Complete $\mathcal{O}(\alpha)$ massive corrections to Bhabha scattering; automatization with aİTALC

1st ECFA–Study WORKSHOP

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Work done in collaboration with

TORD RIEMANN

Contents

Motivation

Why Bhabha again?

- **No physical need to study massive Bhabha**

but . . .

1. **Untested Higgs and "its unphysical sector" ϕ.**
2. **Last piece with s-t crossing symmetry in $2 \to 2$ fermions.**
3. **Comparison precision requires m_e for $>$ 10 digits.**
   (Available numerical cross-checking with other codes)
4. **To test our *new* tool:** aİȚALC

Motivation: Bhabha Physics

Process: $e^+e^- \to e^+e^-(\gamma)$

- **Highest single channel production due to elastic γ exchange**
- **Bhabha scattering is used for monitoring luminosity spectrum**

  Low angle at high energies machines (LEP, LC)

  Large angle for e^+e^- flavour factories

- **Some *New Physics* may show up indirectly in such process:**

  e.g. virtual graviton exchange, extra dimensions, contact interactions, ...

  Essential to analyze future deviations with SM predictions

  $\longrightarrow$ Need lower theoretical uncertainties for next machines

Motivation: Calculations

An incomplete historical summary . . .

Who?	When?	What?
Bhabha	1935	Tree–level QED
Readhead	1953	Leading 1–loop QED
Berends, Gaemers and Gastmans	1974	Complete 1–loop QED with hard γ
Consoli	1979	Complete 1–loop Electro–weak
Böehm, Denner, Hollik and Sommer	1984	Numerical 1–loop Electro–weak
Bardin, Hollik and Riemann	1991	Leading weak 2–loop at Z-resonance
Jadach and others	90's	Precise Monte Carlo
Bern, Dixon and Ghinculov	2000	2–loops QED
Fleischer et al.	2002	$(1\text{–loop})^2$ massive QED

Tree–level example: Observables

θ = **Scattering angle between both e^-,** $\qquad s = E_{\rm CM}^2$

Main Observable: Differential Cross Section

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{32\pi}\frac{1}{s}\Theta(s-4m_e^2)\sum_{\rm conf}|\mathcal{A}(\cos\theta)|^2$$

$$= \text{Energy}^{-2} \times \text{Threshold} \times \text{Amplitude}^2$$

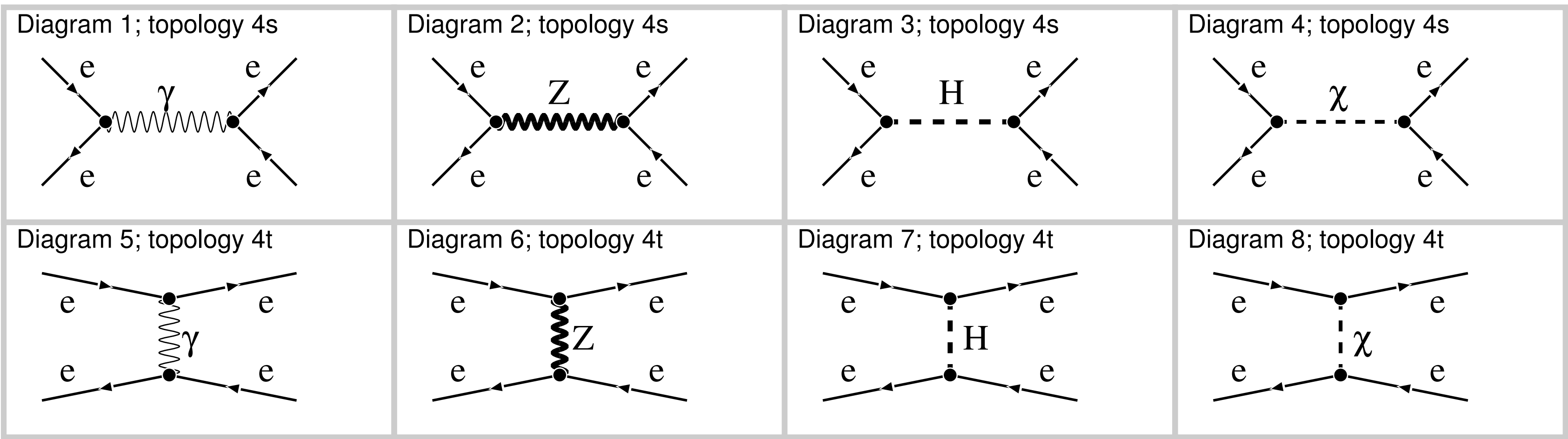
Other Observables:

