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SEMI-ANALYTICAL APPROACH TO OFF-SHELL W AND Z PAIR PRODUCTION [a]

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We give a short review of the results for four fermion production, which have been obtained by the semi-analytical approach. The angular degrees of freedom (typically five or more) are integrated over analytically while the integrations over invariant fermion pair masses (typically two or more) remain to be performed by numerical methods. In addition to doubly resonating cross sections from virtual two boson production, QED corrections and background contributions were determined. However, a large variety of final state topologies has not been treated so far.

1 Introduction

Among the most interesting processes to be studied at a high energy linear collider is pair production of gauge and Higgs bosons. Since these heavy particles are instable, one has to study experimentally their decay products, namely four final state fermions:

$$e^+e^- \to (W^+W^-, ZZ, Z\gamma, \gamma\gamma, ZH, \ldots) \to f_1\bar{f}_2 f_3\bar{f}_4. \tag{1}$$

After integration over five angular variables, the total cross section for reactions (1) may be generically written as follows:

$$\sigma^{res}(s) \;=\; \int ds_1 \rho_B(s_1) \int ds_2 \rho_B(s_2)\;\; \sigma_0(s; s_1, s_2) \tag{2}$$

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where $s_1 = (p_1 + p_2)^2$ and $s_2 = (p_3 + p_4)^2$. The bosons' Breit-Wigner densities

$$\rho_B(s_i) \quad = \quad \frac{1}{\pi} \frac{M_B \Gamma_B}{|s_i - M_B^2 + i M_B \, \Gamma_B|^2} \times \mathrm{BR} \tag{3}$$

attain the following narrow width limit:

$$\rho_B(s_i) \stackrel{\Gamma_B \to 0}{\longrightarrow} \delta(s_i - M_B) \times \mathrm{BR}. \tag{4}$$

The expressions for $\sigma_0(s; s_1, s_2)$ have been derived for off-shell W^+W^- production in [1], for $ZZ, Z\gamma, \gamma\gamma$ production in [2], for ZH production in [3] (see also references therein). In section 3 we will give explicit examples for the basic cross section $\sigma_0(s; s_1, s_2)$.

Naturally, the four fermion final states in (1) are produced not only by doubly resonant amplitudes, but also by many singly resonant and non-resonant tree level background amplitudes, which are characterized by different intermediate states. In addition, radiative corrections must be accounted for. Here, we will concentrate on results which have been obtained by use of the semi-analytical method: QED initial state radiative corrections and background contributions.

1.1 Background

The doubly resonant diagrams yield the dominant cross section contributions even far above threshold; nevertheless, only together with background gauge invariance may be achieved. The number of Feynman diagrams for a given process depends on the topology of the final state. A classification has been introduced in [2]:

CC Processes with final states of type $f_1^u \bar{f}_1^d f_3^u \bar{f}_3^d$ are called *charged current processes*: `CC11`, `CC10`, `CC09`; `CC20`, `CC18`.

NC Processes with final states of type $f_1 \bar{f}_1 f_3 \bar{f}_3$ are called *neutral current processes*: `NC32`; `NC24`, `NC4·16`, `NC4·12`, `NC4·03`, `NC10`, `NC06`; `NC48`, `NC20`, `NC21`, `NC19`, `NC12`, `NC4·36`, `NC4·09`.

mix Processes which may be considered as both CC and NC types are called *mixed processes*: `mix43`, `mix19`; `mix56`. An example for a mixed process is the production of $u\bar{d}d\bar{u} \equiv u\bar{u}d\bar{d}$.

In addition, there are Feynman diagrams with Higgs bosons in the NC and mixed cases. The simplest background processes are those of the `CC20` and the

NC24 classes. These have been studied in detail in [4] and [5,3], respectively. As an example, we discuss the CC11 process in section 3.

Tree level cross sections including backgrounds may be generically written as

$$\sigma^{\rm Born}(s) \;=\; \int ds_1 \int ds_2 \;\; \frac{\sqrt{\lambda}}{\pi s^2} \cdot \sum_k \frac{d^2\sigma_k(s,s_1,s_2)}{ds_1 ds_2} \tag{5}$$

with $\lambda \equiv \lambda(s,s_1,s_2)$, $\lambda(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc$ and

$$\frac{d^2\sigma_k}{ds_1 ds_2} \;=\; \mathcal{C}_k(s,s_1,s_2)\cdot \mathcal{G}_k(s,s_1,s_2) \quad , \tag{6}$$

where $\mathcal{C}_k$ represents coupling constants and off-shell boson propagators, while $\mathcal{G}_k$ is a kinematical function obtained after analytical angular integration. The index k labels cross section contributions with different coupling structure *and* different Feynman topology.

