

# Precision calculations for $H \rightarrow WW/ZZ \rightarrow 4\text{fermions}$ with PROPHECY4F

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*PROPHECY4f* is a Monte Carlo event generator for precise simulations of the Higgs-boson decay  $H \rightarrow ZZ/WW \rightarrow 4\text{fermions}$ , supporting leptonic, semileptonic, and four-quark final states. Both electroweak and QCD corrections are included. Treating the intermediate gauge bosons as resonances, the calculation covers the full Higgs-boson mass range above, near, and below the gauge-boson pair thresholds. In this article we pay particular attention to the recently implemented option of *PROPHECY4f* to generate unweighted events.

## 1 Introduction

The decay of a Standard Model Higgs boson into weak-boson pairs with a subsequent decay into four fermions,  $H \rightarrow ZZ/WW \rightarrow 4f$ , plays an important role both in the Higgs search at the LHC [1] and in precision Higgs physics at the planned International  $e^+e^-$  Linear Collider (ILC). The spin and the CP properties of the Higgs boson could be verified upon studying angular and invariant-mass distributions [2] of the decay fermions. In order to match the estimated experimental precision in predictions, a Monte Carlo generator for  $H \rightarrow ZZ/WW \rightarrow 4f$  including radiative corrections is needed. In the past, only the electroweak  $\mathcal{O}(\alpha)$  corrections to decays into on-shell gauge bosons  $H \rightarrow ZZ/WW$  [3] and some leading higher-order corrections were known. However, in the threshold region the on-shell approximation becomes unreliable. Below the gauge-boson-pair thresholds only the leading order was known until recently.

*PROPHECY4f* [4] is a recently constructed Monte Carlo event generator for  $H \rightarrow ZZ/WW \rightarrow 4f$  that includes electroweak and QCD corrections as well as some higher-order improvements. Since the process with off-shell gauge bosons is consistently considered without any on-shell approximations, the obtained results are valid above, near, and below the gauge-boson pair thresholds. In this note we briefly describe the structure of the underlying calculations and illustrate the new option of *PROPHECY4f* to generate unweighted events by reproducing some of the numerical results presented in Ref. [4].

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## 2 Computational details

The calculation of the complete electroweak  $\mathcal{O}(\alpha)$  and strong  $\mathcal{O}(\alpha_s)$  corrections to the processes  $H \rightarrow 4f$ , which includes both the corrections to the decays  $H \rightarrow ZZ \rightarrow 4f$  and  $H \rightarrow WW \rightarrow 4f$  and their interference, is described in Ref. [4] in detail. Each ingredient of the calculation has been worked out twice, using independent approaches as far as possible.

For the implementation of the finite widths of the gauge bosons we use the “complex-mass scheme”, which was introduced in Ref. [5] for lowest-order calculations and generalized to the one-loop level in Ref. [6]. In this approach the W- and Z-boson masses are consistently considered as complex quantities, defined as the locations of the propagator poles in the complex plane. The scheme fully respects all relations that follow from gauge invariance.

The one-loop amplitudes of the virtual corrections have been generated with *FeynArts*, using the two independent versions 1 [7] and 3 [8]. They have been generated and evaluated both in the conventional 't Hooft–Feynman gauge and in the background-field formalism using the conventions of Refs. [9] and [10], respectively. One version of the algebraic part of the calculation is based on an in-house program implemented in *Mathematica*, another has been completed with the help of *FormCalc* [11]. The one-loop tensor integrals are evaluated as in the calculation of the corrections to  $e^+e^- \rightarrow 4$  fermions [6, 12]. They are recursively reduced to master integrals at the numerical level. The scalar master integrals are evaluated for complex masses using the methods and results of Refs. [13]. Tensor and scalar 5-point functions are directly expressed in terms of 4-point integrals [14]. Tensor 4-point and 3-point integrals are reduced to scalar integrals with the Passarino–Veltman algorithm [15] as long as no small Gram determinant appears in the reduction. If small Gram determinants occur, the alternative reduction schemes of Ref. [16] are applied.

Since corrections due to the self-interaction of the Higgs boson become important for large Higgs masses, we have included the dominant two-loop corrections to the decay  $H \rightarrow VV$  proportional to  $G_\mu^2 M_H^4$  in the large-Higgs-mass limit which were calculated in Ref. [17].

