Electroweak Corrections to Heavy Fermion Pair Production

$$e^+e^- \to \bar{t}t$$

Tord Riemann

DESY Zeuthen

in collaboration with

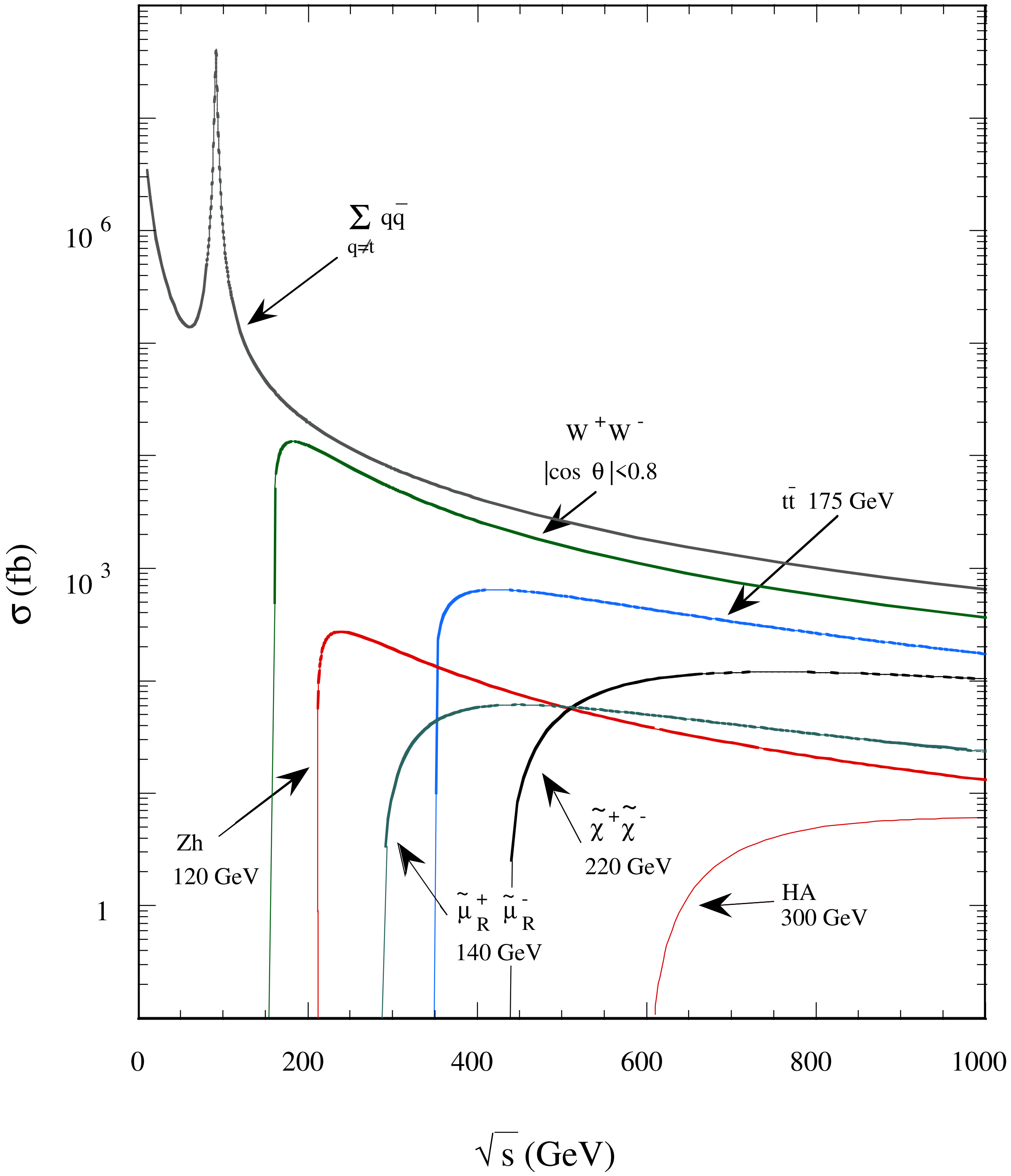
Jochem Fleischer, Univ. Bielefeld
Arnd Leike, LMU Munich
Anja Werthenbach, DESY Zeuthen

Thanks to the Feynarts and Grace teams

See also:

- hep-ph/0202109 (weak corrections, compared to Feynarts)
- hep-ph/0203220 (also hard bremsstrahlung, compared to Grace)
- Talks by L. Kalinovskaya and D. Bardin, this meeting

Cross-sections at TESLA - from TDR (2001)



For $\sigma_T \sim$ 0.5 pbarn at $\sqrt{s} = 500$ GeV
and a luminosity of $\mathcal{L} \sim 300$/fbarn,
we may expect about 10^5 top pairs.

Why study EWRC to $e^+e^- \to \bar{t}t$?

Building Block for Experimental Analysis

$\longrightarrow$ susy and other new physics

$\longrightarrow$ QCD + higher order calculations

$\longrightarrow$ 4-, 5- or 6-fermion production

Numerical Checking of Available Codes

Status of March 2001:

- Fujimoto,Shimizu (1988): **SM, QED**
- Hollik,Hahn et al. (1991-1999): **SM, MSSM**
- Bardin et al. (2000): **SM**

Nontrivial Testing for Complicated Calculations

We were exploring a tool for a semi-automated calculation of Feynman diagams – DIANA – and wanted to start with something familiar to us

See also talk of Anja Werthenbach in LOOPVEREIN session

For Nogueira's qgraph see:

ftp://gtae2.ist.utl.pt/pub/qgraf/

Its output: a list of diagrams, described in a combinatorial fashion.

DIANA – a DIAgramANAlyser

For DIANA (M. Tentyukov, J. Fleischer) see:

http://www.physik.uni-bielefeld.de/∼tentukov/diana.html

What does the program do?

- DIANA reads qgraf output.
  Analyzes the diagrams and produces graphical representation of them.
- Is interpreter of a special text manipulating language (TM).
  Executes the TM-program to insert explicit expressions for the vertices, propagators etc.
  The TM-program produces input text for FORM.

See also http://www-zeuthen.desy.de/~alorca/

From here DIANA 2.24 is now downloadable for linux systems, together with a simple example how to use it.

We are creating a new home page

DESY Zeuthen "Loops'n Legs" Home Page

for a more systematic sampling of loop calculational tools.

DIANA

We added a lot of further routines for general usage in order to get a semi-automated input for a Fortran program.

Our aim is not to fully automate, but to get enabled to safely handle huge amounts of complicated diagrams in a finite time.

Comment on no automatization and on full automatization.

amount of relevant feynman diagrams grows *eg*: $e^+e^- \to t\bar{t}$

$m_e = 0$, 1 fermion family, feynman gauge

$m_e = 0$, 1 fermion family, feynman gauge

\# diagrams, # topologies:

Born: 2 1

1 loop: 279 15

2 loop: 41540 219

DIANA: file = create.OneLoop

```
SET _processname         = OneLoop
SET _qgrafname           = "qgraf2"
SET _TOPOLOGYEDITORNAME = tedi
=========================================================
\Begin(model,Standard.model)        \End(model)

\excludevertex(E,e,H;E,e,G0;NU,e,Gp;E,nu,Gm)
=========================================================
\Begin(process)

  ingoing  e(; p1),E(; p4);
  outgoing t(;-p2),T(;-p3);

     loops = 1;                          \End(process)
=========================================================
\Begin(topology,topol.Born)          \End(topology)
=========================================================
\Begin(qgrafoptions)  options=onepi;  \End(qgrafoptions)
=========================================================
\Begin(output) .........

   g Rq =\integrand();                \End(output)
```

Run this file:

diana -c create.OneLoop
diana -c config.OneLoop

create this way the files:

OneLoop .ps

OneLoopInfo.ps

OneLoop.out contains form expressions for ALL diagrams

Further manipulations with OneLoop.out as INPUT to user-defined form file, one file for each diagram:

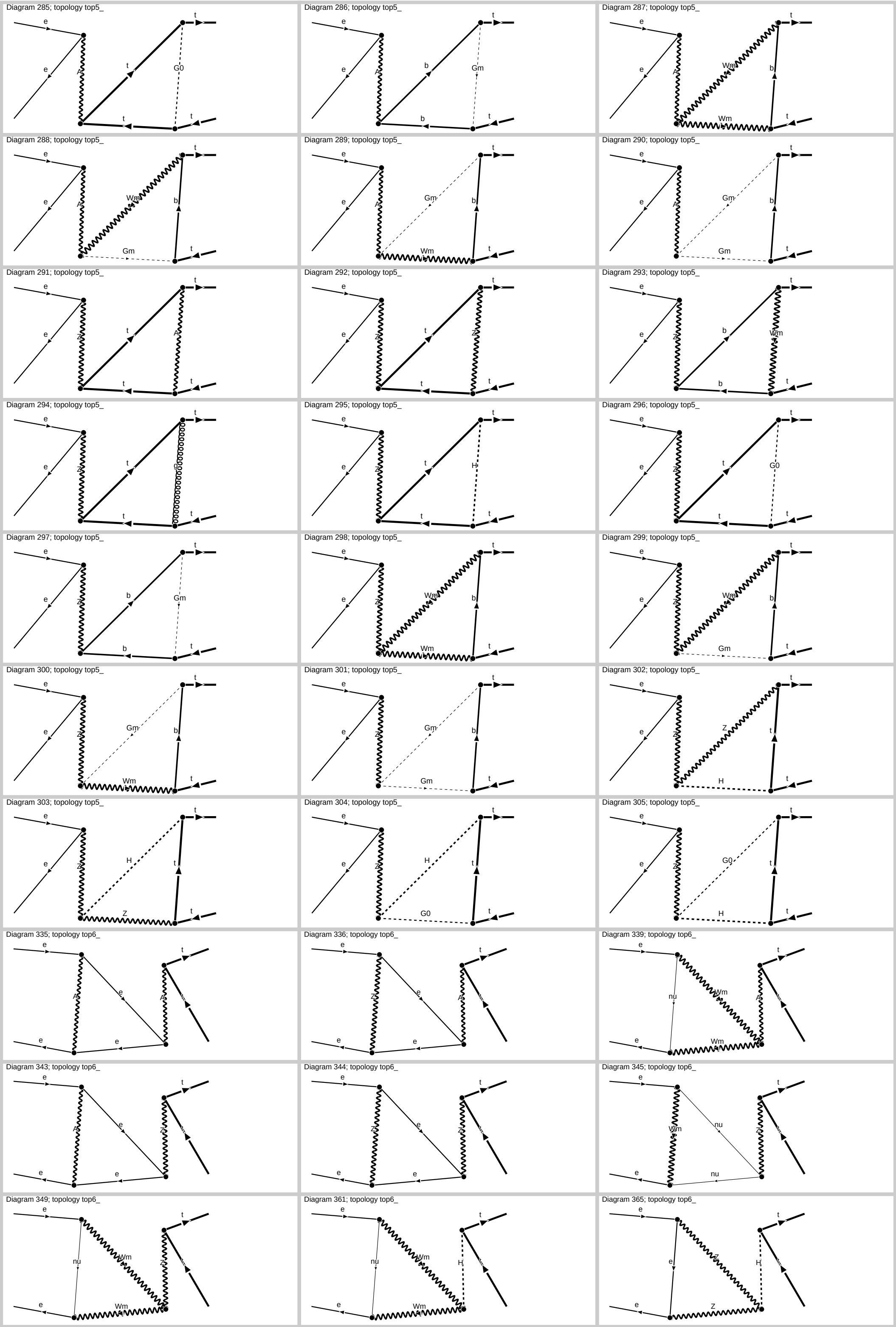
OUT/OneLoop#1.out . . . (binary)
. . .
OUT/OneLoop#last.out (binary)

Further manipulations with these binary files as INPUT to a user-defined form file:

form manipulate_OneLoop.frm

etc.

