12. Blast waves and supernova remnants

12.1 Self-similarity and scales

In galaxies one finds a many sources of shock waves and the interstellar medium is significantly influenced by the shocks. These shocks can arise from strong stellar winds and from stellar explosions, supernova.

Supernovae are caused by run-away thermonuclear reactions that occur when stellar cores collapse. A type I supernova involves a white dwarf that exceeds the Chandrasekhar mass limit on account of accretion from another star in close orbit. A type II supernova happens for massive stars when the iron core, for which no more energy gain by fusion is possible. A typical kinetic explosion energy is 10^{51} ergs or $\sim\ 10^{-3}\ M_{\odot}c^2$. In both cases the outer layers of the star are expelled or ejected with high velocity, which creates a strong shock when the ejecta meet the circumstellar medium. The latter may be dilute gas for a type I supernova or hot, magnetized stellar wind material for a type II supernova.

Much of the astrophysics modeling of supernova explosions and their remnants derives from the nuclear bomb research programs in the East and West. The problem is essentially one of a point release of an enormous amount of energy, E, into a static homogeneous medium of mass density ρ_u. In the initial phase of the expansion the impact of the external medium will be small, for the mass of the ambient medium, that is overrun and taken along, is still small compared with the ejecta mass. The supernova remnant is said to expand adiabatically.

After some time the mass swept up by the outwardly moving shock wave will significantly exceed the mass of the initial ejecta. The ram pressure, $\rho_u\, U_{\rm sh}^2$ of the matter that enters the shock wave may still be much larger than the thermal pressure of the upstream medium, P_u. Let us assume that the radiative energy loss is much smaller than the initial available energy E at this stage. The supernova remnant is said to produce a blast wave.

We expect a spherically-symmetric evolution of the blast wave in space r and time t. Neglecting the pressure of the external medium, P_u, for a moment, we have only the explosion energy, E, and the external density, ρ_u, as parameters of the problem. The hydrodynamical equations can be written in non-dimensional variables using scales, for example a radial scale r_0 for the radius coordinate. Scales in the solution are often mathematically obvious, because the arguments of many functions such as the exponential must be non-dimensional. Here we are concerned with scales in the governing equations.

How could a dimensionless radius variable be composed, if only the radius, time, the energy, and a density are at our disposal? If we set

$$x \equiv r\, t^l\, \rho_u^m\, E^n \tag{12.1}$$

a dimension analysis yields

$$1 = L\,T^l\;M^m\,L^{-3m}\;M^n\,L^{2n}\,T^{-2n} = L^{1-3m+2n}\;T^{l-2n}\;M^{m+n} \tag{12.2}$$

which requires

$$l = -\frac{2}{5} \qquad m = \frac{1}{5} \qquad n = -\frac{1}{5} \qquad \Rightarrow \quad x = r\,t^{-2/5}\,\rho_u^{1/5}\,E^{-1/5} \tag{12.3}$$

We note that through the variable x time and location are coupled. In fact we have not only made the hydrodynamic equations dimensionless, but also embarked on a similarity analysis. We expect the flow at any location and any time to look the same as it did at some other location and an earlier time. The flow is said to be self-similar.

How can I infer observable quantities from my similarity analysis? Suppose the shock resides at some fixed value x_0.

$$r_s(t) = x_0\,\left(\frac{E\,t^2}{\rho_u}\right)^{\frac{1}{5}} \tag{12.4}$$

The velocity of the shock wave then is

$$U_s(t) = \frac{dr_s}{dt} = \frac{2}{5}\,x_0\,\left(\frac{E}{\rho_u\,t^3}\right)^{\frac{1}{5}} \tag{12.5}$$

The blast wave thus decelerates and disappears after some time.

A typical supernova has an explosion energy of the order of 10^{51} erg and expells approximately one solar mass, implying an initial ejecta velocity on the order of 10^4 km/sec. The shock velocity in 12.5 was derived assuming that much more gas has been swept up than was initially ejected, and hence applies to a later time. Taking an ambient density of one atom per cubic centimeter or $\rho_u \simeq 2\cdot 10^{-24}$ g/cm^3, solution 12.5 corresponds to the initial ejecta velocity, when

$$t_c = x_0^{5/3}\,(1.7\cdot 10^9\ \mathrm{sec}) \simeq x_0^{5/3}\,(50\ \mathrm{yr}) \qquad\qquad r_s(t_c) = x_0^{5/3}\,(5\cdot 10^{18}\ \mathrm{cm}) \simeq x_0^{5/3}\,(5\ \mathrm{Lyr})$$

$$M = \frac{4\pi}{3}\,\rho_u\,r_s^3 \simeq (0.5\ M_\odot)\,x_0^5 \tag{12.6}$$

This is a general result: the supernova remnant turns from adiabatic expansion to a blast wave after the shock would have swept up a mass similar to the initial electa mass, if it was located at $x_0 \simeq 1$.