- Total Cross Section
- Forward–Backward Asymmetry

Tree–level example: Diagrams

8 Feynman Diagrams …

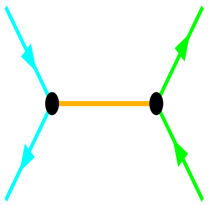
Vector **Scalar**



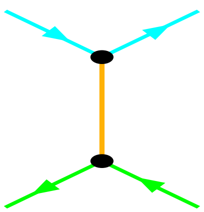
$$|\mathcal{A}|^2 = \mathcal{A}_s^* \mathcal{A}_s + \mathcal{A}_t^* \mathcal{A}_s + \mathcal{A}_s^* \mathcal{A}_t + \mathcal{A}_t^* \mathcal{A}_t$$

Tree–level example: Amplitude

…with their Feynman Rules



$$\begin{aligned}\mathcal{A}_s^{(0)} =\ & \bar{v}_e \; ie\gamma^\mu Q_e \; u_e \frac{-ig_{\mu\nu}}{s+i\varepsilon} \bar{u}_e \; ie\gamma^\nu Q_e \; v_e \\ & + \bar{v}_e ie\gamma^\mu (V_e - A_e\gamma_5) u_e \frac{-ig_{\mu\nu}}{s-m_Z^2+i\varepsilon} \bar{u}_e ie\gamma^\nu (V_e - A_e\gamma_5) v_e \\ & + \bar{v}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W} \; u_e \frac{i}{s-m_H^2+i\varepsilon} \bar{u}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W} \; v_e \\ & + \bar{v}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W}\gamma_5 \; u_e \frac{i}{s-m_Z^2+i\varepsilon} \bar{u}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W}\gamma_5 \; v_e\end{aligned}$$



$$\begin{aligned}\mathcal{A}_t^{(0)} =\ & \bar{u}_e \; ie\gamma^\mu Q_e \; u_e \frac{-ig_{\mu\nu}}{t+i\varepsilon} \bar{v}_e \; ie\gamma^\nu Q_e \; v_e \\ & + \bar{u}_e ie\gamma^\mu (V_e - A_e\gamma_5) u_e \frac{-ig_{\mu\nu}}{t-m_Z^2+i\varepsilon} \bar{v}_e ie\gamma^\nu (V_e - A_e\gamma_5) v_e \\ & + \bar{u}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W} \; u_e \frac{i}{t-m_H^2+i\varepsilon} \bar{v}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W} \; v_e \\ & + \bar{u}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W}\gamma_5 \; u_e \frac{i}{t-m_Z^2+i\varepsilon} \bar{v}_e \; -i\frac{1}{2s_W}\frac{m_e}{m_W}\gamma_5 \; v_e\end{aligned}$$

Coupling Propag. Coupling

1–Loop stuff: Introductory

Going to next perturbation order … 1-Loop

$$|\mathcal{A}|^2 = |\mathcal{A}^{(0)} + \mathcal{A}^{(1)} + \ldots|^2$$
$$= \underbrace{\mathcal{A}^{(0)}\mathcal{A}^{(0)\,*}}_{\mathrm{L}\mathcal{O}(\alpha^2)} + \underbrace{\mathcal{A}^{(0)}\mathcal{A}^{(1)\,*} + \mathcal{A}^{(1)}\mathcal{A}^{(0)\,*}}_{\mathrm{NL}\mathcal{O}(\alpha^3)} + \ldots$$

