Search for Lepton Flavour Violation in Z decays — at TESLA-Giga$Z$

Tord Riemann (DESY)

A DESY/Univ. Granada collaboration together with:

Jose Illana (DESY & Granada U.) & Mark Jack (DESY)

DESY Zeuthen & Granada University

1. Introduction
2. Neutrino generalities
3. General expressions for $Z \to \ell_1 \ell_2$ with massive neutrinos
4. Contributions from *light neutrinos*
5. Contributions from *heavy neutrinos*
6. Conclusions

From Oxford LCWS '99 (I)

change couplings and/or particle content: get similar numbers ...

Neutrino '82:

m_N	m_N^4/M_W^4	Branching
100 eV	2×10^{-36}	$B_Z < 2\times10^{-40}$
20 GeV	3×10^{-3}	$B_Z < 3\times10^{-7}$

$B_Z <$ Since: $\rho_{\mu N} \cdot \rho^{+}_{N\tau}$

This Table demonstrates the impossibility of detecting $\boxed{Z \to \mu + \tau}$ etc.

in the Standard Model and in any model with only light neutrinos, $\boxed{m^2 \ll M_W^2}$

Conclusion:

If one would observe

$\boxed{Z \to \mu + \tau}$

at a rate of $B_Z \gtrsim 10^{-40}$ or so, or even $\gtrsim 10^{-7}$,

this would indicate extremely interesting new physics!

compete with other channels?

models at disposal with e.g. $B_Z > 10^{-10}$?

Ox 7
TR

From Oxford LCWS '99 (II)

Conclusions

(1) Rate for $Z \to \mu\tau,\ \tau e,\ e\mu$:

$$B = \frac{\Gamma(Z \to e_1 + \bar{e}_2)}{\Gamma(Z \to e + e)} \sim \underbrace{\boxed{\left(\frac{\alpha}{\pi}\right)^2}}_{\text{small}} \cdot \left| \sum_i \underbrace{P_{1i}\, P^+_{i2}}_{\text{potentially small}} V(m_i) \right|^2$$

$$B \sim 10^{-4} \cdot |V(m_N) - V(0)|^2 \cdot P_{1N} \cdot P^+_{N2}$$

if only one, ν_N, is heavy

(2) $B \sim \left(\frac{m_N}{M_W}\right)^4$ big? $\qquad V(m_N) \sim \frac{m_N^2}{M_W^2}$

(3) One may imagine that there are models where

$$B \gtrsim 10^{-6} \quad \text{may be}$$

realized in Nature

OX 10 TR

Motivation for this study

Best direct limits at 95% c.l.:

Process	LEP 1 *[PDG '98]*	TESLA Giga–Z *[G. Wilson '98]*
$\mathrm{BR}(Z \to e^{\mp}\mu^{\pm}) <$	1.7×10^{-6}	2×10^{-9}
$\mathrm{BR}(Z \to e^{\mp}\tau^{\pm}) <$	9.8×10^{-6}	$f \times 6.5 \times 10^{-8}$
$\mathrm{BR}(Z \to \mu^{\mp}\tau^{\pm}) <$	1.2×10^{-5}	$f \times 2.2 \times 10^{-8}$

$f = 0.2 \div 1.0$

Assumptions

- Respect known experimental constraints from flavour diagonal and other flavour nondiagonal processes
- Try to get results independent of neutrino mass models

Two scenarios

(I) Observed neutrinos are massive $\Rightarrow$ flavour mixing; what is the size of LFV in $Z \to \ell_1\ell_2$ assuming

1. three light neutrino flavours (νSM)

(II) Adding heavy neutrinos can explain the lightness of the observed ones (seesaw; predictions from GUTs and string-inspired models); what is the effect on LFV/LNV in $Z \to \ell_1\ell_2$, assuming

2. one heavy $\mathrm{SU(2)}_L\otimes\mathrm{U(1)}_Y$ sequential Dirac neutrino
3. one heavy isosinglet Majorana neutrino
4. two heavy isosinglet Majorana neutrinos

Neutrinos in the SM

Facts: Neutral, weakly interacting spin-$\frac{1}{2}$ fermions
There are three light flavours [Z lineshape]
Lepton Number is Conserved [no $(2\beta)_{0\nu}$]

$\Rightarrow$SM: Neutrinos taken massless: economical solution
[end-point of Kurie-plot of β decay, $\nexists\ \nu_\alpha \to \nu_\beta\gamma$]
although no symmetry implies zero mass (unlike γ).
Consequences:
– RH neutrinos are sterile and therefore not needed, since they serve only to build Dirac mass terms
– Majorana mass terms $\overline{\psi_L^c}\psi_L$ ($|\Delta L = 2|$) not possible with one Higgs doublet, even at the loop-level or including nonperturbative effects (instantons), since $B - L$ is a good symmetry.

Neutrinos beyond the SM

New facts: Neutrinos oscillate!

They must be massive and mix. Two possibilities, since they are neutral:

- Neutrinos are Dirac fermions ν
  $\psi \equiv \nu = \nu_L + \nu_R$
  like quarks or charged leptons
- Neutrinos are Majorana fermions $\chi = \eta^* \chi^c$
  $\psi \equiv \chi = \chi_L + \eta \chi_L^c$
  with $\mathcal{CP}$ parity η^*

$\Rightarrow$ SM must be extended

- Extend the Higgs sector: introduce Higgs triplets and allow LNV

– Impact on ρ parameter
– Spontaneous LNV $\Rightarrow$ "triplet Majoron" ruled out by LEP

$\surd$ • Extend the lepton sector with new neutrinos: 3 $\nu_L + \ldots$
(# of $SU(2) \otimes U(1)$ isosinglet ν's not limited by gauge symmetry)

i. νSM: natural extension of the SM
introduce 3 RH isosinglet ν_R to form 3 light Dirac ν's
$\Rightarrow$ LNC, but possible LFV (like in quark sector)

ii. String-inspired models $\Rightarrow$ heavy Dirac isosinglets: LNC

iii. GUT's $\Rightarrow$ heavy RH Majorana isosinglets: LNV

Dirac and Majorana mass terms

$$
\begin{aligned}
-\mathcal{L}_m^\nu &= \overbrace{\overline{\nu_R^0}\, \mathbf{m}_D\, \nu_L^0}^{\text{Dirac}} + \overbrace{\frac{1}{2}\, \overline{\nu_L^{0c}}\, \mathbf{m}_L\, \nu_L^0 + \frac{1}{2}\, \overline{\nu_R^{0c}}\, \mathbf{m}_R\, \nu_R^0}^{\text{Majorana}} + h.c. \\
&= \frac{1}{2}\, \overline{\chi_L^{0c}}\, \mathbf{M}\, \chi_L^0 + h.c. = \frac{1}{2}\, \overline{\chi^0}\, \mathbf{M}\, \chi^0
\end{aligned}
$$

with ν_{Li}^0 $(i = 1, \ldots, n_G)$, ν_{Rj}^0 $(j = 1, \ldots, n_R)$, introducing the Majorana fields $\chi^0 = \chi^{0c}$:

$$
\chi_k^0 = \chi_{L\,k}^0 + \chi_{L\ k}^{0c} \equiv \begin{pmatrix} \nu_L^0 + \nu_L^{0c} \\ \nu_R^{0c} + \nu_R^0 \end{pmatrix}_{k=1,\ldots,n_G+n_R}
$$

and **M** complex, symmetrix, $(n_G + n_R)$-dimensional:

$$
\mathbf{M} \equiv \begin{pmatrix} \mathbf{m}_L & \mathbf{m}_D^T \\ \mathbf{m}_D & \mathbf{m}_R \end{pmatrix}
$$

