# 2-Loop corrections to the Bhabha cross-section

$$e^+ e^- \rightarrow e^+ e^-$$

Tord Riemann, DESY Zeuthen

Contributed to the
Workshop on Forward Calorimetry and Luminosity Measurement in the TESLA Detector
A project in collaboration with

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- Introduction and Conclusions

# Why study 2-loop Bhabha scattering now?

- Bhabha scattering is a prominent luminosity-monitoring reaction for $e^+e^-$ colliders
- 2loop Bhabha: Experimental accuracy points to 0.1%
- New Physics reach: $e^+e^-e^+e^-$ four fermion operators
  small angles: t-channel dominates
- Relatively simple in the low-angle case: QED is by far dominating
  Although at TESLA also $Z$ exchange will play a role there
- In theory, we have now an understanding of 2loop problems nobody expected 8 years ago
  1994: 2nd Loops and Legs conference (Teupitz) had first bulk of 2loop talks
  Nevertheless: very demanding concerning person-power, computer-power, brain-power

## Offer by drinking-a-beer Frits Berends, NL, in 1989

**Okey, I need**

**One million Swiss francs**

**and**

**five – or ten? – Russians**

**and I will do this**

Since then (i.e. in 13 years), there are 9 articles in spires with "Bhabha . . . two loop" in the title.

References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Citation Search
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SPIRES HEP



Modify your search! Press ENTER to run search again.

`FIND TITLE BHABHA TWO LOOP AND DATE AFTER 1989`

Searching for an author? Try HEPNAMES.



Result: **9** documents found:

1) **FACTORIZING ONE LOOP CONTRIBUTIONS TO TWO LOOP BHABHA SCATTERING AND AUTOMATIZATION OF FEYNMAN DIAGRAM CALCULATIONS.**
By J. Fleischer, O.V. Tarasov (Bielefeld U.), T. Riemann, A. Werthenbach (DESY, Zeuthen). Oct 2002. 2pp.
Contributed to 8th International Workshop on Advanced Computing and Analysis Techniques in Physics Research (ACAT 2002), Moscow, Russia, 24-28 Jun 2002.
e-Print Archive: **hep-ph/0210180**

References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX
Abstract and Postscript and PDF from arXiv.org (mirrors: au br cn de es fr il in it jp kr ru tw uk za aps lanl )
CERN Library Record
Conference Info

2) **TWO LOOP CORRECTION TO BHABHA SCATTERING.**
By Z. Bern (UCLA), Lance J. Dixon (SLAC), A. Ghinculov (UCLA). SLAC-PUB-8655, UCLA-00-TEP-26, Oct 2000. 22pp.
Published in **Phys.Rev.D63:053007,2001**
e-Print Archive: **hep-ph/0010075**

References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Citation Search
Abstract and Postscript and PDF from arXiv.org (mirrors: au br cn de es fr il in it jp kr ru tw uk za aps lanl )
CERN Library Record
Phys. Rev. D Server
SLAC Document Server

3) **SECOND ORDER CONTRIBUTIONS TO THE ELASTIC LARGE ANGLE BHABHA SCATTERING CROSS-SECTION. 1. ALL EXCEPT TWO LOOP BOX DIAGRAMS.**
By A.B. Arbuzov, E.A. Kuraev, B.G. Shaikhatdenov (Dubna, JINR). Jun 1998. 14pp.
Published in **Mod.Phys.Lett.A13:2305-2316,1998**
e-Print Archive: **hep-ph/9806215**

References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Citation Search
Abstract and Postscript and PDF from arXiv.org (mirrors: au br cn de es fr il in it jp kr ru tw uk aps lanl )
Mod.Phys.Lett.A Server

4) **SMALL ANGLES BHABHA SCATTERING: TWO LOOP APPROXIMATION.**
By Victor S. Fadin (Novosibirsk, IYF), E.A. Kuraev (Dubna, JINR), L.N. Lipatov (St. Petersburg, IN N.P. Merenkov (Kharkov, FTI), L. Trentadue (CERN). JINR-E2-92-577, Dec 1992. 20pp.
Submitted to Nucl.Phys.B

References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords
Scanned Version (KEK Library)

5) **TWO LOOP LADDER DIAGRAM CONTRIBUTIONS TO BHABHA SCATTERING. 3: T PHI**3 LIMIT OF QED.**
By K.S. Bjoerkevoll, P. Osland (Bergen U.), G. Faeldt (Uppsala U.). 1992.
Published in **Nucl.Phys.B391:591-620,1993**

LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Citation Search
Nuclear Physics Electronic

6) **TWO LOOP VIRTUAL CONTRIBUTIONS TO BHABHA SCATTERING.**
By K.S. Bjoerkevoll, P. Osland (Bergen U.), G. Faeldt (Uppsala U.). 1992.
Preprint - Bjoerkevoll, K.S. (92,rec.Jul.) 8 p.B.

LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords

7) **TWO LOOP LADDER DIAGRAM CONTRIBUTIONS TO BHABHA SCATTERING. 2: ASYMPTOTIC RESULTS FOR HIGH-ENERGIES.**
By K.S. Bjoerkevoll, P. Osland (Bergen U.), G. Faeldt (Uppsala U.). 1992.
Published in **Nucl.Phys.B386:303-342,1992**

LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Citation Search
Nuclear Physics Electronic

8) **TWO LOOP LADDER DIAGRAM CONTRIBUTIONS TO BHABHA SCATTERING. 1: STRUCTURE OF THE MATRIX ELEMENTS.**
By K.S. Bjoerkevoll, P. Osland (Bergen U.), G. Faeldt (Uppsala U.). 1991.
Published in **Nucl.Phys.B386:280-302,1992**

LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Citation Search
Nuclear Physics Electronic

9) **TWO LOOP QED CORRECTIONS TO BHABHA SCATTERING NEAR THE Z0.**
By F. Aversa, M. Greco (Frascati), G. Montagna, O. Nicrosini (INFN, Pavia & Pavia U.). LNF-90-049-PT, Jun 1990. 16pp.
Published in **Phys.Lett.B247:93-100,1990**

# 2-loop Bhabha scattering: What to be done?

$$\begin{aligned}\sigma \quad = \quad & |\text{Born} + \text{1-loop} + \text{2-loop}|^2 \\ & + |(\text{Born} + 1\text{-}\gamma) + (\text{1-loop} + 1\text{-}\gamma)|^2 \\ & + |(\text{Born} + 2\text{-}\gamma)|^2\end{aligned}$$

- Do not include: $|\text{2-loop}|^2$
  $|(\text{1-loop} + 1\text{-}\gamma)|^2$
- New, besides 2-loops: The one-loop 5-point function

We think one should do the calculation with finite electron mass

Why?
A hint from Stashek Jadach, but also the feeling it could help to gain experience for electroweak calculations