1.2 QED initial state radiation

The by far largest radiative corrections are due to QED initial state radiation (ISR). They may be described by the generic formula

$$\sigma^{ISR}(s) \;\;=\;\; \int ds_1 \int ds_2 \;\; \int ds' \;\; \sum_k \frac{d^3\Sigma_k(s,s';s_1,s_2)}{ds_1 ds_2 ds'} \tag{7}$$

with $s' = (p_1+p_2+p_3+p_4)^2$ and

$$\frac{d^3\Sigma_k(s,s';s_1,s_2)}{ds_1 ds_2 ds'} \;\;=\;\;\; \mathcal{C}_k \cdot \left[\beta_e\, v^{\beta_e - 1}\mathcal{S}_k + \mathcal{H}_k\right] \quad , \tag{8}$$

where $\beta_e = 2\,\frac{\alpha}{\pi}\left[\ln(s/m_e^2)-1\right]$ and $v = (1-s'/s)$.

The soft+virtual and hard corrections $\mathcal{S}_k$ and $\mathcal{H}_k$ separate into a universal, factorizing, process-independent and a non-universal, non-factorizing, process-dependent part. They are given by [6,7]

$$\begin{aligned}
\mathcal{S}_k(s,s';s_1,s_2) \;&=\; \left[1+\bar{S}(s)\right]\sigma_{k,0}(s';s_1,s_2) + \;\sigma_{\hat{S},k}(s';s_1,s_2) \quad , \\
\mathcal{H}_k(s,s';s_1,s_2) \;&=\; \underbrace{\bar{H}(s,s')\,\sigma_{k,0}(s';s_1,s_2)}_{Universal\ Part} + \underbrace{\sigma_{\hat{H},k}(s,s';s_1,s_2)}_{Non-universal\ Part}
\end{aligned} \tag{9}$$

with $\sigma_{k,0}(s';s_1,s_2) \equiv \left[\sqrt{\lambda}/(\pi s'^2)\right]\cdot\mathcal{G}_k(s',s_1,s_2)$ and the universal $\mathcal{O}(\alpha)$ soft+virtual and hard radiators $\bar{S}$ and $\bar{H}$

$$\bar{S}(s) \;=\; \frac{\alpha}{\pi}\left[\frac{\pi^2}{3}-\frac{1}{2}\right]+\frac{3}{4}\beta_e \qquad\qquad \bar{H}(s,s') \;=\; -\frac{1}{2}\left(1+\frac{s'}{s}\right)\beta_e \quad . \tag{10}$$

If the index k is associated with s-channel e^+e^- annihilation diagrams only, only universal ISR contributions are present, because non-universal ISR originates from the angular dependence of initial state t- and u-channel propagators. Non-universal ISR contributions are suppressed with respect to universal ones, because they do not contain the leading logarithm β_e. They are, however, analytically very complex.

2 Example 1: The NC8 process with complete initial state corrections

The NC8 process $e^+e^- \to (ZZ, Z\gamma, \gamma\gamma) \to f_1\bar{f}_1 f_2\bar{f}_2 \quad (f_1 \neq f_2 \ , \ f_i \neq e^\pm, \overset{(-)}{\nu}_e)$ is described by only one kinematical function [2,5],

$$\mathcal{G}_{\mathtt{NC8}}(s; s_2, s_2) = s_1 s_2 \left[\frac{s^2 + (s_1+s_2)^2}{s - s_1 - s_2} \mathcal{L}(s; s_2, s_2) - 2 \right] \ , \tag{11}$$

$$\mathcal{L}(s; s_1, s_2) = \frac{1}{\sqrt{\lambda}} \ln \frac{s - s_1 - s_2 + \sqrt{\lambda}}{s - s_1 - s_2 - \sqrt{\lambda}} \ . \tag{12}$$

Numerical results are presented in figure 1 [7,8]. As is seen from the figure, ISR corrections increase the cross section considerably. Universal contributions are of $\mathcal{O}(+20\%)$, non-universal contributions of $\mathcal{O}(+3\%)$. From the inset of the figure one concludes that most important cross section contributions involve Z bosons *and* photons. We mention that the doubly resonant Z pair cross section is easily isolated by cuts on s_1 and s_2.