Diagonalization: $\mathbf{U}^T\, \mathbf{M}\, \mathbf{U} = \hat{\mathbf{M}}$

– Eigenvectors: $\chi_L^0 = \mathbf{U}\, \chi_L$

$\chi_L^{0c} = \mathbf{U}^*\, \chi_L^c$

– Eigenvalues: m_i complex in general

⇒ phases to be absorbed by field rephasing (fully if $\mathcal{CP}$ is conserved)

⇒ physical states: $\xi_i \equiv \chi_{Li} + \eta_i\, \chi_{Li}^c$ with $\mathcal{CP}$ parity η^* namely $\mathbf{U} \to \sqrt{\boldsymbol{\eta}}\, \mathbf{U}$ with $\sqrt{\boldsymbol{\eta}}$=**diag**(η_i)

⇒ physical masses: $|m_i| = \eta_i m_i$

Two Majorana neutrinos

$n_G = n_R = 1$

$$\mathbf{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \qquad \mathbf{U} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$m_{1,2} = \frac{1}{2}\left\{m_R + m_L \mp \sqrt{(m_R - m_L)^2 + 4m_D^2}\right\}$$

$$\tan 2\theta = \frac{2m_D}{m_R - m_L}, \quad \cos 2\theta = \frac{m_R - m_L}{\sqrt{(m_R - m_L)^2 + 4m_D^2}}$$

A. Two Majorana neutrinos: Dirac limit

$$m_L = m_R = 0$$

$$\theta = \frac{\pi}{4} \Rightarrow \begin{cases} m_1 = -m_D < 0 & \rightsquigarrow \quad \eta_1 = -1 \\ m_2 = m_D > 0 & \rightsquigarrow \quad \eta_2 = +1 \end{cases}$$

$$\mathbf{U} \to \mathbf{U}\,\sqrt{\boldsymbol{\eta}} = \mathbf{U} \begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}$$

$$\xi_1 = \chi_{1L} + \eta_1\ \chi_{1L}^c = \frac{1}{\sqrt{2}}[(\nu_R^{0c} - \nu_L^0) + \eta_1(\nu_R^0 - \nu_L^{0c})]$$

$$\xi_2 = \chi_{2L} + \eta_2\ \chi_{2L}^c = \frac{1}{\sqrt{2}}[(\nu_L^0 + \nu_R^{0c}) + \eta_2(\nu_L^{0c} + \nu_R^0)]$$

so that

$$-\mathcal{L}_m^\nu = \frac{m_D}{2}\,(-\overline{\chi_1}\chi_1 + \overline{\chi_2}\chi_2) = \frac{m_D}{2}\,(\overline{\xi_1}\xi_1 + \overline{\xi_2}\xi_2)$$

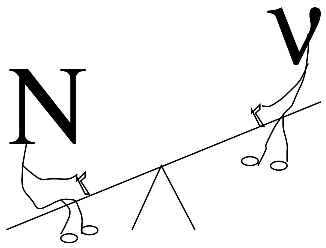
$$= m_D\,(\overline{\nu_R^0}\nu_L^0 + \overline{\nu_L^0}\nu_R^0)$$

$\Rightarrow$ Two Majorana neutrinos of equal mass and opposite $\mathcal{CP}$ parities form a Dirac neutrino

B. Two Majorana neutrinos: seesaw

$$\boldsymbol{m_L = 0,\ m_D \ll m_R}$$

(e.g. Higgs sector consists of Higgs doublets)



Light neutrino ν: Heavy neutrino N:

$$m_\nu \simeq \frac{m_D^2}{m_R} \ll 1 \qquad m_N \simeq m_R \gg m_\nu$$

$$(\eta_1 = -1) \qquad (\eta_2 = +1)$$

Light-heavy mixing:

$$\theta \simeq \frac{m_D}{m_R} \simeq \sqrt{\frac{m_\nu}{m_N}} \ll 1$$

Provides an explanation of the lightness of the known neutrinos:
There is a heavy scale m_R where Lepton Number is violated

Minimal seesaw

[T. Yanagida '79; M. Gell-Mann, P. Ramond, R. Slansky '79; R.N. Mohapatra, G. Senjanovic '80]

$$\left.\begin{array}{l} m_D \sim m_\ell \\ m_R \lesssim m_{\rm GUT} \end{array}\right\} \leadsto m_{\nu_\ell} \simeq \frac{m_\ell^2}{m_N} \sim \text{eV}$$

at the price of a tiny light-heavy mixing $\theta_\ell \simeq \sqrt{\frac{m_{\nu_\ell}}{m_N}}$

$n_G + n_R$ Majorana neutrinos

Interaction Lagrangians in SU(2)$_L\otimes$U(1)$_Y$, with ordinary charged leptons and n_G LH isodoublets and n_R RH isosinglet neutrinos

$$\ell^0_{Li} = \sum_{j=1}^{n_G} \mathbf{U}^{\ell}_{ij}\ \ell_j = (e, \mu, \tau, \ldots)$$

$$\nu^0_{Li} = \sum_{j=1}^{n_G+n_R} \mathbf{U}_{ij}\ \chi_{Lj} = (\nu_e, \nu_\mu, \nu_\tau, \ldots)$$

($n_G \neq n_R$ in general)

Interfamily seesaw

[W. Buchmüller, C. Greub '92; A. Pilaftsis '92]

Several heavy RH neutrinos with general Majorana mass terms $\mathbf{m}_R$ may produce not so small light-heavy mixings

CC and NC Lagrangians: B and C mixings

Interaction Lagrangians in SU(2)$_L \otimes$U(1)$_Y$, with ordinary charged leptons and n_G LH isodoublets and n_R RH isosinglet neutrinos

$$
\begin{aligned}
\ell^0_{Li} &= \sum_{j=1}^{n_G} \mathbf{U}^{\ell}_{ij}\ \ell_j &&= (e, \mu, \tau, \ldots) \\
\nu^0_{Li} &= \sum_{j=1}^{n_G+n_R} \mathbf{U}_{ij}\ \chi_{Lj} &&= (\nu_e, \nu_\mu, \nu_\tau, \ldots)
\end{aligned}
$$

▷ Charge Currents:

$$
\begin{aligned}
-\mathcal{L}_{CC} &= \frac{g}{\sqrt{2}} W_\mu\ \overline{\ell^0_{Li}} \gamma^\mu P_L \nu^0_{Li} + h.c. \\
&= \frac{g}{\sqrt{2}} W_\mu\ \mathbf{B}_{ij}\ \overline{\ell_{Li}} \gamma^\mu P_L \chi_{Lj} + h.c.
\end{aligned}
$$

with $\mathbf{B}_{ij} \equiv \sum_{k=1}^{n_G} \mathbf{U}^{\ell *}_{ki} \mathbf{U}_{kj}$ a rectangular $n_G \times (n_G + n_R)$ matrix

