06 Nov 2000: Partial reconstruction of:

CERN-TH. 6447/92

Radiative Corrections at HERA

H. Spiesberger[a], A. Akhundov[b], H. Anlauf[c], D. Bardin[d], J. Blümlein[e], H. D. Dahmen[f], S. Jadach[g], L. Kalinovskaya[d], A. Kwiatkowski[h], P. Manakos[c], T. Mannel[c,i], H.-J. Möhring[j], G. Montagna[k], O. Nicrosini[l], T. Ohl[c,i], W. Płaczek[g], T. Riemann[e], L. Viola[k]

[a] II. Institut für Theoretische Physik, Universität Hamburg, FRG
[b] Academy of Sciences of Azerbaijan, Azerbaijan
[c] Technische Hochschule Darmstadt, FRG
[d] Joint Institute of Nuclear Research, Dubna, Russia
[e] DESY – Institut für Hochenergiephysik, Zeuthen, FRG
[f] Universität Siegen, FRG
[g] Institute of Physics, Jagellonian University, Kraków, Poland
[h] Institut für Theoretische Teilchenphysik, Universität Karlsruhe, FRG
[i] Deutsches Elektronen–Synchrotron DESY, Hamburg, FRG
[j] Sektion Physik, Universität Leipzig, FRG
[k] Dipartimento di Fisica Nucleare e Teorica, Universitá di Pavia, Italy
[l] Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Italy

Abstract: Some basic issues of radiative corrections in deep–inelastic scattering are discussed. Programs for their calculation are presented and compared with respect to their physics input as well as their numerical results.

Contribution to the Workshop on Physics at HERA, October 1991, DESY – to appear in the Proceedings –

CERN-TH. 6447/92
March 1992

Contents

1 Introduction

This article summarizes the basic theoretical background needed for an understanding of radiative corrections in deep-inelastic lepton–nucleon scattering at HERA. Programs for their calculation are reviewed and compared with one another, from the point of view of both their basic physics input and their numerical output. The aim of this article is to describe the present status of the available programs with respect to their range of application, their limitations, and their technical reliability.

In the following first section we will define our notation, discuss the kinematics of radiative deep–inelastic scattering, and repeat some basic formulas. We will review the characteristic features of radiative corrections in deep–inelastic scattering and give a classification of them. Then we will present a comparative inventory of programs as far as they have been presented and discussed during the workshop on HERA physics, held at DESY in 1991. Finally, we will collect numerical results and discuss their technical and theoretical uncertainties.

2 Features of Radiative Corrections

2.1 Kinematics

We consider the processes

$$e(l) + p(p) \to e'(l') + X(p_X), \tag{1}$$

$$e(l) + p(p) \to e'(l') + \gamma(k) + X(p_X), \tag{2}$$

where the momenta are given in parentheses.

The traditional way to define kinematical variables for deep–inelastic lepton scattering is to use the momentum of the final–state lepton:

$$Q_l^2 = -(l - l')^2, \quad x_l = \frac{Q_l^2}{2p \cdot (l - l')}, \quad y_l = \frac{p \cdot (l - l')}{p \cdot l}, \tag{3}$$

with $Q_l^2 = x_l y_l s$ and $s = (p + l)^2$. For the non-radiative process (1) without emission of a photon from the lepton line, Q_l^2 determines the momentum transferred to the hadronic system and x_l can be interpreted, in the parton model, as the fraction of the proton momentum entering the hard scattering process.

For the radiative process (2) and when the photon is emitted from the lepton line (see fig. 1) one has, however, to take into account the fact that the emission of momentum leads to a shift of the kinematical variables. The kinematics at the hadronic vertex has to be described rather by

$$Q_h^2 = -(p_X - p)^2, \quad x_h = \frac{Q_h^2}{2p \cdot (p_X - p)}, \quad y_h = \frac{p \cdot (p_X - p)}{p \cdot l}. \tag{4}$$

It is easy to show that

$$x_h \geq x_l, \qquad y_h \leq y_l. \tag{5}$$

Figure 1: *Notation for radiative lepton–nucleon scattering*

The hadronic momentum transfer Q_h^2 can be larger or smaller than the leptonic Q_l^2 and it is possible that Q_h^2 becomes very small, $Q_h^2 \ll Q_l^2$. Since the magnitude of the neutral–current cross section is determined by the photon propagator $1/Q_h^2$, photon emission with $Q_h^2 \ll Q_l^2$ leads to an enhancement, compared with the photon propagator of the Born cross section $1/Q_l^2$. This is the main reason why radiative corrections to the cross section as a function of leptonic variables can get large, whereas corrections as a function of the true hadronic variables remain of moderate size.

The hadronic variables eq. (4) are not accessible to a direct experimental determination since most of the final–state hadrons contributing to p_X are lost in the beam pipe. In [**?**] it was proposed to use

$$y_{JB} = \frac{p \cdot (\sum_h p_h - p)}{l \cdot p}, \qquad Q_{JB}^2 = \frac{(\sum_h \vec{p}_{T,h})^2}{1 - y_{JB}}, \qquad x_{JB} = \frac{Q_{JB}^2}{y_{JB} s}, \tag{6}$$

where the sum $\sum_h$ extends over all hadrons in the final–state. The determination of x and Q^2 from these equations is not sensitive to an inaccurate measurement of longitudinal momenta in the direction of the incoming proton. Therefore we will not consider in the following the complication due to losses of hadrons in the beam pipe. Then, for non-radiative events, the measurement according to eq. (6) gives results which agree with eq. (3). In the case of radiative events, the measurement following eq. (6) allows a more direct access to the hadronic variables. Of course, it may happen that the photon appears close to the jet of outgoing hadrons without being identified, and it will therefore not be possible to exclude it from the sum over all final–state hadrons. Then the variables in eqs. (6) agree with those of eqs. (3), still assuming that no particle is lost in the beam pipes. In most cases, however, the photon is lost in the beam hole or cannot be separated from the final–state electron. Then one has always $y_{JB} = y_h$, but $Q_{JB}^2 \neq Q_h^2$ in general. For events with a photon collinear to the incoming electron, and parametrizing the photon momentum by $k = (1-z)l$, one finds

$$Q_{JB}^2 = \frac{1 - y_h/z}{1 - y_h} Q_h^2 \leq Q_h^2.$$

Consequently, when fixing the Jaquet–Blondel variables, emission of initial state photons cannot lead to a reduction of the true Q^2 and thus there is no enhancement of radiative corrections.