DIANA OneLoop_page5.ps



5

DIANA OneLoopInfo1037.ps

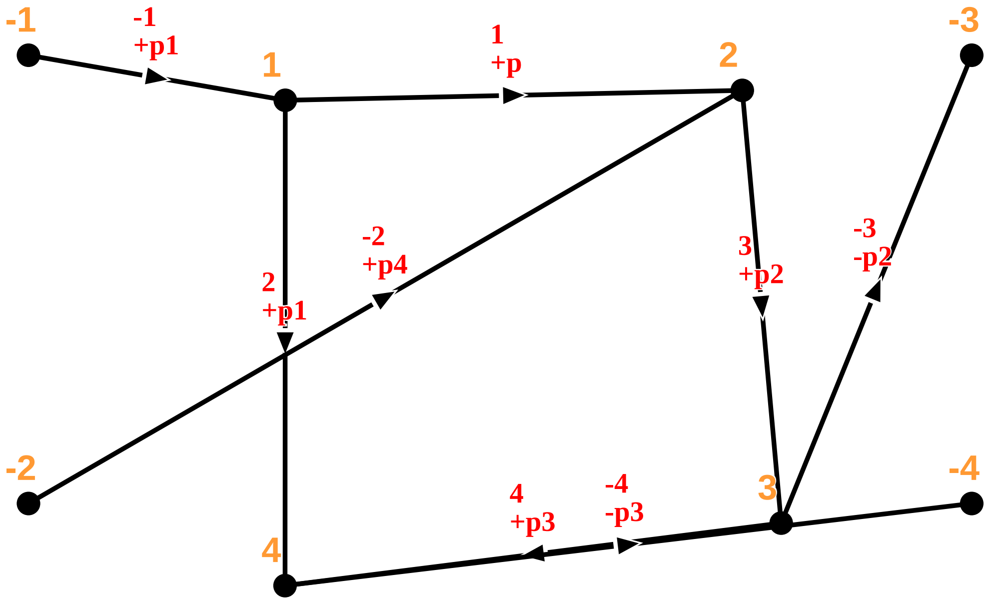
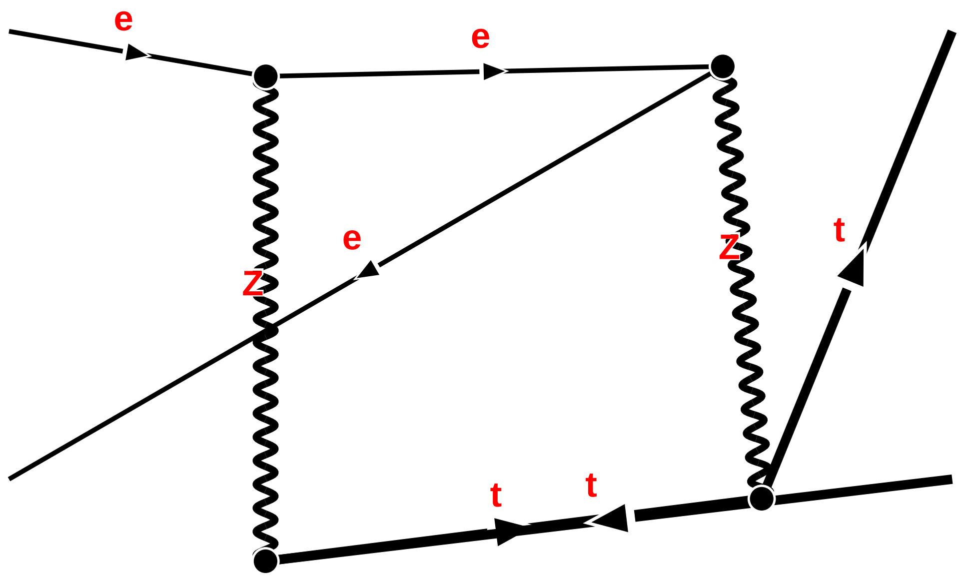


Diagram 1037 topology top16_ (unique top16_) momentaset 1 (of 1)



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DIANA OneLoopInfo1037.ps

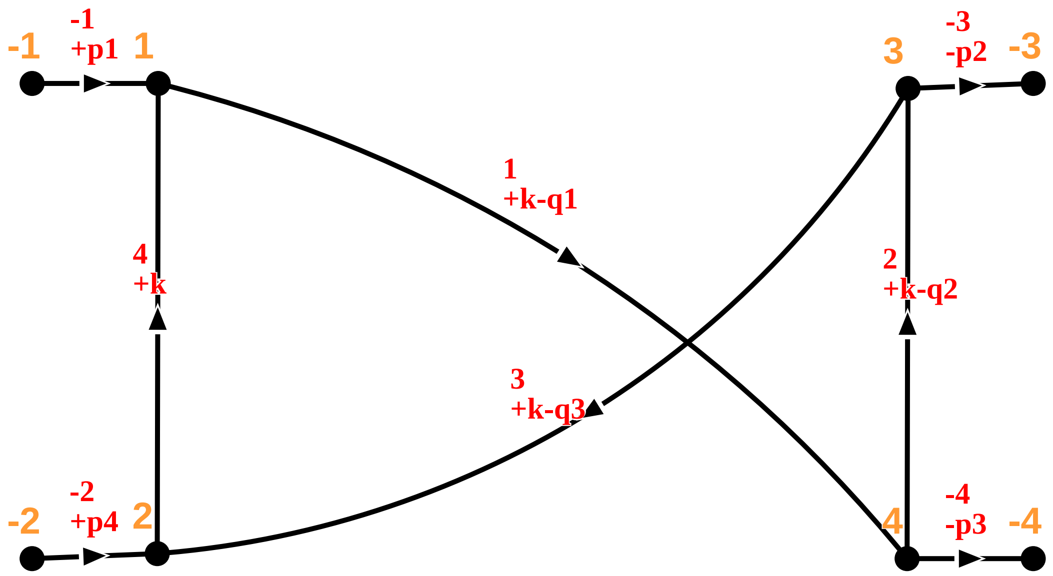
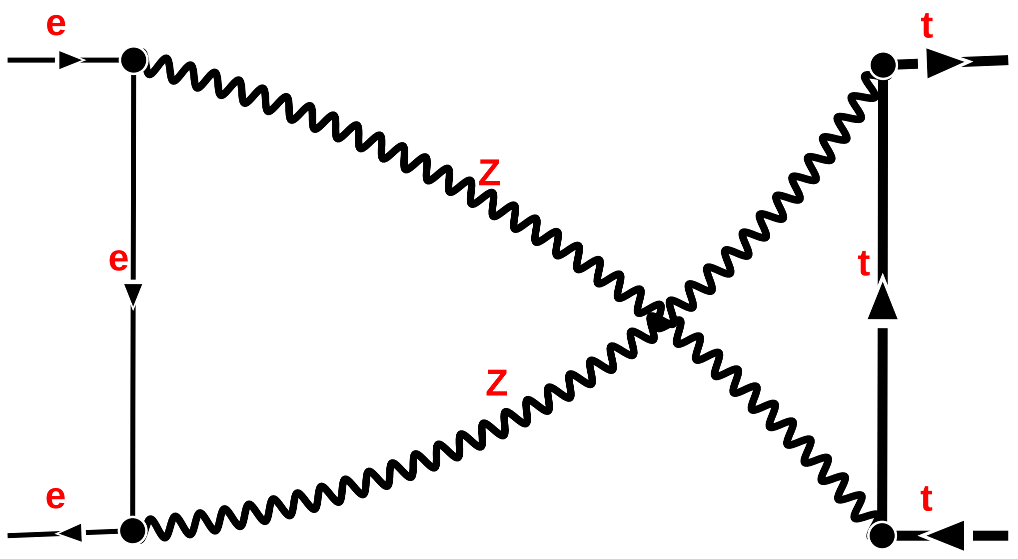


Diagram 1037 topology top15_ (unique top15_) momentaset 1 (of 1)



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file = BORN.out

```
*This file is automatically generated by DIANA.
*--#[ n1:
****(qgraf number 1)
**** d1
#define mm1 "0"
#define xx1 "X0"
#define  m1 "0"
*particle1=A

#define LINE "1"
#define FERMIONLINE "2"
#define TOPOLOGY "1"
#define TNAME "top"

g Rq =  1*F(2,1,mu1,1,0,1)*(-i_)*e*Qt
         *F(1,2,mu,1,0,1)*(-i_)*e*Qe
         *VV(1,mu,mu1,+p,0)*i_;
*--#] n1:
=============================================
*--#[ n2:
****(qgraf number 2)
**** d2   ...........
g Rq =
     1*F(2,1,mu1,GVt,-GAt,1)*(+i_)*e/2/s/c
      *F(1,2,mu,GVe,-GAe,1)*(+i_)*e/2/s/c
      *VV(1,mu,mu1,+p,1)*i_;
*--#] n0:
```

DIANA: file = Standard.model

```
* O.Veretin, J.Fleischer, A.Werthenbach, T.Riemann
.....
*  fermion PROPAGATOR
*  FF( 1. num
*      2. fnum  (fermion line number)
*      3. mass                       )

[nu,NU ;n ; FF(num,fnum,vec, 0 )*i_; 0]
[e,E   ;e ; FF(num,fnum,vec, me)*i_; mme]
[b,B   ;b ; FF(num,fnum,vec, mb)*i_; mmb]
[t,T   ;t ; FF(num,fnum,vec, mt)*i_; mmt]

*-------------------------------------------------------
*  fermion-vector-fermion VERTEX
*    F( 1. num
*       2. fnum
*       3. ind
*       4. GV  (vector coupling)
*       5. GA  (axial coupling)
*       6.                        )

*  neutral weak currents (6)
*  GV,GA --- vector and axial couplings

[NU,nu,Z;; F(num,fnum,ind:3,GVnu,-GAnu,1)*(i_)*e/2/s/c]
[E,e,Z  ;; F(num,fnum,ind:3,GVe ,-GAe ,1)*(i_)*e/2/s/c]
[T,t,Z  ;; F(num,fnum,ind:3,GVt ,-GAt ,1)*(i_)*e/2/s/c]
```