- **More complicated Kinematic structure** `(Matrix Elements)`
- **Much more complicated Dynamic interactions** `(Form Factors)`

$$\mathcal{A} = \sum_{i}^{\substack{\text{complete}\\ \text{set}}} \mathbf{M}_i \left(\mathbf{F}_i^{(0)} + \mathbf{F}_i^{(1)} + \ldots\right)$$

1–Loop stuff: Matrix Elements Definition

We use 9×4 elements for each channel basis $(s,t) = 72$
Example for the fermionic $s-$channel.

$$
\begin{array}{lllllllll}
\hline
\mathbf{M_s}_{\;1,j} & = & \bar{v}_e & \{\mathbb{1},\gamma_5\}_j & u_e & \otimes & \bar{u}_e & \{\mathbb{1},\gamma_5\}_j & v_e \\
\mathbf{M_s}_{\;2,j} & = & \bar{v}_e & \not{p}_a\{\mathbb{1},\gamma_5\}_j & u_e & \otimes & \bar{u}_e & \{\mathbb{1},\gamma_5\}_j & v_e \\
\mathbf{M_s}_{\;3,j} & = & \bar{v}_e & \{\mathbb{1},\gamma_5\}_j & u_e & \otimes & \bar{u}_e & \not{p}_b\{\mathbb{1},\gamma_5\}_j & v_e \\
\mathbf{M_s}_{\;4,j} & = & \bar{v}_e & \not{p}_a\{\mathbb{1},\gamma_5\}_j & u_e & \otimes & \bar{u}_e & \not{p}_b\{\mathbb{1},\gamma_5\}_j & v_e \\
\hline
\mathbf{M_s}_{\;5,j} & = & \bar{v}_e & \gamma^\mu\{\mathbb{1},\gamma_5\}_j & u_e & \otimes & \bar{u}_e & \gamma_\mu\{\mathbb{1},\gamma_5\}_j & v_e \\
\mathbf{M_s}_{\;6,j} & = & \bar{v}_e & \gamma^\mu\not{p}_a\{\mathbb{1},\gamma_5\}_j & u_e & \otimes & \bar{u}_e & \gamma_\mu\{\mathbb{1},\gamma_5\}_j & v_e \\
\mathbf{M_s}_{\;7,j} & = & \bar{v}_e & \gamma^\mu\{\mathbb{1},\gamma_5\}_j & u_e & \otimes & \bar{u}_e & \gamma_\mu\not{p}_b\{\mathbb{1},\gamma_5\}_j & v_e \\
\mathbf{M_s}_{\;8,j} & = & \bar{v}_e & \gamma^\mu\not{p}_a\{\mathbb{1},\gamma_5\}_j & u_e & \otimes & \bar{u}_e & \gamma_\mu\not{p}_b\{\mathbb{1},\gamma_5\}_j & v_e \\
\hline
\mathbf{M_s}_{\;9,j} & = & \bar{v}_e & \gamma^\mu\gamma^\nu\{\mathbb{1},\gamma_5\}_j & u_e & \otimes & \bar{u}_e & \gamma_\mu\gamma_\nu\{\mathbb{1},\gamma_5\}_j & v_e \\
\end{array}
$$

p_a **and** p_b **are external momenta.** $j = 11, 15, 51, 55$

1–Loop stuff: Calculation

At 1–loop in the SM[a] we have:

15 Topologies and 804 Diagrams! $(148 = 0)$

- ☻ **No tadpoles (renorm. conditions)**
- ☻ **No external legs loops (field ren.)**

} **Precomputed s.e. for ren. parameters**

- ☺ **Automatically generated counter-terms diagrams**
- ☻ **No tensor integral reduction (numerical evaluation)**
- ✓ **Code is UltraViolet and InfraRed finite**

All in 7'15" with Pentium IV at 2.4 GHz (and 2 years Ph.D.)