In [**?**] it was shown that a leptonic measurement of x and y is only feasible in the range $y_l \geq 0.1$. For smaller y and for the charged–current reaction, one has to base the measurement on the hadronic final–state. More recent studies [?] have shown that in a certain kinematical range one obtains the smallest experimental errors when the measurement of the leptonic final–state is combined with that of the hadron flow. It was proposed [?] to consider a mixed definition of kinematic variables where the momentum transfer is determined from the lepton measurement and y from the hadronic final–state, thus defining so–called mixed variables:

$$Q_m^2 = Q_l^2, \qquad y_m = y_h, \qquad x_m = \frac{Q_m^2}{y_m s}. \tag{7}$$

This possibility is simple enough to be treated analytically and has been implemented in several analytical programs. Eventually, more refined prescriptions will be necessary, but those can be treated only with the help of Monte Carlo programs.

The distinction between leptonic and hadronic variables, and a precise definition of how the measurement of kinematic variables are performed is essential in the discussion of leptonic radiative corrections and has substantial influence on the numerical results. In most cases we will specify explicitly which definition has to be used. It is only in cases where the difference between various definitions is not relevant, that we will use the generic notation x, y and Q^2 without indices l, h, m, or JB.

2.2 Lowest–order cross sections

In order to define our notation, we repeat here the Born-level expressions for the NC and CC cross sections. For scattering of electrons or positrons via the neutral–current, the cross section can be written in the following form:

$$\frac{d^2\sigma_{NC}^{\pm}}{dxdy} = \frac{2\pi\alpha^2}{xyQ^2}\left(Y_+\mathcal{F}_2(x,Q^2) \mp Y_- x\mathcal{F}_3(x,Q^2) - y^2\mathcal{F}_L(x,Q^2)\right), \tag{8}$$

with

$$Y_\pm = 1 \pm (1-y)^2, \tag{9}$$

and

$$\mathcal{F}_L(x,Q^2) = \mathcal{F}_2(x,Q^2) - 2x\mathcal{F}_1(x,Q^2). \tag{10}$$

Upper and lower signs refer to positron and electron scattering, resp. The structure functions $\mathcal{F}_{1,2,L}(x,Q^2)$ coincide with the usual electromagnetic ones for low Q^2, but for large momentum transfer they contain contributions from the weak neutral current described by Z exchange diagrams and the structure function $\mathcal{F}_3$ is no longer negligible:

$$\begin{aligned}
\mathcal{F}_{1,2}(x,Q^2) &= \sum_{i,j=\gamma,Z} \chi^i(Q^2)\chi^j(Q^2)(\lambda_V^{eij} - P_L\lambda_A^{eij})F_{1,2}^{ij}(x,Q^2), \\
\mathcal{F}_3(x,Q^2) &= \sum_{i,j=\gamma,Z} \chi^i(Q^2)\chi^j(Q^2)(\lambda_A^{eij} - P_L\lambda_V^{eij})F_3^{ij}(x,Q^2).
\end{aligned} \tag{11}$$

P_L is the degree of longitudinal polarization of the leptons and as abbreviations we used

$$\chi^\gamma = 1, \qquad \chi^Z(Q^2) = \frac{Q^2}{Q^2 + M_Z^2}, \tag{12}$$

and

$$\lambda_V^{fij} = v_f^i v_f^j + a_f^i a_f^j, \qquad \lambda_A^{fij} = v_f^i a_f^j + a_f^i v_f^j; \tag{13}$$

v_f^i and a_f^i are the vector and axial vector coupling constants of the fermions f to the Z–boson, given by their charge Q_f and isospin I_3^f:

$$\begin{aligned} v_f^\gamma &= -Q_f, \quad v_f^Z = \frac{I_3^f - 2s_W^2 Q_f}{2s_W c_W}, \\ a_f^\gamma &= 0, \qquad a_f^Z = \frac{I_3^f}{2s_W c_W}. \end{aligned} \tag{14}$$

The weak mixing angle is determined by the gauge–boson masses:

$$c_W = \cos\theta_W = \frac{M_W}{M_Z}, \quad s_W^2 = 1 - c_W^2. \tag{15}$$

One can also parametrize the Z–boson couplings in terms of the μ decay constant G_μ using in lowest–order the relation

$$\frac{1}{2s_W c_W} = \sqrt{\frac{G_\mu M_Z^2}{2\sqrt{2}\pi\alpha}}. \tag{16}$$

For a consistent treatment of electroweak radiative corrections one has to take into account also corrections to the relations between electroweak parameters. This will modify eq. (16) by the inclusion of radiative corrections to the μ decay constant, parametrized with the help of Δr [?]. The prescription for the calculation of radiative corrections based on (16) is often called the 'modified on-mass-shell scheme' (MOMS).

In the parton model, the structure functions are expressed with the help of quark distribution functions:

$$\begin{aligned} F_2^{ij}(x,Q^2) &= x\sum_f (q_f(x,Q^2) + \bar{q}_f(x,Q^2))\lambda_V^{fij}, \\ F_3^{ij}(x,Q^2) &= \sum_f (q_f(x,Q^2) - \bar{q}_f(x,Q^2))\lambda_A^{fij}, \end{aligned} \tag{17}$$

and the longitudinal structure function vanishes:

$$\mathcal{F}_L(x,Q^2) = 0. \tag{18}$$

The lowest–order cross section for the charged–current reaction reads, in the parton model:

$$\frac{d^2\sigma_{CC}^\pm}{dxdy} = \frac{\pi\alpha^2}{4s_W^4}\frac{(\chi^W(Q^2))^2}{xyQ^2}\left\{Y_+ W_2^\pm(x,Q^2) \mp Y_- xW_3^\pm(x,Q^2)\right\}. \tag{19}$$

Here,

$$\chi^W(Q^2) = \frac{Q^2}{Q^2 + M_W^2}, \tag{20}$$

and the parton distribution functions enter in the combinations

$$\begin{aligned} W_2^+ &= d + s + b + \bar{u} + \bar{c}, \\ W_2^- &= u + c + \bar{d} + \bar{s} + \bar{b}, \\ xW_3^+ &= -(d + s + b) + \bar{u} + \bar{c}, \\ xW_3^- &= -(u + c) + \bar{d} + \bar{s} + \bar{b}. \end{aligned} \tag{21}$$

A possible contribution from W_L is neglected here, as is the effect of quark mixing. The normalization of the charged–current cross section can also be rewritten with the help of the muon decay constant using (in lowest–order)

$$\frac{\pi\alpha^2}{4s_W^4} = \frac{G_\mu^2 M_W^4}{2\pi}. \tag{22}$$

2.3 Classification of radiative corrections

The diagrams describing the $\mathcal{O}(\alpha)$ corrections to the NC cross section are shown in fig. 2 (virtual one-loop corrections) and fig. 3 (real single–photon bremsstrahlung). We are not going to present the complete set of formulas for the $\mathcal{O}(\alpha)$ radiative corrections. They can be found in the literature [**?**]–[**?**]. Instead we only discuss some important features [**?**]:

In the case of NC scattering, a **classification** of the various contributions to the radiative corrections can be obtained in terms of Feynman diagrams:

Figure 2: *One–loop diagrams for neutral–current lepton–quark scattering*

Figure 3: *Single–photon bremsstrahlung diagrams for neutral current lepton–quark scattering*

i) The *leptonic corrections* are described by diagrams containing an additional photon attached to the lepton line, i.e. the photonic lepton vertex correction combined with the self energies of the external fermion lines, fig. 2a, and the photon emission from the lepton line, fig. 3a.

ii) The *quarkonic corrections*, represented by diagrams with an additional photon at the quark line analogous to *i*) (diagrams of figs. 2b, 3b). This part is proportional to the square of the quark charge.

iii) The *lepton–quark interference* part consisting of the $\gamma\gamma$ and γZ box diagrams (fig. 2c) and the interference of the bremsstrahlung diagrams of figs. 3a with those of figs. 3b. This contribution is proportional to the quark charge. Finally we have

iv) the *purely weak corrections*, described by all the other diagrams that do not contain an additional photon. This part is infra-red finite and contains the diagonal γ and Z self energies (figs. 2d, e), the γZ mixing (fig. 2f), the weak lepton and quark vertex corrections (fig. 2g), and the boxes with two heavy gauge–bosons (fig. 2h).

Each loop diagram contributing to the $\mathcal{O}(\alpha)$-corrected matrix element can be written in terms of **form factors** $F_i(s,x,y)$ [**?**]. Using the common short-hand notation for the products of lepton and quark currents such as $\gamma_\mu \otimes \gamma^\mu = \bar{u}(l')\gamma_\mu u(l) \cdot \bar{u}(q')\gamma^\mu u(q)$, the matrix elements can be written as

$$\mathcal{M} \simeq F_1\gamma_\mu \otimes \gamma^\mu + F_2\gamma_\mu \otimes \gamma^\mu\gamma_5 + F_3\gamma_\mu\gamma_5 \otimes \gamma^\mu + F_4\gamma_\mu\gamma_5 \otimes \gamma^\mu\gamma_5. \tag{23}$$

The form factors F_i are gauge–invariant and depend on the process and on kinematic variables. Their presence does not change significantly the structure of the Born cross section. They can be used to define an improved or dressed Born cross section which would then include electroweak loop corrections. There are several ways of grouping these corrections together in the cross-section formula; e.g. one could sum several pieces by writing $(1+\sum_i \delta_i)$ or, instead, one could multiply different correction factors $\Pi_i(1+\delta_i)$ introducing thereby additional terms which are formally of higher order. Numerically, higher–order terms of this type, from purely weak one–loop diagrams, may reach the level of a few per mille at HERA.

The **self-energy diagrams** (fig. 2d–f) contain loop diagrams which are built with all particle degrees of freedom that couple to the gauge bosons. Therefore they contain information on the whole theory. They depend on the top mass, the Higgs mass, and on the masses and couplings of eventually existing other unknown particles.

The self energies are dominated by **fermion loops**. This contribution is sometimes refered to as a QED part. The fermion loops of the photon self energy (the vacuum polarization) can be taken into account by the use of the running fine-structure constant which sums the leading logarithms to all orders:

$$\alpha(Q^2) = \frac{\alpha(0)}{1-\Pi_f^\gamma(Q^2)}; \tag{24}$$

$\Pi_f^\gamma = \hat{\Sigma}_f^\gamma/Q^2$ is the fermionic part of the vacuum polarization derived from the renormalized photon self energy $\hat{\Sigma}^\gamma$ and $\alpha(0) = 1/137.036$. Its hadronic part is connected to the total hadronic cross section in e^+e^- annihilation. At $Q^2 \simeq M_Z^2$ the value of Π_f^γ, including hadronic and leptonic contributions, is $\Pi_f^\gamma \simeq 0.06$.

In the amplitude for the photon-exchange diagrams, $\alpha(Q^2)$ has to be used; but of course not in the contributions of the weak amplitudes[1]. There one can combine the fermionic contributions of the Z and W self energies $\Pi_f^Z = \hat{\Sigma}_f^Z(Q^2)/(Q^2+M_Z^2)$ and $\Pi_f^W = \hat{\Sigma}_f^W(Q^2)/(Q^2+M_W^2)$ with the correction $1-\Delta r$ (coming from rewriting the coupling constants in terms of the μ decay constant) into a running ρ parameter. For the NC reaction one defines

$$\rho_{NC} = \frac{1-\Delta r}{1-\Pi_f^Z(Q^2)}, \tag{25}$$

and, similarly, the charged–current (CC) amplitude is modified by the factor

$$\rho_{CC} = \frac{1-\Delta r}{1-\Pi_f^W(Q^2)}. \tag{26}$$

For more details, especially with respect to the treatment of leading higher–order corrections connected to a heavy top quark, see [?].

The **photonic corrections** include contributions from the bremsstrahlung process (2) which has to be integrated over that phase–space region where the additional photon cannot be identified experimentally. In practice, the definition of the relevant phase–space boundaries is very complicated and requires a detailed understanding of detector properties. For simplicity one can assume that no attempt at all is made in the first stage of a physics analysis to single out radiative events. Therefore we consider the integration over the complete phase–space of process (2).

The **leptonic radiation**, i.e. the bremsstrahlung of photons from the lepton line, can be described in a model-independent framework using structure functions $\mathcal{F}_{1,2,3}$ as input [?]. For radiative corrections to the differential cross section defined in terms of leptonic variables, the corresponding contribution can be written in the form

$$\frac{d^2\sigma_{lep}}{dx_l dy_l} = \frac{\alpha^3 y_l}{\pi}\int \frac{dx_h}{x_h}\frac{d\phi_1 d\phi_2}{\sqrt{-\Delta_4}}\left\{\mathcal{F}_1(x_h,Q_h^2)R_1 + \mathcal{F}_2(x_h,Q_h^2)R_2 \mp \mathcal{F}_3(x_h,Q_h^2)R_3\right\}; \tag{27}$$

ϕ_1 and ϕ_2 are internal phase–space variables which can be either two of the following: $t' = -Q_h^2$, $z_1 = 2l\cdot k$, or $z_2 = 2l'\cdot k$. The Gram determinant Δ_4 has to be expressed in terms of ϕ_1 and ϕ_2 accordingly. The coefficient functions R_i describe the emission of a photon from the lepton line and depend on the kinematic variables ϕ_i as well as on x_l, y_l, and x_h.

As is seen from eq. (27), the phase–space integration involves explicitly the arguments of the structure functions (note that $Q_h^2 = Q_l^2 + z_1 - z_2$). Therefore only one integration in (27) can be performed analytically. This complication can be avoided if x_h and Q_h^2 are used as fixed external variables, which applies to the calculation of corrections to the cross section $d^2\sigma/dx_h dy_h$, defined in terms of hadronic variables.