DIANA: file = feynmanrules.prc

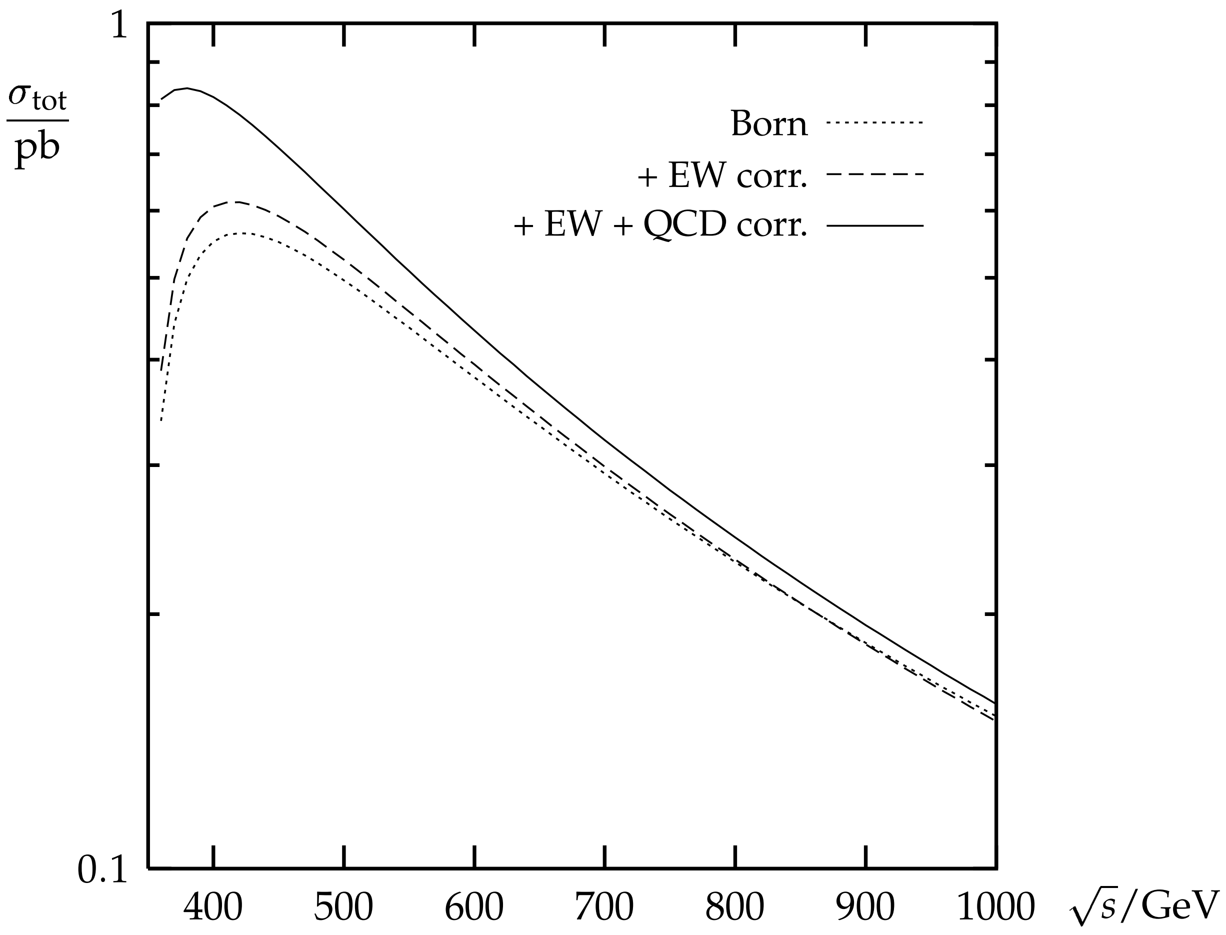
```
#procedure feynmanrules()

* PROPAGATORS:

  #do i=1,'LINE'
  id SS('i',0)                 = 1/c'i';
  id SS('i',c?!{,0})           = 1/c'i';
  id FF('i',c?,p1?,mt?)        = (gamma(c,p1)+mt)/c'i';
  id VV('i',mu1?,mu2?,q?,3) = (-d_(mu1,mu2))/c'i';
  #enddo

* FERMION-(AXIAL)VECTOR-FERMION VERTICES:

id F(s?,c?,mu1?,GVe?,GAe?,1) =

                 gamma(c,mu1)*(GVe + GAe*gamma5(c));

id 1/sqrt2^2 = 1/2;
id mW^2 = mmW;
.sort
#endprocedure feynmanrules
;
```

DIANA: file = manipulateBorn.log

```
   FORM version 2.3 Apr 24 1997

Born1 loaded
Born2 loaded
               g bornA=Born1;
               g bornZ=Born2;

   id c^-2*c1^-1 = c^-2/denZ;
   id      c1^-1 =    1/denA;

   .......

   print;
   .end

  bornA = 4 *
         gamma(1,mu1)*gamma(2,mu1)*i_*e^2
       * denA^-1    * ( Qe*Qt );

  bornZ =
         gamma(1,mu1)*gamma(2,mu1)*i_*e^2*s^-2*c^-2
       * denZ^-1
       * ( GVe*GVt - GVe*GAt - GVt*GAe + GAe*GAt )

      +  ..........
```

.

Radiative Corrections for Top Pair Production – from TESLA TDR (2001) –



Comment from a String Theorist

The study is another one

on a long list of studies

of the standard model.

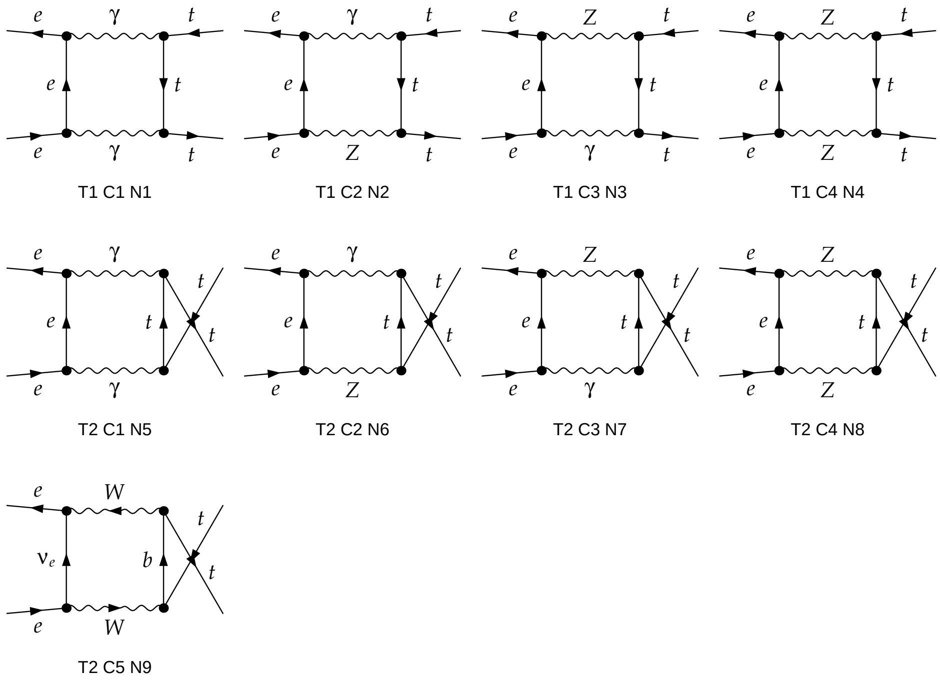
Unfortunately there is little

physical and theoretical motivation

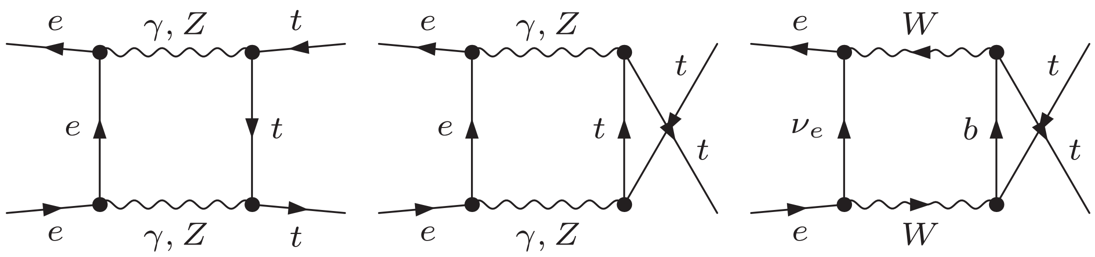
for this study

if any at all.

$$e \quad e \;\rightarrow\; t \quad t$$



Box diagrams:



Comment from a Born Calculator

The study is another one
on a long list of loop studies
of the standard model.

Hopefully there is little
physical and theoretical motivation
for this study
if any at all.

Structure of Virtual Corrections

The matrix element is a sum of many terms:

$$M = \sum_{a=all\ diagrams}^{1037} \sum_{i=1,4} \sum_{H_t,h_e} M_i^{H_t,h_e} \times \mathcal{F}_i^{H_t,h_e}(a)$$

Four Born-like struktures:

$$M_1^{\pm\pm'} = [\bar{u}_t(-p_2)\gamma^\mu \omega_\pm v_t(-p_3)] \times [\bar{v}_e(p_4)\gamma_\mu {\omega'}_\pm u_e(p_1)]$$

with

$$\omega_\pm = \frac{1 \pm \gamma_5}{2}$$

Additional Amplitudes from virtual corrections

Additional 12 structures appear:

$$
\begin{aligned}
{M_2}^{H_t,h_e} &= [\bar{u}_t(-p_2)\not{p}_4\omega_\pm v_t(-p_3)] \times [\bar{v}_e(p_4)\not{p}_2\omega_{\pm'}u_e(p_1)] \\
{M_3}^{H_t,h_e} &= [\bar{u}_t(-p_2)1\omega_\pm v_t(-p_3)] \times [\bar{v}_e(p_4)\not{p}_2\omega_{\pm'}u_e(p_1)] \\
{M_4}^{H_t,h_e} &= [\bar{u}_t(-p_2)\gamma^\mu\not{p}_4\omega_\pm v_t(-p_3)] \times [\bar{v}_e(p_4)\gamma_\mu\not{p}_2\omega_{\pm'}u_e(p_1)]
\end{aligned}
$$

For $m_e = 0$, but $m_{top} \neq 0$ only 6 amplitudes out of 16 are independent.

For $m_e = 0$ *and* $m_{top} = 0$, the 4 Born amplitudes remain.

Our form factors and the effective mixing angle

The Born matrix element:

$$M_1^{Born} \quad = \quad \bar{u}_t \gamma^\mu [2\omega_- - 4|Q_t| \sin^2\theta_W] v_t \times \bar{v}_e \gamma_\mu [2\omega_- - 4|Q_e| \sin^2\theta_W] u_e$$

with

$$\omega_\pm \quad = \quad \frac{1 \pm \gamma_5}{2}, 1 = \omega_- + \omega_+$$

The general structure M_1 has 4 form factors:

$$\begin{aligned} M_1 \quad = \quad & 4\rho\gamma^\mu\omega_- \otimes \gamma^\mu\omega_- \\ & - 8\rho \sin^2\theta_W |Q_e| \kappa_e \gamma^\mu \omega_- \otimes \gamma^\mu(\omega_- + \omega_+) \\ & - 8\rho \sin^2\theta_W |Q_t| \kappa_t \gamma^\mu (\omega_- + \omega_+) \otimes \gamma^\mu\omega_- \\ & + 16\rho \sin^4\theta_W |Q_e||Q_t| \kappa_{et} \gamma^\mu(\omega_- + \omega_+) \otimes \gamma^\mu(\omega_- + \omega_+) \end{aligned}$$

Or in another notation:

$$M_1 \quad = \quad F_1^{--}\gamma^\mu\omega_- \otimes \gamma^\mu\omega_- + F_1^{-+}\gamma^\mu\omega_- \otimes \gamma^\mu\omega_+ + F_1^{+-}\gamma^\mu\omega_+ \otimes \gamma^\mu\omega_- + F_1^{++}\gamma^\mu\omega_+ \otimes \gamma^\mu\omega_+$$

Relation of commonly used form factor sets:

$$
\begin{aligned}
\rho &= \frac{1}{4}[F_1^{--} - 2F_1^{-+} - 2F_1^{+-} - F_1^{++}] \\
\kappa_t &= -\frac{1}{4\rho\sin^2\theta_W|Q_t|}F_1^{+-} \\
\kappa_e &= -\frac{1}{4\rho\sin^2\theta_W|Q_e|}F_1^{-+} \\
\kappa_{et} &= \frac{1}{16\rho\sin^4\theta_W|Q_t||Q_e|}F_1^{++}
\end{aligned}
$$

The Fortran program topfit

Authors: J. Fleischer A.Leike T. Riemann A. Werthenbach

Features of v.0.91 (06 March 2002):

– Exactly one-loop

– SM weak loops and/or pure QED

– Input: $\alpha_{qed}(0)$, all masses

– u, c, t, ν: full weak + QED, μ, τ, d, s, b: presently only QED

Typical results from a calculation:

– six weak form factors

– $\frac{d\sigma}{d\cos\theta}, \sigma_T, A_{FB}$ with hard bremsstrahlung (some simple cuts)

– $\frac{d\sigma}{d\cos\theta}, \sigma_T, A_{FB}$ without photonic diagrams for use in a Monte Carlo

Special: Two independent calculations
using either package FF or package LoopTools
with dim.reg. for IR-divergency or finite photon mass

All eight structures $M_2^{\pm}$ and $M_4^{\pm}$ may be replaced, and two of the $M_3^{\pm}$.

