Consistent Treatment of Imaginary Contributions to Higgs-Boson Masses in the MSSM

T. Hahn[1], S. Heinemeyer[2], W. Hollik[1], H. Rzehak[3] and G. Weiglein[4]

1- Max-Planck-Institut für Physik, Föhringer Ring 6, D–80805 Munich, Germany

2- Instituto de Fisica de Cantabria (CSIC-UC), Santander, Spain

3- Paul Scherrer Institut, Würenlingen und Villigen, CH–5232 Villigen PSI, Switzerland

4- IPPP, University of Durham, Durham DH1 3LE, UK

We show how the imaginary parts of the Higgs-Boson self-energies in the MSSM are consistenly taken into account in the Higgs-Boson mass determination. In a numerical example we find effects of 5 GeV in the mass difference of the two heavy neutral Higgs bosons. The imaginary contributions have been included into the code `FeynHiggs`.

1 Introduction

A striking prediction of models of supersymmetry (SUSY) [2] is a Higgs sector with at least one relatively light Higgs boson. In the Minimal Supersymmetric extension of the Standard Model (MSSM) two Higgs doublets are required, resulting in five physical Higgs bosons: the light and heavy $\mathcal{CP}$-even h and H, the $\mathcal{CP}$-odd A, and the charged Higgs bosons $H^{\pm}$. The Higgs sector of the MSSM can be expressed at lowest order in terms of M_Z, M_A and $\tan\beta \equiv v_2/v_1$, the ratio of the two vacuum expectation values. All other masses and mixing angles can therefore be predicted. Higher-order contributions give large corrections to the tree-level relations. The limits obtained from the Higgs search at LEP (the final LEP results can be found in Refs. [3, 4]), place important restrictions on the parameter space of the MSSM.

For the MSSM with real parameters (rMSSM) the status of higher-order corrections to the masses and mixing angles in the Higgs sector is quite advanced. The complete one-loop result within the rMSSM is known [5, 6, 7, 8]. The computation of the two-loop corrections has meanwhile reached a stage where all the presumably dominant contributions are available, see Refs. [9, 10] and references therein. Leading three-loop corrections have recently been obtained in Ref. [11]. The remaining theoretical uncertainty on the lightest $\mathcal{CP}$-even Higgs boson mass has been estimated to be below ~ 3 GeV [9, 12, 13]. The public code `FeynHiggs` [9, 14, 15, 16] is based on the results obtained in the Feynman-diagrammatic (FD) approach [9, 14, 17, 18]; it includes all available corrections in the FD approach. For the MSSM with complex parameters (cMSSM) the full one-loop result in the FD approach has been obtained in Ref. [16], and the corresponding leading $\mathcal{O}(\alpha_t\alpha_s)$ corrections can be found in Ref. [19].

2 Imaginary Contributions to Higgs-boson self-energies

The propagator matrix of the neutral Higgs bosons h, H, A can be written as a 3×3 matrix, $\Delta_{hHA}(p^2)$. The 3×3 propagator matrix is related to the 3×3 matrix of the irreducible vertex functions by

$$\Delta_{hHA}(p^2) = -\left(\hat{\Gamma}_{hHA}(p^2)\right)^{-1}, \qquad (1)$$

where

$$\hat{\Gamma}_{hHA}(p^2) = i\left[p^2 \mathbb{1} - \mathbf{M}_{\rm n}(p^2)\right], \tag{2}$$

$$\mathbf{M}_{\rm n}(p^2) = \begin{pmatrix} m_h^2 - \hat{\Sigma}_{hh}(p^2) & -\hat{\Sigma}_{hH}(p^2) & -\hat{\Sigma}_{hA}(p^2) \\ -\hat{\Sigma}_{hH}(p^2) & m_H^2 - \hat{\Sigma}_{HH}(p^2) & -\hat{\Sigma}_{HA}(p^2) \\ -\hat{\Sigma}_{hA}(p^2) & -\hat{\Sigma}_{HA}(p^2) & m_A^2 - \hat{\Sigma}_{AA}(p^2) \end{pmatrix}. \tag{3}$$