In the parton model there is also a bremsstrahlung contribution described by **radiation from the quark line**. Its calculation reveals the appearance of quark–mass singularities, very much as is the case for gluonic corrections. These singular pieces can be factorized and have to be absorbed into the quark distribution functions [?]. The remaining parts are known exactly to order $\mathcal{O}(\alpha)$ and are small. In the leading–logarithmic approximation the quarkonic corrections lead to an additional contribution

[1] Note also that the additional factor α coming with real photonic corrections has to be read as $\alpha(0)$ since there the characteristic momentum squared is that of the real photon.

to the Altarelli–Parisi splitting functions and result in a small modification of the Q^2 dependence of parton distribution functions [?, ?].

The **lepton–quark-interference** of radiation from the lepton and the quark line, combined with the $\gamma\gamma$ and γZ box diagrams, is non-singular and in general small. This contribution is however dominant in the corrections to the charge asymmetries, since it is odd under charge conjugation.

There is a similar classification of radiative corrections to the **charged–current** reaction. In this case, the separation of leptonic, quarkonic, and interference parts in terms of Feynman diagrams is possible in a physical gauge only. The separation can be understood most easily by investigating the dependence on the charges of external fermions. A gauge–invariant way to separate photonic and weak corrections has been described in [?].

In the **leading–logarithmic approximation**, leptonic QED corrections can be further separated into contributions coming from the integration over specific phase–space regions: the region with photons collinear with the initial lepton, $k \cdot l \to 0$, leads to a contribution from 'initial-state radiation', whereas 'final–state radiation' is connected to the situation where photons are emitted collinearly with the final–state lepton, $k \cdot l' \to 0$. There is a third contribution, the so-called Compton part [?, ?], which is characterized by quasi-on-shell virtual photons $Q_h^2 \to 0$. Due to the photon propagator $1/Q_h^2$ and since the lower kinematical limit for Q_h^2 with fixed leptonic variables x_l, Q_l^2 is of the order of the proton mass m_p, the magnitude of this latter contribution is determined by the potentially large logarithm $\ln(Q_l^2/m_p^2)$. The Compton part is especially important when corrections are calculated as a function of the leptonic variables. It is an important source of uncertainty in the predictions of radiative corrections, since the experimental knowledge of the proton structure functions for $Q_h^2 \to 0$ is not fully exploited in most of the commonly used parametrizations[2]. The Compton part is characterized by a clean experimental signature. In the limit $Q_h^2 \to 0$, the transverse momentum of the emitted photon is balanced by that of the scattered lepton. Therefore it is hoped that one will have a direct access to its magnitude also by a measurement at HERA.

The leptonic leading logarithms are universal and can therefore actually also be used for an estimate of corrections to processes other than deep–inelastic scattering, to the extent that these processes proceed via one-boson exchange.

3 Programs for the Calculation of Radiative Corrections

3.1 Standards for radiative corrections programs

The best program for the calculation of radiative corrections that can be imagined would have a number of superlative properties: it would be complete with respect to processes that can be described (NC and CC deep–inelastic scattering, jet production, heavy–flavour production, photoproduction, processes beyond the standard model, etc.) and complete

[2]Only recently a set of structure function parametrizations became available, which take into account also data from very low Q_h^2 and from photoproduction [?]; however, in most of the radiative–corrections programs, these parametrizations are not yet implemented.

with respect to radiative corrections (photonic corrections, weak corrections, higher–order contributions, QCD corrections). The 'best' program should combine high speed with high precision, and it should be flexible with respect to input, options, experimental cuts, etc. while being at the same time simple in use. These requirements obviously conflict with one another, and such a program clearly does not exist. Each of the programs described in the following may meet a subset of the above-mentioned properties, but none of them is complete. Each of them has a certain range of application where it is superior to others and in total they complement each other.

There are two types of programs: first we will describe those programs which could be called semi-analytical. They are based on formulas obtained analytically, including an analytical one- or twofold integration over the photon phase–space, and work with some numerical-integration algorithm to obtain the fully integrated radiative cross section. As already mentioned, this final integration involves an integration over the structure functions and can thus only be performed numerically. The second type of programs are Monte Carlo event generators simulating the fully differential cross section. The semi-analytical programs are usually much faster and can reach much higher precision than the Monte Carlo programs, which are more flexible with respect to phase–space cuts.

The program descriptions given below concentrate on the most important aspects relevant for their mutual comparison. More details can be found in the separate manuals contained in volume III of the HERA Workshop Proceedings.

3.2 Analytical programs

3.2.1 TERAD91

TERAD91 is a semi-analytic code for QED and weak corrections to deep–inelastic NC and CC scattering at HERA. Version 2.10 was released on 3 Oct. 1991. The source of TERAD91 originates from four different codes: TERAD, DISEPNC, DISEPCC, and DIZET, which will be discussed in what follows.

TERAD [?]
The first investigations of the TERAD-approach to QED may be found in refs. [?, ?]. The roots of the present code date back to 1986 (TERAD86 [?]: leptonic model-independent QED corrections to fixed-target DIS in leptonic variables with γ-exchange only).
Some of its very recent developments are: the additional possibility $i)$ to use mixed and hadronic variables, including also Z-exchange, and $ii)$ to apply some cuts. The output produced by TERAD is the cross section as chosen by flags (see [?, ?]):

$$\frac{d^2\sigma}{d\mathcal{E}} \sim \frac{2\alpha^3}{\pi}\int d^2\mathcal{I}\,\frac{d^4\Sigma(\mathcal{E},\mathcal{I})}{dy_l dQ_l^2 dy_h dQ_h^2}\frac{1}{Q_h^4} = \frac{2\alpha^3}{\pi}\int d^2\mathcal{I}\,\frac{S(\mathcal{E},\mathcal{I})}{Q_h^4}. \tag{28}$$

Here, $\mathcal{E}$ is a set of two variables on which $d^2\sigma$ finally depends. In case of leptonic, mixed, hadronic variables:

$$\mathcal{E} = \{y_l, Q_l^2\}, \{y_h, Q_l^2\}, \{y_h, Q_h^2\}. \tag{29}$$

$\mathcal{I}$ is a corresponding set of two variables to be integrated over:

$$\mathcal{I} = \{y_h, Q_h^2\}, \{y_l, Q_h^2\}, \{y_l, Q_l^2\}. \tag{30}$$

Within TERAD, two (for leptonic variables) or only one (for mixed and hadronic variables) integrations are left to be performed numerically[3]. The function $S(\mathcal{E},\mathcal{I})$ is the result of a one-dimensional analytic integration,

$$\begin{aligned} S(\mathcal{E},\mathcal{I}) &\equiv \frac{1}{\pi}\int \frac{dz_{1,2}}{\sqrt{-\Delta_4}} S(\mathcal{E},\mathcal{I},z_{1,2}) \\ &= A_1(x_h,Q_h^2)S_1(\mathcal{E},\mathcal{I}) + A_2(x_h,Q_h^2)S_2(\mathcal{E},\mathcal{I}) + A_3(x_h,Q_h^2)S_3(\mathcal{E},\mathcal{I}), \end{aligned} \tag{31}$$

where the variables $z_1 = 2l\cdot k$ and $z_2 = 2l'\cdot k$ are related to a photon angle; Δ_4 is the Gram determinant. The $A_i(x_h,Q_h^2)$ are combinations of structure functions F_i^{mn} describing the electroweak interactions of $m,n = \gamma, Z$ with nucleons as introduced in section 2.2 [see eqs. (11) and (17)].