The soft and collinear singularities appearing in the real corrections are treated both in the dipole subtraction approach [18] and in the phase-space slicing method. For the calculation of non-collinear-safe observables we use the extension of the subtraction method introduced in Ref. [19]. Final-state radiation off charged leptons beyond  $\mathcal{O}(\alpha)$ , which is relevant if bare lepton momenta enter the event selection, is supported for weighted events only. These corrections [4] are sizeable only in regions where the lowest-order prediction is relatively small and can amount to 4% for muons and up to about 10% for electrons.

## 3 Event generation

*PROPHECY4f* employs a multi-channel Monte Carlo generator similar to *RacoonWW* [5, 20] and *Coffer $\gamma\gamma$*  [19, 21]. The results obtained this way have been checked using the adaptive integration program *VEGAS* [22]. In its default version *PROPHECY4f* generates weighted events, which are not positive definite.

As a new option, the program now supports the generation of unweighted events in its “phase-space-slicing” branch, applying a hit-and-miss algorithm similar to the one used by *RacoonWW*. Each time an unweighted event is generated, a Fortran subroutine is called where information about the event is provided in the format of the Les Houches Accord [23] (Fortran common block *HEPEUP*). This subroutine can be modified by the user in order to read out the events.

In the unweighting procedure also negative events occur. Although their number is reduced by using only the sum of the tree-level, the virtual, and the soft endpoint contribution, they cannot be avoided completely. In *PROPHECY4f* the remaining negative events are treated in the same way as the positive events, i.e. they can be read out by the user in a subroutine. Their contribution ranges from less than a per mille to slightly more than one per cent of all events, depending on the Higgs-boson mass.

The price for generating unweighted events is an increase of CPU time by about a factor  $10^2$  up to some  $10^3$  w.r.t. weighted-event generation, depending on the chosen  $4f$  final state and the Higgs-boson mass. The results compared below are obtained with  $5 \times 10^5$  unweighted and  $5 \times 10^7$  weighted events. The generation of these unweighted events requires about 2 days on a AMD Opteron 252 2.6GHz CPU. However, one should keep in mind that such unweighted decay events could be generated once for a chosen setup and stored in a database. Simulations of Higgs production at the LHC or ILC could then just randomly pick events for the Higgs decays from the database.

## 4 Numerical results

The input parameters and the details of the setup in our numerical evaluation are provided in Ref. [4], where a comprehensive survey of numerical results is presented. The results shown in the following are obtained without applying photon recombination, i.e. invariant masses and angles are derived from bare lepton momenta.

In this brief article we focus only on the decay  $H \rightarrow e^- e^+ \mu^- \mu^+$  and show the distributions in the invariant masses of the decay leptons and the angle between the Z-decay planes in Figs. 1 and 2, respectively. These distributions play an important role in the verification of the discrete quantum numbers of the Higgs boson [2]. Since the radiative corrections significantly distort the distributions, they have to be taken into account if these observables are used to set bounds on non-standard couplings. Neglecting the corrections could result in faking new-physics effects. A detailed discussion of the corrections to these distributions can be found in Ref. [4]. Here we merely emphasize the agreement between the results obtained with weighted and unweighted events generated with *PROPHECY4f*.

## 5 Conclusions

The generator *PROPHECY4f*, which simulates the Higgs decays  $H \rightarrow ZZ/WW \rightarrow 4f$  including electroweak and QCD corrections at the state of the art, is extended by an option for the generation of unweighted events. The consistency of the new option is illustrated in invariant-mass and angular distributions.