Examples are:

$$
\begin{aligned}
{M_2}^{++} &= -m_t\ {M_3}^{-+} - \frac{1}{2}\left(m_t^2\ {M_1}^{-+} + t\ {M_1}^{++}\right) \\
{M_4}^{-+} &= -m_t\ \left({M_1}^{++} + {M_1}^{-+}\right) - 2\ {M_3}^{-+} \\
{M_3}^{++} &= {M_3}^{-+}
\end{aligned}
$$

$$
t = m_t^2 - T < 0
$$

Since $M_3^{\pm}$ becomes small for small m_t, there are only the four Born-like structures $M_1^{\pm}$ in that limit.

$$
\frac{d\sigma}{d\cos\vartheta} = const \times \sqrt{1 - 4 * m_t^2/s} \times 2\ \Re e \left(M_1^{\dagger} M_1 + M_1^{\dagger} M_3\right)
$$

Remember:

Weak corrections for $m_f << s$ are contained in four form factors only ($- = L$, $+ = R$):

$$G_F^{eff} = G_F \times \rho(s,t) = G_F \times M_1{}^{--}$$

$\sin_W^{2,eff}\theta = \sin_W^2\theta \times \kappa(s,t)$, but need three of them:

$$\kappa_e(s,t),\ \kappa_f(s,t),\ \kappa_{ef}(s,t)$$

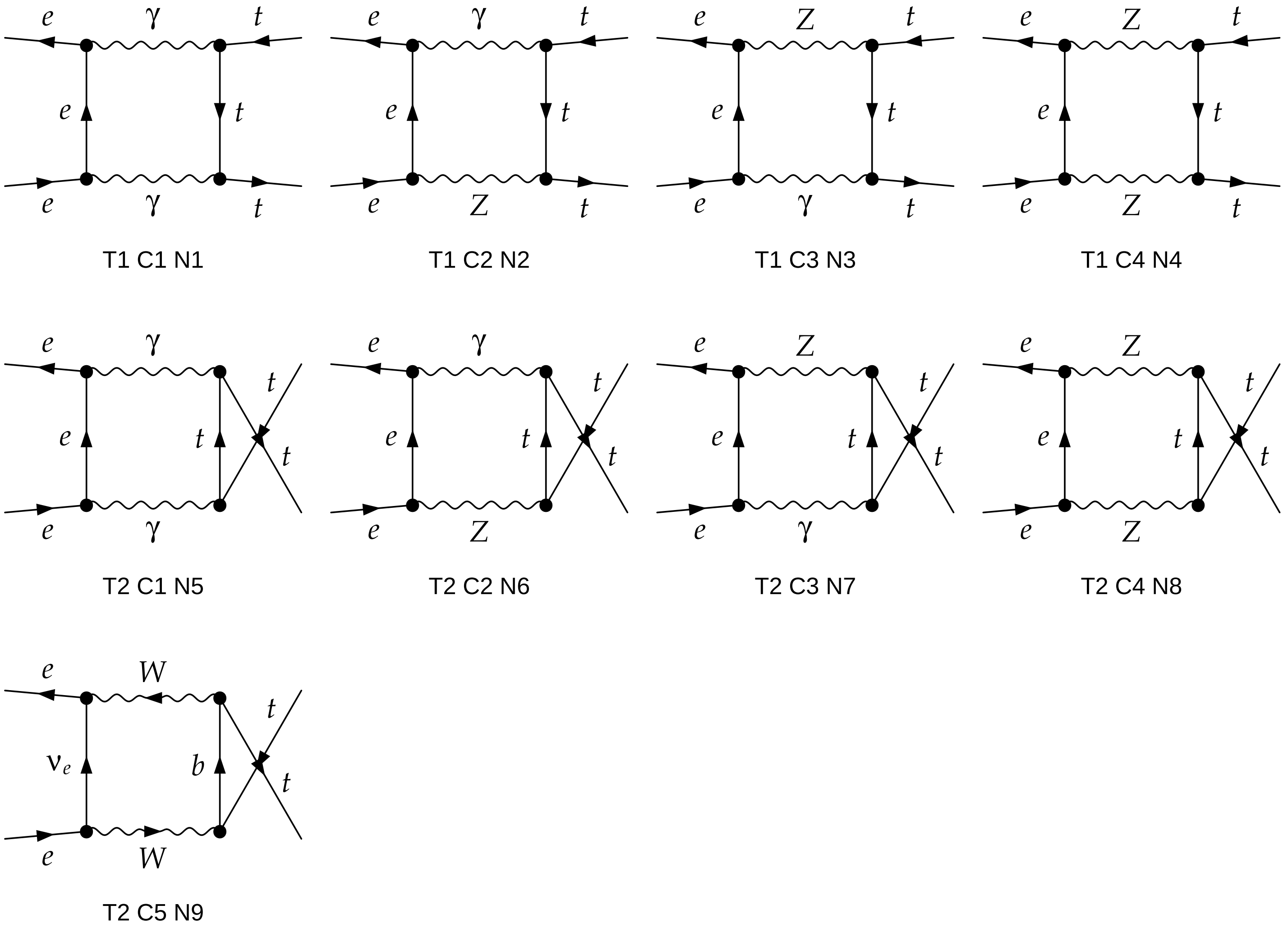
$$\frac{d\sigma}{d\cos\vartheta} = const \times \sqrt{1 - 4 * m_t^2/s} \times 2\ \Re e \left(M_1^\dagger M_1 + M_1^\dagger M_3 \right)$$

$$\begin{aligned} M_1^\dagger M_1 &= 4 * (U^2) * (M1LL1LL + M1RR1RR) \\ &+ (m_t^2 * S) * (M1LL1RL + M1RL1LL + \ldots) \\ &+ (T^2) * (M1RL1RL + M1LR1LR) \end{aligned}$$

$$M_1^\dagger M_3 = 2 * m_t * (u * t - m_t^4) * \ (M3LL1LL + M3LL1RL + \ldots)$$

Box diagrams for top pair production

$$e \quad e \quad \to \quad t \quad t$$



Real Photonic Bremsstrahlung

The Phase Space

We want access to the fermion production angle and need s' as integration variable.

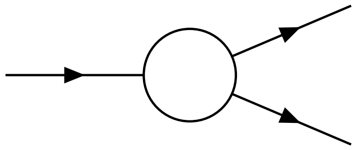
Three-particle phase space:

$$d\Phi_3(1,2,3) = \frac{d^3\vec{p}_1}{2p_1^0}\frac{d^3\vec{p}_2}{2p_2^0}\frac{d^3\vec{p}_3}{2p_3^0}\delta^4(p_{123}-p_{12}-p_3)$$

$$d\Phi_3(1,2,3) = d\Phi_2(1,2)\times ds'\times d\Phi_2(12,3).$$

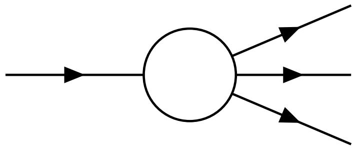
This finally delivers as differential phase volume in the new variables:

$$\begin{aligned}
\rightarrow \quad d\Phi_3(3) &= \frac{1}{(2\pi)^5}\frac{\pi}{16s}\,d\varphi_\gamma\,dV_2\,ds'\,d\cos\vartheta \\
&= \frac{1}{2}\frac{s}{(4\pi)^4}\,d\varphi_\gamma\,d\left(\frac{V_2}{s}\right)\,d\left(\frac{s'}{s}\right)\,d\cos\vartheta \\
&= \frac{1}{2}\frac{s}{(4\pi)^4}\,d\varphi_\gamma\,dx\,dR\,d\cos\vartheta.
\end{aligned}$$

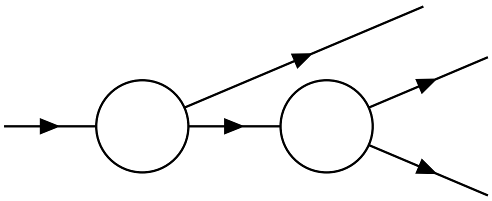


$$= \ d\Phi_2(1,2)$$

The 2-particle phase space.



$$=$$



$$= \ d\Phi_3(1,2,3)$$

A sequential parametrization of the 3-particle phase space.