The double–differential NC cross section may be obtained in leptonic, mixed and hadronic variables. TERAD does not necessarily rely on the quark parton model and calculates QED radiation from the leptonic legs to order α. Soft–photon exponentiation is included. For the cross section in leptonic variables a simultaneous cut on energy and polar angle of the emitted photon may be applied.

Weak corrections can be taken into account through definite modifications of coupling constants in a similar way as in the DISEP part. Longitudinal polarization of the lepton beam is allowed.

DISEPNC [?], DISEPCC [?]
This part calculates the complete electroweak radiative corrections – i.e. QED plus weak loop corrections – to NC and CC scattering in the quark parton model. Output of the DISEP branches are the cross sections as chosen by flags:

$$\frac{d^2\sigma}{dx_l dy_l} = \frac{2\pi\alpha^2 s x_l}{Q_l^4} \sum_B \sum_b \sum_{Q,\bar{Q}} c_b K(B)[V(B)R_b^V(B) \pm A(B)R_b^A(B)], \tag{32}$$

where the different sums extend over: quarks Q and anti-quarks $\bar{Q}$; photon and Z exchange, $B = \{(\gamma\gamma),(\gamma Z),(ZZ)\}$; Born cross section ($b = 0$), and bremsstrahlung from leptons, lepton–quark interference, and quarks ($b = \{e,i,q\}$). For the NC cross section from the (γ,Z) part of lepton radiation, e.g. the formula contains the following pieces:

$$c_e = Q_e^2, \quad V(\gamma,Z) = v_{eQ}(s,Q_l^2) - P_L v_e(s,Q_l^2)a_Q(s,Q_l^2) \sim v_e v_Q - P_L v_e a_Q, \tag{33}$$

$$K(\gamma,Z) = 2|Q_eQ_Q|F_A(Q_l^2)\chi(Q_l^2)\rho(s,Q_l^2). \tag{34}$$

The weak corrections, contained in ρ, v_e, v_Q, v_{eQ} and the running α, obtained from F_A, are delivered from DIZET. The longitudinal polarization of the electron beam is P_L. The QED (or photonic) corrections are contained in the functions $R_b^{V,A}(B)$, e.g.:

$$R_e^V(\gamma,Z) = R_e(\gamma,Z;1,1-y_l) + R_e(\gamma,Z;y_l-1,-1), \tag{35}$$

$$\begin{aligned} R_e(B;a,b) = \frac{\alpha}{\pi}\Bigg\{ & S_e q(x_l,Q_l^2) \\ & + \int_{x_l}^1 dx_h \left[T_e \frac{x_h q(x_h,Q_l^2) - x_l q(x_l,Q_l^2)}{x_h - x_l} + U_e \frac{x_l}{x_h} q(x_h,Q_l^2) \right] \Bigg\}. \end{aligned} \tag{36}$$

[3]In principle, for hadronic variables one could perform the complete threefold integration analytically.

The functions S_e, T_e, and U_e depend on B, a, b. They are results of a twofold analytic integration over photonic angles, and q are the quark distribution functions.

The DISEP branches take into account the QED corrections to the double-differential NC and CC cross sections in leptonic variables, for the NC case with soft–photon exponentiation. The quark parton model is used. Besides leptonic leg radiation, also quark-leg radiation and the interference are included. No cuts may be applied.

Note that in the DISEP approach the factorization scale used in the parton distribution functions is taken to be the lepton momentum transfer. This was chosen in order to enable an analytic treatment of two of the phase–space integrations. Only one integration has to be performed numerically, which makes the DISEP branch of the program very fast. One has to keep in mind, however, that from considerations of higher–order corrections, only the choice Q_h^2 is justified for the part describing leptonic radiation. Numerical differences between the TERAD and DISEP branches will therefore occur and are non-negligible at large y_l and small x_l.

Weak corrections library DIZET v.4.04 [?]
Both branches of the package may use the recently updated weak library DIZET. It allows the calculation of Δr and the complete $O(\alpha)$ corrections (with the inclusion of some leading higher–order terms connected with the top-quark) to both the Z, W widths and the weak form factors for cross sections in the s- and t-channels. These form factors depend on the kinematics and on the fermion type of the scattering particles.

3.2.2 EPRC91

EPRC91 is a package of programs for the calculation of the complete $\mathcal{O}(\alpha)$ electromagnetic and weak corrections to NC and CC deep–inelastic $e^{\pm}p$ scattering. It is based on earlier publications [?, ?, ?] and was developed further during the Workshop. The prescriptions realized in EPRC91 rely on the parton-model approach. The kinematic variables are defined from the final–state lepton momentum. In EPRC91, a twofold numerical integration over the full photon phase–space is performed, allowing for Q^2-dependent parton distribution functions. For the charged–current reaction, exponentiation of soft–photon corrections can be included. The polarization of the lepton beam can be non-zero.

The package includes routines for the evaluation of NC and CC cross sections, the ratio $R_{NC/CC}$ of NC to CC cross sections, and charge or polarization asymmetries, including total or partial electroweak radiative corrections. Routines for the evaluation of leptonic QED corrections in the leading–logarithmic approximation are available. In this case, the user can choose among the leptonic, mixed, hadronic, or Jaquet–Blondel definition of kinematic variables. The package includes also routines for the calculation of the leading–logarithmic QED corrections to second order [?].

The weak corrections library of EPRC91 has become a part of the Monte Carlo generator HERACLES.

Those parts of EPRC91 needed for the calculation of the complete $\mathcal{O}(\alpha)$ QED corrections are rather time-consuming and are now used for test purposes only. The program package may be useful, however, for the study of purely weak corrections, e.g. the dependence of radiative corrections on electroweak parameters such as the top mass or the Higgs mass. Moreover, the leading–log routines are helpful in obtaining a fast estimate of radiative corrections to $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha^2)$.

