19. The interpretation of thermal spectra

Thermal bremsstrahlung is one of the two important continuum radiation processes of radio astro nomy, where it tells about the presence of ionized gas in the interstellar medium at locations that a re inaccessible in the Optical on account of strong absorption. The emissivity depends on the squa re of the plasma density, so one preferentially sees dense clouds of ionized gas. These are normally only found near strong heating sources, e.g. young massive stars with a flux of photoionizing radiat ion. Clouds with a lot of thermal bremsstrahlung emission can therefore be used as tracer of the star formation activity in a galaxy.
For a thermal gas of temperature T the emission parameters are

$$j_\nu \simeq \frac{1}{4\pi}\int_{v_{min}} dv\ P_\nu\, n_e = \frac{\sqrt{2}\, n_i^2\, \alpha\, \sigma_T\, \hbar\, c}{\pi^{3/2}} \sqrt{\frac{mc^2}{kT}}\ \ln\left(\frac{\beta\, E_e}{\alpha\, E_{ph}}\right)\ \exp\left(-\frac{h\nu}{kT}\right) \tag{18.10}$$

where we have used the definitions

$$\sigma_T = \frac{8\pi}{3}\frac{e^4}{m^2\, c^4} \qquad\qquad \alpha = \frac{e^2}{\hbar\, c}$$

The absorption coefficient follows as the ratio of the emission coefficient and a Planckian.

$$\alpha_\nu = \frac{c^2}{2\,\nu^3\, h}\, j_\nu \left[\exp\left(\frac{h\nu}{kT}\right) - 1\right] = \frac{2^{1/2}\, n_i^2\, \alpha\, \sigma_T\, c^3}{\pi^{5/2}\, \nu^3}\sqrt{\frac{mc^2}{kT}}\ \ln\left(\frac{\beta\, E_e}{\alpha\, E_{ph}}\right)\left[1 - \exp\left(-\frac{h\nu}{kT}\right)\right]$$

$$\propto \nu^{-2}\, T^{-3/2} \qquad \text{for } h\nu \ll kT \tag{18.11}$$

The absorption process associated with thermal bremsstrahlung is often named free-free absorption. The energy loss rate is approximately proportional to

$$\dot{T} \propto \int d\nu\ j_\nu \propto \sqrt{T} \tag{18.12}$$

It dominates the cooling function only at temperature above 10^8 K where line emission is inefficient.

19.1 HII regions

In the Milky Way and in other nearby galaxies one often observes small gas clouds in the vicinity of young, hot stars, that show a continuous emission spectrum throughout the radio band and the infrared up to the Optical, superimposed on which are hydrogen lines like Ly α. Apparently these are clouds of ionized gas that is emitting thermal bremsstrahlung. We call them HII regions, where HII denotes singly ionized hydrogen.

Let us have a closer look at an HII region which may be described by the following parameters:

$$r = 5 \quad \text{pc} \qquad\qquad n_i = 100 \quad \text{cm}^{-3} \qquad\qquad T = 10^4 \quad \text{K} \tag{19.1}$$

Then the absorption coefficient at $h\nu \ll kT$ is (in cgs units)

$$\alpha_\nu = 2.2 \cdot 10^{-17} \frac{n_i^2}{\nu^2} \left(\frac{mc^2}{kT}\right)^{3/2} \ln\left(\frac{\beta\, E_e}{\alpha\, E_{ph}}\right) \simeq 10^{-3}\ \nu^{-2} \tag{19.2}$$

since the Coulomb logarithm

$$\ln\left(\frac{\beta\, E_e}{\alpha\, E_{ph}}\right) \simeq 15 \qquad \text{für} \qquad \nu = 10^8\ \text{Hz} \tag{19.3}$$

The optical depth can be estimated using the radius as mean length of the line-of-sight, so

$$\tau \simeq r\,\alpha_\nu \simeq 1.4 \cdot 10^{16}\,\nu^{-2} \simeq \left(\frac{\nu}{100\ \text{MHz}}\right)^{-2} \tag{19.4}$$

