

Factorizing one-loop contributions to two-loop Bhabha scattering and automatization of Feynman diagram calculations

J. Fleischer, O.V. Tarasov
Fakultät für Physik, Universität Bielefeld
and
T. Riemann, A. Werthenbach
DESY Zeuthen

Abstract

In higher order calculations a number of new technical problems arise: one needs diagrams in arbitrary dimension in order to obtain their needed ε -**expansion**, **zero Gram determinants** appear, renormalization produces diagrams with ‘dots’ on the lines, i.e. **higher order powers of scalar propagators**. All these problems cannot be accessed by the ‘standard’ Passarino-Veltman approach: there is not available what is needed for higher loops. We demonstrate our method of how to solve these problems.

We are moving in the direction of **two-loop** Bhabha scattering, which is extremely important, in particular at higher energies, for the luminosity determination of the coming accelerators.

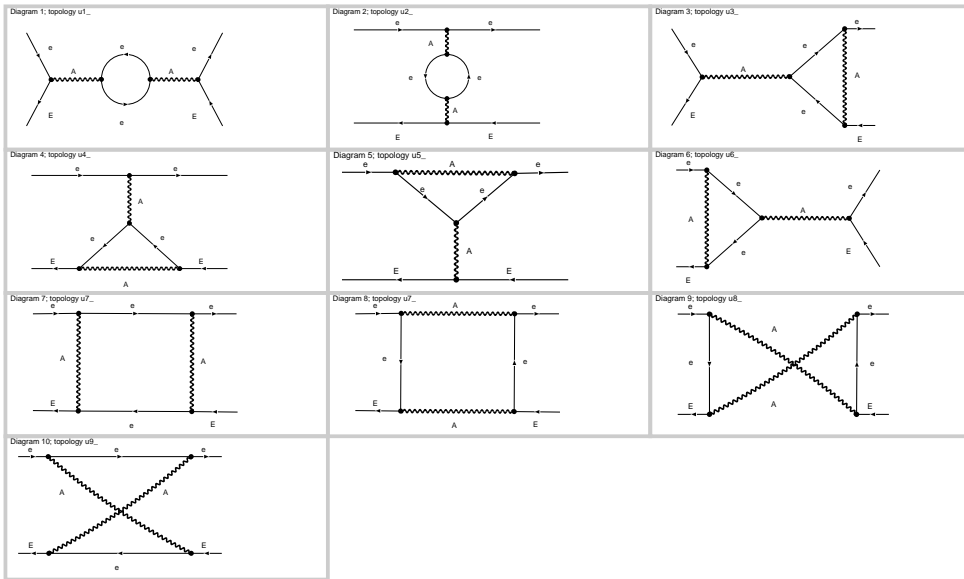
Factorizing one-loop contributions are obtained directly by ‘squaring’, e.g.:

$$1\text{-loop} \times 1\text{-loop} = 2\text{-loop contribution}$$

and via **renormalization**. As an example mass-renormalization: in any Feynman diagram we replace

$$\frac{1}{k^2 - (m^2 + \delta(m^2))} \rightarrow \frac{1}{k^2 - m^2} \left(1 + \frac{\delta(m^2)}{k^2 - m^2}\right),$$

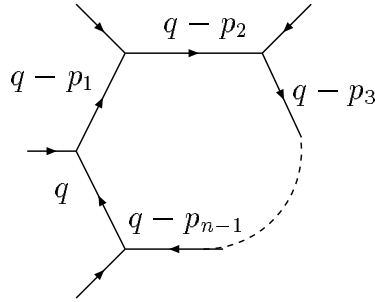
thus obtaining ‘dotted’ diagrams.



Calculation of 1-loop integrals

Ref.

J.F. , F. Jegerlehner and O.V. Tarasov, Nucl.Phys. B566 (2000) p.423



One-loop diagram with n external legs.

Tensor integrals

$$I_{n,r}^{(d)} = \int \frac{d^d q}{\pi^{d/2}} \prod_{j=1}^n \frac{q_{\mu_1} \cdots q_{\mu_r}}{c_j^{\nu_j}},$$

$$= T_{\mu_1 \dots \mu_r}(\{p_s\}, \{\partial_j\}, \mathbf{d}) I_n^{(d)}$$

where $\nu_j =$ 'index'

$$c_j = (q - p_j)^2 - m_j^2 + i\epsilon \quad \text{for } j < n \quad \text{and} \quad c_n = q^2 - m_n^2 + i\epsilon.$$

Tensor operator:

$$T_{\mu_1 \dots \mu_r}(\{p_s\}, \{\partial_j\}, \mathbf{d}^+) = \frac{1}{i^r} \prod_{j=1}^r \frac{\partial}{\partial a_{\mu_j}} \exp \left[i \left(\sum_{k=1}^{n-1} (ap_k) \alpha_k - \frac{1}{4} a^2 \right) \rho \right] \Bigg|_{\substack{\alpha_j=0 \\ \alpha_j=i\partial_j \\ \rho=i\mathbf{d}^+}}.$$

with

$$\partial_j = \frac{\partial}{\partial m_j^2}, \quad \mathbf{d}^+ I^{(d)} = I^{(d+2)},$$

Only scalar integrals remain to be evaluated !

Integrals with non-zero Gram determinants

$$\Delta_n = \begin{vmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{12} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{1n} & Y_{2n} & \dots & Y_{nn} \end{vmatrix}.$$

where

$$Y_{ij} = -(p_i - p_j)^2 + m_i^2 + m_j^2,$$

“Modified Cayley determinant” of the diagram with internal lines $1 \dots n$

$$()_n \equiv \begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & Y_{11} & Y_{12} & \dots & Y_{1n} \\ 1 & Y_{12} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & Y_{1n} & Y_{2n} & \dots & Y_{nn} \end{vmatrix},$$

“Signed minors”

$$\begin{pmatrix} j_1 j_2 \dots \\ k_1 k_2 \dots \end{pmatrix}_n$$

will be labeled by the rows j_1, j_2, \dots and columns k_1, k_2, \dots excluded from $()_n$. E.g. we have

$$\Delta_n = \begin{pmatrix} 0 \\ 0 \end{pmatrix}_n$$

Recursion Relations:

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}_n \nu_j \mathbf{j}^+ I_n^{(d)} = \sum_{k=1}^n \begin{pmatrix} 0j \\ 0k \end{pmatrix}_n \left[d - \sum_{i=1}^n \nu_i (\mathbf{k}^- \mathbf{i}^+ + 1) \right] I_n^{(d)}.$$

The operators \mathbf{j}^\pm etc. shift the indices $\nu_j \rightarrow \nu_j \pm 1$.