3.2.3 HELIOS

HELIOS is a program for the calculation of the $\mathcal{O}(\alpha)$ electromagnetic corrections to NC and CC deep–inelastic $e^{\pm}$–nucleon scattering in the leading–logarithmic approximation. It is based on already published work [?, ?], and a detailed description of the program is given in [?].

The contributions from initial– and final–state lepton bremsstrahlung to the leading–logarithmic corrections are derived from

$$\frac{d^2\sigma_e^{(1)}}{dx_i dy_i} = \frac{\alpha}{2\pi} \ln\left(\frac{Q_l^2}{m_e^2}\right) \int_0^1 dz \frac{1+z^2}{1-z} \left\{ \theta(z-z_i^0) |J(x_i, y_i, s)| \left. \frac{d^2\sigma_0}{dx_i dy_i} \right|_{x_i=\hat{x}_i, y_i=\hat{y}_i, s=\hat{s}} - \frac{d^2\sigma_0}{dx_i dy_i} \right\} \tag{37}$$

where $d\sigma_0/dx_i dy_i$ denotes the respective Born cross section and $(x_i,\ y_i)$ is one of the sets of kinematic variables of eqs. (3), (6), (7). The lower limit z_i^0 of the z integration is a function of x_i, y_i and depends on the choice of the kinematic variables; $J(x_i, y_i, s)$ is the Jacobian relating the variables $\hat{x}_i$, $\hat{y}_i$ to x_i, y_i [?, ?]. The Compton contribution to the NC cross section is described by

$$\frac{d^2\sigma^C}{dx_l dy_l} = \frac{\alpha^3}{S x_l} \sum_f \ln\left(\frac{Q_l^2}{\Lambda^2}\right) \int_{x_l}^1 \frac{dz}{z^3} z \left[q_f(z, Q_l^2) + \bar{q}_f(z, Q_l^2) \right] \frac{z^2 + (x_l - z)^2}{x_l(1-y_l)} (1 + (1-y_l)^2). \tag{38}$$

Here it is assumed that the photon content of the proton factorizes into quark distribution functions $q_f(x, Q^2)$ and the $q \to q + \gamma$ splitting function (the Altarelli–Parisi splitting function); Λ is the QCD scale; the Compton term can also be parametrized by $\ln s/m_p^2$ instead of $\ln Q_l^2/\Lambda^2$.

The program offers the optional calculation of the leading–logarithmic QED corrections to $d^2\sigma/dxdy$ for collisions of unpolarized and polarized electrons or positrons with protons and deuterons. The parton distributions may be chosen in a wide variety of parametrizations [?].

For the calculation of $d^2\sigma/dxdy$, the contribution of the longitudinal structure functions for NC and CC reactions may be included in first–order QCD also [?]. Furthermore, also the full first order QCD corrections in the DIS scheme are available [?]. In order to account for the typical behaviour of the scattering cross sections as $Q^2 \to 0$ two options, following [?, ?], are accessible.

One may choose different definitions of the kinematical variables x_i and y_i for the radiatively corrected cross section: *i)* measurement of the kinematical variables from the scattered lepton, *ii)* from the full electromagnetic final–state for NC reactions (i.e. excluding final–state radiation and the Compton–peak contribution), *iii)* from a measurement of the hadronic final–state using the Jaquet–Blondel method, and *iv)* using the mixed variables (Q_m^2, y_m).

Since only lepton-leg corrections are treated, any $e^{\pm}N$ reaction proceeding via the exchange of one gauge–boson may be investigated with this program on the basis of user–supplied differential CC and NC scattering cross sections. As an illustration the program includes the QED corrections in the leading–logarithmic approximation to neutral–current $Q\bar{Q}$ production in the Weizsäcker–Williams approximation.

3.2.4 APHRODITES

In APHRODITES, version 1.00 [?], higher–order QED leptonic corrections to deep–inelastic electron–proton scattering at HERA energies are implemented according to the structure-function formalism. In this approach the neutral–current corrected cross-section can be obtained by folding the corresponding Born cross section with the electron structure functions as [?]:

$$\begin{aligned} \frac{d\sigma}{dx_l dy_l}(x_l, y_l) = & \int_\Omega dz_1 dz_2 D(z_1, Q^2)[D(z_2, Q^2) \cdot C(E_f/z_2; \delta)] \\ & \times \frac{d\sigma_0}{d\hat{x}_l d\hat{y}_l} \left(\hat{x}_l(z_1, z_2), \hat{y}_l(z_1, z_2)\right) J(z_1, z_2)\Theta(cuts). \end{aligned} \tag{39}$$

We list here only the meaning of the terms entering eq. (39). For their explicit expressions see [?].

(*i*) $d\sigma_0/d\hat{x}_l d\hat{y}_l$ is the lowest–order cross section evaluated with the reduced variables $\hat{x}_l$ and $\hat{y}_l$; the latter depend on the longitudinal momentum fractions z_1 and z_2 of the incoming and outgoing electrons, respectively, after photon radiation. The fractions z_1 and z_2 range inside the radiation phase–space Ω delimited by kinematical and experimental cuts ($\Theta(cuts)$).

(*ii*) J is the Jacobian of the tranformation from reduced $(\hat{x}_l, \hat{y}_l)$ to observable (x_l, y_l) variables.

(*iii*) $D(z, Q^2)$ are the QED structure functions for the initial– and final–state electron giving the probability of finding, inside a parent electron, an electron with longitudinal momentum fraction z and virtualness Q^2. For $D(z, Q^2)$ we use the expression obtained in [?]. They include the following QED corrections:

- Complete $\mathcal{O}(\alpha)$ leptonic radiation;
- Soft–photon exponentiation;
- Isotropic hard bremsstrahlung at $\mathcal{O}(\alpha^2)$ in the leading–log approximation.

For the scale Q^2 entering the electron structure functions the leptonic squared momentum transfer Q_l^2 is chosen.

(*iv*) The function $C(E_f; \delta)$ describe hard collinear photons emitted by the final electron of energy E_f within a cone of half-opening angle δ and which cannot be resolved in a calorimetric experiment because of the finite angular resolution. APHRODITES uses for C an exponentiated form of the expression derived in [?].

Compton subprocesses are implemented in APHRODITES, as explained in [?].

APHRODITES is installed on the VAX 6410 at the University of Pavia and is available on request from the authors. A more complete version of the program, including weak loops and the QED corrections due to fermion–pair production, is planned. The extension to CC scattering is also under consideration.

3.3 Monte Carlos

The Monte Carlo programs presented in the following have been developed in order to solve the specific problems related to radiative corrections at HERA. They do not focus on a description of the hadronic final–state in deep–inelastic scattering. Early versions of them were based on the parton model and generated events at the parton level, leaving the final–state quark unhadronized. Only later, for two of them, have interfaces been developed; these call routines from LEPTO and JETSET for the fragmentation and hadronization of the scattered quark and the proton remnant. Recent versions of HERACLES and KRONOS have an option to calculate the leptonic QED corrections in a model–independent framework where the cross section is described by structure functions $\mathcal{F}_{1,2,3}$. With these options, the programs are suited for a simulation of an electron-inclusive measurement and applicable also for very small x_l and Q_l^2, where a Monte Carlo treatment of the hadronic final state is not straightforward.