## Approaches to High Energy Bhabha Scattering

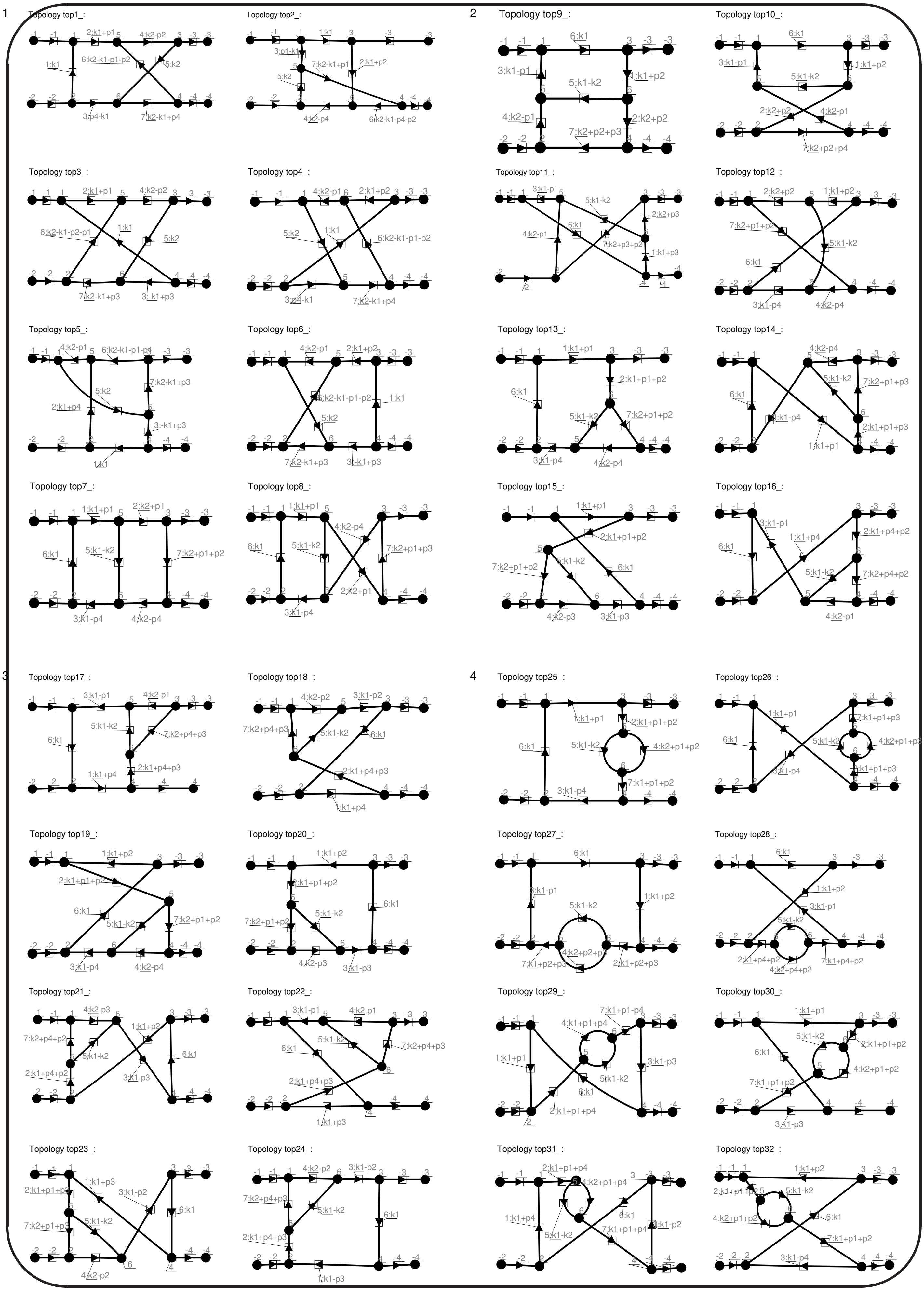
1. Calculate Bhabha scattering using massive electrons and a massive photon as an IR regulator.
   In principle this is a good way, because it would match on to the previous lower order calculations. Finite masses are however more complicated to deal with. Not done.

2. Calculate Bhabha scattering using massless electrons and dimensional regularization as an IR regulator.
   Here one needs a structure function approach to deal with initial state singularities. Bonus is that it is most similar to what we need in analogous QCD calculations.

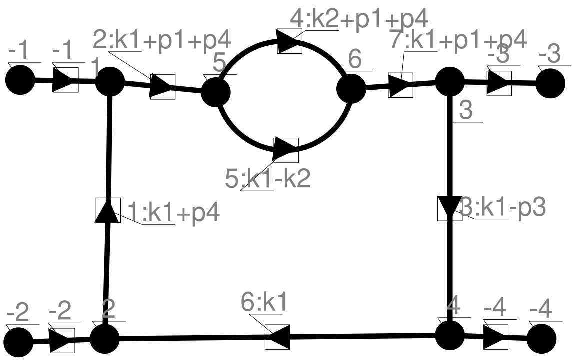
   Because of the availability of integration methods we chose the second option.

From talk of Z. Bern, LoopFest 2002

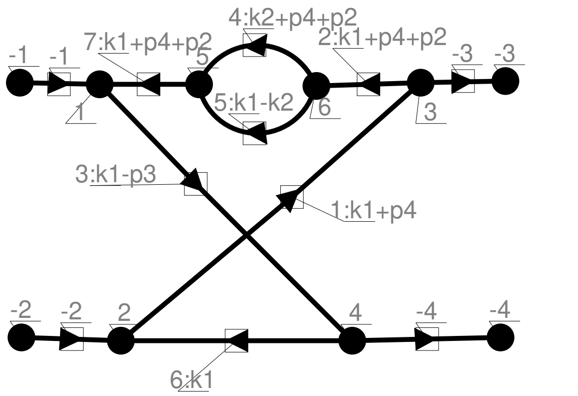
# Two-loop Topologies for Bhabha Scattering