This relation reduces the indices and leaves the dimension.

The following one reduces simultaneously indices and dimension:

$$()_n \nu_j \mathbf{j}^+ I_n^{(d+2)} = \left[- \begin{pmatrix} j \\ 0 \end{pmatrix}_n + \sum_{k=1}^n \begin{pmatrix} j \\ k \end{pmatrix}_n \mathbf{k}^- \right] I_n^{(d)},$$

A relation reducing the space time dimension is given by:

$$(d - \sum_{i=1}^n \nu_i + 1) ()_n I_n^{(d+2)} = \left[\binom{0}{0}_n - \sum_{k=1}^n \binom{0}{k}_n \mathbf{k}^- \right] I_n^{(d)}.$$

Integrals with zero kinematic determinants

$$(d - \sum_{i=1}^n \nu_i + 1) ()_n I_n^{(d+2)} = - \sum_{k=1}^n \binom{0}{k}_n \mathbf{k}^- I_n^{(d)}.$$

To increase again the dimension d one uses relation

$$\sum_{j=1}^n \nu_j \mathbf{j}^+ I_n^{(d+2)} = -I_n^{(d)}.$$

As an example we show the relation for the three-point integral C_0 with zero Gram determinant:

$$(s - 4m_e^2) C_0(m_e, 0, m_e, m_e^2, m_e^2, s) = -\frac{2(d-3)}{(d-4)} B_0(m_e^2, m_e^2, s) - \frac{(d-2)}{m_e^2(d-4)} A_0(m_e^2)$$

Decomposition of the diagrams into Amplitudes (diagram 7)

$$\begin{array}{llllll}
O_1 = & \bar{U}(-p_2) & I & U(p_1) \cdot \bar{V}(p_4) & I & V(-p_3) \\
O_2 = & \bar{U}(-p_2) & \hat{p}_4 & U(p_1) \cdot \bar{V}(p_4) & I & V(-p_3) \\
O_3 = & \bar{U}(-p_2) & I & U(p_1) \cdot \bar{V}(p_4) & \hat{p}_2 & V(-p_3) \\
O_4 = & \bar{U}(-p_2) & \hat{p}_4 & U(p_1) \cdot \bar{V}(p_4) & \hat{p}_2 & V(-p_3) \\
O_5 = & \bar{U}(-p_2) & \gamma_\mu & U(p_1) \cdot \bar{V}(p_4) & \gamma_\mu & V(-p_3) \\
O_6 = & \bar{U}(-p_2) & \gamma_\mu \hat{p}_4 & U(p_1) \cdot \bar{V}(p_4) & \gamma_\mu & V(-p_3) \\
O_7 = & \bar{U}(-p_2) & \gamma_\mu & U(p_1) \cdot \bar{V}(p_4) & \gamma_\mu \hat{p}_2 & V(-p_3) \\
O_8 = & \bar{U}(-p_2) & \gamma_\mu \gamma_\nu & U(p_1) \cdot \bar{V}(p_4) & \gamma_\nu \gamma_\mu & V(-p_3) \\
O_9 = & \bar{U}(-p_2) & \gamma_\mu \gamma_\nu \hat{p}_4 & U(p_1) \cdot \bar{V}(p_4) & \gamma_\nu \gamma_\mu & V(-p_3) \\
O_{10} = & \bar{U}(-p_2) & \gamma_\mu \gamma_\nu & U(p_1) \cdot \bar{V}(p_4) & \gamma_\nu \gamma_\mu \hat{p}_2 & V(-p_3) \\
O_{11} = & \bar{U}(-p_2) & \gamma_\mu \gamma_\nu \hat{p}_4 & U(p_1) \cdot \bar{V}(p_4) & \gamma_\nu \gamma_\mu \hat{p}_2 & V(-p_3) \\
O_{12} = & \bar{U}(-p_2) & \gamma_\mu \gamma_\nu \gamma_\rho & U(p_1) \cdot \bar{V}(p_4) & \gamma_\rho \gamma_\nu \gamma_\mu & V(-p_3)
\end{array}$$

For diagram 8:

$$\bar{U}(-p_2) \cdot V(-p_3) \quad \bar{V}(p_4) \cdot U(p_1)$$

The on-shell diagram reads

$$\text{Diagram} = \sum_{j=1}^{12} \mathbf{A}_j \mathbf{O}_j.$$

with amplitudes \mathbf{A}_j .

Crossing relations (e.g. diagram 8 \leftrightarrow diagram 9)

$$\begin{aligned}
A_1^{(9)} &= A_1^{(8)} - 2m_e A_7^{(8)} - 4m_e A_9^{(8)} + 2d A_8^{(8)} \\
A_2^{(9)} &= -A_2^{(8)} + 4A_7^{(8)} - 2(d-4)A_9^{(8)} - 8m_e A_{11}^{(8)} \\
A_3^{(9)} &= -A_3^{(8)} + 2A_6^{(8)} - 2(d-2)A_{10}^{(8)} - 4m_e A_{11}^{(8)} \\
A_4^{(9)} &= A_4^{(8)} - 2(d-2)A_{11}^{(8)} \\
A_5^{(9)} &= -A_5^{(8)} + 2m_e A_6^{(8)} + 4m_e A_7^{(8)} + 8m_e A_9^{(8)} + 4m_e A_{10}^{(8)} - 12m_e^2 A_{11}^{(8)} - (6d-4)A_{12}^{(8)} \\
A_6^{(9)} &= -4m_e A_{11}^{(8)} + A_6^{(8)} \\
A_7^{(9)} &= -4m_e A_{11}^{(8)} + A_7^{(8)} \\
A_8^{(9)} &= -A_8^{(8)} \\
A_9^{(9)} &= A_9^{(8)} \\
A_{10}^{(9)} &= A_{10}^{(8)} \\
A_{11}^{(9)} &= -A_{11}^{(8)} \\
A_{12}^{(9)} &= A_{12}^{(8)}
\end{aligned}$$

and **exchanging t \leftrightarrow u**. These relations also hold for crossing diagram 7 \leftrightarrow diagram 10 (s \leftrightarrow u). For the crossing 7 \leftrightarrow 8 (s \leftrightarrow t) only a change of sign occurs in the amplitudes. **Thus:** we need to calculate **only one** SE, vertex and box.