At frequencies below $\nu_c \simeq$100 MHz our HII region should be optically thick.
In the optically thin part of the spectrum a frequency dependence of the intensity arises only from the Coulomb logarithm. Often one approximates that frequency dependence by a power-law with a view to characterize the spectrum by a power-law index, the so-called *spectral index.* At the transition frequency ν_c we have $\ln\Lambda \simeq 15$. If we increase the frequency by a factor of 10, the logarithm changes by 2.3 or 15%. In the next higher frequency decade the relative change would be 18%. The corresponding power-law approximation would have an index -0.08, since $10^{-0.08} \simeq 0.83$. Generally on sets the frequency dependence as $\nu^{-0.1}$. Then we obtain for the total spectral energy distribution of our HII region

$$I_\nu \simeq \begin{cases} \nu^2 & \nu \ll \nu_c \\ \nu^{-0.1} & \nu_c \ll \nu \ll kT/h \\ \exp(-h\nu/kT) & \nu \gtrsim kT/h \end{cases} \tag{19.5}$$

Often absorption by dust prevents the observation of optical hydrogen lines. If then one observes an HII region only in the optically thin part of the spectrum, the identification can be difficult. In these cases one may search for transitions between Rydberg states of hydrogen. One characteristic line for example is H 109α at 5008.89 MHz, which corresponds to a transition from n=110 to n=109.
The radiative cooling time is very short for HII regions, about 10 years for our example, so they can exist only in the vicinity of hot stars that provide a sufficient intensity to photoionize and heat the gas.

19.2 Clusters of galaxies

In many clusters of galaxies we observe a diffuse X-ray emission, that is composed of a continuum and many bright lines of heavy elements, oxygen, silicon, iron, etc. The spectral form suggests thermal emission and the continuum component is likely bremsstrahlung.

An exact determination of the temperature is difficult on account of limited spatial resolution and the length of the line-of-sight. One would always see spectra averaged over regions of different density, temperature, and metallicity (abundance of heavy elements). Another problem is photoelectric absorption along the line-of-sight, which is absorption by photoionization of cold gas. The cross section for photoelectric absorption by atomic hydrogen is

$$\sigma_{pa} \simeq (5 \cdot 10^{-22}\ \mathrm{cm}^2) \left(\frac{E_X}{\mathrm{keV}}\right)^{-3} \Theta(E_X - 0.0136\ \mathrm{keV}) \tag{19.6}$$

$$\Rightarrow\ \tau_{pa} \simeq 1 \left(\frac{n_H}{\mathrm{atoms\ cm}{-3}}\right) \left(\frac{L}{700\mathrm{pc}}\right) \left(\frac{E_X}{\mathrm{keV}}\right)^{-3} \Theta(E_X - 0.0136\ \mathrm{keV}) \tag{19.7}$$

so the X-ray spectra of extragalactic objects can be significantly modified up to keV energies.

It appears likely that the plasma in the gravitational well of a galaxy cluster is in a hydrostatic equilibrium, i.e. the thermal pressure would just equilize the gravitational force. Assuming adiabatic conditions we therefore have

$$P = \frac{k}{\mu}\rho T = C\rho^\gamma \qquad \wedge \qquad \frac{1}{\rho}\frac{\partial P}{\partial r} = \gamma\,\frac{k}{\mu}\,T\,\frac{\partial \rho}{\partial r} = -G\,\frac{M(r)}{r^2} \tag{19.8}$$

We note that the temperature and density distribution is a function of the distribution of mass in the cluster, and thus the plasma is a beacon that tells the mass distribution.

There is evidence that the dominant part of the mass is not carried by the dilute gas and all the visible galaxies that make up the cluster, but rather by *dark matter* that we observe only by its gravitational effect. Alternative explanations involve deviations from the Newtonian law for large distances or small accelerations. In any case we can use Eq.19.8 to infer the distribution of mass and gravitational acceleration. I remind you that here we have used two assumptions:

– the plasma follow an adiabatic relation.

– the plasma is in hydrostatic equilibrium.