All programs apply a rejection technique to the cross section written in terms of variables in which there is a smooth dependence. The most important differences between the programs result from different choices and ordering of the independent variables. HERACLES starts the event generation by choosing the leptonic variables x_l and Q_l^2. Therefore the phase space can be restricted by cuts on these variables without losing efficiency and the program is suited for a calculation of radiative corrections to the differential cross section in terms of leptonic variables. In contrast to this, LESKO-F and KRONOS start with hadronic variables x_h and Q_h^2. The latter programs are therefore better suited for a calculation of corrections as a function of the hadronic variables, but they lose efficiency when they are used for a calculation of corrected differential cross sections in terms of leptonic variables, very much like HERACLES would become less efficient when used for a calculation of corrections in terms of hadronic variables.

3.3.1 HERACLES

The event generator HERACLES, for the simulation of NC and CC deep–inelastic lepton–nucleon scattering, includes the leptonic corrections, as well as radiation from the quark line and interference of leptonic and quarkonic radiation, and the complete one-loop virtual corrections using the results of refs. [?, ?, ?]. Version 4.1 of HERACLES is described in these proceedings [?] (see also [?]).

The most important capabilities of the generator that are relevant for the discussion of radiative corrections are:

- It allows integration of the differential cross sections for $\ell p \to \ell' X$ and $\ell p \to \ell' \gamma X$ over kinematical regions which can be given in terms of the leptonic variables x_l, y_l, Q_l^2 and the mass of the hadronic final–state. Also, in the calculation of the cross section for $\ell p \to \ell' \gamma X$ it is possible to limit the phase–space by requiring a minimal value for the photon energy E_γ. These kinematical boundaries are implemented in such a way that no efficiency is lost. Of course, any other phase–space restrictions can be applied. However, since these other kinematical restrictions are realized by rejecting events, the efficiency of the algorithm is reduced.

- The structure of the program allows a separate treatment of the Born term and several parts of the QED corrections (comprising soft and hard bremsstrahlung and the corresponding virtual corrections). These parts describe, for the neutral–current reaction, leptonic initial–state radiation, leptonic final–state radiation, a contribution called Compton part, and quarkonic radiation. In the case of the charged–current reaction, there is at present only one radiative channel describing radiation from the initial lepton.

- Several switches allow the $\mathcal{O}(\alpha)$ electroweak virtual corrections (self energies, vertex corrections, and box diagrams) to be included into the non-radiative cross section. If only a moderate accuracy is required, one can turn off the weak contributions thereby saving some CPU-time. There is also a switch that determines whether the electroweak parameters are calculated from fixed Z and W boson masses M_Z, M_W, or from fixed M_Z and the μ decay constant G_μ.

- The program describes electron as well as positron scattering, and allows for polarized leptons.

- The user can choose from a set of parametrizations for input quark distributions used in the parton-model approach or for structure functions in a model-independent description of leptonic corrections. In particular, a non-zero longitudinal structure function F_L can be included. Note that in this case it is not possible to include quarkonic radiation and its interference with leptonic radiation. The quark distribution functions may be Q^2–dependent and thus include leading–logarithmic QCD corrections. Other $\mathcal{O}(\alpha_S)$ corrections are not included, however.

- HERACLES itself generates events at the parton level. The interface DJANGO [?] can be used to call routines from LEPTO 5.2 and JETSET 6.3 [?], for the fragmentation and hadronization of the scattered quark and the proton remnant.

The program has been tested and gives reliable results in the range $10^{-4} \leq x_l \leq 0.9$ and $0.01 \leq y_l \leq 0.99$, excluding the corner of large x_l and small y_l. Since exponentiation of soft photonic corrections is not yet included, the program is restricted to not too small values of y_l and not too large x_l. In the present version, x_l and y_l must lie in the region

$$y_l(1 - x_l)^2 \geq 0.004. \tag{40}$$

The bremsstrahlung cross section is split into a soft and a hard part. The soft bremsstrahlung is treated analytically and included as a correction in the non-radiative cross section. It describes emission of photons with an energy smaller than a cut–off E_γ^{soft}, which is chosen by the program below the experimental threshold for the detection of photons. E_γ^{soft} depends on x and y and is of the order of a few 10 MeV. The hard bremsstrahlung contribution describes events with photons of an energy above E_γ^{soft}. For the NC process, the hard bremsstrahlung is split into four contributions. Two of them correspond to the collinear peaks from initial– and final–state leptonic radiation. The third one is dominated by the kinematical situation where the electron scatters off an almost real photon, collinear to the incident quark (Compton contribution). These three parts have been obtained by partial a fractioning of the expression describing emission of a photon from the lepton line. Finally, there is a channel for the radiation of a photon from the quark line. This channel is also needed when interference of leptonic and quarkonic radiation is requested. Since the interference contribution is not positive definite, it cannot be treated separately, but it has to be combined with the other channels. Therefore, it is necessary to turn on *all* channels when lepton–quark–interference is requested. Quark–mass singularities originating from photon radiation off the quark line are absorbed into the parton distribution functions. The separation of these singularities is performed with the help of cuts on the angles between the photon and the incoming and the outgoing quarks. Details of the treatment of these contributions can be found in ref. [?].

3.3.2 LESKO-F

LESKO-F is a Monte Carlo program for deep–inelastic $e^{\pm}p \to e^{\pm}X$ scattering at HERA, including QED bremsstrahlung from the lepton line [?]. It is an upgrade of the previous version, with collinear bremsstrahlung from the initial lepton state, called LESKO-C [?]. The present version 3.0 includes the whole $\mathcal{O}(\alpha)$ QED bremsstrahlung from the initial and final lepton states for the neutral–current $(\gamma + Z)$ deep–inelastic scattering process at HERA. The incoming lepton may have non-zero spin polarization. Vacuum polarization corrections to the photon propagator, being significant at HERA energies, are implemented in the program as well. The leptonic part of the vacuum polarization corrections has been taken from [?], while the hadronic part is calculated using the dispersion relation from [?]. The Z coupling constants are calculated with the help of the muon decay constant G_μ. Corrections to the muon decay, Δr, are included in the above calculation; they are taken from [?].