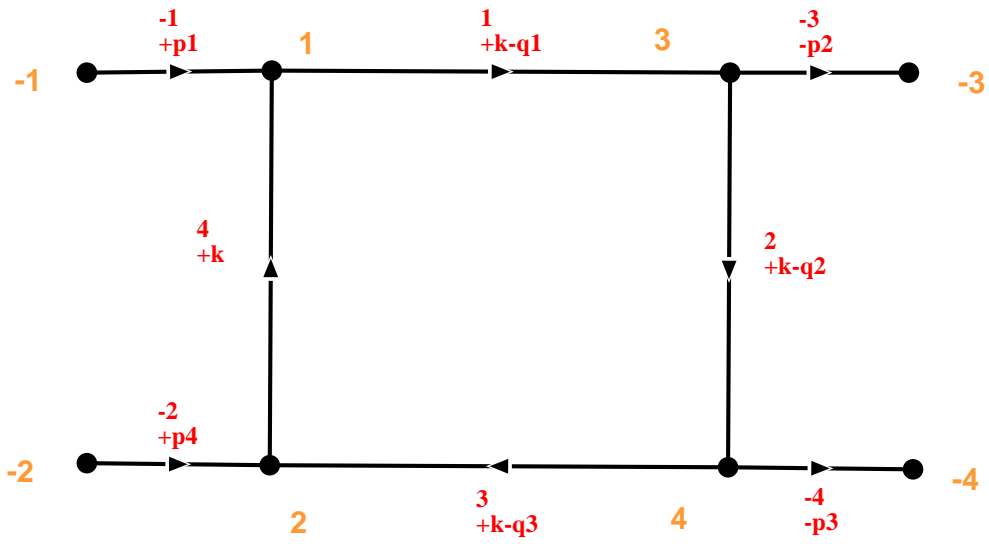
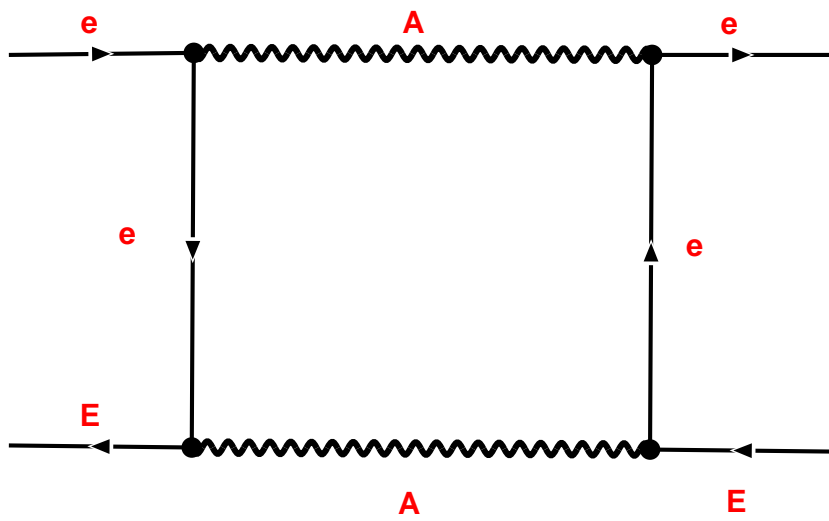
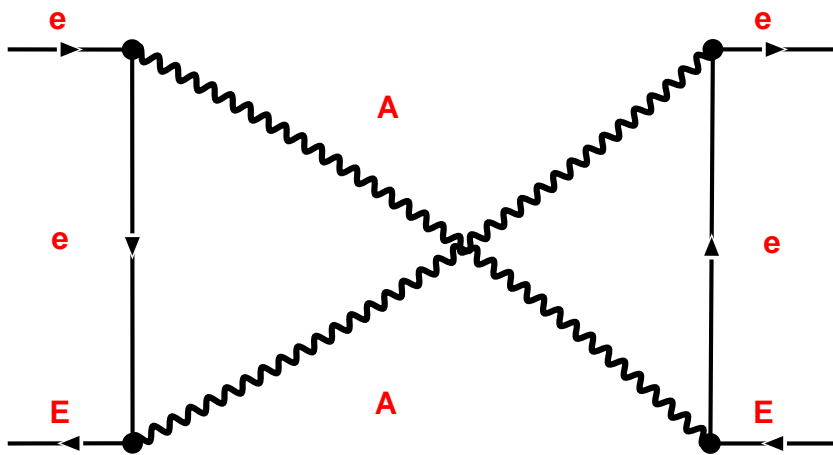
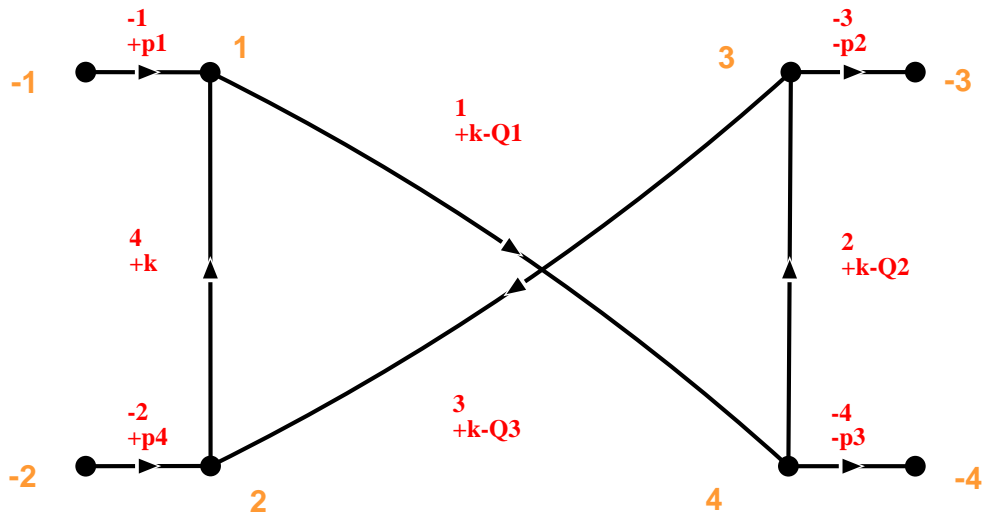


Diagram 8 topology u7_ (unique u7_) momentaset 1 (of 1)





Further relations between amplitudes of **one** diagram (obtained by solving a system of equations):

$$A_3 = A_2$$

$$A_7 = A_6$$

$$A_{10} = A_9$$

$$A_8 = \frac{1}{4}A_1 - \frac{m_e}{2}A_2 + (A_{12})$$

$$A_9 = \frac{1}{4m_e}A_1 - \frac{1}{2}A_2 - \frac{1}{2}A_6$$

$$A_{11} = \frac{1}{4m_e^2}A_1 - \frac{1}{2m_e}A_2$$

Thus: there are only **6 independent amplitudes**.

