracy of $\alpha_s(M_Z)$ [15]. Additionally, we compare the predictions obtained within the above mentioned approach with those based on the leading-logarithmic (LL) approximation suggested in Ref. [2].

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2 Evaluation of $\alpha_s(\mu_{\rm GUT})$ from $\alpha_s(M_Z)$

The aim of this study is to compute α_s at a high-energy scale $\mu \simeq \mathcal{O}(\mu_{\rm GUT})$, starting from the strong coupling constant at the mass of the Z boson M_Z. We denote this parameter $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$ to specify that the underlying theory is QCD with five active flavours and $\overline{\rm MS}$ is the renormalization scheme . The value of $\alpha_s(\mu_{\rm GUT})$ is the strong coupling constant in a supersymmetric theory, renormalized in the $\overline{\rm DR}$-scheme. The relation between the two parameters requires the consistent combination of the following ingredients.

- The renormalization group evolution of α_s.
The energy dependence of the strong coupling constant is governed by the RGE. In QCD with n_f quark flavours, the β function is known through four loops both in the $\overline{\rm MS}$ [16, 17] and the $\overline{\rm DR}$-scheme [18]. In SUSY-QCD, the β function has been evaluated in the $\overline{\rm DR}$-scheme through three loops [12].

- The transition from the $\overline{\rm MS}$- to the $\overline{\rm DR}$-scheme.
For the three-loop running analysis we are focusing on, one needs to evaluate the dependence of α_s values in the $\overline{\rm DR}$-scheme from those in $\overline{\rm MS}$-scheme through two loops [18]

$$\alpha_s^{\overline{\rm MS}} = \alpha_s^{\overline{\rm DR}}\left[1 - \frac{\alpha_s^{\overline{\rm DR}}}{4\pi} - \frac{5}{4}\left(\frac{\alpha_s^{\overline{\rm DR}}}{\pi}\right)^2 + \frac{\alpha_s^{\overline{\rm DR}}\alpha_e}{12\pi^2}\, n_f + \ldots\right], \qquad (1)$$

where $\alpha_s^{\overline{\rm DR}} \equiv \alpha_s^{\overline{\rm DR},(n_f)}(\mu)$ and $\alpha_s^{\overline{\rm MS}} \equiv \alpha_s^{\overline{\rm MS},(n_f)}(\mu)$. $\alpha_e \equiv \alpha_e^{(n_f)}(\mu)$ is one of the so-called evanescent coupling constants that occur when $\overline{\rm DR}$ is applied to non-supersymmetric theories (QCD in this case). In particular, it describes the coupling of the 2ε-dimensional components (so-called ε-scalars) of the gluon to a quark.

- The transition from five-flavour QCD to the full SUSY theory.
For mass independent renormalization schemes like $\overline{\rm MS}$ or $\overline{\rm DR}$, the decoupling of heavy particles has to be performed explicitely. In practice, this means that intermediate effective theories are introduced by integrating out the heavy degrees of freedom. One may separately integrate out every particle at its individual threshold ("step approximation"), a method suited for SUSY models with a severely split mass spectrum. But the intermediate effective theories with "smaller" symmetry raise the problem of introducing new couplings, each governed by its own RGE. To overcome this difficulty, for SUSY models with roughly degenerate mass spectrum at the scale $\tilde{M}$, one can consider the MSSM as the full theory that is valid from the GUT scale $\mu_{\rm GUT}$ down to $\tilde{M}$, which we assume to be around 1 TeV. Integrating out all SUSY particles at this common scale, one directly obtains the SM as the effective theory, valid at low energies. The transition between the two theories can be done at an arbitrary decoupling scale μ:

$$\alpha_s^{\overline{\rm DR},(n_f)}(\mu) = \zeta_s^{(n_f)}\,\alpha_s^{\overline{\rm DR},({\rm full})}(\mu)\,, \quad \alpha_e^{(n_f)}(\mu) = \zeta_e^{(n_f)}\alpha_e^{({\rm full})}(\mu)\,. \qquad (2)$$

ζ_s and ζ_e depends logarithmically on the scale μ, which is why one generally chooses $\mu \sim \tilde{M}$. In Eq. (2), $n_f = 6$ means that only the SUSY particles are integrated out, while for $n_f = 5$ at the same time the top quark is integrated out. This procedure, also known as "common scale approach" [19], is implemented in most of the present codes computing the SUSY spectrum [8, 7, 9] by applying the one-loop approximation of Eq. (2) and setting $n_f = 5$ and $\mu = M_Z$.

In the following, we will assume that QCD is obtained by integrating out the heavy degrees of freedom (squarks and gluinos) from SUSY-QCD. Due to SUSY, the evanescent couplings in SUSY-QCD can be related to the gauge coupling α_s as follows:

$$\alpha_e^{(\text{full})}(\mu) = \alpha_s^{\overline{\text{DR}},(\text{full})}(\mu)\,. \tag{3}$$

The evanescent couplings in n_f-flavour QCD, i.e. $\alpha_e^{(n_f)}$ are then obtained by decoupling relations analogous to Eq. (2).