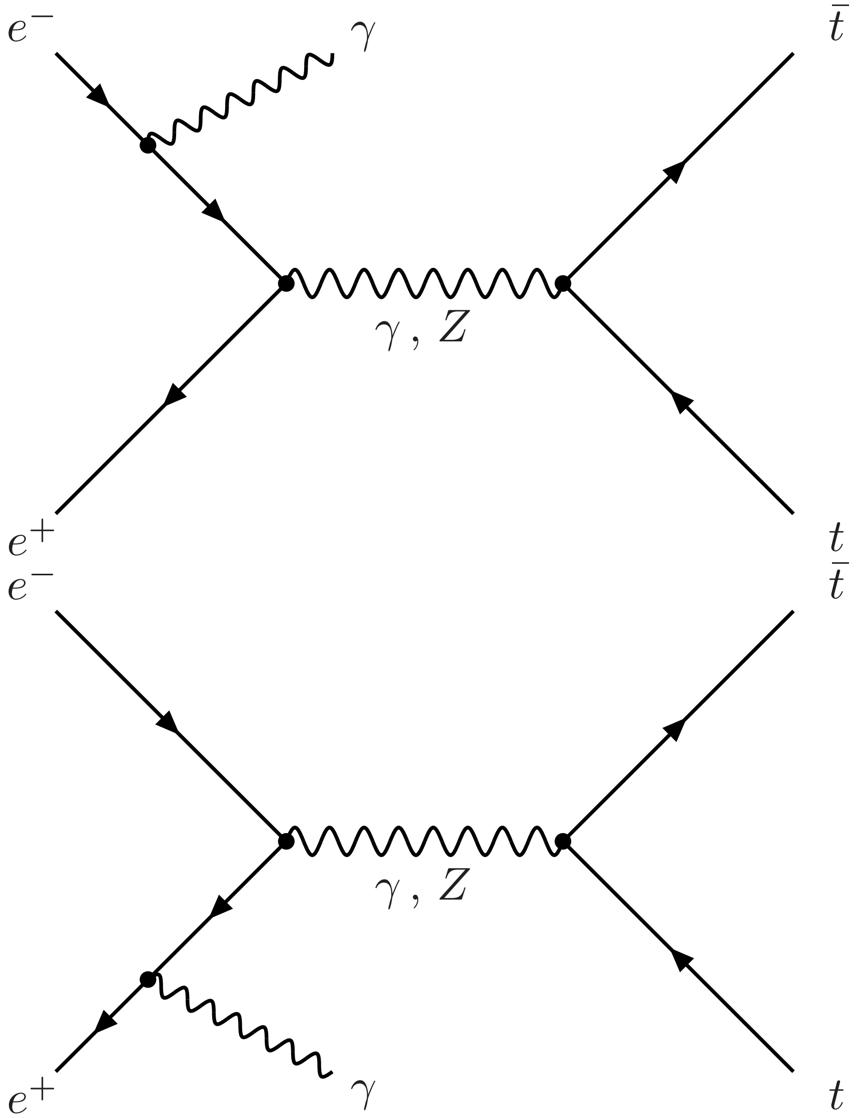


Figure 1: Feynman diagrams of initial-state radiation

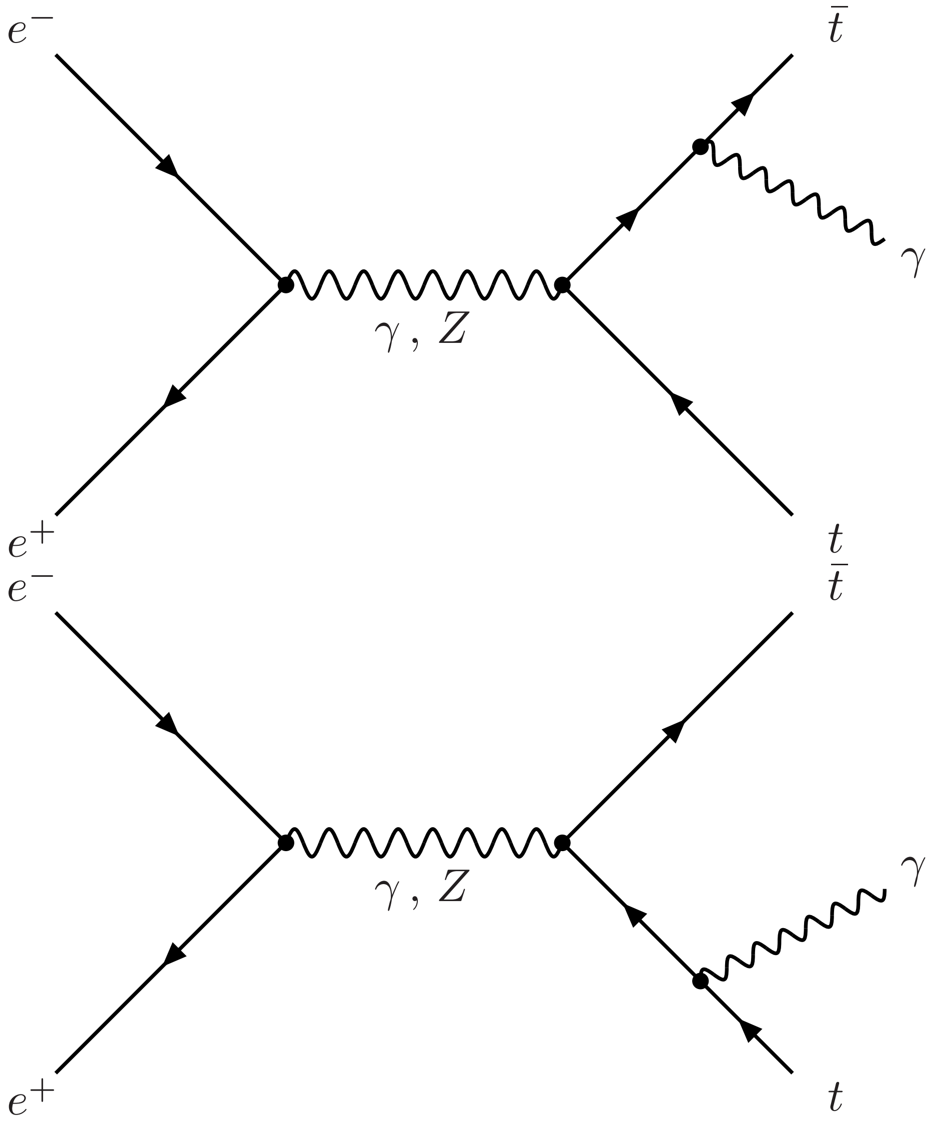


Figure 2: Feynman diagrams of final-state radiation

Kinematical Variables

cms energy: $s = (k_1 + k_2)^2$
invariant mass of the top pair $s' := (p_1 + p_2)^2$

Z_1 and Z_2 come from the propagators of the electron and positron in the diagrams of the initial-state radiation,

$$\begin{aligned} Z_1 &= 2p \cdot k_1 = -\left[(k_1 - p)^2 - m_e^2\right] \\ Z_2 &= 2p \cdot k_2 = -\left[(p - k_2)^2 - m_e^2\right] . \end{aligned}$$

Similarly V_1 and V_2 come from the propagators of the final top and anti-top of the final-state radiation,

$$\begin{aligned} V_1 &= 2p \cdot p_1 = \left[(p + p_1)^2 - m_f^2\right] \\ V_2 &= 2p \cdot p_2 = \left[(p + p_2)^2 - m_f^2\right] \end{aligned}$$

Differential cross section of the bremsstrahlung:

$$d\sigma = \frac{1}{2s\beta_0} \mid M_{ini} + M_{fin} \mid^2 d\Phi_3,$$

where $d\Phi_3$ is the differential phase space of three outgoing particles; $\beta_0 = \sqrt{(}1 - \frac{4me^2}{s})$

One-loop vertex corrections:

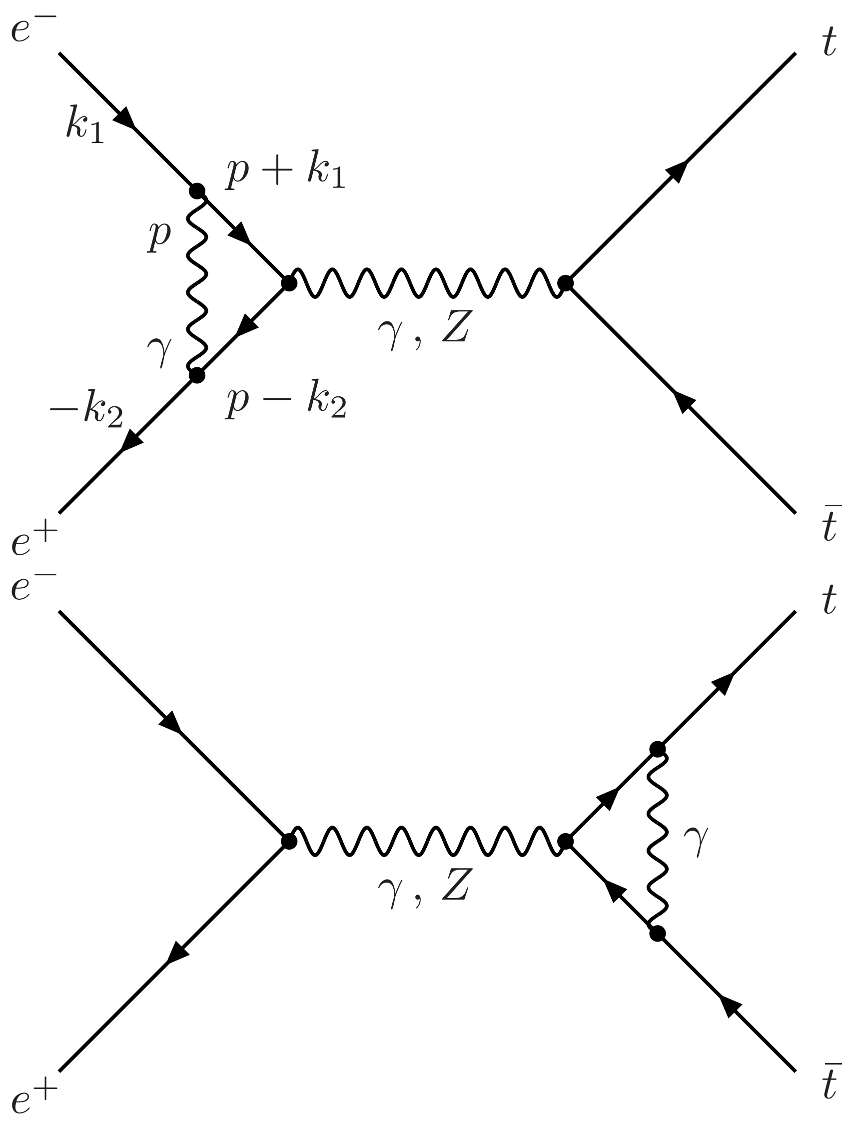


Figure 3: The photonic vertex corrections.

Interference between Born and initial state v.d. cancels
IR-divergences of $\mid M_{ini} \mid^2$

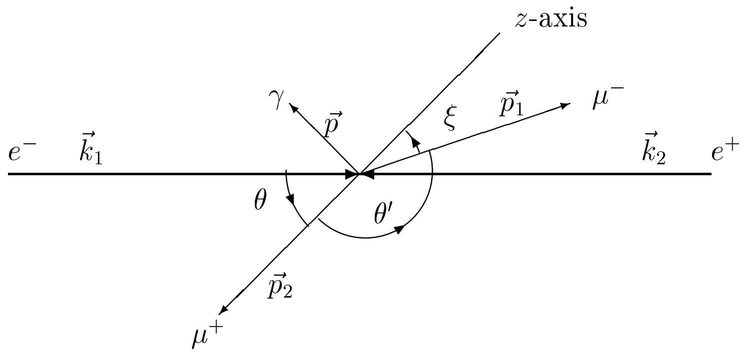
Interference between Born and final state v.d. cancels
IR-divergences of $\mid M_{fin} \mid^2$

Interference between M_{ini} and M_{fin} is cancelled by box- diagrams.

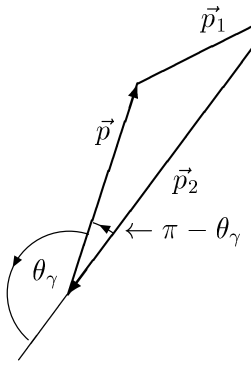
Independent kinematical variables:

- $s' := (p_1 + p_2)^2$ as invariant mass squared of the top pair,
- $\cos\vartheta$ as cosine of the scattering angle of $\bar{t}$ with respect to the e^- beam axis in the c.m.s.,
- $\cos\theta_\gamma$ as cosine of the polar angle between the three-momenta of $\bar{t}$ and the photon in the c.m.s.,
- φ_γ as azimuthal angle of the photon in the rest frame of (t, γ) (z-axis defined by $\vec{p}_2$ of $\bar{t}$ in the c.m.s.).

The different angles of phase space are shown in Fig. **??**.


z-axis
γ
$\vec{p}$
ξ
$\vec{p}_1$
μ^-
e^-
$\vec{k}_1$
$\vec{k}_2$
e^+
θ
θ'
$\vec{p}_2$
μ^+

next figure:


$\vec{p}_1$
$\vec{p}$
$\vec{p}_2$
$\leftarrow \pi - \theta_\gamma$
θ_γ

Angles of phase space and photon angle θ_γ.

ξ = acollinearity angle.

$$d\Phi_2(1,2) \quad = \quad \frac{d^3\vec{p}_1}{2p_1^0}\frac{d^3\vec{p}_2}{2p_2^0}\delta^4(p_{12}-p_1-p_2)$$

Limits of integration:

Azimuthal photon angle φ_γ: integration can be performed analytically.

$$\varphi_\gamma \,\epsilon\, [0; 2\pi].$$

1. Cuts on the minimum and maximal scattering angle $\cos\vartheta$ (acceptance cut):

$$-\, c \le \cos\vartheta \le c.$$

2. A cut on the minimum fermion energies: $E_{\min} = E_{\min}^{t,\bar{t}}$.

$$\left.\begin{array}{llll} \text{For } t & : \quad p_1^0 \ge E_{\min} & , & p_1^0 = \dfrac{s - V_2}{2\sqrt{s}} \\ \text{For } \bar{t} & : \quad p_2^0 \ge E_{\min} & , & p_2^0 = \dfrac{s' + V_2}{2\sqrt{s}} \end{array}\right\} \longrightarrow$$