Contribution to the differential cross section

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \sum_{j=1}^{12} \left[\frac{B_j(s,t)}{4} A_j(s,t) - \frac{B_j(t,s)}{4} A_j(t,s) \right]$$

$$B_1 = -\frac{4}{t}s^2 + (2d - 8 + \frac{24}{t}m_e^2)s - \frac{32}{t}m_e^4 - 8dm_e^2 - 32\frac{m_e^2}{s}(t - 2m_e^2),$$

$$B_2 = \frac{4}{t}m_e(s^2 + 8m_e^4) + (2(4 - d)m_e - \frac{24}{t}m_e^3)s - 4(d + 2)m_e t + 8(d - 2)m_e^3 - 16\frac{m_e}{s}(t - 2m_e^2)^2,$$

$$B_3 = B_2,$$

$$B_4 = -\frac{4}{t}(m_e^2 + t)s^2 + ((-4 - 2d)t + 2dm_e^2 + \frac{24}{t}m_e^4)s - 2(d + 4)t^2 + 8(1 + d)m_e^2 t - 8(d - 4)m_e^4 - \frac{32}{t}m_e^6 - \frac{8}{s}(t - 2m_e^2)^3,$$

$$B_5 = -\frac{1}{m_e}B_6 + 4td + \frac{8(st + 2t^2 + 2m_e^2s - 4m_e^2t)}{s},$$

$$\frac{B_6}{m_e} = -4\frac{(d - 2)}{t}(s^2 - 2tm_e^2) + (2(d - 2)(d - 10) + 8(d - 4)\frac{m_e^2}{t})s + 32\frac{m_e^4}{t} + 32\frac{m_e^2}{s}(t - 2m_e^2),$$

$$B_7 = B_6,$$

$$B_8 = -4\frac{(d - 2)^2}{t}s^2 + (2(d - 2)(d - 4)(d - 10) + 8(3d^2 - 8d + 8)\frac{m_e^2}{t})s - 8(d - 2)(d - 4)t - 8(d - 2)(d^2 - 10d + 20)m_e^2 - 32(d^2 - 2d + 2)\frac{m_e^4}{t} - 32d\frac{m_e^2}{s}(t - 2m_e^2),$$

$$\frac{B_9}{m_e} = 4(d - 2)^2\frac{s^2}{t} + (2(2 - d)(d - 4)(d - 10) - 8(3d^2 - 8d + 8)\frac{m_e^2}{t})s - 4(d^3 + 34d - 10d^2 - 28)t + 8(d^3 - 14d^2 + 52d - 60)m_e^2 + 32(d^2 - 2d + 2)\frac{m_e^4}{t} - 16d\frac{(t - 2m_e^2)^2}{s},$$

$$B_{10} = B_9,$$

$$B_{11} = (-4(d - 4)^2t - 4(d - 2)^2m_e^2)s^3 + (2(4 - d)(d^2 - 4d - 2)t^2 + 2(d^3 + 20d - 12d^2 + 32)m_e^2t + 8(3d^2 - 8d + 8)m_e^4)s^2 + ((20d^2 - 68d + 64 - 2d^3)t^3 + 8(d - 4)(d^2 - 8d + 10)m_e^2t^2 - 8(d - 2)(d^2 - 14d + 28)m_e^4t - 32(d^2 - 2d + 2)m_e^6)s - 8(d - 2)t^3(t - 6m_e^2) - 32(3d - 4)m_e^4t^2 + 64dm_e^6t,$$

$$B_{12} = 4(d - 2)^3s^3 + (-2(d - 2)(d^3 - 20d^2 + 110d - 208)t - 8(24d - 16 + d^3 - 12d^2)m_e^2)s^2 + ((12d^3 - 120d^2 - 352 + 400d)t^2 - 8(3d^3 - 42d^2 + 148d - 160)m_e^2t - 32(4 - 6d + 3d^2)m_e^4)s + 16(3d - 2)t(t^2 - 4m_e^2t + 4m_e^4).$$

Contribution to the differential cross section

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \sum_{j=1}^{12} \left[\frac{B_j(s,t)}{4} A_j(s,t) - \frac{B_j(t,s)}{4} A_j(t,s) \right]$$

$$B_1 = -\frac{4}{t}(s^2 - 6m_e^2 s + 8m_e^4) - 32m_e^2 - 32\frac{m_e^2}{s}(t - 2m_e^2),$$

$$B_2 = 4\frac{m_e}{t}s^2 - 24\frac{m_e^3}{t}s - 24m_e t + 16m_e^3 + 32\frac{m_e^5}{t} - 16\frac{m_e}{s}(t - 2m_e^2)^2,$$

$$B_3 = B_2,$$

$$B_4 = -4\left(\frac{m_e^2}{t} + 1\right)s^2 + (-12t + 8m_e^2 + 24\frac{m_e^4}{t})s - 16t^2 + 40m_e^2 t - 32\frac{m_e^6}{t} - \frac{8}{s}(t - 2m_e^2)^3,$$

$$B_5 = 8\frac{s^2}{t} + 24s + 24t - 32\frac{m_e^4}{t} + 16\frac{(t - 2m_e^2)^2}{s},$$

$$B_6 = -8\frac{m_e s^2}{t} - 24m_e s + 16m_e^3 + 32\frac{m_e^5}{t} + 32\frac{m_e^3}{s}(t - m_e^2),$$

