S-matrix approach to the Z resonance

or: The Z resonance without weak loops

or: The SMATASY/ZFITTER approach to the Z resonance

Tord Riemann, DESY **Thanks to: M. Grünewald + S. Riemann**

talk held at workshop “Matter To The Deepest”
XXXIX International Conference on Recent Developments In Physics Of Fundamental Interactions
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http://indico.if.us.edu.pl/conferenceOtherViews.py?view=nicecompact&confId=2

Present interest in precision approaches to the Z boson

Belle-II

$\sqrt{s} \sim 10$ GeV
→ **Belle-II** will measure 10^9 $\mu^+\mu^-$ events
See e.g. T.Ferber [1]

Fcc-ee

$\sqrt{s} \sim M_Z$
→ **Fcc-ee** expects 10^{13} events at the Z resonance
See e.g. A. Freitas [2].
Much work on weak two-loop contributions to the Z resonance has been done by Hollik et al., Czakon et al., Freitas et al.
see [3] and many refs. therein.

Model-independent alternative: How to do?
→ Request by the Fcc-ee physics study group
→ **S-matrix approach** a la SMATASY/ZFITTER
See T.Riemann [4] 2015.

massive

Outline

S-matrix approach to the Z line shape

- Developed as a model-independent analysis tool of $e^+e^- \to (\gamma, Z) \to f^+f^-$ **around the Z boson resonance**
- Aim: determinations of M_Z and Γ_Z
- $\to \sigma_T$: Leike/TR/Rose 1991 [5]
  $\to A_{FB,LR,pol}$: TR 1992 [6],
  $\to$ `SMATASy` code: Kirsch/TR 1994 [7]
- First application: LEP/L3 1993 [8], also: Tristan/TOPAZ,VENUS, LEP/OPAL, . . .

Introduction

boson
The reaction

$$e^+e^- \to (\gamma, Z) \to f^+f^- \; + \; (n\,\gamma) \tag{1}$$

allows to study the Z boson, its mass M_Z, its width Γ_Z, its couplings, and potentially deviations from the Standard Model.

Need correct “model”

See experiences with *constant* and *s-dependent* Z width:

$$\frac{1}{[s - M_Z^2 + iM_Z\,\Gamma_Z(s)]} \quad \textbf{versus} \quad \frac{1}{[s - M_Z^2 + iM_Z\,\Gamma_Z]} \tag{2}$$

To a very good accuracy, it holds: $\Gamma_Z(s) \approx s/M_Z^2 \times \Gamma_Z$

see next slide, → Bardin/Leike/Riemann/Sachwitz 1988 [9]; also: Berends/Burgers/Hollik/v.Neerven 1988 [10]

Need correct unfolding ..

.. of *Realistic Observables* in order to get *Pseudo Observables*. → e.g.: Borrelli/Consoli/Maiani/Sisto

Lesson: The model influences numerical results

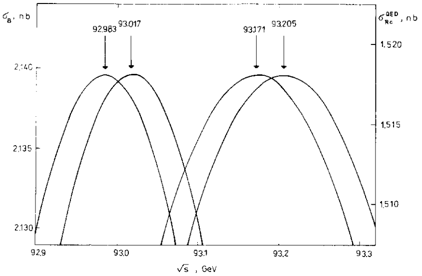


Fig. 1. Total cross sections σ_B, σ_{RC}^{QED} for the reactions $e^+e^- \to \mu^+\mu^-(\gamma)$ in Born approximation (left scale) and including $O(\alpha)$ QED corrections right scale). Peaks of σ with energy-dependent width $\Gamma_Z(s)$ are shifted by 34 MeV to the left.

Total cross section for $e^+e^- \to \mu^+\mu^-$ production at LEP …

…without (left) and with (right) QED corrections. Both sample data produced with an energy-dependent Z width.
The assumptions on the Z-propagator in the fit formulas influence the location of the peak, but not the “experimental errors”.
Fig.: from [9], license Number: 3557090997554.