⇒ Lepton Flavour Change due to mixing

Generalization of the $n_G \times n_G$ LCKM matrix $\mathbf{V}_{ij} \equiv \sum_{k=1}^{n_G} \mathbf{U}^{\ell *}_{ki} \mathbf{U}^{\nu}_{kj}$

▷ Neutral Currents:

$$
\begin{aligned}
-\mathcal{L}^Z_{NC} &= \frac{g}{2c_W} Z_\mu\ [\overline{\nu^0_{Li}} \gamma^\mu P_L \nu^0_{L_i} - \overline{\nu^{0c}_{L}}_i \gamma^\mu P_R \nu^{0c}_{L_i}] \\
&= \frac{g}{2c_W} Z_\mu\ \overline{\chi_i}\ [\mathbf{C}_{ij} P_L - \mathbf{C}^*{}_{ij} P_R]\ \chi_j
\end{aligned}
$$

with $\mathbf{C}_{ij} \equiv \sum_{k=1}^{n_G} \mathbf{U}^*_{ij} \mathbf{U}_{kj}$ a quadratic $(n_G + n_R)^2$ matrix

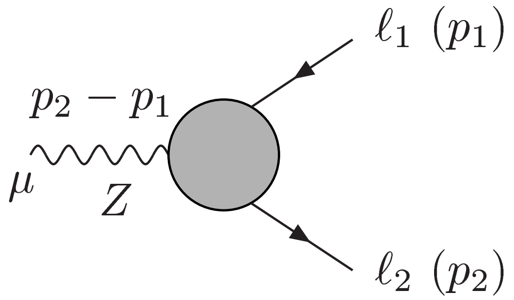
⇒ Flavour Changing Neutral Currents at tree level

⇒ Both L and R chiralities interact with the Z boson

⇒ Lepton Number Violation through $\nu - \nu^c$ mixing

Branching Ratio $Z \to \ell_1 \ell_2$

Born amplitude is zero



To one loop:

vertex + fermion self–energy graphs

$$\mathcal{M} = -\frac{ig\alpha_W}{16\pi c_W} \mathcal{V}(Q^2)\ \varepsilon_Z^\mu\ \bar{u}_{l_2}(p_2)\ \gamma_\mu (1-\gamma_5)\ u_{l_1}(-p_1)$$

$$\alpha_W = \frac{g^2}{4\pi}, \qquad Q^2 = (p_2 - p_1)^2$$

$$\mathrm{BR}(Z \to l_1^\mp l_2^\pm) = \underbrace{\frac{\alpha_W^3 M_Z}{192\pi^2 c_W^2 \Gamma_Z}}_{\approx 10^{-6}} |\mathcal{V}(M_Z^2)|^2$$

Dirac

($i \to n_G$ ν's with isodoublet L and isosinglet R components):

$$\mathcal{V}_D(Q^2) = \sum_{i=1}^{n_G} \mathbf{V}_{l_1 i} \mathbf{V}^*{}_{l_2 i}\ \mathcal{F}_D(i)$$

$$\mathcal{F}_D(i) = v_W(i) + v_\phi(i) + v_{WW}(i) + v_{\phi\phi}(i) + v_{W\phi}(i) + v_\Sigma(i)$$

Majorana

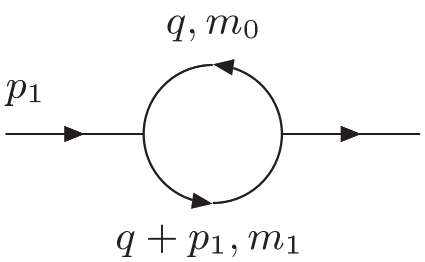
($i \to n_G$ isodoublet ν_L's + n_R isosinglet ν_R's):

$$\mathcal{V}_M(Q^2) = \sum_{i,j=1}^{n_G+n_R} \mathbf{B}_{l_1 i} \mathbf{B}^*{}_{l_2 j}\ \mathcal{F}_M(i,j)$$

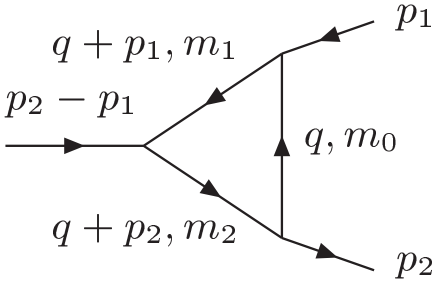
$$\mathcal{F}_M(i,j) = v_W(i,j) + v_\phi(i,j) + v_{WW}(i) + v_{\phi\phi}(i) + v_{W\phi}(i) + v_\Sigma(i)$$

One–loop integrals

[G. Passarino, M. Veltman '79]



$$\frac{i}{16\pi^2}\{B_0, B^\mu\}(p_1^2; m_0^2, m_1^2) = \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{\{1, q^\mu\}}{\mathcal{D}_0 \mathcal{D}_1}$$



$$\frac{i}{16\pi^2}\{C_0, C^\mu, C^{\mu\nu}\}(p_1^2, Q^2, p_2^2; m_0^2, m_1^2, m_2^2) = \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{\{1, q^\mu, q^\mu q^\nu\}}{\mathcal{D}_0 \mathcal{D}_1 \mathcal{D}_2}$$

in D dimensions, with $Q^2 = (p_2 - p_1)^2$ and

$$\begin{aligned}
\mathcal{D}_0 &= q^2 - m_0^2 + i\epsilon \\
\mathcal{D}_1 &= (q + p_1)^2 - m_1^2 + i\epsilon \\
\mathcal{D}_2 &= (q + p_2)^2 - m_2^2 + i\epsilon
\end{aligned}$$

Lorentz structure decomposition:

$$\begin{aligned}
B^\mu &= p^\mu B_1 \\
C^\mu &= p_1^\mu C_{11} + p_2^\mu C_{12} \\
C^{\mu\nu} &= p_1^\mu p_1^\nu C_{21} + p_2^\mu p_2^\nu C_{22} + (p_1^\mu p_2^\nu + p_2^\mu p_1^\nu) C_{23} + g^{\mu\nu} M_W^2 C_{24}
\end{aligned}$$

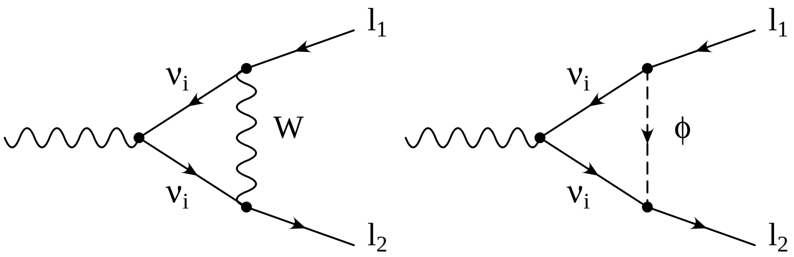
Abbreviations for dimensionless functions: UV–behaviour:

$$\begin{array}{lcl||rcr}
\mathbb{B}_1 & \equiv & B_1(0, Q^2, 0; m_i^2, M_W^2) & (D-4)\ \mathbb{B}_1 & = & 1 \\
\mathbb{C}_{..} & \equiv & M_W^2\ C_{..}(0, Q^2, 0; M_W^2, m_i^2, m_j^2) & (D-4)\ \mathbb{C}_{24} & = & -\frac{1}{2} \\
\bar{\mathbb{C}}_{..} & \equiv & M_W^2\ C_{..}(0, Q^2, 0; m_i^2, M_W^2, M_W^2) & (D-4)\ \bar{\mathbb{C}}_{24} & = & -\frac{1}{2}
\end{array}$$