$$B_7 = B_6,$$

$$B_8 = -16\frac{s^2}{t} + 192\frac{m_e^2}{t}s + 64m_e^2 - 320\frac{m_e^4}{t} - 128\frac{m_e^2}{s}(t - 2m_e^2),$$

$$B_9 = 16\frac{m_e s^2}{t} - 192\frac{m_e^3 s}{t} - 48m_e t - 96m_e^3 + 320\frac{m_e^5}{t} - 64\frac{m_e}{s}(t - 2m_e^2)^2,$$

$$B_{10} = B_9,$$

$$B_{11} = -16m_e^2 s^3 - 32m_e^2(t - 8m_e^2)s^2 - 16(t^3 - 12m_e^4 t + 0m_e^6)s - 16t^4 + 96m_e^2 t^3 - 256m_e^4 t(t - m_e^2),$$

$$B_{12} = 32s^3 + 96(t + 4m_e^2)s^2 + (96t^2 + 384m_e^2 t - 896m_e^4)s + 160t(t - 2m_e^2)^2.$$

‘Undotted’ Diagrams; common factor $\frac{e^2}{(4\pi)^{d/2}}$ (normalized to Born term)

Selfenergy - diagrams:

$$A_5 = A_0 \frac{4}{s^2} \left[\frac{1}{d-1} - 1 \right] + B_e \frac{2}{s} \left[\frac{1}{d-1} (1 - z^2) - 1 \right]$$

Vertex - diagrams:

$$A_2 = A_0 \frac{2}{m_e s^2} x^2 \left[2 \frac{1}{d-3} - (d-4) \right] + B_e \frac{4m_e}{s^2} x^2 [1 - (d-4)]$$

$$A_5 = 2 \left(-A_0 \frac{1}{m_e^2 s} \left[2 \frac{1}{d-3} - x^2 z^2 \right] + B_e \frac{1}{s} \left[x^2 (1 + z^2) + d - 4 \right] + F_s \frac{2}{(d-4)s} [1 + x^2] \right)$$

‘Dotted’ Diagrams; common factor $\delta(m_e)/m_e e^4$

Selfenergy - diagrams:

$$A_5 = A_0 \frac{4}{s^2} (d-2) x^2 z^2 - B_e \frac{2}{s} z^2 [1 - (d-3) x^2]$$

Vertex - diagrams:

$$A_2 = -A_0 \frac{4}{m_e s^2} x^2 \left[\frac{1}{d-3} x^2 + (d-4) \left(1 + (d-4) \left(1 - x^2/2 \right) \right) - 3 + 2x^2 \right] \\ - B_e \frac{4m_e}{s^2} x^2 \left[(d-4) \left(x^2 - (d-4) x^2 z^2 \right) + 2x^2 z^2 \right] + F_s \frac{8m_e}{(d-4)s^2} x^2 [1 + x^2]$$

$$A_5 = 2 \left(A_0 \frac{2}{s^2} x^2 \left[\left(\frac{1}{d-3} + 6 \frac{1}{d-5} \right) - (d-4) (2x^2 - 1 + (d-4)) + 7 - 4x^2 \right] \right. \\ \left. + A_0 \frac{2}{m_e^2 s} \left[1 + 3 \frac{1}{d-5} \right] - B_e \frac{1}{s} x^2 z^2 \left[(d-4) (2x^2 + (d-4)) + 2x^2 \right] - F_s \frac{2}{(d-4)s} x^2 z^2 \right)$$

Notation:

$$A_0 = A_0(m_e), B_t = B_0(m_e, m_e, t), B_0 = B_0(0, 0, s), B_e = B_0(m_e, m_e, s)$$

$$F_s = A_0(m_e)/m_e^2 + B_0(m_e, m_e, s), F_t = A_0(m_e)/m_e^2 + B_0(m_e, m_e, t),$$

$$C_0 = C_0(0, m_e, 0, m_e^2, m_e^2, s), D_0 = D_0(m_e, 0, m_e, 0, m_e^2, m_e^2, m_e^2, m_e^2, t, s).$$

$$F_s, F_t: \text{ only with factor } \frac{1}{d-4} !$$

Further abbreviations:

$$r_t = t/s, \quad r_u = u/s$$

$$\text{and } x^2 = 1/(1 - 4m_e^2/s), \quad y^2 = 1/(1 - 4m_e^2/t), \quad z^2 = \frac{4m_e^2}{s} (x^2 - 1 = x^2 z^2).$$

‘Undotted’ Box-Diagrams; common factor $\frac{e^2}{(4\pi)^{d/2}}\mathbf{r}_t^{-2}\mathbf{r}_u^{-2}$

$$\begin{aligned}
A_1 = & A_0 \frac{2r_t}{s^2} \left(-\frac{1}{(d-3)} (1 + r_u + 2r_t r_u x^2 + r_u y^2 - z^2) - (r_u + 2r_t + 2r_t r_u x^2 + (1 + 5r_u) y^2) \right) - \\
& B_t r_t \frac{z^2}{s} (2r_t + (1 + 5r_u) y^2) - B_0 r_t \frac{z^2}{s} (1 + 2r_t x^2) r_u + F_t r_t \frac{z^2}{s(d-4)} (1 - 2r_t - (1 + 4r_u) y^2 - z^2) \\
& - C_0 r_t z^2 \left(\frac{1}{(d-3)} (r_t + r_u - z^2 (1 + r_t + r_u - z^2)) - (r_t + r_u + 2r_t^2 + 6r_t r_u - 2z^2 r_t (1 - r_u x^2)) \right) / 2 \\
& + D_0 r_t s z^2 \left(\frac{1}{(d-3)} (r_t + z^2 (r_u - z^2 (1 + r_t + r_u - z^2))) \right) \\
& + (r_t (1 + 2r_t + 2r_u + 2r_t^2 + 6r_t r_u + 4r_u^2) - 2z^2 r_t (1 + 2r_t + 3r_u - z^2)) / 4,
\end{aligned}$$