	Born		Born + QED	
from fit: →	M_Z	Γ_Z	M_Z	Γ_Z
$\Gamma_Z(s)$	93.000 ± 0.013	2.498 ± 0.009	93.000 ± 0.016	2.498 ± 0.011
Γ_Z	92.966 ± 0.013	2.498 ± 0.009	92.966 ± 0.016	2.498 ± 0.011

Introduction

Stuart 1991 [16], S-Matrix ansatz for $e^+e^- \longrightarrow Z \longrightarrow f^+f^-$

$$M = \frac{R}{s - s_0} + F(s), \qquad s_0 = M_Z^2 - iM_Z\Gamma_Z \tag{3}$$

Allows to study:

- Mass M_Z and width Γ_Z → Leike/Riemann/Rose 1991 [5]
- How many independent degrees of freedom? → Leike/Riemann/Rose 1991 [5], Kirsch/S.Riemann, L3 [17, 8]
- But also: How to define mass and width of the Z boson at higher orders of perturbation theory? → Denner 2014 [18], Freitas 2014 [3] and Fcc-ee [2], Degrassi FCC-ee [19] and refs. therein

Total cross sections

There are immediate questions, from an experimental point:

- What about the photon exchange?
- What about QED corrections, e.g. the $2 \to 3$ part of the cross sections?
- What about asymmetries, besides σ_{tot}?

We have to describe

$$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow f^+f^-(\gamma) \tag{4}$$

Ansatz in the complex energy plane, for **four helicity matrix elements**:

$$\mathcal{M}^i(s) = \frac{R^i_\gamma}{s} + \frac{R^i_Z}{s - s_Z} + F^i(s), \quad i = 1, \ldots 4. \tag{5}$$

Beware: Eqn. (5) is mathematically not consistent → Böhm/Sato 2004 [20]

The poles of $\mathcal{M}$ have complex residua R_Z and R_γ, the latter corresponding to the photon, and the background $F(s)$ is an analytic function without poles:

$$F^i(s) = \sum_{n=0}^{\infty} F^i_n (s - s_0)^n \tag{6}$$

Comment on the photon term (3 Feb 2015)

$$
\begin{aligned}
\frac{R_\gamma^i(s)}{s} &= \frac{\sum_{n=0}^{\infty} R_n^i (s-s_0)^n}{s} \\
&= \frac{\sum_{n=0}^{\infty} R_n^i (s-s_0)^n}{s_0-(s_0-s)} \\
&= \sum_{n=0}^{\infty} R_n^i (s-s_0)^n \, \frac{1}{s_0} \, \frac{1}{1-\frac{s_0-s}{s_0}} \\
&= \sum_{n=0}^{\infty} R_n^i (s-s_0)^n \, \frac{1}{s_0} \left[1 + \frac{s_0-s}{s_0} + \left(\frac{s_0-s}{s_0}\right)^2 \cdots \right]
\end{aligned}
\qquad (7)
$$

The term $R_\gamma^i(s)/s$ is part of the the background term $F(s)$.

- *It is useful to sum up a selected part of the photonic background of the Z resonance in order to take explicit notice of physically known pieces of the input expressions.*
- Compare: *It is useful to sum up a selected part of self-energy insertions in the propagators in order to derive the Breit-Wigner resonance form.*

Ansatz for realistic applications

The analysis of the Z line shape will be based here on the cross section

$$\sigma(s) = \sum_{i=1}^{4} \sigma^{i}(s) = \frac{1}{4} \sum_{i=1}^{4} s|\mathcal{M}^{i}(s)|^2, \tag{8}$$

where the sum must be performed over four helicity amplitudes with different residua R_Z^i and functions $F^i(s)$. The result is, with QED corrections folded in:

$$\sigma_T(s) = \frac{4}{3}\pi\alpha^2 \int \frac{ds'}{s} \left[\frac{r^\gamma}{s} + \frac{sR + (s - M_Z^2)J}{(s - {M_Z}^2)^2 + M_Z^2{\Gamma_Z}^2} + \cdots \right] \rho_{ini}\left(\frac{s'}{s}\right). \tag{9}$$

The radiation connected with initial-final state interferences can be taken into account by an analogue formula to (9) with a slightly more complicated structure [21, 22]:

$$\sigma_{\rm int}(s) = \int ds' \sigma(s, s') \rho_{\rm int}(s'/s). \tag{10}$$

ZFITTER

The Standard Model analysis tool for the Z resonance: ZFITTER (D. Bardin et al.)

- Complete electroweak radiative corrections;
- QED corrections by convolution:
  with some $\sigma_0(s')$
  beware: for initial-final state interferences with some $\sigma_0(s, s')$;
- **semi-analytical QED** integrations;
- free choice of $\sigma_0(s')$ by **user interfaces**;
- Standard Model interfaces: four weak form factors $\rho, \kappa_e, \kappa_f, \kappa_{ef}$.

ZFITTER is **well-tested**, **flexible**, **accurate** and **fast**.

