Mass effects in the polarized virtual photon structure

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1. Introduction and Motivation

Polarized e^+e^- collision

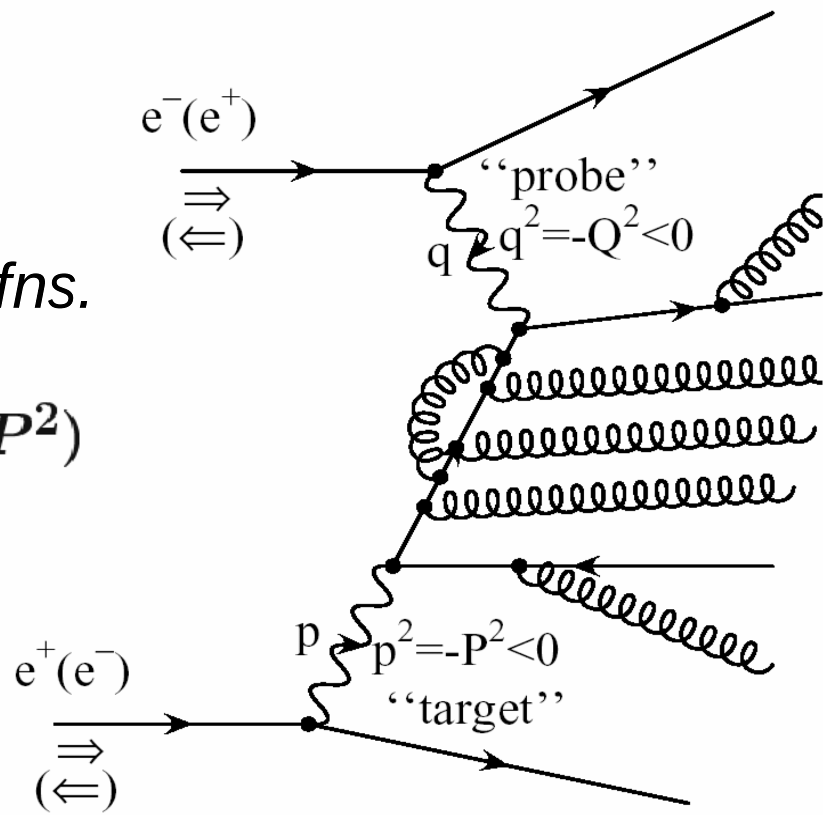
⇨ *Photon's spin structure fns.*

$$g_1^\gamma(x, Q^2, P^2) \ \& \ g_2^\gamma(x, Q^2, P^2)$$

In the kinematic region:

$$\Lambda^2 \ll P^2 \ll Q^2$$

structure fns. g_1^γ *and* g_2^γ
perturbatively calculable

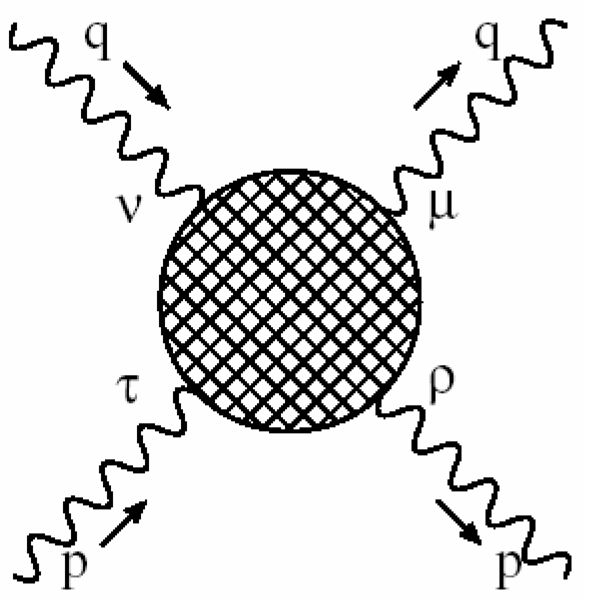


Forward virtual photon-photon scattering

Structure tensor

$$W_{\mu\nu\rho\tau}$$

anti-symmetric part



$$W^{A}_{\mu\nu\rho\tau} = \epsilon_{\mu\nu\lambda\sigma} q^{\lambda} \epsilon_{\rho\tau}{}^{\sigma\beta} p_{\beta} \frac{1}{p \cdot q} g_1^{\gamma}$$
$$+ \epsilon_{\mu\nu\lambda\sigma} q^{\lambda} (p \cdot q \; \epsilon_{\rho\tau}{}^{\sigma\beta} p_{\beta} - \epsilon_{\rho\tau\alpha\beta} p^{\beta} p^{\sigma} q^{\alpha}) \frac{1}{(p \cdot q)^2} g_2^{\gamma}$$

