Different vacua in 2HDM

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The potential of Two Higgs Doublet Model (2HDM) can have extrema with different physical properties. We found explicit equations for extremum energies via parameters of potential if it has explicitly CP conserving form. These equations allow to pick out extremum with lower energy – vacuum state and to look for change of extrema (phase transitions) with the variation of parameters of potential. Our goal is to find general picture here to apply it for description of early Universe.

■ **Lagrangian**. The spontaneous electroweak symmetry breaking via the Higgs mechanism is described by the Lagrangian[a]

$$\mathcal{L} = \mathcal{L}_{gf}^{SM} + \mathcal{L}_H + \mathcal{L}_Y \;\; with \;\; \mathcal{L}_H = T - V, \quad \varphi_i = \begin{pmatrix} \varphi_i^+ \\ \varphi_i^0 \end{pmatrix}. \tag{1}$$

Here $\mathcal{L}_{gf}^{SM}$ describes the $SU(2)\times U(1)$ Standard Model interaction of gauge bosons and fermions, $\mathcal{L}_Y$ describes the Yukawa interactions of fermions with Higgs scalars and $\mathcal{L}_H$ is the Higgs scalar Lagrangian; T is the Higgs kinetic term and V is the Higgs potential.

The most general renormalizable Higgs potential is the sum of the operator $-V_2$ of dimension 2 and the operator V_4 of dimension 4. In the 2HDM

$$\begin{gathered} V = -V_2(x_i) + V_4(x_i)\,; \quad V_2(x_i) = M_i x_i \equiv \left[m_{11}^2 x_1 + m_{22}^2 x_2 + \left(m_{12}^2 x_3 + h.c.\right)\right]/2\,, \\ V_4(x_i) = \frac{\Lambda_{ij}x_i x_j}{2} \equiv \frac{\lambda_1 x_1^2 + \lambda_2 x_2^2}{2} + \lambda_3 x_1 x_2 + \lambda_4 x_3 x_3^\dagger + \left[\frac{\lambda_5 x_3^2}{2} + \lambda_6 x_1 x_3 + \lambda_7 x_2 x_3 + h.c.\right], \\ x_1 = \varphi_1^\dagger \varphi_1, \quad x_2 = \varphi_2^\dagger\varphi_2, x_3 = \varphi_1^\dagger\varphi_2\,, \quad x_{3^*} \equiv x_3^\dagger = \varphi_2^\dagger \varphi_1 \;\; (i,\, j = 1,\,2,\,3,\,3^*). \end{gathered} \tag{2}$$

Here $\Lambda_{ij} = \Lambda_{ji}$, λ_{1-4} and m_{ii}^2 are real while λ_{5-7} and m_{12}^2 are generally complex.

■ **Extrema of potential**. The extrema of the potential define the values $\langle\varphi_{1,2}\rangle$ of the fields $\varphi_{1,2}$ via equations:

$$\partial V/\partial \varphi_i|_{\varphi_i=\langle\varphi_i\rangle} = 0\,, \qquad \partial V/\partial \varphi_i^\dagger|_{\varphi_i=\langle\varphi_i\rangle} = 0\,. \tag{3}$$

These equations have the electroweak symmetry conserving (EWc) solution $\langle\varphi_i\rangle = 0$ and could have several electroweak symmetry breaking (EWSB) solutions. Below e.g. $\langle F\rangle_N$ means numerical value of the operator F in N-th extremum.

We consider also the values y_i of operators x_i at the extremum points

$$y_{i,N} \equiv \langle x_i\rangle_N = \langle\varphi_a\rangle_N^\dagger \langle\varphi_b\rangle_N \qquad \text{for} \qquad x_i = \varphi_a^\dagger \varphi_b\,.$$

In each extremum point these values obey inequalities following from definition and Cauchy inequality, written for important auxiliary quantity Z:

$$y_1 > 0\,, \quad y_2 > 0\,, \qquad Z = y_1 y_2 - y_3^* y_3 \geq 0\,. \tag{4}$$

- **Classification of EWSB extrema**. It is useful to define quantities

$$T_a \equiv \langle \partial V/\partial x_a\rangle = \Lambda_{ai} y_i - M_a \quad (a = 1,\,2,\,3,\,3^*)\,, \quad T_{1,2} \;\; are \;\; real, \qquad T_{3^*} = T_3^*\,. \tag{5}$$