References

- ZFITTER has been published in CPC in 1990 [23], 2001 [24], 2006 [25].
- The actual Fortran package is v.6.44; version 6.42 is public in CPC program library, with CPC-licence.
- Beware: **Gfitter/GSM (2007-2011) is an illegal clone of ZFITTER**, available at
  `http://zfitter-gfitter.desy.de/` and
  `http://fh.desy.de/projekte/gfitter01/Gfitter01.htm`.
  See also: `http://zfitter.education`, `http://zfitter.com`.

Some details

See [26]

Born Asymmetries

On a Sunday in Summer 1992, I had a discussion with Luciano Maiani in the CERN library.
He had doubt that, **for asymmetries**, an analogue to the model-independent ansatz for σ_{tot} might be usefully formulated, especially in view of the QED corrections.

I believed one can do that, and I followed the rule "The proof of the pudding is in the eating" [6]. The result:

$$A^{Born}(s) = A_0 + A_1 \left(\frac{s}{M_Z^2} - 1 \right) + A_2 \left(\frac{s}{M_Z^2} - 1 \right)^2 + \ldots \tag{11}$$

$$A_{FB} = \frac{\sigma_{FB}}{\sigma_T}, \qquad A_{pol} = \frac{\sigma_{pol}}{\sigma_T}. \tag{12}$$

The A_{FB} and A_{pol} are helicity combinations as also σ_T is, i.e. have also S-matrix ansatzes.

The parameters in $A^{Born}(s)$ are in [QED-]Born approximation:

$$A_0 = \frac{R_A}{R_T}, \tag{13}$$

and

$$A_1 = \left[\frac{J_A}{R_A} - \frac{J_T}{R_T} \right] A_0. \tag{14}$$

Some details

See [26]

QED corrections for asymmetries

QED corrections to asymmetries lead to few simple correction factors [6, 7]:

$$A_{LR}^{QED}(s) = A_{0,LR}^{Born} + c_{1,T}(s)\, A_{1,LR}^{Born} \left(\frac{s}{M_Z^2} - 1 \right) + \cdots \quad (15)$$

$$A_{FB}^{QED}(s) = c_{0,FB}(s)\, A_{0,FB}^{Born} + c_{1,FB}(s)\, A_{1,FB}^{Born} \left(\frac{s}{M_Z^2} - 1 \right) + \cdots \quad (16)$$

The A_0 and A_1 are constant, and the same as in Born approximation.
The QED corrections are contained in the model-independent factor $C(s)$.

$$c_{0,FB}(s) = \frac{C_{FB}^R}{C_T^R}, \qquad c_{0,T}(s) = 1 \quad (17)$$

$$c_{1,A}(s) = c_{0,A}\, \frac{C_T^J}{C_T^R} \quad (18)$$

Sample QED factor

$$C_{T,FB}^R(s) = \int dk\, \rho_{T,FB}(s'/s)\, \frac{s'R}{sR}\, \frac{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}{(s'-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \quad (19)$$

Scetch of derivation of the expression for $A_{FB}(s)$ with QED corr's

See [26]

QED corrections for asymmetries

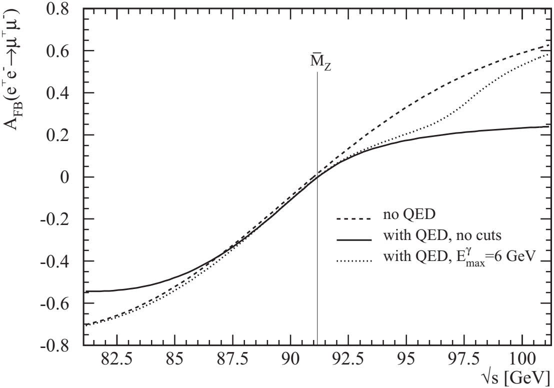


Figure 1 : The forward-backward asymmetry for the process $e^+e^- \to \mu^+\mu^-$ near the Z boson peak. From Kirsch/Riemann 1994 [7], license Number: 3557090997554.

Applications

In Leike/S.Riemann/Riemann 1992 [27] correlations are discussed.

For the Z **peak position** s_{peak}, one may derive the relation:

$$\Delta\sqrt{s_{peak}} = \Delta M_Z + \frac{1}{4}\frac{\Gamma_Z^2}{M_Z}\,\Delta\left(\frac{J_T}{R_T}\right) + \dots \tag{20}$$

between an uncertainty in M_Z and an uncertainty in the γZ interference.
The latter also influences A_1.