For the evaluation of $\alpha_s^{\overline{\text{DR}},(\text{full})}(\mu_{\text{GUT}})$ from $\alpha_s^{\overline{\text{MS}},(n_f)}(\mu_{M_Z})$ we propose the following method:

$$\begin{aligned} \alpha_s^{\overline{\text{MS}},(n_f)}(M_Z) \quad &\stackrel{(i)}{\to} \quad \alpha_s^{\overline{\text{MS}},(n_f)}(\mu_{\text{dec}}) \quad \stackrel{(ii)}{\to} \quad \alpha_s^{\overline{\text{DR}},(n_f)}(\mu_{\text{dec}}) \\ &\stackrel{(iii)}{\to} \quad \alpha_s^{\overline{\text{DR}},(\text{full})}(\mu_{\text{dec}}) \quad \stackrel{(iv)}{\to} \quad \alpha_s^{\overline{\text{DR}},(\text{full})}(\mu_{\text{GUT}})\,. \end{aligned} \tag{4}$$

The individual steps require: (i) $\beta(\alpha_s)$ in QCD through three loops, (ii) the $\overline{\text{MS}}$–$\overline{\text{DR}}$ relation through order α_s^2, (iii) decoupling of the SUSY particles through order α_s^2, and (iv) $\beta(\alpha_s)$ through three loops in SUSY-QCD. The advantage of this procedure as compared to a multi-scale approach is that the RGEs are only one-dimensional and that for α_e one can apply Eq. (2).

2.1 Numerical results

The result for $\alpha_s^{\overline{\text{DR}}}(\mu_{\text{GUT}} = 10^{16}$ GeV), obtained using $M_Z = 91.1876$ GeV and $m_t = 170.9 \pm 1.9$ GeV , $\alpha_s^{\overline{\text{MS}}}(M_Z) = 0.1189$, $\tilde{M} = m_{\tilde{q}} = m_{\tilde{g}} = 1000$ GeV as input parameters is shown if Figure 1. The dotted, dashed and solid line are based on Eq. (4), where n-loop running is combined with $(n-1)$-loop decoupling, as it is required for consistency ($n = 1, 2, 3$, respectively). We find a nice convergence when going from one to three loops, with a very weakly μ_{dec}–dependent result at three-loop order. For comparison, we show the result (the dash-dotted line) obtained from the formula given in Eq. (21) of Ref [2]. It corresponds to the resummed one-loop contributions originating from both the change of scheme and the decoupling of heavy particles. However, the difference between our three-loop result with two-loop decoupling (upper solid line) and the one-loop formula given in Ref. [2] exceeds the experimental uncertainty by almost a factor of four for sensible values of μ_{dec}. This uncertainty is indicated by the hatched band, derived from $\delta\alpha_s(M_Z) = \pm 0.001$ [15]. The formulae of Ref. [2] should therefore be taken only as rough estimates.

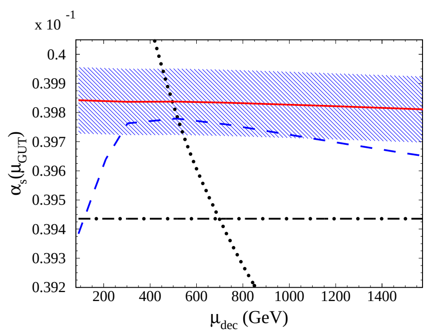

Figure 1: $\alpha_s(\mu_{\text{GUT}})$ as a function of μ_{dec}.

In Figure 2 we show $\alpha_s(\mu_{\rm GUT})$ as a function of $\tilde{M}$ where $\mu_{\rm dec} = \tilde{M}$ has been adopted. Dotted, dashed and full curve correspond again to the one-, two- and three-loop analysis and the uncertainty form $\alpha_s(M_Z)$ is indicated by the hatched band. One observes a variation of 10% as $\tilde{M}$ is varied between 100 GeV and 10 TeV. This shows that the actual SUSY scale can significantly influence the unification, respectively, the non-unification behaviour of the three couplings at the GUT scale.

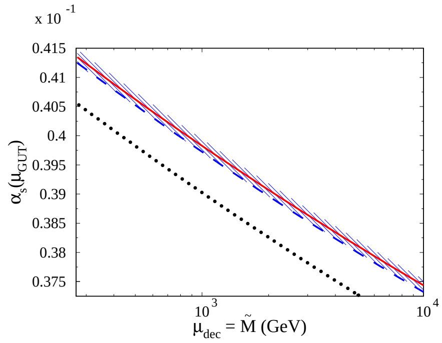

Figure 2: $\alpha_s(\mu_{\rm GUT})$ as a function of $\tilde{M}$.

3 Conclusions

We have used recent three- and four-loop results for the β functions, and the decoupling coefficients in order to derive $\alpha_s^{\overline{\rm DR}}(\mu_{\rm GUT})$ from $\alpha_s^{\overline{\rm MS}}(M_Z)$ at three- and four-loop level, respectively.

It turns out that the three-loop terms are numerically significant. The dependence on where the SUSY spectrum is decoupled becomes particularly flat in this case. The theoretical uncertainty is expected to be negligible w.r.t. the uncertainty induced by the experimental input values. In consequence, we recommend that phenomenological studies concerning the implications of low energy data on Grand Unification should be done at three-loop level.

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