In these terms system (3) can be transformed to equations for y_i:

$$\begin{gathered} \langle\varphi_1\rangle^\dagger \langle\partial V/\partial\varphi_1^\dagger\rangle = y_1 T_1 + y_3 T_3 = 0\,, \quad \langle\varphi_2\rangle^\dagger\langle\partial V/\partial\varphi_1^\dagger\rangle = y_3^* T_1 + y_2 T_3 = 0\,, \\ \langle\varphi_2\rangle^\dagger\langle\partial V/\partial\varphi_2^\dagger\rangle = y_2 T_2 + y_3^* T_{3^*} = 0\,, \quad \langle\varphi_1\rangle^\dagger\langle\partial V/\partial\varphi_2^\dagger\rangle = y_3 T_2 + y_1 T_{3^*} = 0\,. \end{gathered} \tag{6}$$

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[a] Notations and main definitions follow [1], we use some equations from [2].

One can consider each pair of these equations as a system for calculation of quantities T_i via y_i. The determinant of these systems are precisely $Z = y_1y_2 - y_3^*y_3$. Therefore, it is natural to distinguish two types of extrema, with $Z \neq 0$ (*charged extrema* with $T_i = 0$) and with $Z = 0$ (*neutral extrema* with $T_i \neq 0$).

• For each EWSB extremum one can choose the z axis in the weak isospin space so that the most general electroweak symmetry violating solution of (3) can be written in a form with real v_1 and complex v_2:

$$\langle\varphi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\varphi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 \end{pmatrix} \qquad \textit{with}\;\; v_1 = |v_1|,\; v_2 = |v_2|e^{i\xi}\,. \tag{7}$$

At $u \neq 0$ we have $Z > 0$ – charged extremum, at $u = 0$ we have $Z = 0$ – neutral extremum.

• **The distances from some extremum and between two extrema** are useful conceptions for discussions below, they are defined as

$$\begin{aligned}\mathcal{D}(\varphi, N) &= \left(\varphi_1\langle\varphi_2\rangle_N - \varphi_2\langle\varphi_1\rangle_N\right)^\dagger\left(\varphi_1\langle\varphi_2\rangle_N - \varphi_2\langle\varphi_1\rangle_N\right) \equiv x_1y_2 + x_2y_1 - x_3y_{3^*} - x_3^\dagger y_3\,,\\ \mathcal{D}(I, II) &= \left(\langle\varphi_1\rangle_I\langle\varphi_2\rangle_{II} - \langle\varphi_2\rangle_I\langle\varphi_1\rangle_{II}\right)^\dagger\left(\langle\varphi_1\rangle_I\langle\varphi_2\rangle_{II} - \langle\varphi_2\rangle_I\langle\varphi_1\rangle_{II}\right).\end{aligned} \tag{8}$$

• **Decomposition around EWSB extremum**. Our potential can be rewritten as a sum of extremum energy and two polynomials in x_i of first ans second order. The form of second order polynomial is fixed by a quartic terms of potential, it can be only $V_4(x_i - y_i)$. The residuary first order polynomial in x_i must be proportional to $\mathcal{D}(\varphi, N)$. Therefore

$$V = \mathcal{E}_N^{ext} + V_4(x_i - y_{i,N}) + \mathcal{R}\cdot\mathcal{D}(\varphi, N)\,. \tag{9a}$$

Let us define $\mathcal{R}$. The differentiation of (9a) gives for T_i:

$$T_1 = y_2\mathcal{R}\,, \quad T_2 = y_1\mathcal{R}\,, \quad T_3 = -y_3^*\mathcal{R}\,, \quad T_{3^*} = -y_3\mathcal{R}\,.$$

For the *charged extremum* $T_i = 0$, and we have from here $\mathcal{R}_{ch} = 0$.