$$\begin{aligned}
A_2 = & A_0 \frac{2r_t}{m_e s^2} \left(-\frac{1}{(d-3)} (1 + r_t + r_u + r_t r_u x^2 - z^2) - (r_t + r_u + r_t r_u x^2 + (1 + 2r_u) y^2) \right) \\
& - B_t \frac{4m_e r_t}{s^2} (r_t + (1 + 2r_u) y^2) - B_0 \frac{4m_e r_t}{s^2} (r_u + r_t r_u x^2) + F_t \frac{4m_e r_t}{s^2 (d-4)} (1 - (1 + 2r_u) y^2 - z^2) \\
& - C_0 \frac{2m_e r_t}{s} \left(\frac{1}{(d-3)} ((1 + r_t)(r_u + r_t) - z^2 (1 + r_u + 2r_t - z^2)) - (r_t + r_t^2 \right. \\
& \left. + (1 + 3r_t) r_u - z^2 r_t (1 - x^2 r_u)) \right) + D_0 m_e r_t \left(\frac{1}{(d-3)} (r_t + r_t^2 + z^2 (r_t^2 + (1 + r_t) r_u \right. \\
& \left. - z^2 (1 + r_u + 2r_t - z^2))) + r_t (1 + 3r_t + 2r_t^2 + 2r_u (2 + 3r_t + 2r_u) - z^2 (3 + 4r_t + 6r_u - 2z^2)) \right),
\end{aligned}$$

$$\begin{aligned}
A_4 = & A_0 \frac{2r_t}{m_e^2 s^2} \left(-\frac{1}{(d-3)} (1 - 2r_t (1 + r_t) + r_u (1 - 2r_t) + 2r_t r_u x^2 - r_u y^2 - z^2 (1 - 2r_t)) \right) \\
& - (r_u + 2r_t + 2r_t r_u x^2 + (1 - r_u) y^2) - B_t \frac{4r_t}{s^2} (2r_t + (1 - r_u) y^2) - B_0 \frac{4r_t}{s^2} (r_u + 2r_t r_u x^2) \\
& + F_t \frac{4r_t}{s^2 (d-4)} (1 - 2r_t r_u - 4r_t - 2r_t^2 - y^2 - z^2 (1 - 2r_t)) - C_0 \frac{2r_t}{s} \left(\frac{1}{(d-3)} (r_u + r_t (1 - 2r_t \right. \\
& \left. - 2r_t^2 - 2r_u (1 + r_t)) - z^2 (1 + r_u (1 - 2r_t) - r_t (1 + 4r_t) - z^2 (1 - 2r_t)) \right) - (r_u + r_t + 2r_t^2) \\
& + 2z^2 r_t (1 - r_u x^2) \left. \right) + D_0 r_t \left(\frac{1}{(d-3)} (r_t (1 - 2r_t - 2r_t^2) + z^2 (r_u (1 - 2r_t - 2r_t^2) - 2r_t^3 \right. \\
& \left. - z^2 (1 - r_t - 4r_t^2 + r_u (1 - 2r_t) - z^2 (1 - 2r_t))) \right) + r_t (1 + 2r_u + 2r_t + 2r_t^2) \\
& - 2z^2 r_t (1 + 2r_t - z^2),
\end{aligned}$$

$$\begin{aligned}
A_5 = & A_0 \frac{r_t}{m_e^2 s} \left(-\frac{1}{(d-3)} (2r_t(1 - r_t r_u - r_t^2 + r_t r_u x^2) + z^2(r_t r_u - r_t + 3r_t^2 - r_u y^2 - z^2 r_t)) \right. \\
& - (2r_t(1 + r_t + 2r_u + r_t r_u x^2) + 2z^2(1 + r_u)y^2) - B_t \frac{4r_t}{s} (r_t(1 + r_t + r_u) + z^2(1 + r_u)y^2) \\
& - B_0 \frac{4r_t}{s} (r_t r_u(1 + r_t x^2)) - F_t \frac{2r_t}{s(d-4)} (2r_t(r_t + r_t^2 + (2 + r_t)r_u) + z^2(r_t(1 - r_u - 3r_t) \\
& + (2 + 3r_u)y^2 + z^2 r_t)) - C_0 r_t^2 \left(\frac{1}{(d-3)} (2(r_t - r_t^3 + (1 - r_t^2)r_u) - z^2(2 + r_t + r_u \right. \\
& - 3r_t r_u - 5r_t^2 - z^2(1 - r_u - 4r_t + z^2))) - 2(r_t + r_t^2 + (1 + 2r_t)r_u - z^2 r_t(1 - r_u x^2)) \\
& + D_0 r_t^2 s \left(\frac{1}{(d-3)} (r_t - r_t^3 + z^2(r_t + 3r_t^2 - 2r_t^3 + 2(1 - r_t^2)r_u - z^2(2(1 + r_t) - 5r_t^2 \right. \\
& + (1 - 3r_t)r_u - z^2(1 - r_u - 4r_t + z^2))) / 2) + r_t(1 + r_t)(1 + r_t + 2r_u) \\
& \left. - z^2(4r_t(1 + r_t) - (r_u - 4r_t)r_u - 2z^2 r_t) / 2 \right),
\end{aligned}$$

$$\begin{aligned}
A_6 = & A_0 \frac{r_t}{m_e s^2} \left(\frac{1}{(d-3)} (2(r_t - r_t r_u x^2 - r_u y^2)) - 2(r_t + r_t r_u x^2 + 2r_u y^2) \right) \\
& - B_t \frac{4m_e r_t}{s^2} (r_t + 2r_u y^2) - B_0 \frac{4m_e r_t^2}{s^2} r_u x^2 - F_t \frac{2m_e r_t}{s^2(d-4)} (4r_t + 2r_u y^2) \\
& + C_0 \frac{2m_e r_t^2}{s} \left(\frac{1}{(d-3)} (r_t + r_u - z^2) + r_t + 2r_u - z^2(1 - r_u x^2) \right) \\
& - D_0 m_e r_t^2 \left(\frac{1}{(d-3)} (r_t + z^2(r_t + r_u - z^2)) + (r_t + r_u - z^2) \right),
\end{aligned}$$

$$\begin{aligned}
A_{12} = & -A_0 \frac{r_t^2}{m_e^2 s(d-3)} r_u / 2 + F_t \frac{r_t^2 r_u}{s(d-4)} - C_0 r_t^2 \left(\frac{1}{(d-3)} ((r_t + r_u)r_u - z^2 r_u) \right) / 2 \\
& + D_0 r_t^2 s \left(\frac{1}{(d-3)} (r_t r_u + z^2((r_t + r_u)r_u - z^2 r_u)) \right) / 4.
\end{aligned}$$