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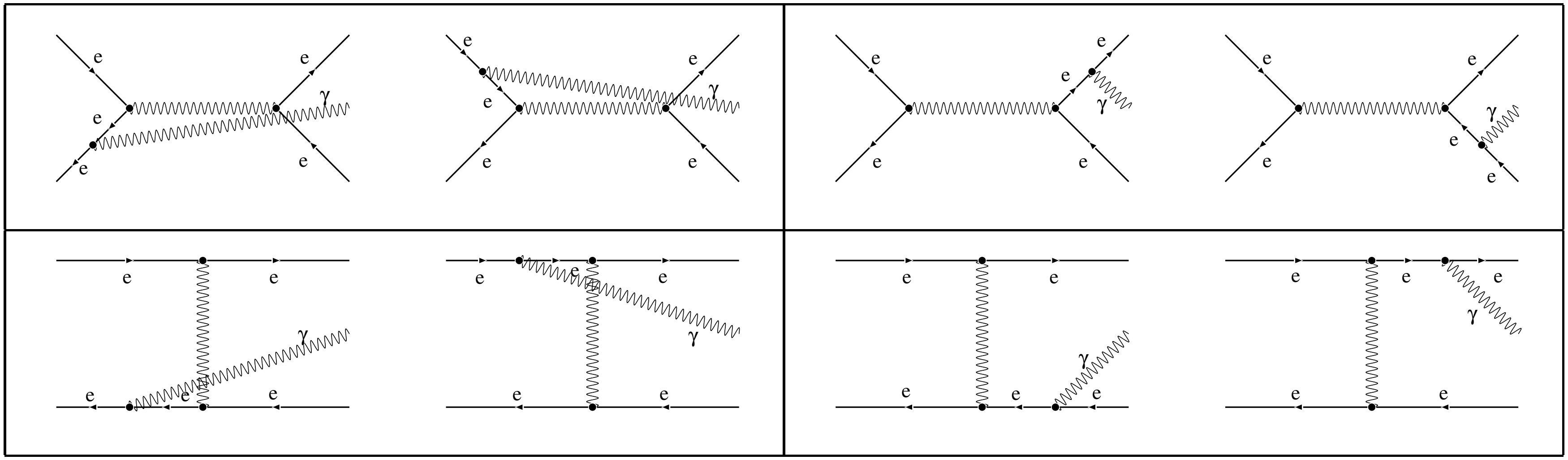
[a] ☞On-Shell renormalization scheme, Feynman–'t Hooft Gauge

$\mathcal{O}(\alpha)$ corrections: Photon emission

- ✗ **Real detectors cannot observe soft photons**
- ☞ **Photon emission mixes incoherently with pure Bhabha**
- ✓ **Remove IR singularities from ext. s.e., vertices and boxes in 1–loop**

Initial State Radiation | **Final State Radiation**



$$\frac{d\sigma}{d\Phi_3} \propto |\mathcal{A}_\gamma^{\text{ini}} + \mathcal{A}_\gamma^{\text{fin}}|^2$$

with Φ_3 = Phase-Space 3 part…

$$\left.\frac{d\sigma}{d\cos\theta}\right|_{\text{Soft}} = \text{Soft}_\gamma\text{Factor} \times \left.\frac{d\sigma}{d\cos\theta}\right|_{\text{Born}}$$

Computing Tools

Full calculation performed with *a*İTALC

an Integrated Tool for Automating Loop Calculations (A.L and T. Riemann)

Under development. **Expected launching time:** ***29th February 2004***

http://www-zeuthen.desy.de/∼alorca/aitalc

It runs under Make environment:

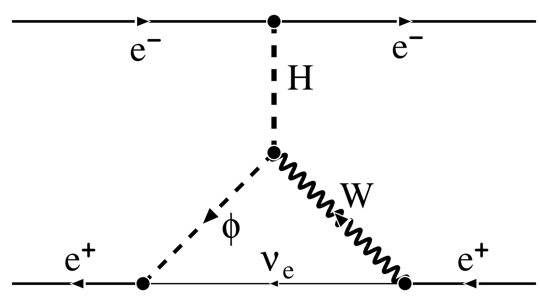
- **Generating and sorting Feynman Diagrams: DIANA 2.35**
  DIagram ANAlizer based on QGRAF2
- **Extracting the Form Factors and Matrix Elements: kitForm3**
  Our own 1–Loop library in FORM 3
- **Getting the numbers: kitFortran** (Evolution of TOPFIT)
  FORTRAN **program with** LOOPTOOLS **package (based on** FF)