The main feature of LESKO-F is that it focuses on the description of QED radiative corrections for the deep–inelastic scattering region, where the parton model can be applied. This is guaranteed by the requirement that the hadronic transverse momentum p_T must be greater than some $p_{T,min}$, which is the main cut implemented in the program. Such a cut could also be used as an experimental trigger for deep–inelastic events. It ensures that all the accepted events have $Q^2 > p^2_{T,min}$. Since the parton model is not valid for Q^2 below $\mathcal{O}(1\ \mathrm{GeV}^2)$, the value of $p_{T,min}$, which is one of the steering parameters of the program, should be greater than $\mathcal{O}(1\ \mathrm{GeV})$. The recent version of LESKO-F (version 3.0) gives the possibility to impose additional cuts for the hadronic variables x_h and Q^2_h. Other cuts and/or event selection criteria can be imitated by rejection.

Owing to the p_T-cut, it is possible to use hadronic variables (e.g. x_h and Q^2_h) in the event generation. The rejection rate is reasonably small, usually less than 20%. The program is stable for the whole DIS range at HERA. A typical running time is about 1 ms per event for the IBM 3090 computer. The program does not need any external routines or numerical libraries and any extra data sets, so it can be installed on any computer with a FORTRAN 77 compiler.

For each generated event, LESKO-F provides momenta and flavours of final particles at the parton level. They are stored in the standardized HEPEVT event record. An interface for LESKO-F and routines from the program LEPTO 5.2 [?] for parton cascades and fragmentation, called FRANEQ [?], is available. Interfaces for other hadronization programs will be developed in the future.

LESKO-F is simple in use and easy to connect with other programs. It can be run by calling only one routine in three modes: initialization, event generation and post-generation. In the final mode, which is optional, the program prints some useful output and provides the value of the total integrated cross section corresponding to a generated event sample.

The present version of the program is sufficient to describe the deep inelastic scattering process with a precision of better than 5%. In order to improve the above precision higher–order QED radiative corrections must be included. It is planned to include soft–photon exponentiation and to implement the second–order leading–logarithmic corrections in future releases of the program. Another planned improvement is the inclusion of the longitudinal structure function.

3.3.3 HEKTOR

A Japanese group has developed the Monte Carlo program HEKTOR for the calculation of $\mathcal{O}(\alpha)$ QED radiative corrections to the neutral–current process at HERA. The program includes leptonic and quarkonic radiation but not yet the lepton–quark interference and no weak corrections. The leptonic corrections may be described in the model-independent approach based on a structure-functions input. Basic kinematical variables are the hadronic ones eq. (4) and the program is therefore suited for the calculation of corrections as a function of x_h and y_h. The work has not yet been published and is still in progress, but preliminary numerical results have been communicated [**?**] which agree with those obtained with TERAD91.

3.3.4 KRONOS

KRONOS was designed for the study of higher–order QED corrections. The program is described in [**?**, **?**].

Features of KRONOS 1.0
The present version of KRONOS is restricted to a description of photon radiation from the initial-lepton state in the leading- logarithmic approximation. Thus the dominant contributions to QED radiative corrections are included, since radiation from the lepton final–state leads only to small corrections for a calorimetric measurement of the energy of the scattered electron. The following three features are unique to KRONOS: firstly, it treats electromagnetic initial–state radiation in the leading–logarithmic approximation to all orders. Secondly, KRONOS can generate events with an arbitrary number of bremsstrahlung photons and allows the soft–photon cut off to be set as low as desired. Thirdly, KRONOS includes radiative corrections to the QED Compton processes.

In addition, KRONOS shares the following features with other Monte Carlos for HERA physics: An exhaustive set of parton distribution functions is available using the package `PAKPDF` [**?**]. Alternatively, phenomenological parametrizations of the proton structure functions F_2 and F_L can be used.

KRONOS has been used for an estimation of radiative corrections to the QED Compton cross section at HERA [**?**]. Furthermore, a study of multiphoton final states at HERA has been contributed to these proceedings [**?**].

Future releases of KRONOS after version 1.0 are expected to provide additional features, which are currently under study: interfaces to QCD parton shower and fragmentation Monte Carlos, final–state radiative corrections, higher–order radiative corrections to CC scattering, and the optional generation of suitably weighted event samples.

Algorithm for the generation of multiphoton states
The leading–logarithmic corrections can be summed by using renormalization group techniques that are well known from QCD. These methods are directly applicable to QED inclusive radiative corrections where the outgoing photons are not individually detected. In this approach the cross section with inclusive radiative corrections is obtained by folding the Born cross section with a so-called radiator $e(\xi, Q^2)$:

$$\sigma(l) = \int_0^1 d\xi\, e(\xi, Q^2)\sigma_{Born}(\xi l), \tag{41}$$

where l is the momentum of the incoming electron, Q^2 is the characteristic momentum transfer in the process, and ξ is the fraction of the initial electron momentum left after photon radiation. The radiator $e(\xi, Q^2)$ is obtained as the solution of the evolution equation

$$Q^2 \frac{\partial}{\partial Q^2} e(\xi, Q^2) = -\frac{\alpha}{2\pi}\left[\int_0^{1-\epsilon} d\zeta P(\zeta)\right] e(\xi, Q^2) + \frac{\alpha}{2\pi}\int_\xi^{1-\epsilon} \frac{d\zeta}{\zeta} P(\zeta) e\left(\frac{\xi}{\zeta}, Q^2\right) \tag{42}$$

with the initial condition

$$e(\xi, m_e^2) = \delta(1-\xi) \tag{43}$$

and the splitting function

$$P(\zeta) = \frac{1+\zeta^2}{1-\zeta}\,. \tag{44}$$

The infra-red regulator $\epsilon \ll 1$ will drop out of the final results.

KRONOS uses a cascade algorithm similar to space-like parton showers in the context of QCD. The branching of the incoming electron is implemented in the following way. First the number of photons is generated, according to a Poisson distribution with mean

$$\bar{n} = \frac{\alpha}{2\pi} \ln\left(\frac{Q^2}{m^2}\right) \int_0^{1-\epsilon} dz P(z). \tag{45}$$

The energies of the generated photons are determined according to the splitting function (44), while the transverse components of the photon momenta are determined by a probability distribution according to the pole in the electron propagator $1/2l \cdot k$.

The procedure is repeated as many times as the number of photons in the corresponding event, taking into account the appropriate ordering of virtualities to obtain the leading–logarithmic contribution.

3.4 Comparison of the programs

Table 1 contains an overview of the programs for radiative corrections at HERA as described in the preceeding sections. We list there the program names, version numbers, dates of the last release, authors, e-mail contacts, and whether there is a write-up contained in volume III of these proceedings. It is mentioned whether the program needs files for input and output. For details on the use of external files one has to consult the corresponding write-up. Sometimes routines are used which have been taken over from other sources or other authors. These are mentioned explicitly and are put in parentheses when the corresponding source code is provided as a part of the complete program listing. For example, most of the programs use the package PAKPDF [**?**]

Here ends the file of TR. The original has 42 pages.