The three complex poles $\mathcal{M}^2$ of Δ_{hHA}, eq. (1), are determined as the solutions of

$$\mathcal{M}_i^2 - m_i^2 + \hat{\Sigma}_{ii}^{\rm eff}(\mathcal{M}_i^2) = 0\ ,\ \ i = h, H, A\ . \tag{4}$$

The effective self-energy reads (no summation over i, j, k)

$$\hat{\Sigma}_{ii}^{\rm eff}(p^2) = \hat{\Sigma}_{ii}(p^2) - i\frac{2\hat{\Gamma}_{ij}(p^2)\hat{\Gamma}_{jk}(p^2)\hat{\Gamma}_{ki}(p^2) - \hat{\Gamma}_{ki}^2(p^2)\hat{\Gamma}_{jj}(p^2) - \hat{\Gamma}_{ij}^2(p^2)\hat{\Gamma}_{kk}(p^2)}{\hat{\Gamma}_{jj}(p^2)\hat{\Gamma}_{kk}(p^2) - \hat{\Gamma}_{jk}^2(p^2)}, \tag{5}$$

where the $\hat{\Gamma}_{ij}(p^2)$ are the elements of the 3×3 matrix $\hat{\Gamma}_{hHA}(p^2)$ as specified in eq. (2). The complex pole is decomposed as

$$\mathcal{M}^2 = M^2 - iM\Gamma, \tag{6}$$

where M is the mass of the particle and Γ its width, We define the loop-corrected mass eigenvalues according to

$$M_{h_1} \le M_{h_2} \le M_{h_3}. \tag{7}$$

In our determination of the Higgs-boson masses we take into account all imaginary parts of the Higgs-boson self-energies (besides the term with imaginary parts appearing explicitly in eq. (4), there are also products of imaginary parts in $\mathrm{Re}\,\hat{\Sigma}_{ii}^{\rm eff}(M_i^2)$). The effects of the imaginary parts of the Higgs-boson self-energies on Higgs phenomenology can be especially relevant if the masses are close to each other. This has been analyzed in Ref. [20] taking into account the mixing between the two heavy neutral Higgs bosons, where the complex mass matrix has been diagonalized using a complex mixing angle, resulting in a non-unitary mixing matrix. The effects of imaginary parts of the Higgs-boson self-energies on physical processes with s-channel resonating Higgs bosons are discussed in Refs. [20, 21, 22]. In Ref. [20] only the one-loop corrections from the $t/\tilde{t}$ sector have been taken into account for the H–A mixing, analyzing the effects on resonant Higgs production at a photon collider. In Ref. [21] (using the code `CPSuperH` [23]) the full one-loop imaginary parts of the self-energies have been evaluated for the mixing of the three neutral MSSM Higgs bosons. The effects have been analyzed for resonant Higgs production at the LHC, the ILC and a photon collider (however, the corresponding effects on the Higgs-boson masses have been neglected). In Ref. [22] the $\tilde{t}/\tilde{b}$ one-loop contributions (neglecting the t/b corrections) on the H–A mixing for resonant Higgs production at a muon collider have been discussed. Our calculation [16, 24] incorporates for the first time the complete effects arising from the imaginary parts of the one-loop self-energies in the neutral Higgs-boson propagator matrix, including their effects on the Higgs masses and the Higgs couplings in a consistent way.

3 Numerical example

In order to study the impact of the imaginary parts of the Higgs-boson self-energies, it is useful to compare the full result with the "$\mathrm{Im}\,\Sigma = 0$" approximation, which is defined by performing the replacement

$$\mathrm{Im}\,\Sigma = 0 \text{ approximation:} \qquad \Sigma(p^2) \quad \to \quad \mathrm{Re}\,\Sigma(p^2) \tag{8}$$

for all Higgs-boson self-energies in eq. (3). The numerical example has been obtained for the following set of parameters:

$$\begin{aligned} &M_{\mathrm{SUSY}} = 500 \text{ GeV},\ |A_t| = A_b = A_\tau = 1000 \text{ GeV}, \\ &\mu = 1000 \text{ GeV},\ M_2 = 500 \text{ GeV},\ M_1 = 250 \text{ GeV},\ m_{\tilde{g}} = 500 \text{ GeV}, \\ &\mu_{\overline{\mathrm{DR}}} = m_t = 171.4 \text{ GeV [25]}. \end{aligned} \tag{9}$$