$$R_E - R \le x \le 1 - R_E, \quad R_E = \frac{2E_{\min}}{\sqrt{s}}.$$

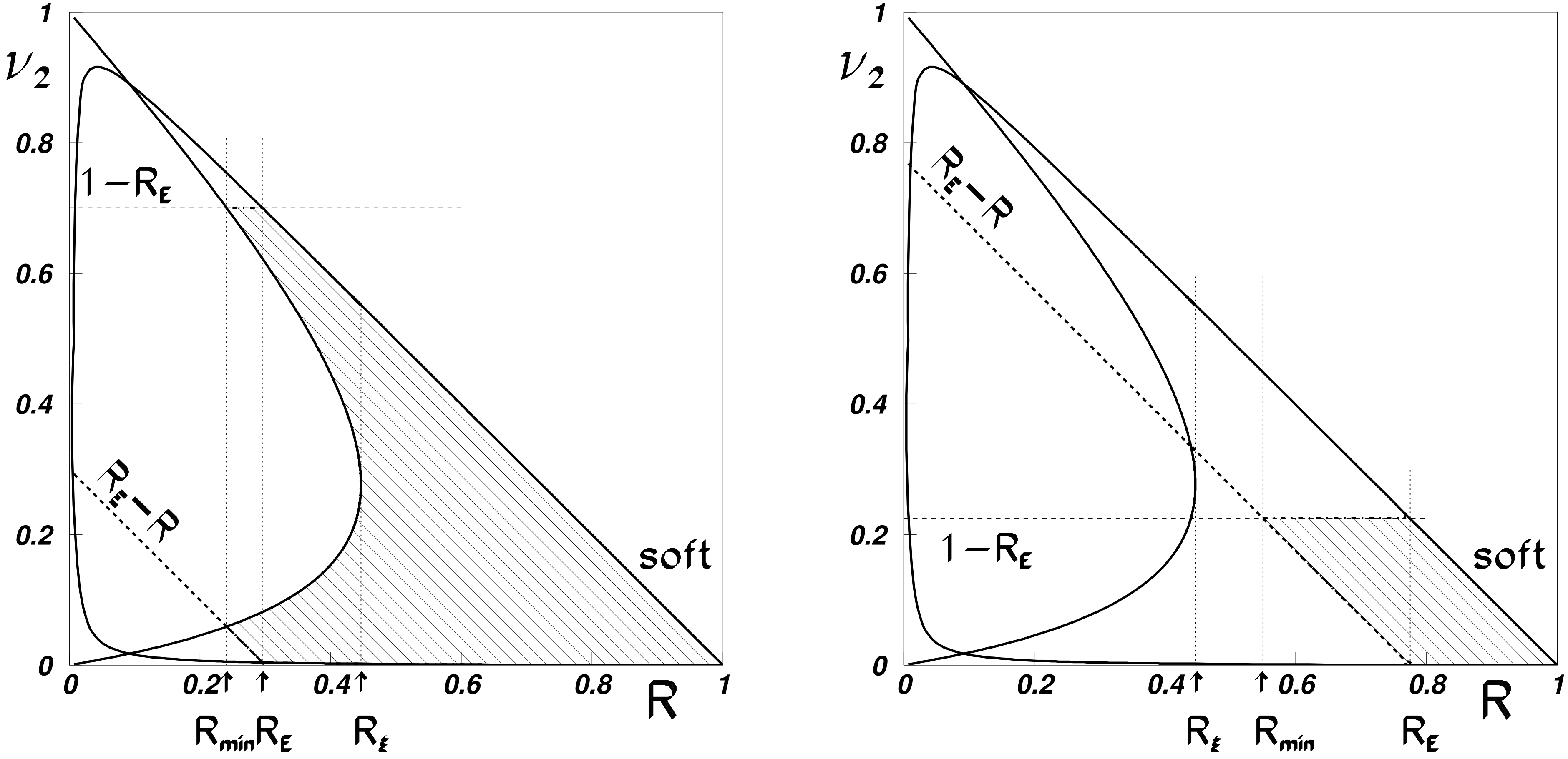


Figure 5: Phase space with acollinearity cut R_ξ and equal top energy cuts R_E: a. $R_{cut} < R_\xi$, b. $R_{cut} \geq R_\xi$; $v_2 := V_2/s \equiv x$.

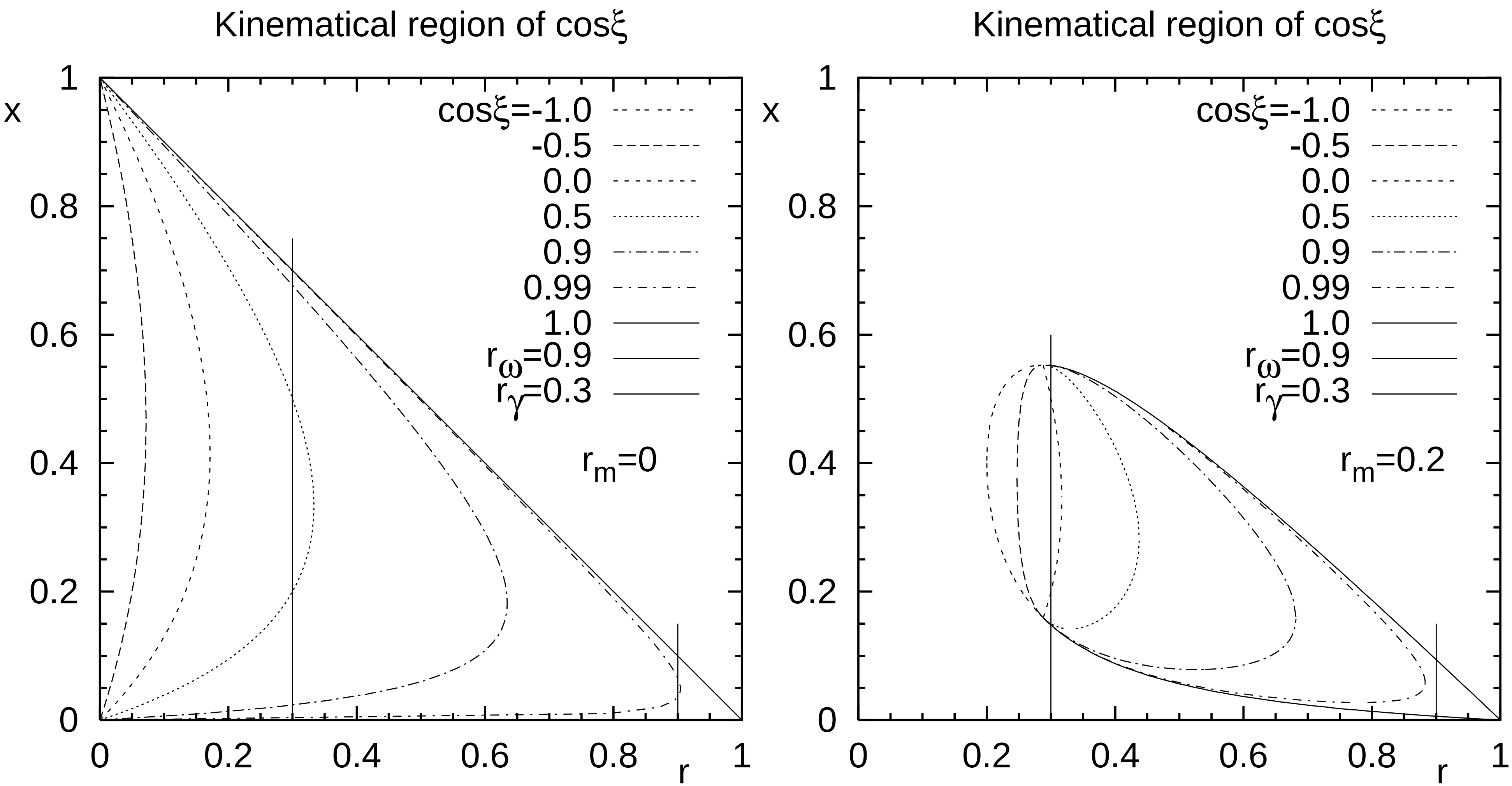
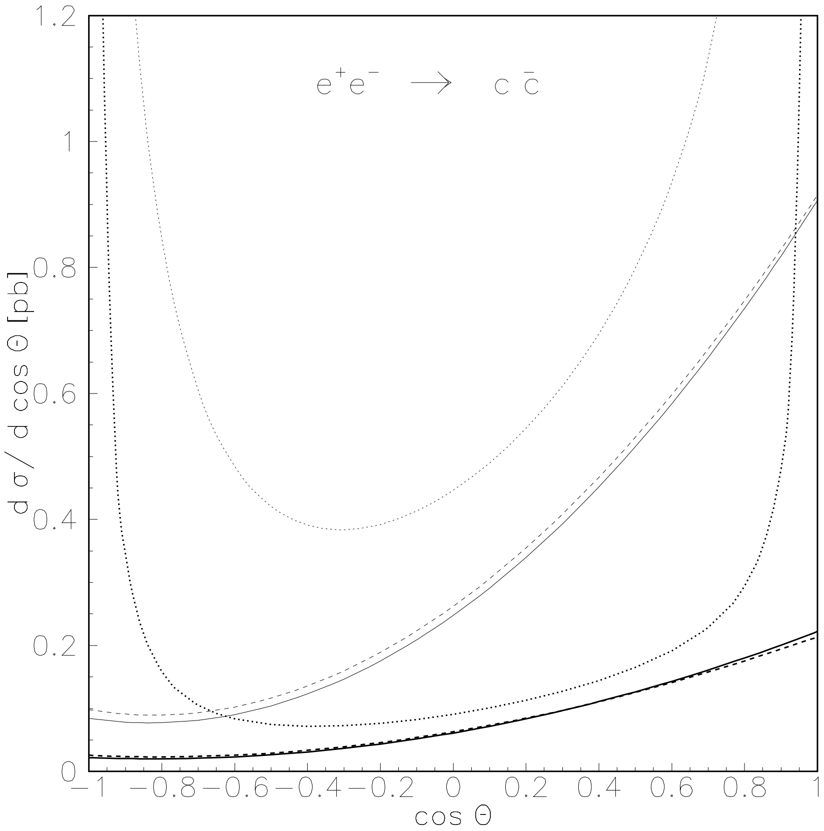


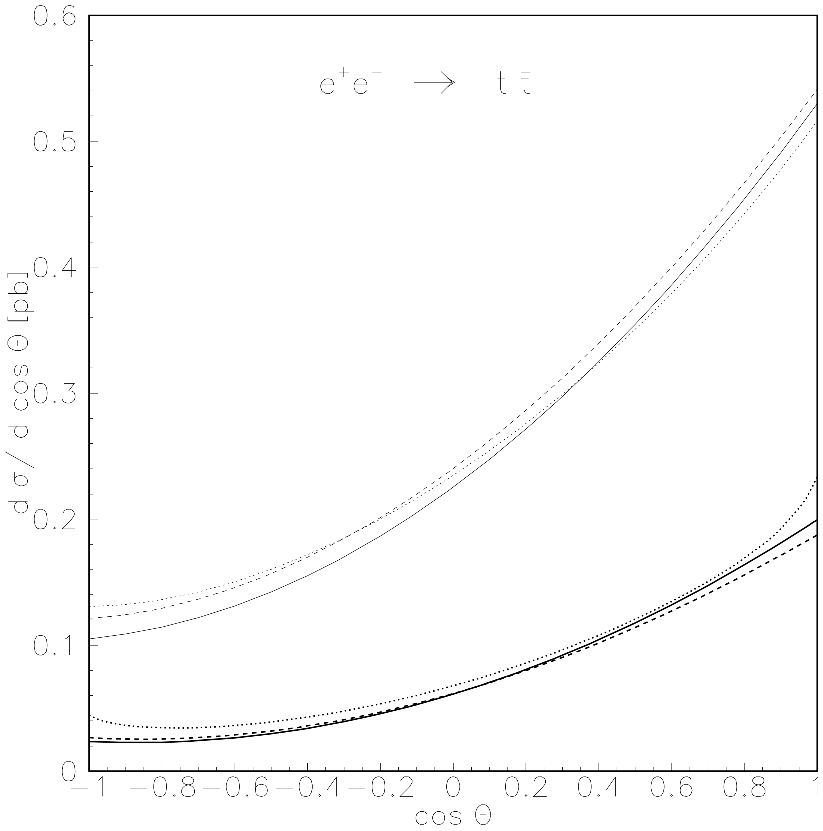
Figure 6: Phase space with acollinearity cut R_ξ; $v_2 := V_2/s \equiv x$.

a. $R_m = 4m_t^2/s = 0$ b. $R_m = 4m_t^2/s = 0.2$

topfit: charm pairs versus top pairs

$\frac{d\sigma}{dcos\theta}$ for $e^+ + e^- \to c + \bar{c} + \gamma$ and $e^+ + e^- \to t + \bar{t} + \gamma$ at TESLA





Solid: Born Dashed: weak Dotted: SM = weak + QED (no cut)

Thin lines: $\sqrt{s} = 500$ GeV, thick lines: $\sqrt{s} = 1000$ GeV.