3 Example 2: The CC11 processes with initial state corrections

The basic charged current four fermion process is described by three kinematical functions [1]:

$$\sigma_0^{\mathtt{CC3}}(s; s_1, s_2) = \frac{\left(G_\mu M_W^2\right)^2}{8\pi s} \left[\mathcal{C}^s_{\mathtt{CC3}} \mathcal{G}^{33}_{\mathtt{CC3}} + \mathcal{C}^{st}_{\mathtt{CC3}} \mathcal{G}^{3f}_{\mathtt{CC3}} + \mathcal{C}^t_{\mathtt{CC3}} \mathcal{G}^{ff}_{\mathtt{CC3}} \right]. \tag{13}$$

The functions $\mathcal{G}_{\mathtt{CC3}}$ depend only on the virtualities:

$$\mathcal{G}^{ff}_{\mathtt{CC3}}(s; s_1, s_2) = \frac{1}{48} \Big[\lambda + 12\, s\, (s_1 + s_2) - 48 s_1 s_2 + 24\,(s - s_1 - s_2)\, s_1 s_2 \cdot \mathcal{L}(s; s_1, s_2) \Big] \ , \tag{14}$$

$$\mathcal{G}^{33}_{\mathtt{CC3}}(s, s_1, s_2) = \frac{\lambda}{192} \Big[\lambda + 12\,(s s_1 + s s_2 + s_1 s_2) \Big] \ , \tag{15}$$

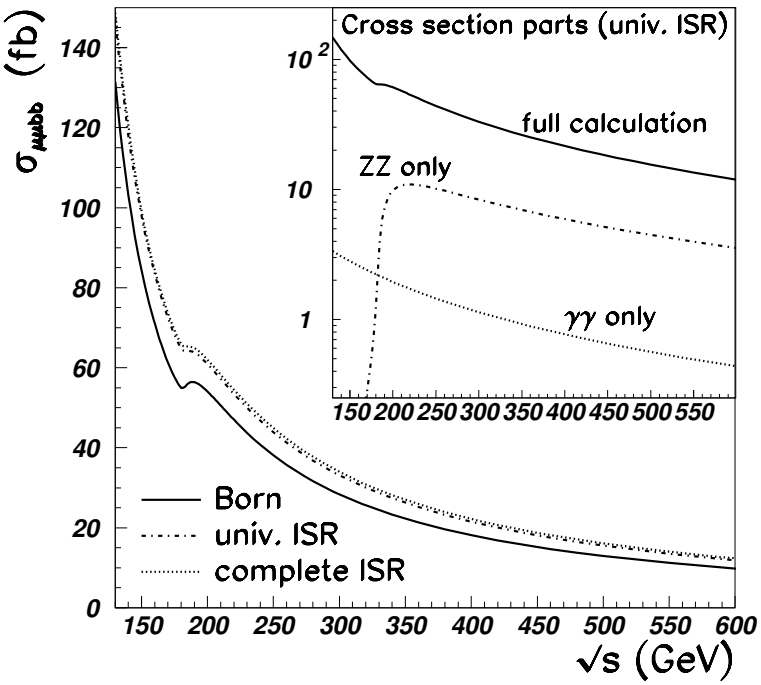

Figure 1: *The cross section for the* NC8 *process with ISR.*

$$\mathcal{G}^{3f}_{\mathrm{CC3}}(s;s_1,s_2) \;=\; \frac{1}{48}\left\{(s-s_1-s_2)\left[\lambda+12s(ss_1+s_1s_2+s_2s)\right]\right.$$
$$\left.-\,24\,(ss_1+ss_2+s_1s_2)\,s_1s_2\cdot\mathcal{L}(s;s_1,s_2)\right\}, \quad (16)$$

Adding terms from background diagrams yields [4]

$$\sigma^{\mathrm{CC11}}(s) \;=\; \int ds_1 \int ds_2 \;\frac{\sqrt{\lambda}}{\pi s^2}\cdot\sum_{k=1}^{15}\mathcal{C}_k\cdot\mathcal{G}_k(s,s_1,s_2)\;. \quad (17)$$