1–loop contributions to $Z \to \ell_1\ell_2$

('t Hooft-Feynman gauge)

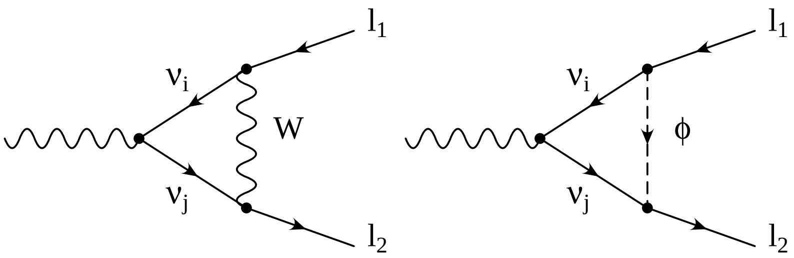
Dirac:



$$
\begin{aligned}
v_W(i) &= -(v_i + a_i)\ \left[\lambda_Q\ (\mathbb{C}_0 + \mathbb{C}_{11} + \mathbb{C}_{12} + \mathbb{C}_{23}) - 2\ \mathbb{C}_{24} + 1\right] \\
&\quad -(v_i - a_i)\ \lambda_i\ \mathbb{C}_0 \\
v_\phi(i) &= -(v_i + a_i)\ \frac{\lambda_i^2}{2}\ \mathbb{C}_0 \\
&\quad -(v_i - a_i)\ \frac{\lambda_i}{2}\ \left[\lambda_Q\ \mathbb{C}_{23} - 2\ \mathbb{C}_{24} + \frac{1}{2}\right]
\end{aligned}
$$

$$
v_i = I_3^{i_L} - 2e_i s_W^2, \quad a_i = I_3^{i_L}, \quad \lambda_i = \frac{m_i^2}{M_W^2}, \quad \lambda_Q = \frac{Q^2}{M_W^2}
$$

Majorana (non–diagonal $Z\nu_i\nu_j$):

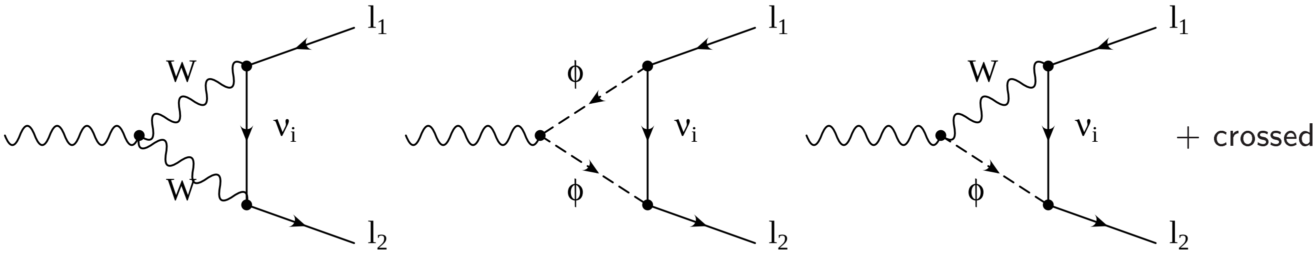


$$
\begin{aligned}
v_W(i,j) &= -\mathbf{C}_{ij}\ \left[\lambda_Q\ (\mathbb{C}_0 + \mathbb{C}_{11} + \mathbb{C}_{12} + \mathbb{C}_{23}) - 2\ \mathbb{C}_{24} + 1\right] \\
&\quad +\mathbf{C}_{ij}^*\ \sqrt{\lambda_i\lambda_j}\ \mathbb{C}_0 \\
v_\phi(i,j) &= -\mathbf{C}_{ij}\ \frac{\lambda_i\lambda_j}{2}\ \mathbb{C}_0 \\
&\quad +\mathbf{C}_{ij}^*\frac{\sqrt{\lambda_i\lambda_j}}{2}\ \left[\lambda_Q\ \mathbb{C}_{23} - 2\ \mathbb{C}_{24} + \frac{1}{2}\right]
\end{aligned}
$$

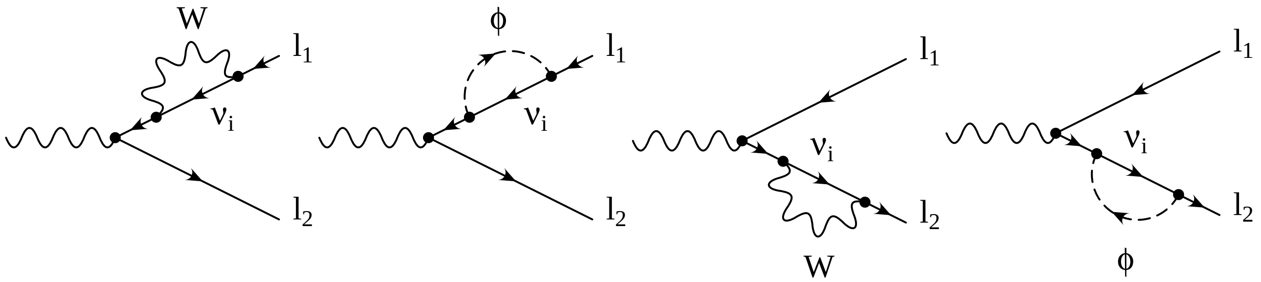
More 1–loop contributions to $Z \to \ell_1\ell_2$

('t Hooft-Feynman gauge)

Dirac or Majorana:



$$
\begin{aligned}
v_{WW}(i) &= 2c_W^2\ (2I_3^{i_L})\left[\lambda_Q\ (\bar{\mathbb{C}}_{11}+\bar{\mathbb{C}}_{12}+\bar{\mathbb{C}}_{23})-6\ \bar{\mathbb{C}}_{24}+1\right]\\
v_{\phi\phi}(i) &= -(1-2s_W^2)\ (2I_3^{i_L})\ \lambda_i\ \bar{\mathbb{C}}_{24},\\
v_{W\phi}(i) &= -2s_W^2\ (2I_3^{i_L})\ \lambda_i\ \bar{\mathbb{C}}_0
\end{aligned}
$$



$$
v_\Sigma(i) = \frac{1}{2}\underbrace{(v_i+a_i-4c_W^2a_i)}_{v_l+a_l}\left[(2+\lambda_i)\mathbb{B}_1+1\right]
$$

The Majorana form factor can be cast into

$$
\mathcal{F}_M(i,j)=\delta_{ij}\ F(\lambda_i)+\mathbf{C}_{ij}\ G(\lambda_i,\lambda_j)+\mathbf{C}_{ij}^*\sqrt{\lambda_i\lambda_j}\ H(\lambda_i,\lambda_j)
$$