g_1^γ $\Rightarrow$ *the 1st moment is related to the axial anomaly*

g_2^γ $\Rightarrow$ *only exists for the virtual photon* $P^2 \neq 0$

the twist-3 effect contributes

(*No* g_2^γ *for real photon* $P^2 = 0$)

In contrast to nucleon target, the mass ² P^2 $\Rightarrow$ *variable*

Possible mass effects

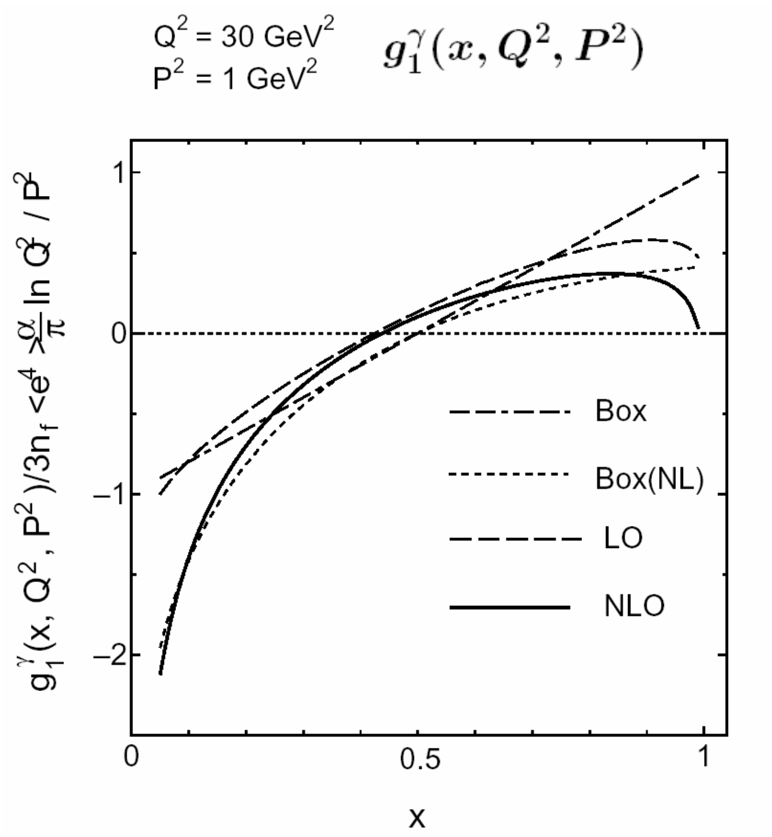
$\Rightarrow$ *Target mass effects: power-correction in* P^2/Q^2