'Dotted' Box-Diagrams; common factor $\frac{\delta(m_e)}{m_e} \frac{e^2}{(4\pi)^{d/2}} \mathbf{r}_t^{-2} \mathbf{r}_u^{-2}$

$$\begin{aligned}
A_1 = & A_0 \frac{1}{s^2} ((d-4)(-2r_t r_u(1+2x^2 r_t + y^2) - 2z^2 y^4(1-3r_u)) - 4r_t r_u(1+2x^2 r_t + y^2) \\
& + 2r_t(1-y^2) - 2z^2(r_t(1-2y^2) + y^4(2-5r_u))) + B_t \frac{z^2}{s} ((d-4)(-z^2 y^4(1-3r_u)) \\
& + r_t(1-y^2) - z^2((1-2y^2)r_t + y^4(1-2r_u))) + B_0 \frac{z^2}{s} r_t ((d-4)((r_t + y^2)(1-2r_u) \\
& - r_u(1+2x^2 r_t)) + ((r_t + y^2)(1-2r_u) - r_u(1+2x^2 r_t))) + F_t \frac{z^2}{s(d-4)} (r_t(1-y^2) \\
& - z^2(r_t(1-2y^2) + y^4 r_u)) + C_0 z^2 2r_t ((d-4)(-(r_t + y^2)(1-2r_u) + r_u(1+2r_t) \\
& + z^2(2x^2 r_t r_u + y^2(1-2r_u))) + z^2(2x^2 r_t r_u + 1 - r_t + r_u - z^2))/4 \\
& + D_0 z^2 s r_t (-(d-4)z^2 y^2(1-2r_u) + 4r_t r_u^2 + z^2((y^2(1-2r_u) - (1-2r_t(1-r_u))) + z^2))/4,
\end{aligned}$$

$$\begin{aligned}
A_2 = & A_0 \frac{1}{m_e s^2} ((d-4)(-2r_t(r_t + r_u + x^2 r_t r_u + y^2(1+r_u)) + 2z^2 y^4 r_u) - \frac{6}{(d-5)} r_t(y^2(1+r_u) \\
& + r_t) + 2r_t(1-3r_t-2r_u-2x^2 r_t r_u - y^2(4+3r_u)) - 2z^2(r_t - 2y^4 r_u)) \\
& + B_t \frac{4m_e}{s^2} (r_t(1-y^2) + z^2((d-4)y^4 r_u - r_t + y^4 r_u)) + B_0 \frac{4m_e}{s^2} r_t (d-3)(r_t - r_u + y^2 \\
& - 2r_t r_u x^2 + z^2 r_t r_u x^2) + F_t \frac{m_e}{s^2(d-4)} (4r_t((1-y^2) - z^2)) + C_0 \frac{2r_t m_e}{s} ((d-4)(2r_t r_u \\
& - (r_t - r_u) - y^2 + z^2(r_t + y^2 + x^2 r_t r_u)) + z^2(1+r_t + (1+x^2 r_t)r_u - z^2)) \\
& - D_0 2m_e r_t ((d-4)z^2(r_t + y^2)/2 + r_t(1+r_t)r_u + z^2(1-y^2 - r_t(1+r_t+2r_u) \\
& - z^2(1-r_t))/2),
\end{aligned}$$

$$\begin{aligned}
A_4 = & A_0 \frac{1}{m_e^2 s^2} ((d-4)(2r_t(2r_t(3+r_t+2r_u-x^2 r_u) + 4y^2 - r_u(1-5y^2)) - 2z^2(2r_t(r_t + y^2) \\
& + (3+5r_u)y^4)) + \frac{12}{(d-5)} r_t(r_t(3+r_t+r_u) + 2(1+r_u)y^2 - z^2 r_t) + 2r_t(6r_t(3+r_t) \\
& - (3-10r_t+4r_t x^2 - 15y^2)r_u + 12y^2) - 2z^2(4r_t^2 - 2r_t r_u y^2 + (6+9r_u)y^4)) \\
& - B_t \frac{4}{s^2} ((d-4)z^2(2r_t y^2 + (3+5r_u)y^4) + r_t r_u(1-y^2) - z^2(2r_t(r_t + y^2(1+r_u)) \\
& - (3+4r_u)y^4)) + B_0 \frac{4}{s^2} r_t ((d-4)(1-y^2 - z^2(1+2x^2 r_t r_u)) + (1-y^2) \\
& - z^2(1+2x^2 r_t r_u)) - F_t \frac{1}{(d-4)s^2} (4r_t r_u(1-y^2) - 4z^2(r_t(2r_t + 4y^2 + 2y^2 r_u) + r_u y^4)) \\
& - C_0 \frac{2r_t}{s} ((d-4)(1-y^2 - z^2(1-2r_t(2(1+r_u) + r_t - x^2 r_u - z^2) - y^2)) + 2z^2 r_t(2+r_t \\
& - x^2 r_u - z^2)) + D_0 r_t z^2 ((d-4)(2r_t(2+r_t+2r_u-z^2) + y^2) - (1+y^2) + z^2),
\end{aligned}$$