Similarly, for a **hypothetical heavy gauge boson** Z', the effects from its virtual exchange transform after a partial fraction decomposition into simple shifts of the γZ interferences [27]:

$$\Delta\left(\frac{J_T}{R_T}\right) = -2\frac{g'^2}{g^2}\,\frac{M_{Z'}^2}{M_{Z'}^2 - M_Z^2}\,\frac{(a_e a_e' + v_e v_e')(a_f a_f' + v_f v_f')}{(a_e^2 + v_e^2)(a_f^2 + v_f^2)}, \tag{21}$$

Correlations: L3 at LEP1, 1993

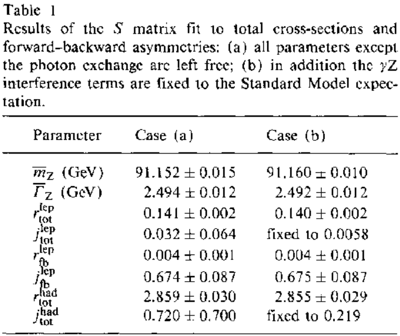


Table 1
Results of the S matrix fit to total cross-sections and forward–backward asymmetries: (a) all parameters except the photon exchange are left free; (b) in addition the γZ interference terms are fixed to the Standard Model expectation.

Parameter	Case (a)	Case (b)
$\overline{m}_Z$ (GeV)	91.152 ± 0.015	91.160 ± 0.010
$\overline{\Gamma}_Z$ (GeV)	2.494 ± 0.012	2.492 ± 0.012
$r_{\rm tot}^{\rm lep}$	0.141 ± 0.002	0.140 ± 0.002
$j_{\rm tot}^{\rm lep}$	0.032 ± 0.064	fixed to 0.0058
$r_{\rm fb}^{\rm lep}$	0.004 ± 0.001	0.004 ± 0.001
$j_{\rm fb}^{\rm lep}$	0.674 ± 0.087	0.675 ± 0.087
$r_{\rm tot}^{\rm had}$	2.859 ± 0.030	2.855 ± 0.029
$j_{\rm tot}^{\rm had}$	0.720 ± 0.700	fixed to 0.219

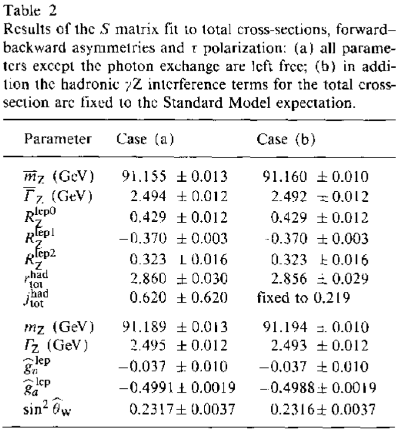


Table 2
Results of the S matrix fit to total cross-sections, forward–backward asymmetries and τ polarization: (a) all parameters except the photon exchange are left free; (b) in addition the hadronic γZ interference terms for the total cross-section are fixed to the Standard Model expectation.

Parameter	Case (a)	Case (b)
$\overline{m}_Z$ (GeV)	91.155 ± 0.013	91.160 ± 0.010
$\overline{\Gamma}_Z$ (GeV)	2.494 ± 0.012	2.492 ± 0.012
$R_Z^{\rm lep0}$	0.429 ± 0.012	0.429 ± 0.012
$R_Z^{\rm lep1}$	-0.370 ± 0.003	-0.370 ± 0.003
$R_Z^{\rm lep2}$	0.323 ± 0.016	0.323 ± 0.016
$r_{\rm tot}^{\rm had}$	2.860 ± 0.030	2.856 ± 0.029
$j_{\rm tot}^{\rm had}$	0.620 ± 0.620	fixed to 0.219
m_Z (GeV)	91.189 ± 0.013	91.194 ± 0.010
Γ_Z (GeV)	2.495 ± 0.012	2.493 ± 0.012
$\widehat{g}_v^{\rm lep}$	-0.037 ± 0.010	-0.037 ± 0.010
$\widehat{g}_a^{\rm lep}$	-0.4991 ± 0.0019	-0.4988 ± 0.0019
$\sin^2 \widehat{\theta}_W$	0.2317 ± 0.0037	0.2316 ± 0.0037

From: L3-Collaboration, "An S matrix analysis of the Z resonance", Phys. Lett. B315 (1993) 494-502.
See also **LEPEWWG et al., Phys. Rept. 2006**, section 2 [28]
and **LEPEWWG et al., Phys. Rept. 2013**, App. A [29]

Correlations: LEP1 + LEP2

A complete analysis of the LEP-2 data in terms of j_{had}^{tot} is lacking.
But see K. Sachs, 2003 [30].