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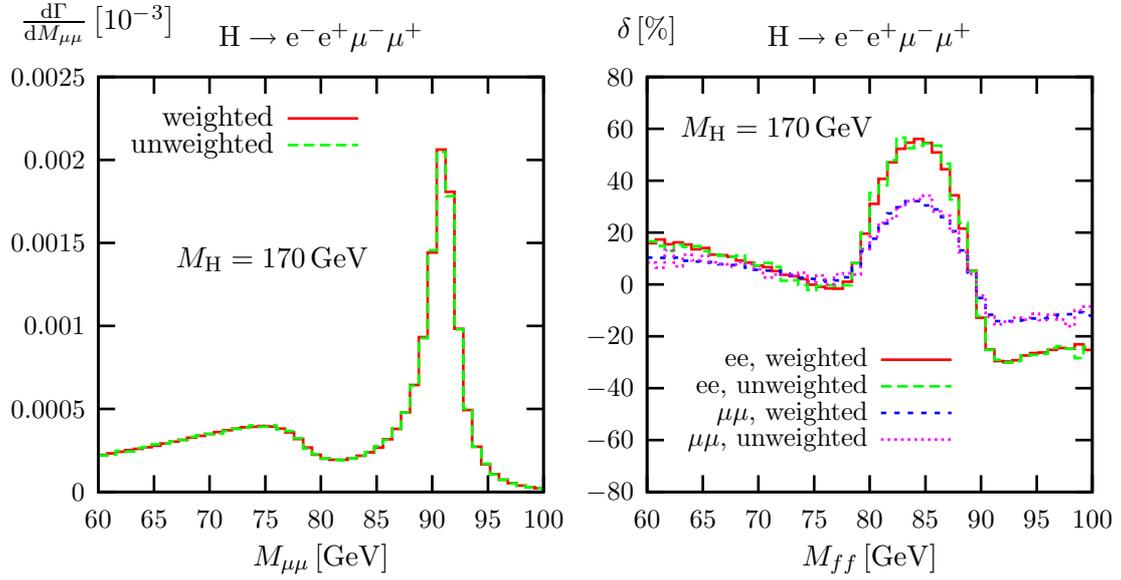


Figure 1: Corrected distribution in the invariant mass of the  $\mu^-\mu^+$  pair (l.h.s.) and relative corrections for  $e^-e^+$  and  $\mu^-\mu^+$  pairs (r.h.s.) in the decay  $H \rightarrow e^-e^+\mu^-\mu^+$ , obtained with weighted and unweighted events.

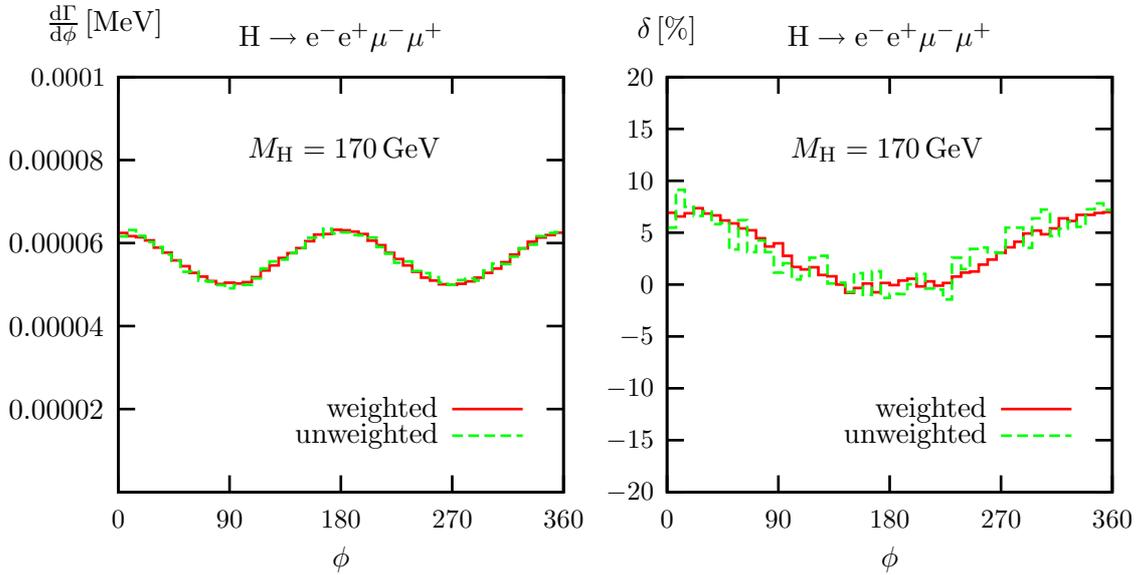


Figure 2: Corrected distribution in the angle between the  $Z \rightarrow l^-l^+$  decay planes in the Higgs rest frame (l.h.s.) and relative corrections (r.h.s.) in the decay  $H \rightarrow e^-e^+\mu^-\mu^+$ , obtained with weighted and unweighted events.

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