$$\begin{aligned}
A_5 = & A_0 \frac{1}{m_e^2 s} ((d-4)(2r_t^2(r_t^2 + 3r_t + 2 + r_u(2 + 2r_t + 2r_u - x^2 r_t)) \\
& - z^2(r_t(3r_t^2 + 5r_t + 2r_t r_u - y^2(1 + 2r_u)r_u) - z^2(r_t(r_t - y^2) - (1 + r_u)y^4))) \\
& + \frac{3}{(d-5)}(2r_t^2(r_t^2 + 3r_t + 2 + r_u(2 + 2r_u + r_t)) - z^2 r_t(3r_t^2 + 3r_t + r_t r_u \\
& - y^2 2(1 + r_u + r_u^2) - z^2 r_t)) + 2r_t^2(3r_t^2 + 9r_t + 5r_t r_u + 6(1 + r_u + r_u^2) - \\
& 2x^2 r_t r_u) + z^2(r_t(1 + r_u - 22r_t - 14r_t^2 - 6r_t r_u + y^2(3 + 17r_u + 12r_u^2))/2 \\
& + z^2(r_t(8r_t - 2 - 4y^2(1 - r_u)) - 4y^4(2 + r_u))/4) - B_t \frac{z^2}{s} (2(d-4)(2r_t(r_t \\
& + y^2(1 + r_u)) + z^2 y^2(r_t + y^2(1 + r_u))) - r_t(1 + 4r_t^2 + (1 + 4r_t)r_u - 5y^2(1 - r_u)) \\
& + z^2(r_t(1 + 2r_t) - 2y^2(r_t r_u - y^2))) - B_0 \frac{1}{s} (d-3)(8r_t^2(1 + r_t + r_u)r_u + z^2 r_t(1 + 3r_t \\
& + r_u(1 - 4r_t) + 2x^2 r_t(1 + 2r_t)r_u + 2y^2(1 + 2r_u + 2r_u^2) - z^2(1 + 2x^2 r_t r_u))) + \\
& F_t \frac{z^2}{(d-4)s} (r_t(1 + r_u + 4r_t(1 + r_u + r_t) - y^2(1 - 9r_u)) - z^2(r_t(1 + 2r_t - 2y^2(1 + r_u)) \\
& - 2y^4 r_u)) + C_0 r_t ((d-4)(4r_t(1 + r_t + r_u)r_u + z^2((1 + r_u - r_t(1 + 10r_u + 8r_u^2 \\
& + r_t(8 + 8r_u + 4r_t)) + 4x^2 r_t^2 r_u + 2y^2(1 + 2r_u + 2r_u^2))/2 - z^2(1 - 4r_t(1 + r_u) - \\
& 6r_t^2 + y^2(2 + 4r_u + 4r_u^2) + 2z^2 r_t)/2)) - z^2(r_t(r_u + 2r_t(1 + r_t) - 2x^2 r_t r_u) - z^2(1 + r_u \\
& + 3r_t + 6r_t^2 - z^2(1 + 2r_t))/2) + D_0 r_t z^2 s ((d-4)(r_t(1 + 2(r_t + (1 + r_t + r_u)r_u) + r_t^2) \\
& - z^2(r_t(2 + 2r_u + 3r_t) - y^2(1 + 2r_u + 2r_u^2) - z^2 r_t)/2) \\
& - r_t(2 + 2r_t + 2r_t r_u + r_u^2 + 3/2 r_u) + z^2(r_t(3 + 2r_u) - y^2(1 + 2r_u + 2r_u^2))/2),
\end{aligned}$$

$$\begin{aligned}
A_6 = & A_0 \frac{1}{m_e s^2} ((d-4)(2r_t(r_t - x^2 r_t r_u + y^2) - 2z^2 y^4(1 - r_u)) + \frac{6}{(d-5)}(r_t(r_t \\
& + y^2(1 + r_u))) + 2r_t(3r_t - 2x^2 r_t r_u + 3y^2) + 2z^2 y^2(2r_t - y^2(2 - r_u))) - \\
& B_t \frac{4m_e}{s^2} ((d-4)z^2 y^4(1 - r_u) + y^2 r_t r_u - z^2 y^2(2r_t - y^2)) - \\
& B_0 \frac{4m_e}{s^2} (d-3)(r_t(r_t + y^2)r_u + z^2 x^2 r_t^2 r_u) - F_t \frac{4m_e}{(d-4)s^2} (r_t r_u y^2 - z^2 y^2(2r_t - r_u y^2)) + \\
& C_0 \frac{2m_e}{s} r_t ((d-4)((r_t + y^2)r_u - z^2(r_t - r_u(r_t x^2 - y^2))) + r_t r_u - z^2(2r_t - x^2 r_t r_u)) + \\
& D_0 r_t m_e ((d-4)(z^2(r_t + y^2 r_u)) + r_t(1 + 2r_u + 2r_t)r_u + z^2(r_t(1 - r_t - 3r_u) - y^2 r_u + z^2 r_t)),
\end{aligned}$$

$$\begin{aligned}
A_{12} = & -A_0 \frac{2}{s^2} r_t r_u y^2 - B_t \frac{z^2}{s} r_t r_u y^2 - F_t \frac{z^2}{(d-4)s} r_t r_u y^2 + C_0 2z^2 r_t^2 r_u / 4 - D_0 z^2 s r_t^2 r_u / 4,
\end{aligned}$$

One loop integrals occurring in Bhabha scattering

1. Two - point integrals

$$I_G^{(d)} = I_2^{(d)} \Big|_{m_1=0, m_2=0, p^2} = \frac{\sqrt{\pi}}{(-p^2)^{(2-\frac{d}{2})}} \frac{\Gamma(2-\frac{d}{2}) \Gamma(\frac{d}{2}-1)}{2^{d-3} \Gamma(\frac{d-1}{2})}$$

$$I_F^{(d)} = I_2^{(d)} \Big|_{m_1=m, m_2=m, p^2=m^2} = (m^2)^{-(2-\frac{d}{2})} \Gamma\left(2-\frac{d}{2}\right) {}_2F_1\left[1, 2-\frac{d}{2}; \frac{3}{2}; \frac{p^2}{4m^2}\right].$$

2. Three - point integrals

For **two zero masses** and final on-shell momenta we have (from **J. F., F. Jegerlehner and O.V Tarasov, unpublished**)

$$\frac{J_G}{\Gamma(2-\frac{d}{2})} = \frac{(m^2)^{\frac{d}{2}-3}}{2(d-3)} {}_2F_1\left[1, 1; 1-\frac{p^2}{4m^2}; \frac{d-1}{2}\right] - \frac{\sqrt{\pi} \Gamma(\frac{d-2}{2}) (-p^2)^{\frac{d-4}{2}}}{2^{d-2} \Gamma(\frac{d-1}{2}) m^2} {}_2F_1\left[1, \frac{d-2}{2}; 1-\frac{p^2}{4m^2}; \frac{d-1}{2}\right],$$

where

$$J_G = -C_0(0, me, 0, m_e^2, m_e^2, p^2).$$

For the other master three-point integral with **two equal masses** and one zero mass and final on-shell momenta the **Gram determinant is zero**. This integral can be expressed in terms of a propagator integral as shown before. A compact result also is

$$J_F = \frac{\Gamma(2-\frac{d}{2})}{2 m^{6-d}} {}_2F_1\left[1, 3-\frac{d}{2}; \frac{3}{2}; \frac{p^2}{4m^2}\right],$$

where

$$J_F = -C_0(m_e, 0, m_e, m_e^2, m_e^2, p^2).$$