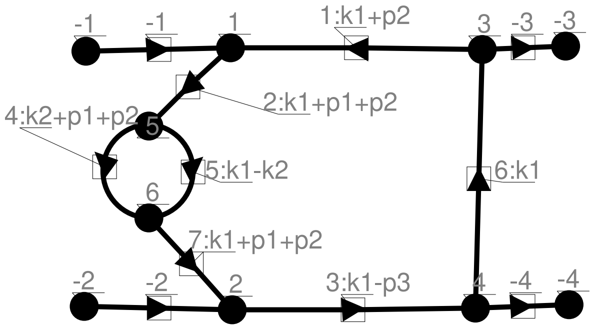
5 Topology top33_:



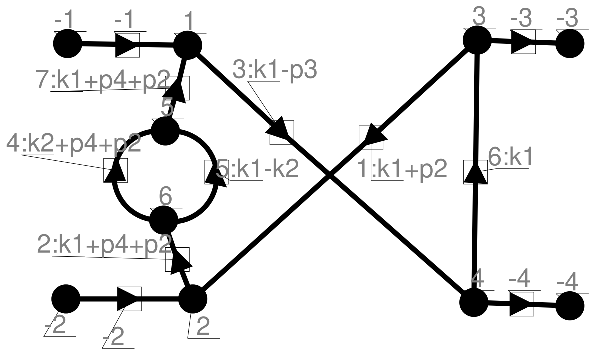

Topology top34_:




Topology top35_:




Topology top36_:




Diagrams from Feynman Diagram Analyzer DIANA (Tentyukov, Fleischer)

# 2-loop Bhabha's: Why we feel well-prepared?

We think that we have some special experiences in several fields, all of them are needed here

- Creation of Feynman diagrams and form input with DIANA
- Treatment of massive loops
- Treatment of complicated multi-loop structures
- Phenomenological experiences from LEP 1 and LEP 2
- Last but not least: Cooperation between Loop specialists and Monte-Carlo specialists (and later: with some of you . . . )
- We are several theorists believing in this project

Hope to have sufficient computer power when needed

From Bern, LoopFest 2002

## Some All-Massless Integrals References

$\mathrm{Xbox}_1$, tri_sym: Tausk, hep-ph/9909506

$\mathrm{Xbox}_{1,2}$, Xtri: Anastasiou et al., hep-ph/0003261

$\mathrm{Pbox}_{1,2}$, Bbox, Dbox: Smirnov, hep-ph/9905323; Smirnov & Veretin, hep-ph/9907385

pentabox, Dbox: Anastasiou, Glover, Oleari, hep-ph/9912251

Bbox: Anastasiou, Glover, Oleari, hep-ph/9907523

Basic identities:

- Integration by parts
  Tkachov, PL100B(1981)65; Chetyrkin & Tkachov, NPB192(1981)159
- Lorentz invariance Gehrmann & Remiddi, hep-ph/9912329

Act in space of integrals with multiple propagators.
Used judiciously, reduce tensor integrals to master integrals plus simpler boundary integrals.

Have to add: Shifted dimensions, O. Tarasov, PRD, NPB (1996)