$$
\begin{aligned}
|M_{ini}|^2 &= (4\pi\alpha)^3 \Bigg\{ \frac{\mathcal{V}(s')}{s s'} \Bigg[\frac{s'}{Z_1 Z_2} \left(2\,T^2 + 2\,U^2 + 2\,{s'}^2 - 2\,(T+U)\,s' + 4\,m_t^2\,s' \right) \\
& \quad + \frac{1}{Z_1} \left(s\,s' - 2\,U\,s' + {s'}^2 + 2\,m_t^2\,(s+s') \right) \\
& \quad + \frac{1}{Z_2} \left(s\,s' - 2\,T\,s' + {s'}^2 + 2\,m_t^2\,(s+s') \right) - 2\,s' - 4\,{m_t}^2 \Bigg] \\
&+ \frac{\mathcal{A}(s)}{s s'} \left[- \frac{2\,(T-U)\,{s'}^2}{Z_1 Z_2} - \frac{s'(s+s'-2\,U)}{Z_1} + \frac{s'(s+s'-2\,T)}{Z_2} \right] \\
&+ \frac{2\,m_t^2}{s s'}\, a_t^2\,(v_e^2 + a_e^2)\ |\,\chi(s')^2\,| \\
& \quad \cdot \left[- \frac{4\,{s'}^2}{Z_1 Z_2} - \frac{2\,(s+s')}{Z_1} - \frac{2\,(s+s')}{Z_2} \right] \Bigg\},
\end{aligned}
$$

where

$$
\begin{aligned}
T &= 2\,k_1 \cdot p_2 = \frac{1}{2} \left(s' + V_2 + \beta_0\, \sqrt{\lambda_2}\, \cos\vartheta \right), \\
U &= 2\,k_2 \cdot p_2 = \frac{1}{2} \left(s' + V_2 - \beta_0\, \sqrt{\lambda_2}\, \cos\vartheta \right)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}(s) &= Q_e^2 Q_t^2 + 2 Q_e Q_t \, v_e \, v_t \, \Re e \, \chi(s') + [v_e^2 + a_e^2] \, [v_t^2 + a_t^2] \, | \, \chi(s') \, |^2, \\
\mathcal{A}(s) &= 2 Q_e Q_t \, a_e \, a_t \, \Re e \, \chi(s') + 4 \, v_e \, a_e \, v_t \, a_t \, | \, \chi(s') \, |^2;
\end{aligned}
$$

$$
\lambda_2 = (s' + V_2)^2 - 4 s m_t^2
$$

and

$$
\chi(s) \equiv \kappa \, \frac{s}{s - m_Z^2 + i \frac{s}{mZ} \Gamma_Z}, \ \kappa = \frac{1}{4 s_W^2 c_W^2}
$$

```
From  riemann@ifh.de Mon Nov 26 12:11:32 2001
Date: Wed, 7 Nov 2001 17:13:41 +0100 (MET)
Subject: born?
 ============================
 SM parameters
 ----------------------------
 1/Alfa =  137.035989
 Alfas(s) =  0.0941216949
 GF =  1.16639E-05
 SW2 =  0.222249945
 ----------------------------
 Gauge-boson masses:
 MW =  80.419
 MZ =  91.1882
 ----------------------------
 Fermion masses:
 ME =  0.00051099907
 MM =  0.105658389
 ML =  1.777
 MU =  0.0538
 MC =  1.5
 MT =  174.3
 MD =  0.0538
 MS =  0.15
 MB =  4.7
 ----------------------------
 Higgs mass:
 MH =  100.
 ============================
 MH=  100.00
 sqrt(s) =  400.  theta =  0.174532925
 sqrt(s) =  400.  theta =  2.35619449
```

The output from the automatic Fortran:

```
MH=  100.00

0.174532925199 0.7466704219937E-01 0.7969866133010E-01
2.356194490192 0.2892227102414E-01 0.3290398685229E-01
```

Questions . . .

```
Zitat Hahn:
erste Spalte ist der Winkel in Radians,
zweite Spalte der WQ auf Born-Niveau in Picobarn und
dritte Spalte der WQ auf 1-loop-Niveau in Picobarn.


Comment t.:
Ich denke, dass lambda und mu etc so fixiert sind, wie immer.
Aber erstmal brauchen wir alpha_qed
leider geben sie die z-breite nicht an. danach frag ich mal....
```

```
==================================================
Date: Wed, 7 Nov 2001 18:41:23 +0100 (CET)
From: Dmitri Bardin <Dmitri.Bardin@cern.ch>
To: <werthenb@ifh.de>   Cc: <riemann@ifh.de>,

Name: "Dubna-Zeuthen_ZZ_box_FF.log"

omega=10   lambda=1     mu=1  cnini,cnint,cnfin = 0 1 0
IBORN= 0   ISOFT =0  IVIRT=1  kappa            = 1.44935078
 mt  = 173.8

  cos  |                  sqrt(s) = 400
-----------------------------------------------------------
 -0.9  |    f1ll= (-7.64523868E-10, 2.93105051E-09)
            F_LL= (-7.64529314E-10, 2.93104968E-09)
                          |                 |
       |    f1rl= (-1.01237479E-09,-3.1369112E-11)
            F_RL= (-1.01237471E-09,-3.13690994E-11)
                            |             |
       |    f1lr= (1.55955319E-10, -5.01846831E-10)
            F_LR= (1.5595716E-10,  -5.01846552E-10)
                        |                 |
       |    f1rr= (-6.57565276E-10, 9.87816662E-10)
            F_RR= (-6.575655E-10,   9.87816626E-10)
                          |                |
       |    f3rl= (-5.612866E-12,   3.5016928E-12)
            F_RD= (-5.61286475E-12, 3.50169303E-12)
                          |                |
       |    f3lr= ( 2.81709442E-12,-1.33818211E-12)
            F_LD= ( 2.81716156E-12,-1.33817193E-12)
                         |               |
-----------------------------------------------------------
etc. etc. etc.
```

Numerical Results Based on

J. Fleischer, T. Riemann, A. Werthenbach

and

T. Hahn, W. Hollik, C. Schappacher

Complete Electroweak One-loop Corrections to Top-pair Production at TESLA - a Comparison

hep-ph/0202109

(weak corrections, topfit compared to Feynarts)

J. Fleischer, A. Leike, T. Riemann, A. Werthenbach

and

J. Fujimoto, T. Ishikawa, Y. Shimizu

One-loop Corrections to the Process $e^+e^- \to t\bar{t}$ Including Hard Bremsstrahlung

hep-ph/0203220

(also hard bremsstrahlung, topfit compared to Grace)

Top-quark Pair-production Cross section

Conventions for comparison with Karlsruhe team T. Hahn, Ch. Schappacher and W. Hollik:

$$
\begin{aligned}
\Gamma_Z &= 0 \\
\alpha = \frac{e^2}{4\,\pi} &= 1/137.03599976 \\
E_\gamma^{\max} &= \sqrt{s}/10 \\
M_w &= 80.4514958 \text{ GeV} \\
M_z &= 91.1867 \text{ GeV} \\
M_h &= 120 \text{ GeV} \\
m_e &= 0.00051099907 \text{ GeV} \\
m_t &= 173.8 \text{ GeV} \\
m_b &= 4.7 \text{ GeV} \\
m_\mu &= 0.105658389 \text{ GeV} \\
m_u &= 0.062 \text{ GeV} \\
m_d &= 0.083 \text{ GeV} \\
m_\tau &= 1.77705 \text{ GeV} \\
m_c &= 1.5 \text{ GeV} \\
m_s &= 0.215 \text{ GeV}
\end{aligned}
$$

Comment from a 2-Loops Person

The study is the first one explaining the hidden secrets of our perfect software like Feynarts, Feyncalc, LoopTools, Formcalc and all that.

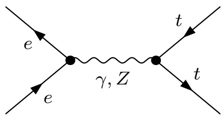
Fortunately there is no need of physical and theoretical motivation for this study if any at all.

Appendix

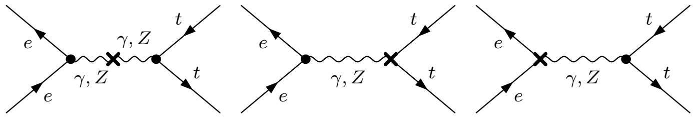
A Feynman diagrams

In cases where several particles are listed for a propagator, there is one diagram for each combination of particles. On fermion lines, an f_i stands for $\{e, \mu, \tau, u, c, t, d, s, b\}$ and a ν_i for $\{\nu_e, \nu_\mu, \nu_\tau\}$.

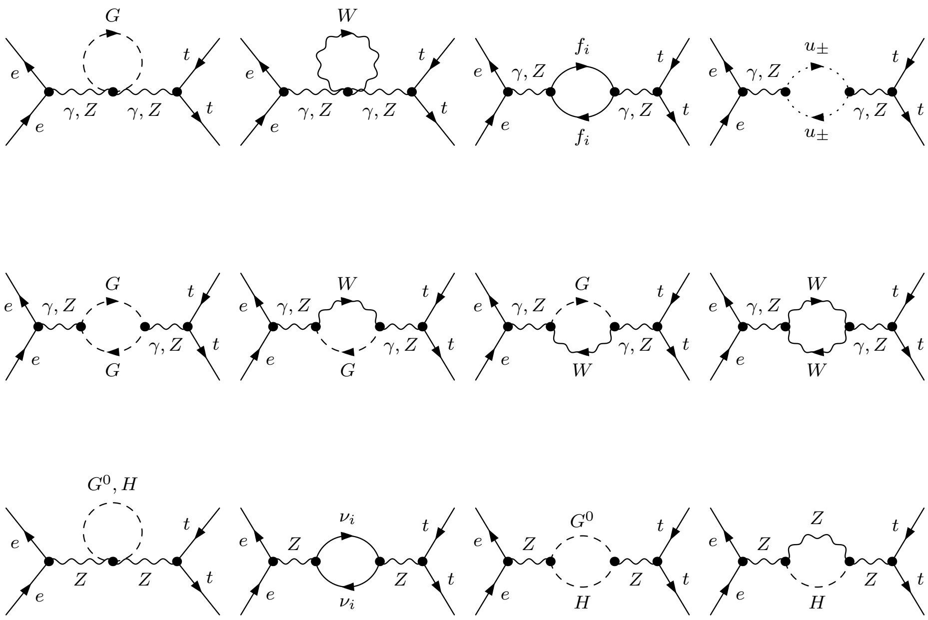
Born diagrams:



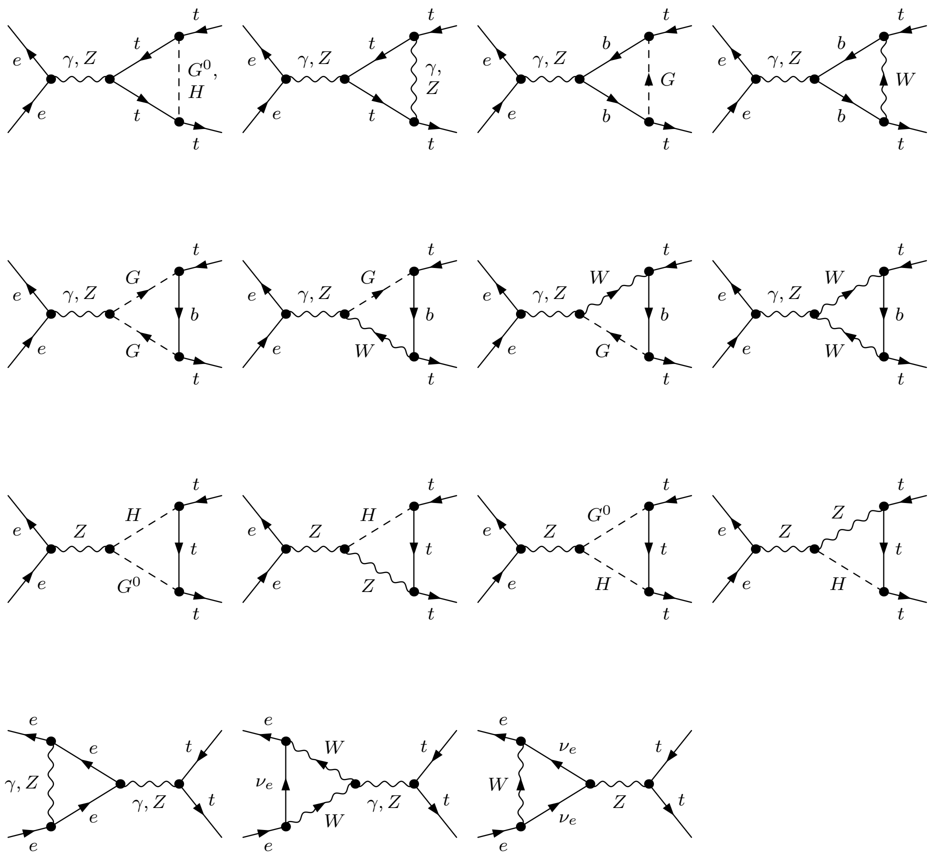
Counter-term diagrams:



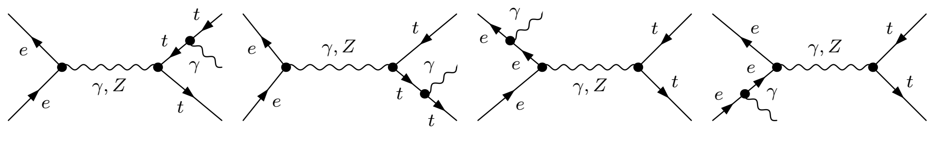
Self-energy diagrams:



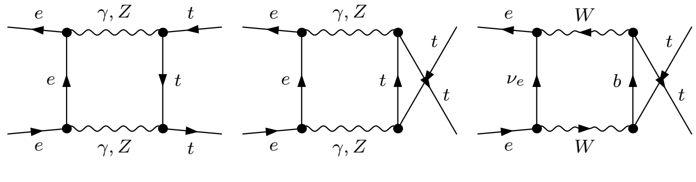
Vertex diagrams:



Real-photon-emission diagrams:



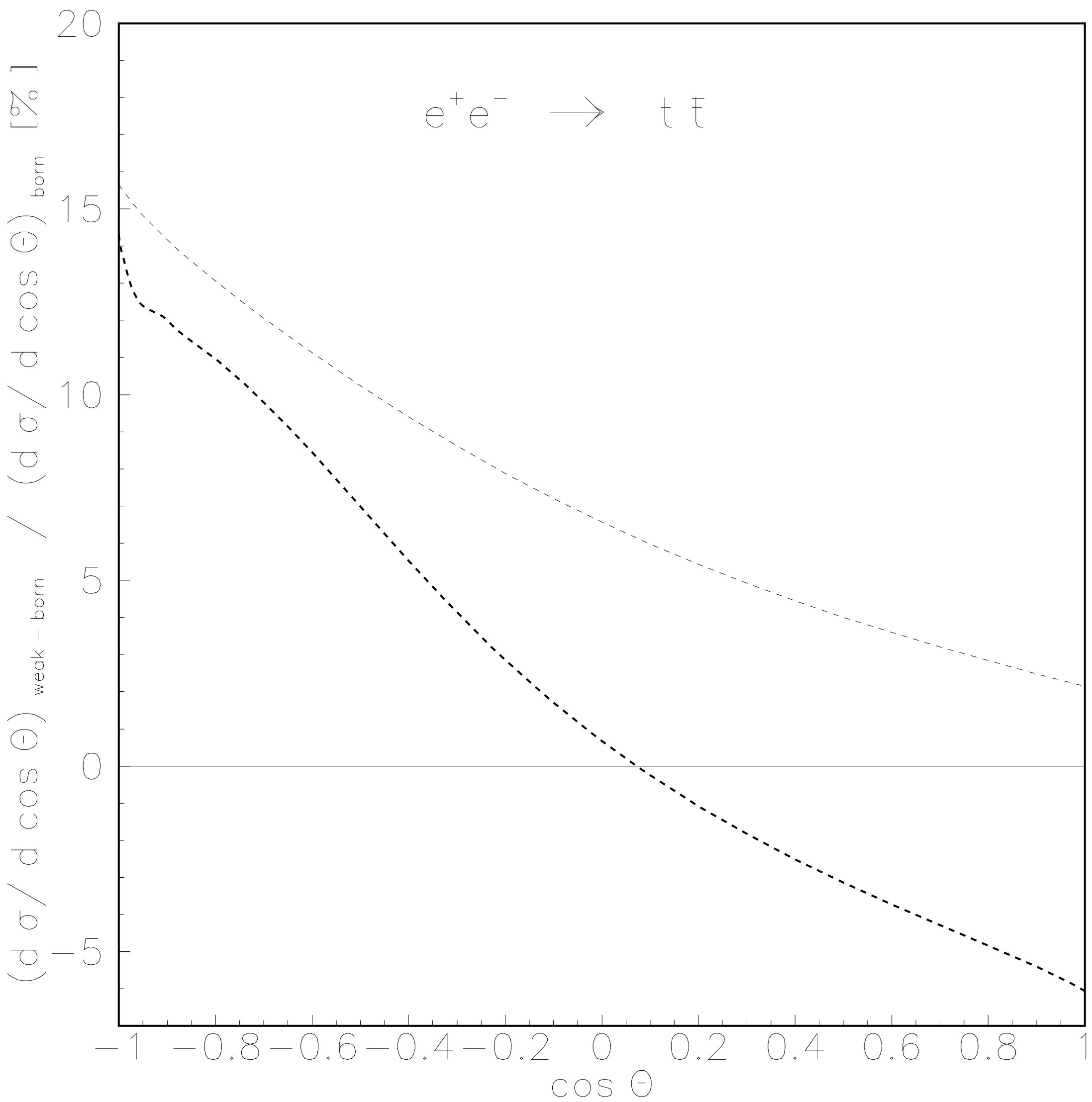
Box diagrams:



10

topfit: top pairs virtual corrections

$\frac{d\sigma}{dcos\theta}$ for $e^+ + e^- \to t + \bar{t} + \gamma$ at TESLA



One-loop corrections = (weak corrs./Born)x100%

Thin line: $\sqrt{s} = 500$ GeV

Thick line: $\sqrt{s} = 1000$ GeV.

Top pair production: Feynarts and topfit

$\sqrt{s} = 500$ GeV: $e^+ + e^- \to \bar{t} + t$ at TESLA

$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{QED}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{weak}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{SM}}$
-0.9	0.1088 3919 4075	0.09866 4253	0.1242 59037 1	0.1140 8410
	0.1088 3919 4075	0.09866 4253	0.1242 59037 6	0.1140 8410
-0.5	0.1422 7506 9392	0.1285 0790	0.1568 48371 9	0.1430 8121
	0.1422 7506 9392	0.1285 0790	0.1568 48371 8	0.1430 8121
0.	0.2254 7046 4032	0.2023 9167	0.2402 6680 4	0.2171 8801
	0.2254 7046 4033	0.2023 9167	0.2402 6680 3	0.2171 8801
0.5	0.3546 6647 0331	0.3151 1724	0.3688 8650 7	0.3293 3727
	0.3546 6647 0332	0.3151 1723	0.3688 8650 5	0.3293 3727
0.9	0.4911 4371 5766	0.4307 1437	0.5033 3751 2	0.4429 0817
	0.4911 4371 5767	0.4307 1437	0.5033 3750 8	0.4429 0816

Agreement confirms accuracy of electroweak calculations of two groups (06 March, 2002):
upper line: Feynarts, lower line: topfit

Top pair production: topfit and Grace, angular distribution

$\cos\theta$	$\omega/\sqrt{s}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{QED}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{SM}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{tot}}$
-0.9	T : 0.1	0.108839194075	+0.098664253	+0.11408410	0.13144
	T : 0.00001	0.108839194075	−0.017474702	−0.002054858	0.13229
	G : 0.00001	0.108839194076		−0.002054859	0.13206(12)
-0.5	T : 0.1	0.142275069392	+0.12850790	+0.14308121	0.15973
	T : 0.00001	0.142275069392	−0.029702340	−0.015129038	0.16029
	G : 0.00001	0.142275069393		−0.015129039	0.16013(13)
$+0.0$	T : 0.1	0.225470464033	+0.20239167	+0.21718801	0.23638
	T : 0.00001	0.225470464033	−0.058010508	−0.043214169	0.23476
	G : 0.00001	0.225470464033		−0.043214168	0.23513(14)
$+0.5$	T : 0.1	0.354666470332	+0.31511723	+0.32933727	0.35651
	T : 0.00001	0.354666470332	−0.109721291	−0.095501257	0.35062
	G : 0.00001	0.354666470332		−0.095501252	0.35104(17)
$+0.9$	T : 0.1	0.491143715767	+0.43071437	+0.44290816	0.48796
	T : 0.00001	0.491143715767	−0.179672655	−0.16747886	0.47768
	G : 0.00001	0.491143715767		−0.16747886	0.47709(21)

Various differential cross sections. The upper and lower numbers correspond to the `topfit` (T) and GRACE (G) approach, respectively, $\sqrt{s} = 500$ GeV.

Top pair production: topfit and Grace

$\sqrt{s}$	$\sigma^0_{\rm tot}$	$A^0_{\rm FB}$	$\sigma_{\rm SM,tot}$	$\sigma_{\rm SM,FB}$	$\sigma_{\rm tot}$	$A_{\rm FB}$
500	T : 0.5122744	0.4146039	−0.1198972	−0.0855551	0.526337	0.362929
	G : 0.5122751	0.4146042	−0.1198973		0.526371	0.363140
1000	T : 0.1559185	0.5641706	−0.0683693	−0.0522582	0.171916	0.488869
	G : 0.1559187	0.5641710	−0.0683695		0.171931	0.488872

Total cross sections (in pbarn) and forward-backward asymmetries.

$\sigma^0_{\rm tot}$ – Born

$\sigma_{\rm SM,tot}$ – elastic (with soft photons)

$\sigma_{\rm tot}$ – also hard photons, $\omega/\sqrt{s} = 0.00001$.

Summary

Hopefully everybody may be happy now with the numerical status reached for the weak corrections to $e^+e^- \to t\bar{t}$.

For hard photons: do better.

We gathered some experience using DIANA and FORM and FF and LoopTools for a complex calculation.

We are collecting some sample programs for everybodies use.

It is very important to make sure that what we do today may be re-animated in 10 or 20 years when a Linear Collider might start working.

Let us hope that then people will have yet linux and Fortran and CD format iso 9660 etc.

Top pair production: topfit and Grace, angular distribution

$\cos\theta$	$\omega/\sqrt{s}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{QED}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{SM}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{tot}}$
-0.9	T : 0.1	0.0227854232732	+0.020365844	+0.023101706	0.036334
	T : 0.00001	0.0227854232732	−0.004756230	−0.002020367	0.036461
	G : 0.00001	0.02278542327319		−0.002020369	0.036582(48)
-0.5	T : 0.1	0.0297821311031	+0.026741663	+0.028823021	0.038888
	T : 0.00001	0.0297821311031	−0.008561495	−0.006480137	0.039055
	G : 0.00001	0.0297821311031		−0.006480139	0.038965(42)
$+0.0$	T : 0.1	0.0611800674224	+0.054539344	+0.054950889	0.067789
	T : 0.00001	0.0611800674224	−0.021532420	−0.021120874	0.067801
	G : 0.00001	0.0611800674225		−0.021120874	0.068039(55)
$+0.5$	T : 0.1	0.117746949888	+0.10311626	+0.099416999	0.12095
	T : 0.00001	0.117746949888	−0.050123708	−0.053822973	0.12051
	G : 0.00001	0.117746949888		−0.053822964	0.12064(07)
$+0.9$	T : 0.1	0.181122097086	+0.15403823	+0.14426232	0.19355
	T : 0.00001	0.181122097086	−0.096682759	−0.10645866	0.19272
	G : 0.00001	0.181122097086		−0.10645866	0.19057(10)

Various differential cross sections. The upper and lower numbers correspond to the `topfit` (T) and GRACE (G) approach, respectively, $\sqrt{s} = 1000$ GeV.

Some Numerical Results

for A^b_{F-B} at LEP 1

```
  At TESLA energies:

 sqrt(s) =       500.
 m_f     =       4.
 Q_f     =      -0.333333333

 AFB_num =          0.150759697
 AFB_flx =          0.151470125
 diff AFB= num-flx -0.000710428057   = 0.7 x 10 E-3




At LEP 1 energies we get something quite different :

 sqrt(s) =       91.1812166
 m_f     =       2.8
 Q_f     =      -0.333333333

 AFB_num =          0.152658028
 AFB_flx =          0.152661305
 diff AFB= num-flx -3.2767968E-06    = 0.003 x 10 E-3
```

Comment from Woody Allen

The project is the first one explaining

Everything You Always Wanted to Know About Loops

Of course I feel little interest

in the

physical and theoretical motivation

of this study

if any at all.