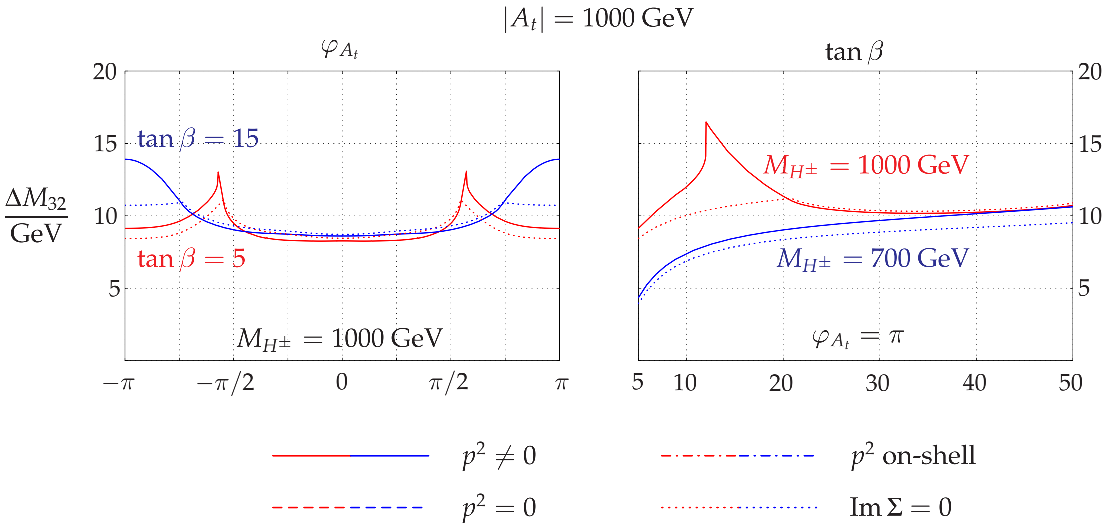

Figure 1: The mass difference $\Delta M_{32} := M_{h_3} - M_{h_2}$ is shown for $\tan\beta = 5, 15$ and $M_{H^\pm} = 1000$ GeV as a function of φ_{A_t} (left) and for $\varphi_{A_t} = \pi$, $M_{H^\pm} = 700, 1000$ GeV as a function of $\tan\beta$ (right). The solid line shows the full result, the dotted line the "$\mathrm{Im}\,\Sigma = 0$" approximation. The other two lines correspond to other Higgs-boson self-energy approximations, see Ref. [16] for details.

In Fig. 1 we show an example of the effects of the imaginary parts of the Higgs-boson self-energies, i.e. the comparison of the full result with the "$\mathrm{Im}\,\Sigma = 0$" approximation as defined in eq. (8). In the left plot we show $\Delta M_{32} := M_{h_3} - M_{h_2}$ as a function of φ_{A_t} for $\tan\beta = 5, 15$ and $m_{H^\pm} = 1000$ GeV. In the right plot we display ΔM_{32} as a function of $\tan\beta$ for $m_{H^\pm} = 700, 1000$ GeV and $\varphi_{A_t} = \pi$. Here φ_{A_t} denotes the angle of the complex valued trilinear coupling A_t. The other parameters are given in eq. (9). As one can see from the plot, the difference between the full result and the approximation with neglected imaginary parts is often ~ 1 GeV and can become as large as about 5 GeV.

4 Acknowledgments

We thank K. Williams for numerous checks and helpful discussions. Work supported in part by the European Community's Marie-Curie Research Training Network under con-

tract MRTN-CT-2006-035505 'Tools and Precision Calculations for Physics Discoveries at Colliders'

References

[1] Slides:
`ilcagenda.linearcollider.org/contributionDisplay.py?contribId=158&sessionId=71&confId=1296`

[2] H. Nilles, *Phys. Rept.* **110** (1984) 1; H. Haber and G. Kane, *Phys. Rept.* **117** (1985) 75; R. Barbieri, *Riv. Nuovo Cim.* **11** (1988) 1.