Using the symmetries of the process, the 15 functions may be expressed by the three functions $\mathcal{G}_{\mathrm{CC3}}$, two of the process NC24 – $\mathcal{G}_{\mathrm{NC8}}, \mathcal{G}_{\mathrm{NC24}}$ [5] – plus only one new kinematical function, which is the most complicated one:

$$\mathcal{G}^{u,d}_{\mathrm{CC11}}(s;s_1;s_2) \;=\; -120s^4\frac{s_1^3s_2^3}{\lambda^3}\mathcal{L}(s_2;s,s_1)\mathcal{L}(s_1;s_2,s)$$

$$-s\left[1+\frac{s(s-\sigma)}{\lambda}+20s^2\frac{s_1s_2}{\lambda^2}-30s^3s_1s_2\frac{s-3\sigma}{\lambda^3}\right]\left[s_1^2\mathcal{L}(s_2;s,s_1)+s_2^2\mathcal{L}(s_1;s_2,s)\right]$$

$$-s(s_1-s_2)\left[\frac{s-\sigma}{\lambda}+10s\frac{s_1s_2}{\lambda^2}-30s^2s_1s_2\frac{s+\sigma}{\lambda^3}\right]\left[s_1^2\mathcal{L}(s_2;s,s_1)-s_2^2\mathcal{L}(s_1;s_2,s)\right]$$

$$-\frac{1}{12}\left\{(s^2-\sigma^2)\left[1+12\frac{s\sigma}{\lambda}-60s^2\frac{s_1s_2}{\lambda^2}\right]\right.$$

$$\left.-8s_1s_2\left[1-\frac{s(4s+5\sigma)}{\lambda}+15s^2\frac{s_1s_2}{\lambda^2}\left(1-6\frac{s(s-\sigma)}{\lambda}\right)\right]\right\}, \quad (18)$$

with $\sigma \equiv s_1 + s_2$.

Numerical results have been produced with programs [4,9,10,11]. Table 1 contains a comparison of the Born cross sections with background in different channels for a wide range of beam energies. At energies above the ZZ threshold, the background modifies σ_{tot}, but for all channels quite similarly. While the changes of the rates are of the percent level, this may be quite different for other processes, e.g. those of the `CC20` type. In table 2, a comparison of cross

Table 1: *σ_{tot} in pbarn for Born 4f production as function of $\sqrt{s}$ (in GeV). For this comparison, the branching ratios are* not *taken into account in the single mode channels.*

$\sqrt{s}$	`CC3`	$L\nu l\nu$	$l\nu qq'$	$QQ'qq'$
30	1.4519×10^{-7}	5.9295×10^{-6}	7.2478×10^{-6}	5.0897×10^{-6}
60	1.9358×10^{-5}	4.0025×10^{-5}	4.6879×10^{-5}	3.5441×10^{-5}
91.189	0.11225	0.021329	0.024551	0.018975
176	16.225	16.242	16.243	16.243
200	18.578	18.586	18.588	18.588
500	7.3731	7.3301	7.3318	7.3334
`WWGENPV`	7.3731(7)			7.3332(5)
1000	2.9888	2.9342	2.9344	2.9348
`WWGENPV`	2.9887(7)			2.9342(5)
2000	1.5702	1.5020	1.5018	1.5016
`WWGENPV`	1.5700(4)			1.5014(4)

sections including universal ISR is performed. The high level of numerical and general precision is clearly demonstrated.

Table 2: *Results from different programs for the total* CC11 *cross sections* $e^+e^- \to 4f$ *and* $e^+e^- \to 4f+\gamma$ *(both in pbarn) as functions of* $\sqrt{s}$ *for the* $QQ'qq'$ *channel.*

	$\sqrt{s}$	500	1000	2000
σ_{Born}	GENTLE	0.81482	0.32609	0.16684
	WWGENPV	0.81480(6)	0.32602(6)	0.16682(7)
σ_{QED}	GENTLE,SF	0.86950	0.36514	0.18247
	WPHACT	0.86956(9)	0.36515(5)	0.18250(4)
	WTO	0.86960(25)		
	WWGENPV	0.86956(14)	0.36530(35)	0.18247(13)

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