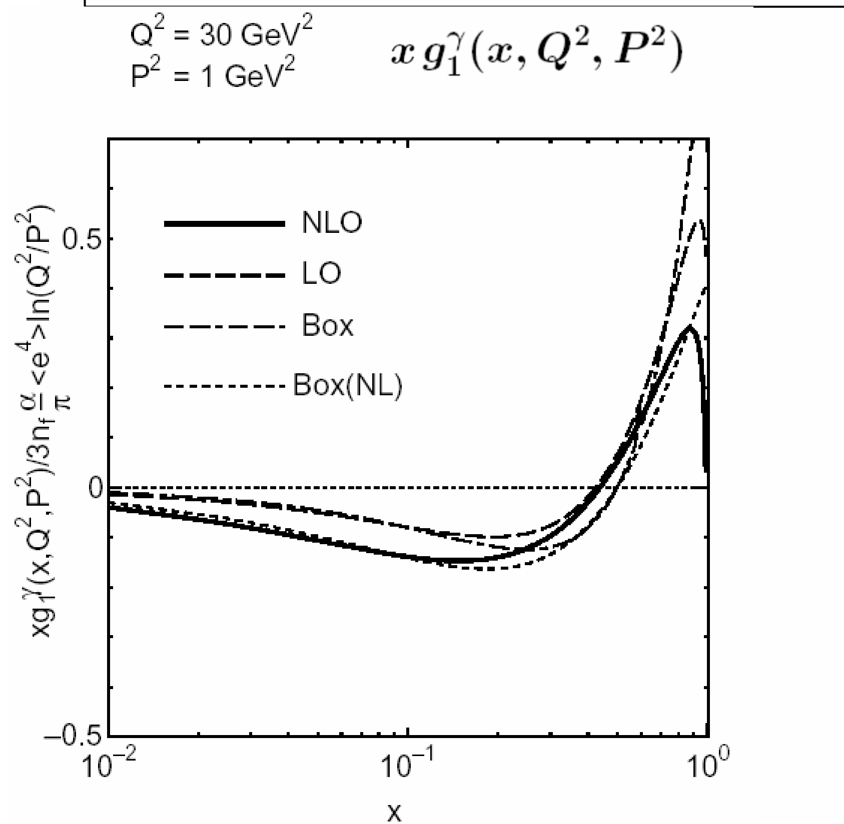
to be discussed here

Quark mass effects:

Polarized photon structure function $g_1^\gamma(x, Q^2, P^2)$

only twist-2 op. contributes

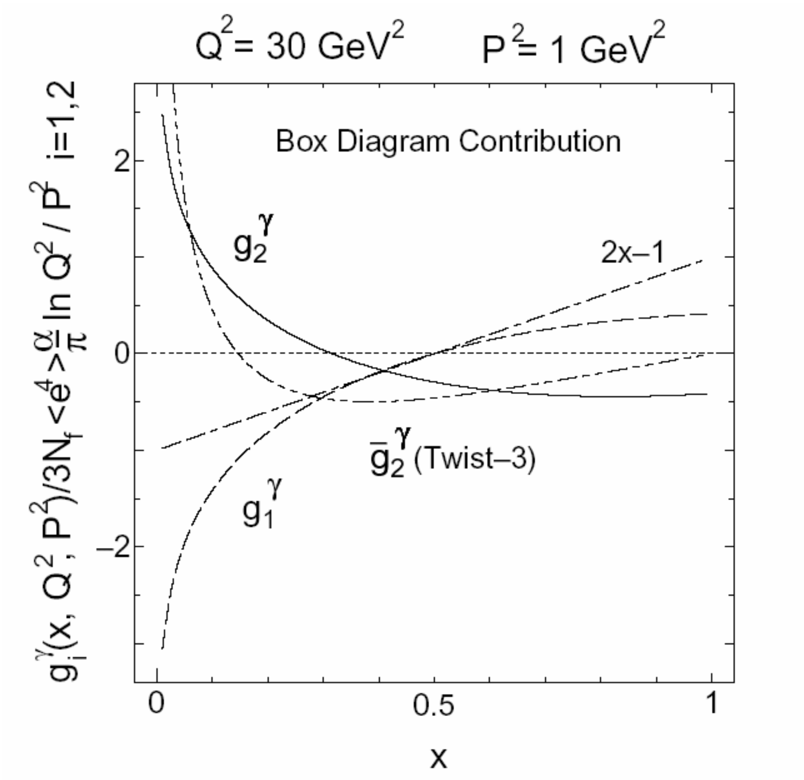


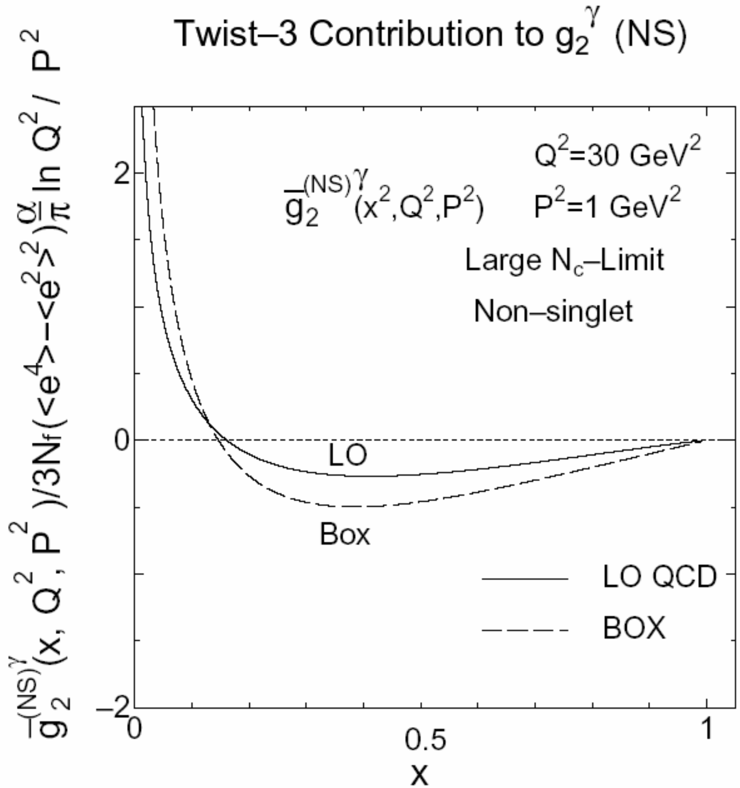


Baba, Sasaki & Uematsu, PRD 65, 1140185

Polarized photon structure function $g_2^\gamma(x, Q^2, P^2)$

Both twist-2 & twist-3 ops. contribute





- For the virtual photon target,

  the maximal value of the Bjorken variable

  $$\Rightarrow \qquad x_{\max} = \frac{1}{1+\frac{P^2}{Q^2}} < 1$$

  due to the constraint $(p+q)^2 \geq 0.$

  In the nucleon case $x_{\max} = 1$

- We study the Target Mass Effects (TME) based on Operator Product Expansion (OPE) taking into account the trace terms of the matrix elements of the operators.
- This amounts to consider Nachtmann moments for g_1^γ and g_2^γ .

  By using the orthogonality in the O(4) expansion we project out the definite spin contributions.

2. Nachtmann moments

Baba, Sasaki & Uematsu, PRD 68, 054025

Deep Inelastic photon-photon scatt. amp.

$$\begin{aligned}
T^{A}_{\mu\nu\rho\tau} &= i\int d^4x e^{iq\cdot x}\langle 0|T(A_\rho(-p)(J_\mu(x)J_\nu(0))^A A_\tau(p))|0\rangle_{\rm Amp} \\
&= -\epsilon_{\mu\nu\lambda\sigma}q^\lambda \sum_{n=1,3,\cdots}\left(\frac{2}{Q^2}\right)^n q_{\mu_1}\cdots q_{\mu_{n-1}} \\
&\qquad \times\left\{\sum_i \underbrace{a^{\gamma,n}_{(2)i}E^n_{(2)i}M^{\sigma\mu_1\cdots\mu_{n-1}}_{(2)\rho\tau}}_{\textit{twist-2}} + \underbrace{a^{\gamma,n}_{(3)i}E^n_{(3)i}M^{[\sigma,\{\mu_1]\cdots\mu_{n-1}\}}_{(3)\rho\tau}}_{\textit{twist-3}}\right\}
\end{aligned}$$

where matrix elements of spin-n traceless twist-2 and -3 operators $R^n_{(2)i}$ and $R^n_{(3)i}$

$$\langle 0|T(A_\rho(-p)R^{\sigma\mu_1\cdots\mu_{n-1}}_{(2)i}A_\tau(p))|0\rangle_{\rm Amp} = -ia^{\gamma,n}_{(2)i}M^{\sigma\mu_1\cdots\mu_{n-1}}_{(2)\rho\tau}\ ,$$

$$\langle 0|T(A_\rho(-p)R^{\sigma\mu_1\cdots\mu_{n-1}}_{(3)i}A_\tau(p))|0\rangle_{\rm Amp} = -ia^{\gamma,n}_{(3)i}M^{[\sigma,\{\mu_1]\cdots\mu_{n-1}\}}_{(3)\rho\tau}$$