For the *neutral extremum* the Higgs fields mass matrix $\partial^2 V/\partial\varphi_{i,a}\partial\varphi_{j,b}|_N$ for the upper ($\pm$) components a, b can be written as

$$M_{++} = \begin{pmatrix} T_1 & T_3 \\ T_{3^*} & T_2 \end{pmatrix} \equiv \begin{pmatrix} y_2 & -y_3^* \\ -y_3 & y_1 \end{pmatrix}\mathcal{R}\,.$$

At $Z = 0$ determinant of this matrix equals to 0. Therefore, one eigenstate of this matrix equals to 0. This massless combination of charged Higgs fields form well known Goldstone state. The second eigenstate of above matrix describes the physical charged Higgs boson with mass $M_{H^\pm}^2 = Tr\ M_{++} = T_1 + T_2 = (y_1 + y_2)\mathcal{R}$. This quantity is positive for the minimum of the potential, it can be negative in other extremes. Finally, we obtain

$$\mathcal{R}_N = M_{H^\pm}^2/(y_1 + y_2)\big|_N \;\; \textit{for neutral extremum}\; N\,, \;\; \textit{and}\;\; \mathcal{R}_{ch} = 0\,. \tag{9b}$$

• **The extremum energy** in each extremum point can be expressed, using the theorem on homogeneous functions:

$$\mathcal{E}_N^{ext} = V(y_{i,N}) = -V_2(y_{i,N}) + V_4(y_{i,N}) = -V_4(y_{i,N}) = -V_2(y_{i,N})/2\,. \tag{10}$$

The global minimum of potential realizes the vacuum state of the model. The direct comparison of extremum energies looks the best way for finding vacuum. More delicate but also important problem is possible existence minima of potential different from vacuum ("false vacuum" or "metastable state"). It can happen if only all eigenvalues of mass matrix near each extremum are positive.

■ **EW symmetry conserving (EWc) point**. The EWc point $\langle\varphi_1\rangle = \langle\varphi_2\rangle = 0$ is extremum of potential. Depending on m_{ij}^2 it has different nature: it is *minimum* if $det|m_{ij}^2| \geq 0$ and $m_{11}^2 < 0$, $m_{22}^2 < 0$, it is *maximum* if $det|m_{ij}^2| \geq 0$ and $m_{11}^2 > 0$, $m_{22}^2 > 0$, or it is *saddle point* in any other case. According to [3] no other extremum can be a maximum of potential.

■ **Charged extremum**. In the case when $Z \neq 0$, eqs. (6) have form $T_i = 0$. This system of linear equations for y_i can have unique solution which is calculated easily. In accordance with (7), it describes an extremum of the original potential (2) if only the obtained values $y_{1,2}$ obey inequalities (4). These inequalities determine the range of possible values of λ_i and m_{ij}^2 where the charged extremum can exist. According to (9), the charged extremum is minimum of the potential if the quadratic form $V_4(x_i - y_{i,ch})$ is positively defined at each classical value of operators x_i, i.e. $V_4(z_i)$ must be positive at arbitrary real z_1, z_2 and complex z_3 (see also [3]). It is more strong condition for potential than positivity constraint ($V_4(z_i)$ must be positive in the corner of z_i space, limited by conditions of form (4)).

■ **Neutral extrema, general case**. Other solutions of the extremum condition (3) obey a condition for $U(1)$ symmetry of electromagnetism, that is solution with $Z = 0 \Rightarrow u = 0$. In the calculation of extremum condition it is essential that in case quantities y_i are not independent.

For the Higgs potential of general form we have no idea about classification of neutral extrema. However, if CP conserving extremum (with no scalar-pseudoscalar mixing) exists, there is a basis in (φ_1, φ_2) space in which potential has explicitly CP conserving form [4], [1] (with all real λ_i, m_{ij}^2). Below we use this very form of potential.

■ **Neutral extrema, case of explicit CP conservation (real $\boldsymbol{\lambda_i}$, $\boldsymbol{m_{ij}^2}$).** In accordance with definitions (7), we have for each solution $y_3 = \sqrt{y_1 y_2}\, e^{i\xi}$. Now the extremum energy (10) is transformed to the form

$$\begin{aligned}\mathcal{E}^{ext} = -\frac{1}{2}\left\{m_{11}^2 y_1 + m_{22}^2 y_2 + 2m_{12}^2\sqrt{y_1 y_2}cos\xi\right\} + \frac{\lambda_1}{2}y_1^2 + \frac{\lambda_2}{2}y_2^2 + (\lambda_3+\lambda_4)y_1 y_2 + \\ +\lambda_5 y_1 y_2 cos2\xi + 2\,(\lambda_6 y_1 + \lambda_7 y_2)\sqrt{y_1 y_2}cos\xi.\end{aligned} \tag{11}$$