Let us look at the cooling rate again. The intracluster gas has a temperature of about 10^7 K, so bremsstrahlung dominates the cooling and the cooling time is approximately

$$\tau \simeq 8 \cdot 10^{10}\ \sqrt{T}\,n_i^{-1} \quad \mathrm{sec} \simeq 2.5 \cdot 10^{14}\ n_i^{-1} \quad \mathrm{sec} \tag{19.9}$$

The Hubble time as a measure of the age of the universe is, depending on the model used, around $\tau_H \approx 3 \cdot 10^{17}$ sec. This implies that intracluster gas with a density $n \gtrsim 10^{-3}$ will

significantly cool within the lifetime of the system. High density means high X-ray luminosity because $j_\nu \propto n^2$, so in particular the very X-ray-luminous clusters would be affected.
What would happen, if the intracluster gas cools? The thermal pressure is reduced and the gas begins to fall towards the cluster center. The density will increase on account of mass conservation, and so will the cooling rate, and the process amplifies itself. The hydrostatic equilibrium is thus unstable and one expects a slow advection of gas, a *Cooling Flow*.

19.3 Stellar winds

It is well know that the sun emits a stream of particles called the solar wind. It is dilute enough to impose negligible free-free absorption at radio frequencies, and so we can study the sun with radio telescopes. For many stars the opposite is true, and their radio emission is determined by amission and absorption in the wind zone. Recall that the plasma density will drop quickly with increasing distance from the star, and so do the radiation coefficients. The resulting spectrum can therefore be very different from that of a H*II* region.
We will not study the properties of stellar winds in detail. It may suffice to presume that the density of the wind plasma scale with the radial distance, r, as $\rho = D\,r^{-s}$, that scattering is irrelevant, the temperature be constant and the material fully ionized. If the wind had a constant speed, mass copnservation would require $s = 2$. Generally, the density profile can be much more complicated than that, and we assume a power-law scaling.
We consider radio emission, ignore the logarithmic terms in the radiation coefficients, a and set

$$j_\nu = A\,\rho^2 \qquad \wedge \qquad \alpha_\nu = F\,\rho^2\,\nu^{-2} \tag{19.10}$$

Obviously the source function is independent of density and hence of location. The formal solution to the radiation transport equation for an arbitrary line of sight then is

$$I_\nu(\tau) = S_\nu\,\exp(-\tau)\int_0^\tau d\tau'\;\exp(\tau') = \frac{A}{F}\,\nu^2\,\left[1 - \exp(-\tau)\right]\;. \tag{19.11}$$

Now consider a line of sight that passes the star with impact parameter b. We integrate the absorption coefficient to obtain the opacity.

$$\begin{aligned}\tau &= \int_{-\infty}^{\infty} dl\;\alpha_\nu = \frac{F}{\nu^2}\int_{-\infty}^{\infty} dl\;\rho^2(r = \sqrt{l^2+b^2})\\ &= \frac{F\,D^2}{\nu^2}\int_{-\infty}^{\infty}\frac{dl}{(l^2+b^2)^s} = \frac{F\,D^2}{\nu^2\,b^{2s-1}}\,B\left(\frac{1}{2};s-\frac{1}{2}\right)\end{aligned} \tag{19.12}$$

where B is a beta function that here depends only on s.
The total emission spectrum of the entire system is found be integration over the apparent surface of the system, $2\pi b\,db$, or over all lines of sight.

$$I = 2\pi\int_0^\infty db\;b\,I_\nu(b) = 2\pi\,\frac{A}{F}\,\nu^2\int_0^\infty db\;b\left[1 - \exp\left(-C\,\frac{b^{1-2s}}{\nu^2}\right)\right]$$

$$\Rightarrow \; I = C'\,\nu^2\,\nu^{\frac{4}{1-2s}} \int_0^\infty dx\; x^{\frac{1+2s}{1-2s}}\,(1-\exp(-x)) \tag{19.13}$$

The remaining integral yields only a number and doesn't influence the spectral form. In fact the spectral form of the total flux from the system only depends on s, the scaling index of the wind density with radial distance from the star. Here are exemplary results for three different numerical values of s.

$$I = \begin{cases} \nu^{0.0} & \text{for} \quad \text{s=1.5} \\ \nu^{0.67} & \text{for} \quad \text{s=2.0} \\ \nu^{1.0} & \text{for} \quad \text{s=2.5} \end{cases} \tag{19.14}$$

Observations of the radio/FIR spectra of a few members of the OB association Cygnus OB2 suggest that their quiescent emission, i.e. discounting flares, is well described by such models of optically thick thermal bremsstrahlung in the stellar winds.