Including more measurements from LEP*II* solves this problem, reducing the correlation. The final result of $M_Z = 91\ 186.9 \pm 2.3$ MeV [8] is in very good agreement with the result of the standard lineshape fit $M_Z = 91\ 187.6 \pm 2.1$ MeV [9] with only slightly increased error.

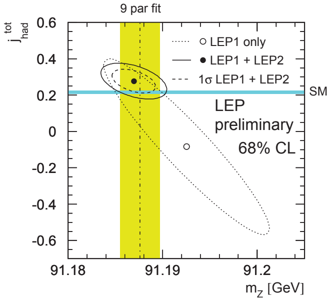
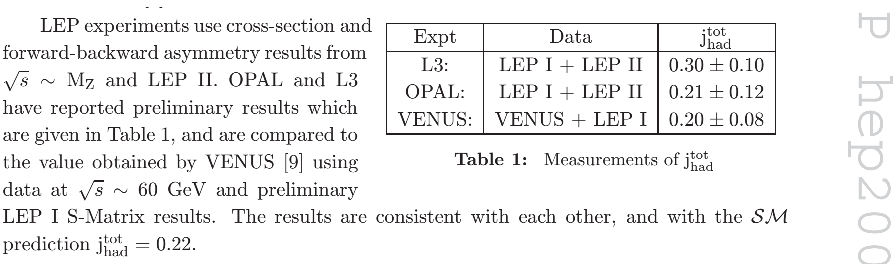


Figure 2: Correlation between the mass of the Z and $j_{\mathrm{had}}^{\mathrm{tot}}$. Results are shown for LEP*I* data only and for a combined fit to LEP*I* and LEP*II* data. The yellow band indicates the 1 σ error from the 9 parameter fit.

Figure 2 : K. Sachs, “Standard model at LEP II”, talk held at Moriond 2001, fig. 2 [30]

Correlations: LEP1 + TRISTAN



LEP experiments use cross-section and forward-backward asymmetry results from $\sqrt{s} \sim \mathrm{M_Z}$ and LEP II. OPAL and L3 have reported preliminary results which are given in Table 1, and are compared to the value obtained by VENUS [9] using data at $\sqrt{s} \sim 60$ GeV and preliminary LEP I S-Matrix results. The results are consistent with each other, and with the $\mathcal{SM}$ prediction $\mathrm{j_{had}^{tot}} = 0.22$.

Expt	Data	$\mathrm{j_{had}^{tot}}$
L3:	LEP I + LEP II	0.30 ± 0.10
OPAL:	LEP I + LEP II	0.21 ± 0.12
VENUS:	VENUS + LEP I	0.20 ± 0.08

Table 1: Measurements of $\mathrm{j_{had}^{tot}}$

Figure 3 : P. Holt, “Fermion pair production above the Z^0 resonance”, talk held at HEP 2001, table 1 [31]

Fortran programs: ZPOLE - ZUSMAT - SMATASY/ZFITTER

ZPOLE – The stand-alone Fortran test package (Leike/Riemann, v.0.5, July 1991) is available on request.
It was used for the numerics of [5].

ZUSMAT . . . the S-Matrix interface of older ZFITTER versions.
ZUSMAT was used for analysing the total cross sections, but could not treat asymmetries.

SMATASY/ZFITTER – With interface package **SMATASY** one has the full functionality of **ZFITTER** corrections [24, 25, 32].

The actual Fortran program for the S-matrix Z line shape approach:

M. Grünewald, S. Kirsch, T. Riemann 1994 [7]
SMATASY v.6.42.01 = SMATA642 (2 June 2005)
available at `https://gruenew.web.cern.ch/gruenew/smatasy.html`

Summary

- The S-matrix approach is absolutely independent of the Standard Model approach.
- The degrees of freedom for σ_{tot} are, at minimum:
  M_Z
  Γ_Z
  R – the residue of the Z resonance, *per scattering channel*
  J – the value of the γZ interference, *per scattering channel*
- So we have at least four degrees of freedom.
  This deserves at least five data points as a function of s.
  See also: M.Grünewald, S.Kirsch 1993 [33].
- Asymmetries may be described as well as σ_{tot}.
- For a exact numerical analysis of data, an accurate description of QED corrections is mandatory.
  This has been realised by combining SMATASY with ZFITTER.
- With so much more statistics at the Fcc-ee compared to LEP-1 and LEP-2:

  The S-matrix approach might gain at the Fcc-ee even more interest as an alternative to the Standard Model approach.

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