Now we find extrema in coordinates y_1, y_2, ξ. We start from the minimization in ξ at fixed y_i. It gives two types of solutions:

$$(A):\quad \cos\xi = \frac{m_{12}^2 - 2(\lambda_6 y_1 + \lambda_7 y_2)}{4\lambda_5\sqrt{y_1 y_2}}\,, \quad (B):\quad \sin\xi = 0\,. \tag{12}$$

- **Spontaneously CP violating extremum**. The extremum point (12A) describes a solution with complex value v_2 at real parameters of the potential. In this case physical neutral Higgs states have no definite CP parity. So this extremum is called the **spontaneously CP violating (sCPv) extremum** [5, 6]. The substitution of $cos\xi$ from (12A) into (11) transform extremum energy to the second order polynomial in y_1, y_2. Minimum condition for this energy become system two linear equations for extremal values of y_1, y_2 with unique solution. Therefore, $\boldsymbol{y_1}$**,** $\boldsymbol{y_2}$ **and** $\boldsymbol{\cos\xi}$ **are described by parameters of the potential unambiguously**. Certainly, this extremum can be realized only in the range of parameters of the potential obeying inequalities $|\cos\xi| < 1\,, \quad y_1 > 0\,,\; y_2 > 0$.

The energy (11) does not changes at the change $\xi \to -\,\xi$ (i. e. $\langle\varphi_2\rangle \to \langle\varphi_2\rangle^*$). Therefore

If $\varphi_1 = \langle\varphi_1\rangle$, $\varphi_2 = \langle\varphi_2\rangle$ is the extremum of potential, then $\varphi_1 = \langle\varphi_1\rangle$, $\varphi_2 = \langle\varphi_2\rangle^*$ is also the extremum and these two extrema are degenerate in energy [5],	(13)

the sCPv extremum is doubly degenerated in the "direction" of CP violation.

Note that the potential (11) is a second order polynomial in $\cos\xi$. The sCPv extremum (if it exist) can be a minimum only if $\lambda_5 > 0$, in accordance with [7].

- **CP conserving extrema**. The solution (12B) describes extrema that correspond to $\xi = 0,\ \pi$. The case $\xi = \pi$ can be obtained from the case $\xi = 0$ if we allow v_2 (i.e. $\tan\beta$) to be negative. Therefore, without loss of generality we consider below the only case with $\xi = 0$. In these cases physical Higgs bosons have definite CP parity (**CP conserving – CPc – extrema**). The extremum condition , written for $v_i = \sqrt{2y_i}$, has form of the system of two cubic equations. Rewriting this system with parametrization $v_1 = v\cos\beta$, $v_2 = v\sin\beta$, we express the quantity v^2 via $t \equiv \tan\beta$ and obtain the equation for t similar to those presented in [9]. This equation can have up to 4 different solutions. Considering nearly degenerated example, one can state that in some cases system can have 2 different CPc minima.

■ **Case of soft Z_2 symmetry violation ($\lambda_6 = \lambda_7 = 0$) at real λ_5, m_{12}^2**. In the mentioned important case many equations become more transparent. We present explicit equations for extremum energy for the charged, sCPv and CPc extrema respectively

$$\begin{gathered}
\mathcal{E}_{ch}^{ext} = -\frac{m_{11}^4\lambda_2 + m_{22}^4\lambda_1 - 2m_{11}^2 m_{22}^2\lambda_3}{8(\lambda_1\lambda_2 - \lambda_3^2)} - \frac{m_{12}^4}{4(\lambda_4+\lambda_5)}\ ;\\
\mathcal{E}_{sCPv} = -\frac{m_{11}^4\lambda_2 + m_{22}^4\lambda_1 - 2m_{11}^2 m_{22}^2\tilde\lambda_{345}}{8(\lambda_1\lambda_2 - \tilde\lambda_{345}^2)} - \frac{m_{12}^4}{8\lambda_5}\ \ where\ \ \tilde\lambda_{345} = \lambda_3+\lambda_4-\lambda_5\ ;\\
\mathcal{E}_{CPc} = -\frac{\left(m_{11}^2 + tm_{12}^2\right)\left(m_{11}^2 + 2tm_{12}^2 + t^2 m_{22}^2\right)}{8(\lambda_1 + \lambda_{345}t^2)},\ \ where\ \lambda_{345} = \lambda_3+\lambda_4+\lambda_5\ \ and\\
\lambda_2 m_{12}^2 t^4 + (\lambda_2 m_{11}^2 - \lambda_{345}m_{22}^2)t^3 + (\lambda_{345}m_{11}^2 - \lambda_1 m_{22}^2)t - \lambda_1 m_{12}^2 = 0\,.
\end{gathered}\tag{14}$$

The results for general case with λ_6, $\lambda_7 \neq 0$ at real λ_i, m_{ij}^2 are presented in [8].