Modules in *a*ITALC: DIANA

What do we ask? DRIVER FILE

```
SET _processname = Bhabha
SET _TOPOLOGYEDITORNAME = "tedi"
---
\Begin(model,EWSM.model)
\End(model)
\Begin(process)
ingoing le(;p1),Le(;p4);
outgoing le(;-p2),Le(;-p3);
loops = 1;
\End(process)
---
options = onshell,notadp;
*\excludevertex(Le,le,H)
---
SET MakeEps = "!"
\setpropline(Wm,arrowWavy, 5, 3)
...
```

What does Diana answer?




```
G Amplitude =
(-1)*F(1,1,1,0,0)*(-i_)*e/2/sw*Mle/MW*F(2,2,1,-1,0)*
(-i_)*e/2/sqrt2/sw*Mle/MW*FF(3,2,+q,Mne)*i_*
F(3,2,mu1,1,-1,1)*(+i_)*e/2/sqrt2/sw*SS(4,0)*i_*
SS(1,2)*i_*VV(2,mu2,mu1,-q-k2,2)*i_*
V(4,mu2,+p1+p2-(+q+k1),1)*(-i_)*e/2/sw;
---
#define COUNTER "626"
#define LINE "4"
#define LOOPTYPE "c"
#define PROTOTYPE "WWnH"
...
```

Generates, sorts and extracts information of the diagrams!

Modules in aITALC: kitForm3

DIANA
(symbolic level)

¿¿¿ 1–Loop Library ???

FORTRAN
(numeric level)

Written in FORM

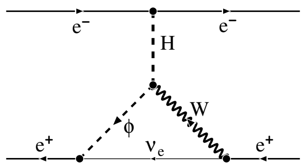
```
#call feynmanrules()
.....................................
#call tracefermiloops()
#call integration()
#call chisholm()
#call dimensionfour()
#call gammaalgebra()
#call onshell()
#call diracequation()
#call massivefofa(,‘i’,15,0)
.end
```

→ **Those procedures are intended to be general**

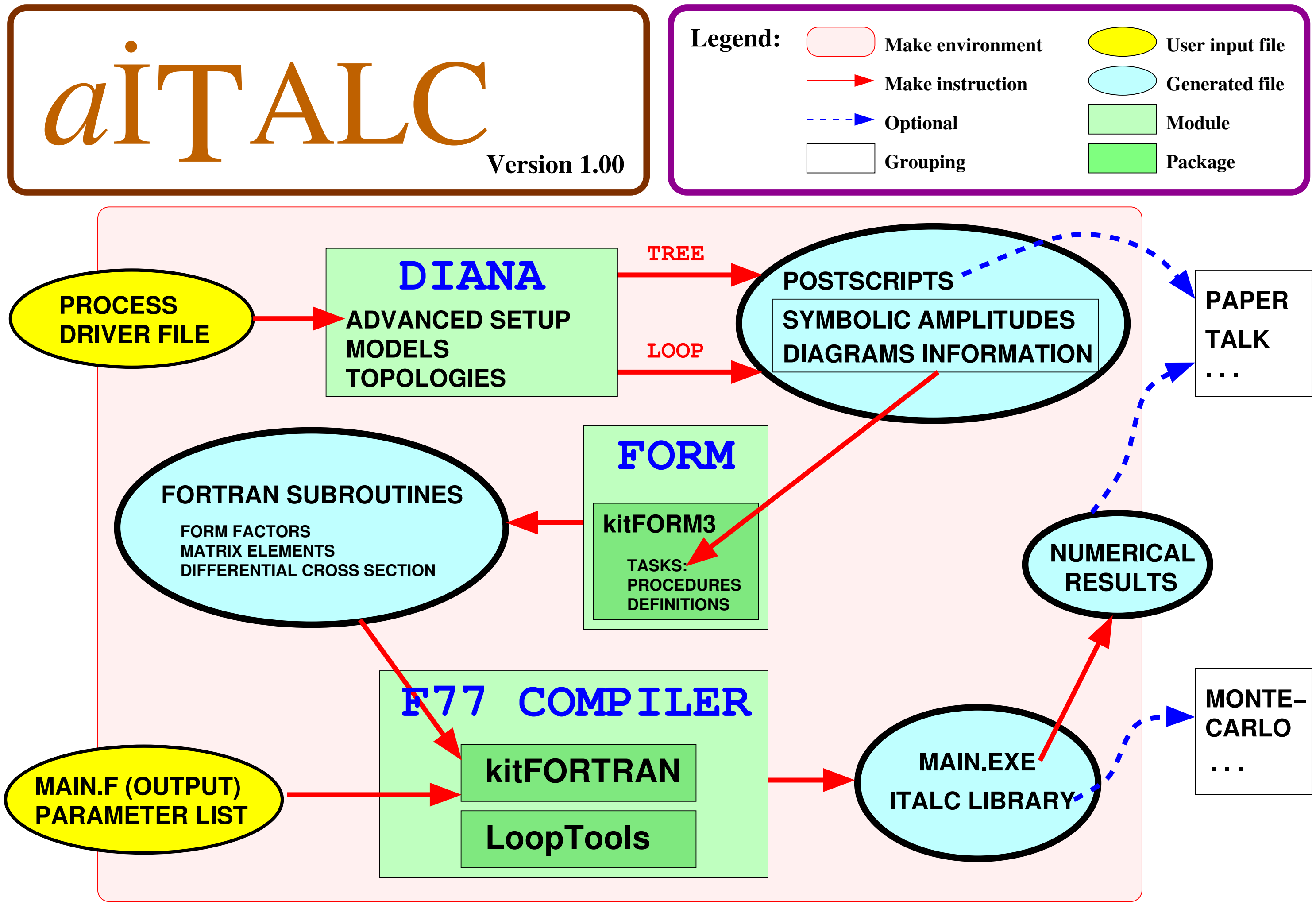
✓ **Write automatically** FORTRAN **subroutines from** DIANA **output**