The Dirac form factor reads

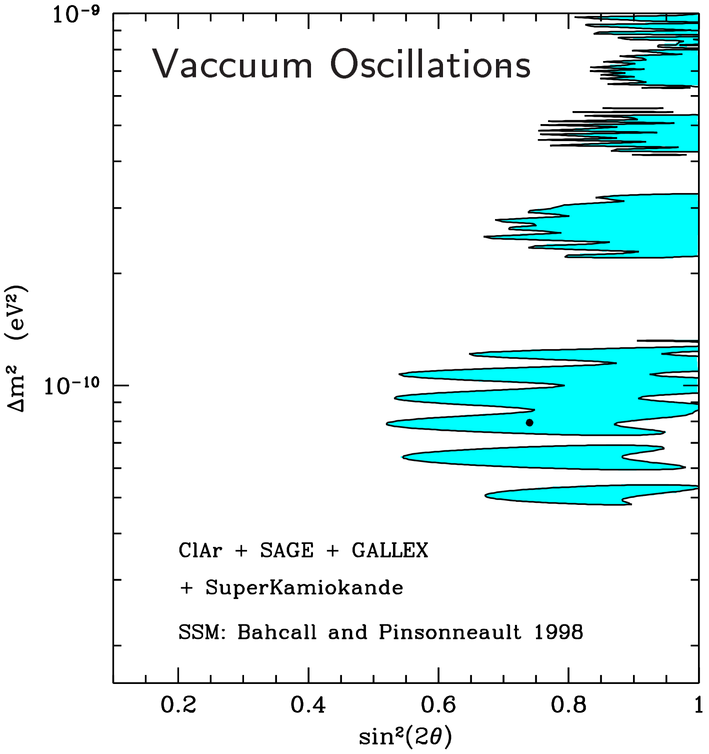
$$
\mathcal{F}_D(i)=F(\lambda_i)+G(\lambda_i,\lambda_i)
$$

Loop integrals numerically evaluated with `LoopTools`

[T. Hahn '99]

Solutions of neutrino puzzles assuming oscillations

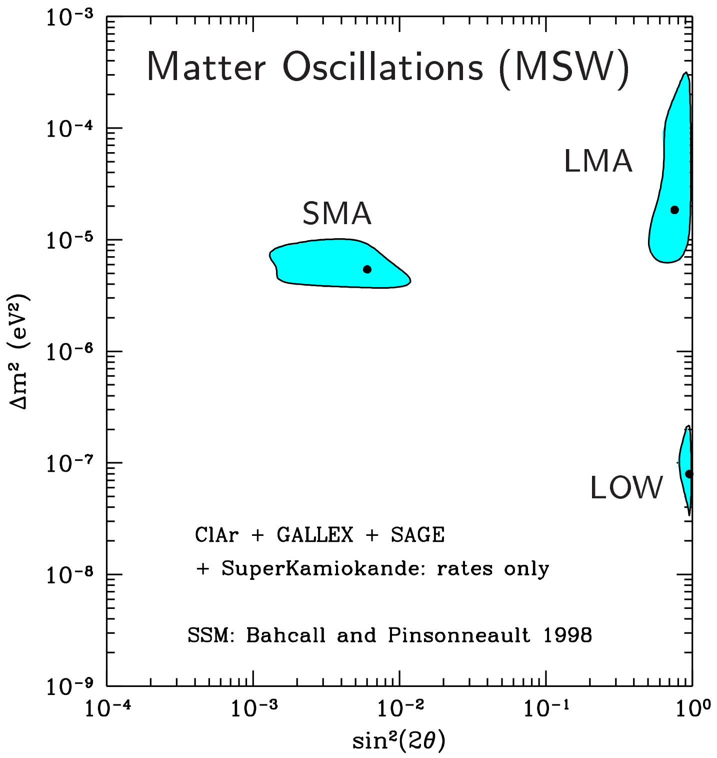
▷ Solar [Homestake (ClAr), SAGE, GALLEX, SuperK]:



VO:

$\Delta m^2_\odot \simeq (0.5 - 1.0) \times 10^{-10}$ eV^2

$\sin^2 2\theta_\odot \simeq 0.6 - 1.0$



SMA:

$\Delta m^2_\odot \simeq (4 - 10) \times 10^{-6}$ eV^2

$\sin^2 2\theta_\odot \simeq (0.1 - 1.0) \times 10^{-2}$

LMA:

$\Delta m^2_\odot \simeq (2 - 20) \times 10^{-5}$ eV^2

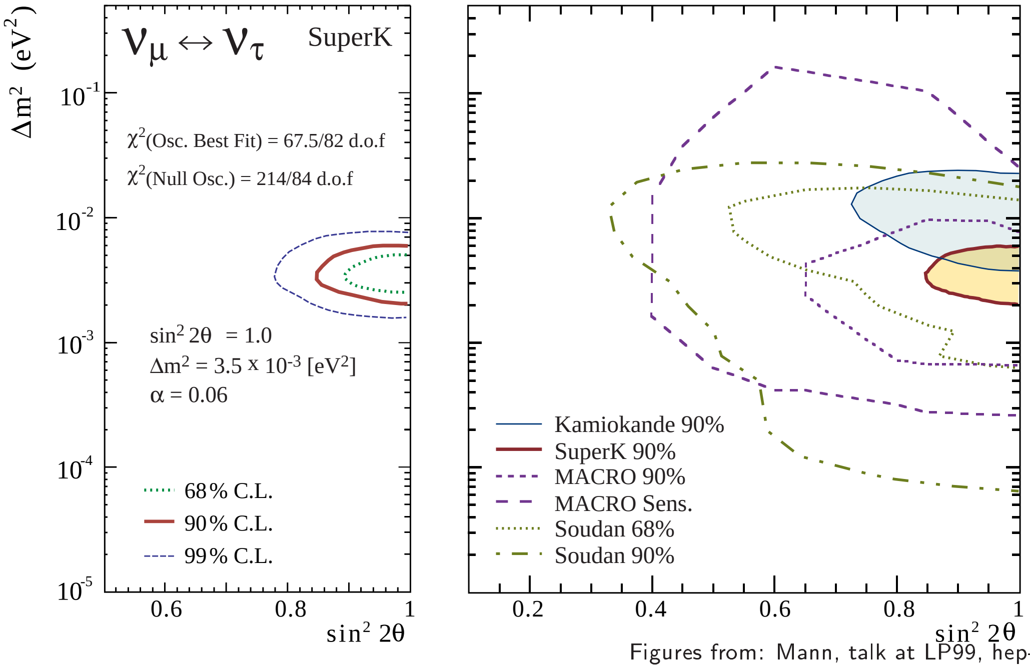
$\sin^2 2\theta_\odot \simeq 0.65 - 0.97$

LOW: very low probability

Figures from: Bahcall et al., PR D58 (1998) 096016

Solutions of neutrino puzzles assuming oscillations

▷ Atmospheric [Kamiokande, SuperK, MACRO, Soudan]:



Figures from: Mann, talk at LP99, hep-ex/9912007

$$\Rightarrow \Delta m^2_{\rm atm} \simeq (2-6)\times 10^{-3}\ \text{eV}^2\ , \quad \sin^2 2\theta_{\rm atm} \simeq 0.82 - 1.0$$