■ **Vacuum**. Now the using of decomposition (9) or direct comparison of general equations for extremum energy like (14) allow to obtain following conclusions.

1. If the EWc extremum ($\langle\varphi_1\rangle = \langle\varphi_2\rangle = 0$) realizes the vacuum state (it can happen only at m_{11}^2, $m_{22}^2 < 0$) all EWSB extrema are saddle points .

2. If the charged extremum realizes the minimum of the potential, all neutral extrema are saddle points.

3. For two neutral minima of potential or a minimum and a saddle point with $M_{H_\pm,N}^2 > 0$, the deeper (a candidate for the global minimum – the vacuum) is the extremum with the larger value of ratio $M_{H_\pm,N}^2/v_N^2$.

For **explicitly CP conserving potential** one can distinguish a CP conserving (CPc) extremum with zero phase difference between the values $\langle\varphi_i\rangle$ at the extremum point and spontaneously CP violating (sCPv) extrema, in which the phase difference between the values $\langle\varphi_i\rangle$ is nonzero, the latter generates neutral Higgs states without definite CP parity. Total number of extrema in this case can be up to 8 (0 or 1 charged extremum, up to 4 CPc extrema, 2 or 0 sCPv extrema, 1 EWc extremum).

4. At $\lambda_5 > 0$ and $\lambda_5 > \lambda_4$ system can have a sCPv minimum, and this minimum is vacuum. This vacuum is doubly degenerate in sign of phase difference between the values of fields at the extremum point. This degeneracy is broken by loop corrections to potential in correspondence with direction of *arrow of time*. In this case other EWSB extrema are saddle points, not minima.

5. System can have more than one CPc local minima, e. g. I and II. In this case the vacuum state is lowest among them. For the important case of softly broken Z_2 symmetry

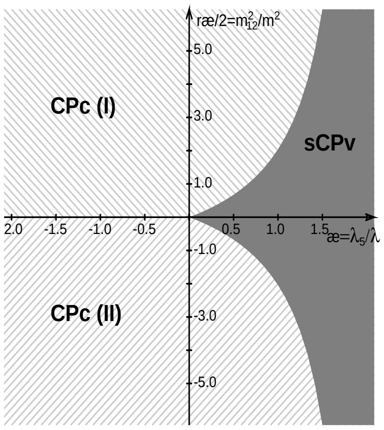
(14) the state I is below state II and can describe vacuum if

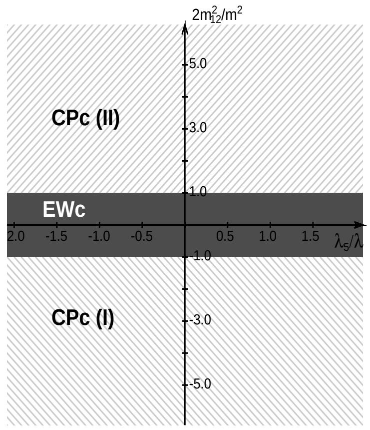
$$m_{12}^2/(v_I^2 sin2\beta_I) - m_{12}^2/(v_{II}^2 sin2\beta_{Ii}) > 0\,. \qquad (15)$$

■ To illustrate our general discussion, we consider a simple toy potential with additional $\varphi_1 \leftrightarrow \varphi_2$ symmetry, where all the extrema can be calculated easily:

$$V_t = \frac{\lambda}{2}\,(x_1+x_2)^2 + \frac{\lambda\varkappa}{2}\left(x_3^2 + x_3^{*2}\right) - \frac{m^2}{2}\,(x_1+x_2) - \frac{m^2\varkappa\, r}{4}\,(x_3+x_3^*)\ . \qquad (16)$$

Below we present the map, representing change of vacuum states with the change of parameters of potential in the plane ($\varkappa r$, $\varkappa$), where $\varkappa r = 2m_{12}^2/m^2$ – vertical axis and $\varkappa = \lambda_5/\lambda$ – horizontal axis. Left plot: $m^2 > 0$, right plot: $m^2 < 0$. Note that in this toy model potential has no minima except vacuum.





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