tools: Diagram analytical output

```
Diagram No. 626\\
FORM by J.Vermaseren,version 3.1(Jan 24 2003) Run at: Tue Nov 11 19:32:33 2003\\
    #-

   BhabhaD626 =

       + ident(1)*ident(2)*den(t,MH2)*im*Miw2*e4*siw4*pi2i * (
          - 1/512*Mde2
          + 1/256*C0i(cc1,Me2,t,Me2,Mn2,MW2,MW2)*Mde4
          - 1/128*C0i(cc2,Me2,t,Me2,Mn2,MW2,MW2)*t*Mde2
          + 1/256*C0i(cc2,Me2,t,Me2,Mn2,MW2,MW2)*Mde4
          + 1/64*C0i(cc00,Me2,t,Me2,Mn2,MW2,MW2)*Mde2
          + 1/256*C0i(cc11,Me2,t,Me2,Mn2,MW2,MW2)*Mde4
          - 1/256*C0i(cc12,Me2,t,Me2,Mn2,MW2,MW2)*t*Mde2
          + 1/128*C0i(cc12,Me2,t,Me2,Mn2,MW2,MW2)*Mde4
          + 1/256*C0i(cc22,Me2,t,Me2,Mn2,MW2,MW2)*Mde4
          )

       + ident(1)*gamma5(2)*den(t,MH2)*im*Miw2*e4*siw4*pi2i * (
          + 1/512*Mde2
          - 3/256*C0i(cc1,Me2,t,Me2,Mn2,MW2,MW2)*Mde4
          + 1/128*C0i(cc2,Me2,t,Me2,Mn2,MW2,MW2)*t*Mde2
          - 5/256*C0i(cc2,Me2,t,Me2,Mn2,MW2,MW2)*Mde4
          - 1/64*C0i(cc00,Me2,t,Me2,Mn2,MW2,MW2)*Mde2
          - 1/256*C0i(cc11,Me2,t,Me2,Mn2,MW2,MW2)*Mde4
          + 1/256*C0i(cc12,Me2,t,Me2,Mn2,MW2,MW2)*t*Mde2
          - 1/128*C0i(cc12,Me2,t,Me2,Mn2,MW2,MW2)*Mde4
          - 1/256*C0i(cc22,Me2,t,Me2,Mn2,MW2,MW2)*Mde4
          );
```