▷ Reactor [CHOOZ]: No signs of $\bar{\nu}_e \to \nu_X$

$$\Rightarrow \Delta m^2 > 7\times 10^{-4}\ \text{eV}^2 \text{ at } \sin^2 2\theta = 1 \text{ (90\% c.l.)}$$

▷ Accelerator:

[LSND]: Signal of $\nu_\mu \to \nu_e$:

$$\Rightarrow 0.2 \lesssim \Delta m^2_{\rm LSND} \lesssim 2\ \text{eV}^2, \quad 2\times 10^{-3} \lesssim \sin^2 2\theta_{\rm LSND} \lesssim 4\times 10^{-2}$$

[KARMEN]: No signal of $\bar{\nu}_\mu \to \bar{\nu}_e$, partly incompatible with LSND

Conclusion

Ignoring LSND, all experiments compatible with 3 light neutrinos:

$$\Delta m^2_{\rm atm} \left\{ \begin{array}{c} m_3^2 \\ \hline \\ m_2^2 \\ \hline \hline \\ m_1^2 \end{array} \right. \quad \} \ \Delta m^2_\odot$$

$$\Delta m^2_{31} = \Delta m^2_{32} \equiv \Delta m^2_{\rm atm}$$
$$\Delta m^2_{21} \equiv \Delta m^2_\odot \ll \Delta m^2_{\rm atm}$$

Note: Absolute neutrino mass scale bound only by ^3H β decay (< 3 eV)

Phenomenological mixing matrix for 3 light neutrinos

$$
U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times
$$

$$
\times \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}
$$

[S. Maki, M. Nakagawa, S. Sakata '62]

where

- $\theta_{12} \equiv \theta_{\odot}$ free [VO, LMA $\rightsquigarrow$ large; SMA $\rightsquigarrow$ small]
- $\theta_{23} \equiv \theta_{\rm atm} \simeq 45^\circ$
- $\theta_{13} \simeq 0$ [CHOOZ]

Oscillations are not sensitive to Majorana-type $\mathcal{CP}$-phases α, β and no information yet on Dirac-type $\mathcal{CP}$-phase δ: we take

- $\alpha = \beta = \delta = 0$

Conclusion:

$$
U \simeq \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}s_{12} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \end{pmatrix}
$$

Results for 3 light neutrinos

The LCKM matrix is $\mathbf{V} = \mathbf{U}$. From its unitarity:

$$\mathrm{BR}(Z \to \ell_1^{\mp}\ell_2^{\pm}) = \frac{\alpha_W^3 M_Z}{192\pi^2 c_W^2 \Gamma_Z} \left| \sum_{i=1}^{3} \mathbf{V}_{\ell_1 i}\mathbf{V}^*_{\ell_2 i}\, \mathcal{F}_D(i)\right|^2$$

$$\simeq 10^{-6} \times \left| \sum_{i=1}^{3} \mathbf{V}_{\ell_1 i}\mathbf{V}^*_{\ell_2 i} [V(\lambda_i) - V(0)] \right|^2$$

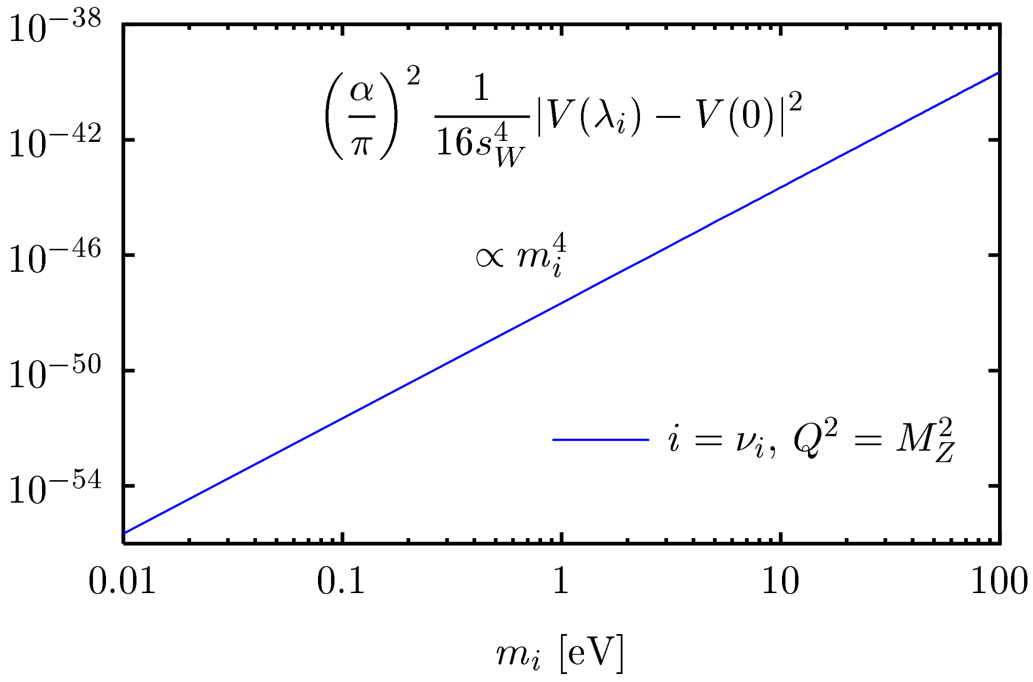
with $V(\lambda_i) - V(0) = a_1(\lambda_Z)\ \lambda_i, \quad |a_1^2| = 11.832$ for $m_i \ll M_Z$

[G. Mann, T. Riemann '82]

Substituting phenomenological mixings θ_{ij} and $\lambda_{ij} = \dfrac{\Delta m^2_{ij}}{M_W^2}$,

$$\begin{aligned} \mathrm{BR}(Z \to e^{\mp}\mu^{\pm}) &\approx \mathrm{BR}(Z \to e^{\mp}\tau^{\pm}) \approx 0.6 \times 10^{-5} c_{12}^2 s_{12}^2 \lambda_{21}^2 \\ &\lesssim 4 \times 10^{-60} \\ \mathrm{BR}(Z \to \mu^{\mp}\tau^{\pm}) &\approx 0.3 \times 10^{-5} |s_{12}^2 \lambda_{21} - \lambda_{32}|^2 \\ &\lesssim (3 \div 30) \times 10^{-55} \end{aligned}$$

[J. Illana, M. Jack, T.R. DESY 99-165]



Expressions in terms of the heavy neutrino masses and the light-heavy mixings only

Contribution to LFV Z decays from n_G light neutrinos negligible, as if they were massless, allows to write the BR's in terms of the heavy masses and light-heavy mixings only:

- ▷ One heavy sequential Dirac neutrino

  From unitarity of LCKM **V** for $n_G + 1$ generations

$$\mathrm{BR}(Z \to \ell_1^{\mp} \ell_2^{\pm}) = \frac{\alpha_W^3 M_Z}{192\pi^2 c_W^2 \Gamma_Z} \left|\mathbf{V}_{\ell_1 N} \mathbf{V}^*_{\ell_2 N}\right|^2 \; |V(\lambda_N) - V(0)|^2$$

- ▷ n_R heavy RH Majorana isosinglet neutrinos

  From unitarity of $\mathbf{U}^{\ell}$ and **U** one gets useful relations for products of **B** and **C** matrix elements

  [J.G. Körner et al. '93]