Our treatment is also valid only as long as radiative losses are not significant. In our discussion of hydrodynamic shocks we used the Rankine-Hugoniot conditions to determine the post-shock temperature of a non-radiative shock to be

$$T_{\mathrm{SNR}} \simeq (2\cdot 10^8\ \mathrm{K})\,\left(\frac{U_s}{3000\ \mathrm{km/sec}}\right)^2 \propto t^{-6/5} \qquad \text{for } \gamma = \frac{5}{3} \tag{12.7}$$

Radiative cooling becomes important, if the age of the system is higher than the cooling timescale of the post-shock gas.

$$\tau \simeq \frac{1}{\gamma-1}\,\frac{kT}{n\,\Lambda} \simeq \left(\frac{n}{\mathrm{atoms\,cm^{-3}}}\right)^{-1} \begin{cases} 5\cdot 10^3\ \mathrm{yr} & \text{for } T = 3\cdot 10^5\ \mathrm{K} \\ 1.5\cdot 10^6\ \mathrm{yr} & \text{for } T = 3\cdot 10^6\ \mathrm{K} \\ 3\cdot 10^7\ \mathrm{yr} & \text{for } T = 10^8\ \mathrm{K} \end{cases} \tag{12.8}$$

For a shock at $x_0 \simeq 1$ and ambient density $n_u = 1\ \mathrm{cm}^-3$ it takes about 50.000 years to reduce the shock velocity sufficiently for a post-shock temperature of $T = 3{\cdot}10^5$ K, but shorter than that to significantly cool the gas, hence the shock would be radiative and our self-similarity desciption no longer valid. At that time the remnant would have a radius $r_s \simeq x_0\,(80\mathrm{Lyr}) \simeq x_0\,(25\mathrm{pc})$

12.2 The Taylor-Sedov solution for a blast wave

Having established a basic understanding of supernova remnants by using scaling arguments we now intend to study the structure of SNR blast waves in more detail. For that purpose we consider a strong shock ($M_u \to \infty$) and use a non-relativistic (Galilei-) transformation to a frame that is fixed to the center of the remnant.

$$\rho_d = \left(\frac{\gamma+1}{\gamma-1}\right)\rho_u \qquad\qquad U_d = V_d + U_s = V_2 - V_1 = \frac{2}{\gamma+1}\,U_s$$

$$P_d = \frac{2\,\gamma}{\gamma+1}\,M_u^2\,P_u = \frac{2\,\gamma}{\gamma+1}\,M_u^2\,\frac{c_{s,u}^2}{\gamma}\,\rho_u = \frac{2}{\gamma+1}\,\rho_u\,U_s^2 \tag{12.9}$$

If we knew r_s and $U_s = \dot r_s$, equations 12.9 would describe the conditions just inside the blast wave. Deep inside the remnant, the normal hydrodynamical equations should apply.

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\,\frac{\partial}{\partial r}\,(r^2\,\rho\,u) = 0 \tag{12.10}$$

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial r} + \frac{1}{\rho}\,\frac{\partial P}{\partial r} = 0 \tag{12.11}$$

$$\epsilon = \frac{1}{\gamma-1}\,P \qquad \Rightarrow \qquad \frac{\partial P}{\partial t} + u\,\frac{\partial P}{\partial r} + \frac{\gamma\,P}{r^2}\,\frac{\partial r^2\,u}{\partial r} = 0 \tag{12.12}$$

Taylor and Sedov independently derived solutions based on the similarity variable x by writing the fluid quantities in self-similar form.

$$\rho(r,t) = \rho_d\,a(x) = \left(\frac{\gamma+1}{\gamma-1}\right)\rho_u\,a(x) \qquad a(x_0) = 1 \qquad x = r\,t^{-2/5}\,\rho_u^{1/5}\,E^{-1/5} \tag{12.13}$$

and corresponding forms for the fluid velocity and pressure with scaling functions $u(x)$ and $p(x)$. One then has to express the temporal and spatial derivatives as derivatives in x and derives equations for the scaling functions $a(x)$, $u(x)$, and $p(x)$, that can be solved.

It turned out that for $\gamma = 5/3$ the position of the blast wave corresponds to $x_0 \simeq 1.17$, indeed very close to unity as we had suspected.

12.3 The collisionless nature of the shock

Is it justified to treat the SNR blast wave as a shock? We have seen that SNR size is between a few and a hundred light-years during the blast-wave phase, if the ambient medium has a density of one atom per cubic centimeter. For a shock velocity of a few thousand km/sec a proton crossing the shock front has a kinetic energy in the range of 0.1–1 MeV with respect to the ambient medium. Its stopping length for neutral collisions is of the order 1 pc, that for ionization is of the order of 10 pc, and the mean free path for isotropization and energy loss by Coulomb collisions is somewhat larger. These length scales are not negligibly small in comparison with the system size.

What we are missing at this point are magnetic fields that would tie the MeV-ish protons to the fluid by the helical gyration orbits with Larmor radius

$$r_g = \frac{m\,c\,v}{e\,B} \simeq (10^{10}\ \mathrm{cm})\,\left(\frac{B}{10\ \mu\mathrm{G}}\right)^{-1} \tag{12.14}$$

The helical motion is further disturbed by magnetic turbulence, so that the shock transition is mediated by chaotic electromagnetic fields rather than collision. The shock is said to be collisionless. The shock thickness is of the order of the mean free path for the isotropization of MeV-ish protons.