Structure of *a*ITALC: Flowchart



Results: Numerical comparison

$e^- e^+ \to e^- e^+ (\gamma)$ **at LC:** $\sqrt{s} = 500$ GeV, $E_{\rm max}(\gamma_{\rm soft}) = \frac{\sqrt{s}}{10}$

$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\rm Born}$ (pb)	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\mathcal{O}(\alpha^3)=\text{Born+QED+weak+soft}}$	Group
−0.9	0.21699 88288 10920 5	0.19344 50785 26863 6	aITALC
−0.9	0.21699 88288 10920 0	0.19344 50785 26862 2	FA + FC + LT
−0.9	0.21699 88288 41513 1	0.19344 50785 62637 9	$m_e = 0$
−0.5	0.26136 04305 85323 6	0.23870 66977 23338 2	aITALC
−0.5	0.26136 04305 85323 4	0.23870 66977 23344 6	FA + FC + LT
−0.5	0.26136 04306 17585 0	0.23870 66977 50854 7	$m_e = 0$
0.0	0.59814 23072 50330 3	0.54667 71794 69423 1	aITALC
0.0	0.59814 23072 50329 4	0.54667 71794 69421 8	FA + FC + LT
0.0	0.59814 23072 88584 4	0.54667 71794 99961 4	$m_e = 0$
0.5	0.42127 29493 91625 6 $\cdot 10^1$	0.38130 07881 78966 1 $\cdot 10^1$	aITALC
0.5	0.42127 29493 91625 1 $\cdot 10^1$	0.38130 07881 78953 9 $\cdot 10^1$	FA + FC + LT
0.5	0.42127 29493 96691 5 $\cdot 10^1$	0.38130 07881 81327 0 $\cdot 10^1$	$m_e = 0$
0.9	0.18916 03223 32270 6 $\cdot 10^3$	0.17292 83490 66507 2 $\cdot 10^3$	aITALC
0.9	0.18916 03223 32270 6 $\cdot 10^3$	0.17292 83490 66508 0 $\cdot 10^3$	FA + FC + LT
0.9	0.18916 03223 31848 5 $\cdot 10^3$	0.17292 83490 61347 4 $\cdot 10^3$	$m_e = 0$

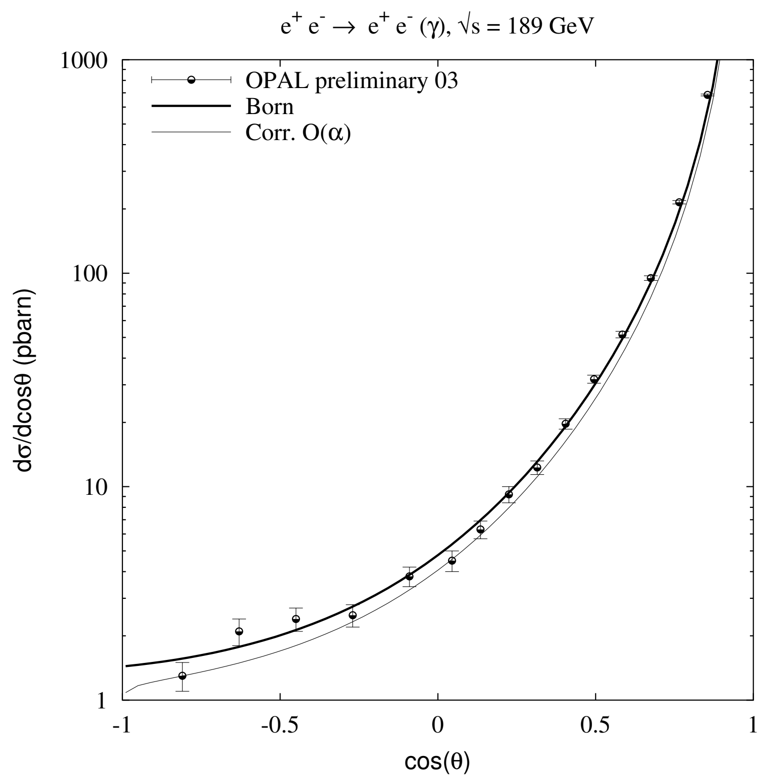
Huge independent agreement! 14 digits : limit in double precision