  Reminding that

$$\mathcal{F}_M(i,j) \equiv \delta_{ij} \; F(\lambda_i) + \mathbf{C}_{ij} \; G(\lambda_i, \lambda_j) + \mathbf{C}^*_{ij} \sqrt{\lambda_i \lambda_j} \; H(\lambda_i, \lambda_j)$$

  one obtains

$$\begin{aligned}\mathrm{BR}(Z \to \ell_1^{\mp} \ell_2^{\pm}) = \frac{\alpha_W^3 M_Z}{192\pi^2 c_W^2 \Gamma_Z} \Bigg| \sum_{i,j=1}^{n_R} & \mathbf{B}_{l_1 N_i} \mathbf{B}^*_{l_2 N_j} \\ \times \Big\{ & \delta_{N_i N_j} [F(\lambda_{N_i}) - F(0) + G(\lambda_{N_i}, 0) + G(0, \lambda_{N_i}) - 2G(0,0)] \\ & + \mathbf{C}_{N_i N_j} [G(\lambda_{N_i}, \lambda_{N_j}) - G(\lambda_{N_i}, 0) - G(0, \lambda_{N_j}) + G(0,0)] \\ & + \mathbf{C}^*_{N_i N_j} \sqrt{\lambda_{N_i} \lambda_{N_j}} \; H(\lambda_{N_i}, \lambda_{N_j}) \Big\} \Bigg|^2\end{aligned}$$

Present experimental constraints on light-heavy mixings

General formalism *[P. Langacker, D. London '88]* to describe

light-heavy mixings $\equiv s^2_{\nu_\ell}$

$\ell = e, \mu, \tau$. They are best constrained by

▷ Flavour diagonal processes (indirect)

√ Neglecting heavy neutrinos in the loops but taking into account the change induced in the light neutrino couplings:

- Nonuniversality of CC couplings
- Deviation of the quark CKM from unitarity
- Change of the invisible width of the Z boson

[E. Nardi et al. '92 & '94; S. Bergmann, A. Kagan '99]

$$\boxed{s^2_{\nu_e} < 0.012, \qquad s^2_{\nu_\mu} < 0.0096, \qquad s^2_{\nu_\tau} < 0.016}$$

- Including heavy neutrino effects in loops:
  - μ decay and Δr *[P. Kalyniak et al. '95]*
  - $Z \to \ell\bar{\ell}$ *[P. Kalyniak et al. '97]*

▷ Flavour nondiagonal processes, no neutrinos in the final state. They are direct and unambiguous signal for LFV

They proceed only through loops

√ $\mu \to e\gamma$, $\tau \to e\gamma/\mu\gamma$ *[P. Langacker, D. London '88]*

$$\boxed{s_{\nu_e} s_{\nu_\mu} < 1.4 \times 10^{-8}}$$ *[MEGA collab '99]*

√ $\mu \to e$ in nuclei and $\mu \to ee^+e^-$ *[J. Bernabeu et al. '95]*

– ...

√ $Z \to \ell_1\bar{\ell}_2$ *[A. Ilakovac, A. Pilaftsis '95; X.Y. Pham '98]* *[J. Illana, M. Jack, T.R. DESY 99-165]*

Information from light-heavy mixing constraints

√ One heavy sequential Dirac

$$s_{\nu_\ell}^2 \equiv |\mathbf{V}_{\ell N}|^2$$

∘ One heavy isosinglet Majorana: $n_R = 1$ (minimal seesaw)

$$s_{\nu_\ell}^2 \equiv |\mathbf{B}_{\ell N}|^2$$

$$\mathbf{B}_{\ell N} = s_{\nu_\ell} \simeq \sqrt{\frac{m_{\nu_\ell}}{m_N}} \ll \text{ present indirect bounds}$$

$$\mathbf{C}_{NN} = \sum_\ell s_{\nu_\ell}^2 \simeq \sum_\ell \frac{m_{\nu_\ell}}{m_N}$$

√ Two heavy isosinglet Majorana: $n_R = 2$ (interfamily seesaw)

$$s_{\nu_\ell}^2 \equiv \sum_{i=1}^{2} |\mathbf{B}_{\ell N_i}|^2$$

Exactly calculable in terms of $s_{\nu_\ell}^2$ and $r \equiv m_{N_2}^2/m_{N_1}^2$

$$\mathbf{B}_{lN_1} = \frac{r^{1/4}}{\sqrt{1+r^{1/2}}}\, s_{\nu_\ell}$$

$$\mathbf{B}_{l_k N_2} = \frac{i}{\sqrt{1+r^{1/2}}}\, s_{\nu_\ell}$$

$$\mathbf{C}_{N_1 N_1} = \frac{r^{1/2}}{1+r^{1/2}} \sum_\ell s_{\nu_\ell}^2$$

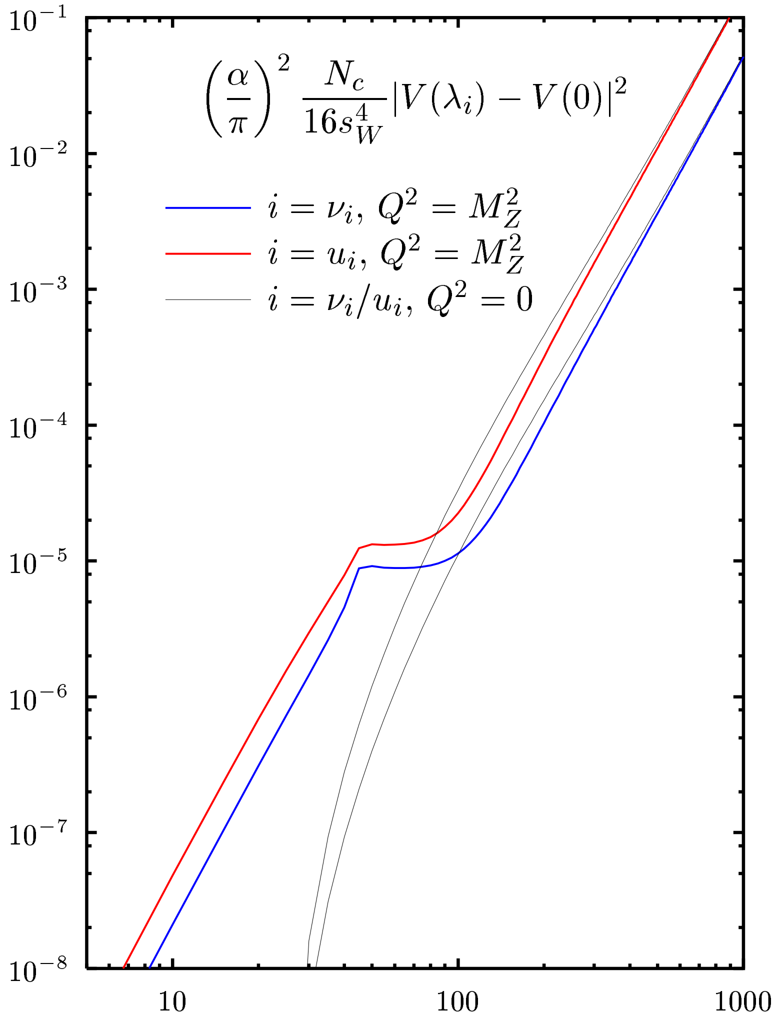
$$\mathbf{C}_{N_2 N_2} = \frac{1}{1+r^{1/2}} \sum_\ell s_{\nu_\ell}^2$$

$$\mathbf{C}_{N_1 N_2} = -\mathbf{C}_{N_2 N_1} = \frac{i r^{1/4}}{1+r^{1/2}} \sum_\ell s_{\nu_\ell}^2$$