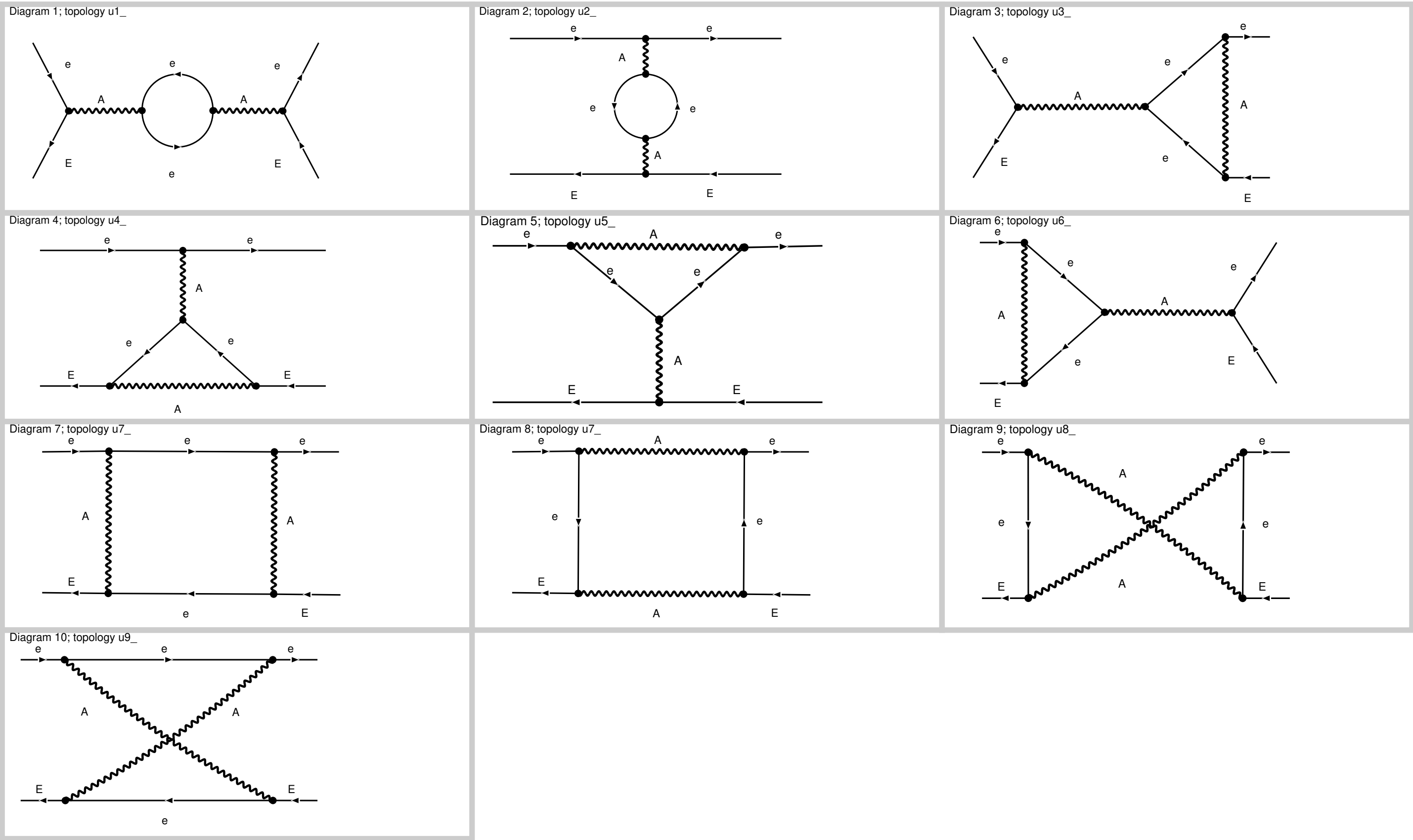



Figure 1: One-loop diagrams to $e^+e^- \to e^+e^-$

Fleischer, Riemann, Tarasov, Werthenbach:

Factorizing one loop contributions to two loop Bhabha scattering and automatization of Feynman diagram calculations – ACAT, Moscow, hep-ph/0210180

Massive two-loop Bhabha scattering - the factorizable subset – Radcor, Banz, hep-ph/0211167

# Example: 3-point function for Bhabha

For **two zero masses** and final on-shell momenta we have e.g. (J. Fleischer, F. Jegerlehner, O. Tarasov)

$$J_G = -C_0(0, me, 0, m_e^2, m_e^2, p^2)$$

$$\frac{J_G}{\Gamma\left(2-\frac{d}{2}\right)} = \frac{(m^2)^{\frac{d}{2}-3}}{2(d-3)} \, {}_2F_1\left[\begin{matrix} 1,1\,; \\ \frac{d-1}{2}\,; \end{matrix} \; 1-\frac{p^2}{4m^2}\right] - \frac{\sqrt{\pi}\,\Gamma\left(\frac{d-2}{2}\right)(-p^2)^{\frac{d-4}{2}}}{2^{d-2}\,\Gamma\left(\frac{d-1}{2}\right)m^2} \, {}_2F_1\left[\begin{matrix} 1,\frac{d-2}{2}\,; \\ \frac{d-1}{2}\,; \end{matrix} \; 1-\frac{p^2}{4m^2}\right],$$

while for usual 1-loop calculations it is sufficient to work with $A_0$, $B_0$, $C_0$, and $D_0$.

These Passarino-Veltman functions lead to $\ln(X)$ and $\mathrm{Li}_2(X)$.

New techniques like recurrence relations, difference equations etc.