[3] [LEP Higgs working group], *Phys. Lett.* **B 565** (2003) 61 [arXiv:hep-ex/0306033].

[4] [LEP Higgs working group], *Eur. Phys. J.* **C 47** (2006) 547 [arXiv:hep-ex/0602042].

[5] J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett.* **B 257** (1991) 83; Y. Okada, M. Yamaguchi and T. Yanagida, *Prog. Theor. Phys.* **85** (1991) 1; H. Haber and R. Hempfling, *Phys. Rev. Lett.* **66** (1991) 1815.

[6] A. Brignole, *Phys. Lett.* **B 281** (1992) 284.

[7] P. Chankowski, S. Pokorski and J. Rosiek, *Phys. Lett.* **B 286** (1992) 307; *Nucl. Phys.* **B 423** (1994) 437 [arXiv:hep-ph/9303309].

[8] A. Dabelstein, *Nucl. Phys.* **B 456** (1995) 25 [arXiv:hep-ph/9503443]; *Z. Phys.* **C 67** (1995) 495 [arXiv:hep-ph/9409375].

[9] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, *Eur. Phys. J.* **C 28** (2003) 133 [arXiv:hep-ph/0212020].

[10] S. Martin, *Phys. Rev.* **D 71** (2005) 016012 [arXiv:hep-ph/0405022]; *Phys. Rev.* **D 71** (2005) 116004 [arXiv:hep-ph/0502168].

[11] S. Martin, *Phys. Rev.* **D 75** (2007) 055005 [arXiv:hep-ph/0701051].

[12] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rept.* **425** (2006) 265 [arXiv:hep-ph/0412214].

[13] B. Allanach, A. Djouadi, J. Kneur, W. Porod and P. Slavich, *JHEP* **0409** (2004) 044 [arXiv:hep-ph/0406166].

[14] S. Heinemeyer, W. Hollik and G. Weiglein, *Eur. Phys. J.* **C 9** (1999) 343 [arXiv:hep-ph/9812472].

[15] S. Heinemeyer, W. Hollik and G. Weiglein, *Comput. Phys. Commun.* **124** (2000) 76 [arXiv:hep-ph/9812320]; see `www.feynhiggs.de` .

[16] M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, R. Rzehak and G. Weiglein, *JHEP* **02** (2007) 047 [arXiv:hep-ph/0611326].

[17] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rev.* **D 58** (1998) 091701 [arXiv:hep-ph/9803277]; *Phys. Lett.* **B 440** (1998) 296 [arXiv:hep-ph/9807423].

[18] G. Degrassi, A. Dedes and P. Slavich, *Nucl. Phys.* **B 672** (2003) 144 [arXiv:hep-ph/0305127].

[19] S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, to appear in *Phys. Lett.* **B**, arXiv:0705.0746 [hep-ph].

[20] S. Choi, J. Kalinowski, Y. Liao and P. Zerwas, *Eur. Phys. J.* **C 40** (2005) 555 [arXiv:hep-ph/0407347].

[21] J. Ellis, J. Lee and A. Pilaftsis, *Phys. Rev.* **D 70** (2004) 075010 [arXiv:hep-ph/0404167]; *Nucl. Phys.* **B 718** (2005) 247 [arXiv:hep-ph/0411379]; *Phys. Rev.* **D 72** (2005) 095006 [arXiv:hep-ph/0507046].

[22] J. Bernabeu, D. Binosi and J. Papavassiliou, *JHEP* **0609** (2006) 023 [arXiv:hep-ph/0604046].

[23] J. Lee, A. Pilaftsis et al., *Comput. Phys. Commun.* **156** (2004) 283 [arXiv:hep-ph/0307377].

[24] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, G. Weiglein and K. Williams, arXiv:hep-ph/0611373.

[25] E. Brubaker et al. [Tevatron Electroweak Working Group], arXiv:hep-ex/0608032, see:
`tevewwg.fnal.gov/top/` .