Previous agreement with FA+FC+LT: 11 digits hep-ph/0307132**, SANC: 10 digits** hep-ph/0207156

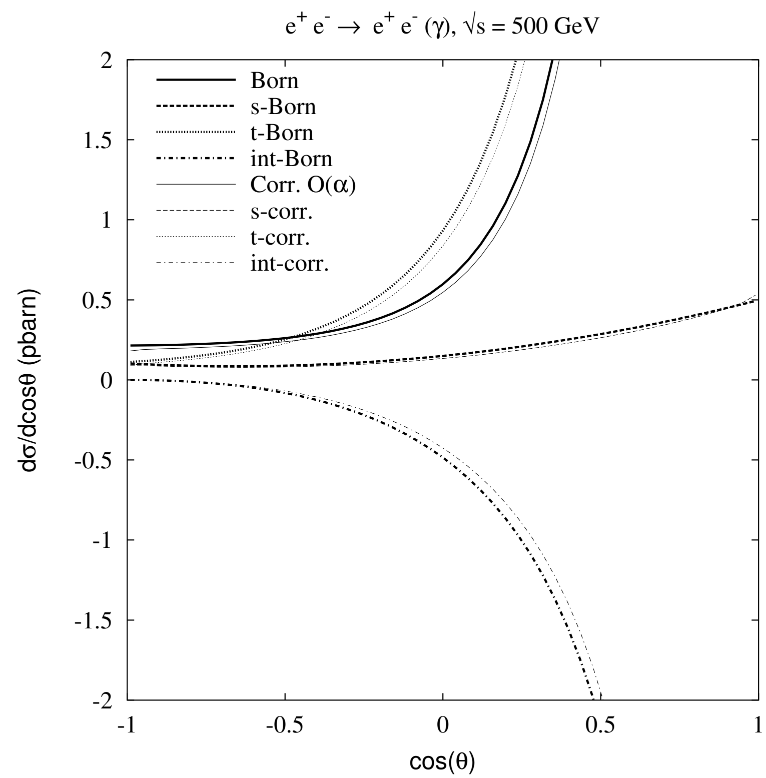
Thanks to T. Hahn, numbers supplied with *FeynArts* + *FormCalc* + *LoopTools*

Results: Some plots

a) Checking some real data…



b) $s - t$ interference effects



$\mathcal{O}(\alpha)$ corrections are about 8 : 12 : 18 % at 0.5 : 1 : 3 TeV

Conclusions

- ☞ **We presented the complete $\mathcal{O}(\alpha)$ corrections to massive Bhabha**

  Building block in such an accurate project

  Exact treatment of m_e brings credibility but no more physics

- ✓ **Well known calculation method. Following**

  Böhm, Spiesberger and Hollik. Fortschr. Phys. 34 (1986) 11

  A. Denner. Fortschr. Phys. 41 (1993) 4

- **14 digits agreement! Fully satisfied, good cross-check for codes**

- ♣ **Collected some experience with *a*iTALC on automatization towards**

  Further testing and launch as public available the *2 to 2* fermion version

  Apply method on realistic physical studies.

Thanks to . . .

M. Tentyukov and J. Fleischer for DIANA**:**

`http://www.physik.uni-bielefeld.de/~tentukov/diana.html`

P. Nogueira for QGRAF**:** `ftp://gtae2.ist.utl.pt/pub/qgraf/`

J. Vermaseren for FORM**:** `http://www.nikhef.nl/~form/`

T. Hahn and G. J. van Oldenborgh for *LoopTools* **and** FF**:**

`http://www.feynarts.de/looptools`
`http://www.xs4all.nl/~gjvo/FF.html`

Free Software Fundation and GNU-project for Make **and** g77

`http://www.gnu.org`

~ FIN ~