Results: sequential Dirac case

- One heavy sequential Dirac

[light-heavy mixing set to one and showing
$\mathrm{B}_Z \equiv \frac{1}{2} \times \mathrm{BR}(Z \to \ell^{\mp}\ell^{\pm})/\mathrm{BR}(Z \to \bar{\nu}\nu)$



[G. Mann, T. Riemann '82]
[J. Illana, M. Jack, T.R. DESY 99-165]

For $\lambda_N = m_N^2/M_W^2 \gg 1$

$$
\begin{aligned}
|V(\lambda_N) - V(0)|^2 &= \frac{1}{4}\,\lambda_N^2 + 1.42\,\lambda_N \ln\lambda_N - 3.50\,\lambda_N \\
&+ 2.03\,\ln^2\lambda_N + \mathcal{O}(\ln\lambda_N).
\end{aligned}
$$

Results: RH singlets added

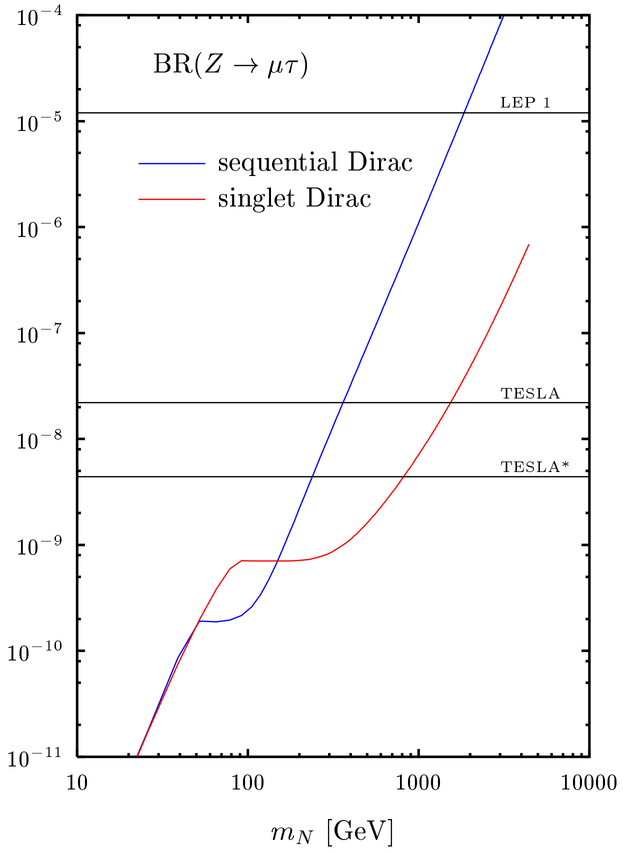
Taking present upper bounds for light-heavy mixings
and showing upper perturbative unitarity mass limits for heavy
Dirac (Majorana) neutrino masses:

$$\Gamma_{N_i} \simeq (2\times)\frac{\alpha_W}{8M_W^2} m_{N_i}^3 \sum_{k=1}^{3} |\mathbf{B}_{l_k N_i}|^2 \leq \frac{1}{2} m_{N_i}$$

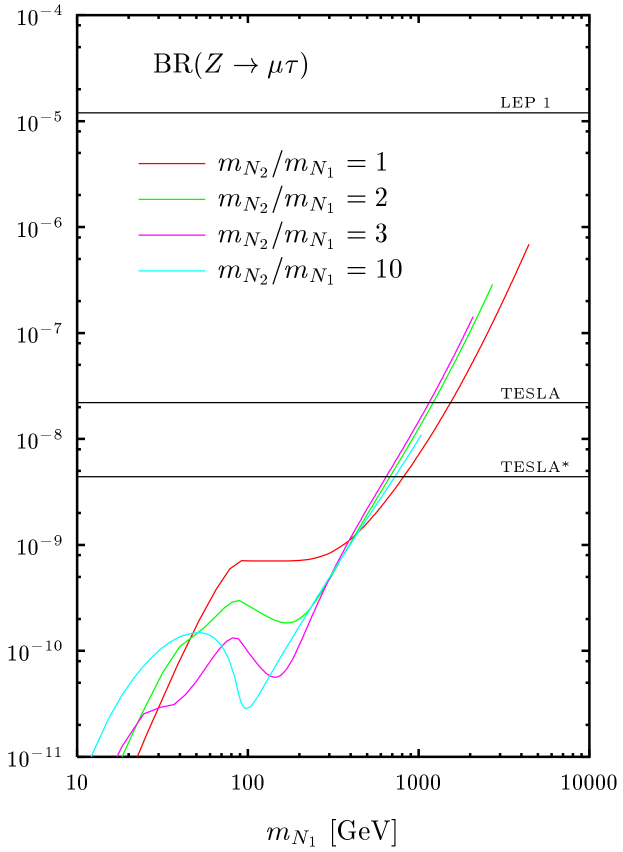
leaves the allowed mass ranges *below* the curves:

One sequential Dirac

One singlet Dirac



Two true Majoranas



[J. Illana, M. Jack, T.R. DESY 99-165]

Conclusions on $Z \to \ell_a \ell_b$ decays $(a \neq b)$

- Not clear yet whether neutrinos are Dirac or Majorana particles (oscillations are insensitive and no signal from neutrinoless 2β)
- For $Z \to \ell_a \ell_b$ decays with only light neutrinos: negligible effects, BR $\lesssim \mathcal{O}(10^{-55})$
- For $Z \to \ell_a \ell_b$ decays with additional heavy neutrinos:
  - Flavour diagonal processes set indirect best bounds on the light-heavy mixings
    $$s_{\nu_\ell}^2 \equiv \begin{cases} \sum\limits_{\text{heavy } N} |\mathbf{V}_{\ell N}|^2 & \text{for Dirac} \\ \sum\limits_{\text{heavy } N} |\mathbf{B}_{\ell N}|^2 & \text{for Majorana} \end{cases} \sim \mathcal{O}(10^{-2})$$
    only improved by direct $\mu \to e\gamma \Rightarrow s_{\nu_e} s_{\nu_\mu} < 1.4 \times 10^{-8}$!!
    They are all (practically) mass independent
  - Relevant mixings are bound by the Schwartz ineq. and are exactly obtainable in cases considered here:
    $$|\sum_N \mathbf{B}_{l_a N} \mathbf{B}^*_{l_b N}|^2 < s_{\nu_a}^2 s_{\nu_b}^2$$
  - There is an interplay between heavy neutrino masses and light-heavy mixings, effectively preventing non-decoupling. One can obtain upper bounds to BR$(Z \to \ell_1 \ell_2)$. With present constraints, for RH singlet cases e.g.:
    BR $\lesssim \mathcal{O}(10^{-6})$ (thus below the LEP range)

$$\boxed{\text{TESLA Giga}-Z \text{ might have a chance!}}$$