tensors are given by

$$M_{(2)\rho\tau}^{\sigma\mu_1\cdots\mu_{n-1}} \equiv \frac{1}{n}[\epsilon_{\rho\tau\alpha}{}^{\sigma}p^{\mu_1}\cdots p^{\mu_{n-1}} + \sum_{j=1}^{n-1} p^{\sigma}p^{\mu_1}\cdots\epsilon_{\rho\tau\alpha}{}^{\mu_j}\cdots p^{\mu_{n-1}}]p^{\alpha}$$
$$-(\text{trace terms})$$
$$M_{(3)\rho\tau}^{[\sigma,\{\mu_1]\cdots\mu_{n-1}\}} \equiv \Big[\frac{n-1}{n}\epsilon_{\rho\tau\alpha}{}^{\sigma}p^{\mu_1}\cdots p^{\mu_{n-1}}$$
$$-\frac{1}{n}\sum_{j=1}^{n-1} p^{\sigma}p^{\mu_1}\cdots\epsilon_{\rho\tau\alpha}{}^{\mu_j}\cdots p^{\mu_{n-1}}\Big]p^{\alpha} - (\text{trace terms})$$

and satisfy the traceless conditions

$$g_{\sigma\mu_i}M_{(k)\rho\tau}^{\sigma\mu_1\cdots\mu_{n-1}} = 0, \qquad g_{\mu_i\mu_j}M_{(k)\rho\tau}^{\sigma\mu_1\cdots\mu_{n-1}} = 0 \qquad (k=2,3)$$

The contraction of the tensors with $q_{\mu_1}\cdots q_{\mu_{n-1}}$

can be expressed in terms of Gegenbauer polynomials

For the twist-2 part

$$q_{\mu_1}\cdots q_{\mu_{n-1}}\widetilde{M}_{(2)\beta}^{\sigma\mu_1\cdots\mu_{n-1}} = \frac{1}{n^2}\left[\delta_\beta{}^\sigma a^{n-1}C_{n-1}^{(2)}(\eta) + q_\beta p^\sigma a^{n-2}\ 2C_{n-2}^{(3)}(\eta)\right]$$
$$+(\text{terms with } p_\beta \text{ or } q^\sigma)$$

For the twist-3 part

$$q_{\mu_1}\cdots q_{\mu_{n-1}}\widetilde{M}_{(3)\beta}^{\sigma\mu_1\cdots\mu_{n-1}} = \delta_\beta{}^\sigma \frac{a^{n-1}}{n^2}\left[(n-1)C_{n-1}^{(2)}(\eta) - (n+1)C_{n-3}^{(2)}(\eta)\right]$$
$$-q_\beta p^\sigma \frac{2a^{n-2}}{n^2}\left[C_{n-2}^{(3)}(\eta) + C_{n-4}^{(3)}(\eta)\right]$$
$$+(\text{terms with } p_\beta \text{ or } q^\sigma)$$

where

$$M_{(2)\rho\tau}^{\sigma\mu_1\cdots\mu_{n-1}} \equiv \widetilde{M}_{(2)\beta}^{\sigma\mu_1\cdots\mu_{n-1}}\epsilon_{\rho\tau\alpha}{}^\beta p^\alpha$$
$$M_{(3)\rho\tau}^{[\sigma,\{\mu_1]\cdots\mu_{n-1}\}} \equiv \widetilde{M}_{(3)\beta}^{\sigma\mu_1\cdots\mu_{n-1}}\epsilon_{\rho\tau\alpha}{}^\beta p^\alpha$$

$$a = -\tfrac{1}{2}PQ,\ \eta = -p\cdot q/PQ$$

$C_n^{(\nu)}(\eta)$: Gegenbauer polynomials

Using the orthogonality property of the Gegenbauer polynomilas we derive the Nachtmann moments

twist-2

$$\begin{aligned} M_2^n &\equiv \sum_i a_{(2)i}^{\gamma,n} E_{(2)i}^n(Q^2,P^2,g) \\ &= \int_0^{x_{\max}} \frac{dx}{x^2}\xi^{n+1}\left[\left\{\frac{x}{\xi}+\frac{n^2}{(n+2)^2}\frac{P^2x\xi}{Q^2}\right\} g_1^\gamma(x,Q^2,P^2)\right. \\ &\qquad \left. +\frac{4n}{n+2}\frac{P^2x^2}{Q^2} g_2^\gamma(x,Q^2,P^2)\right], \\ &\qquad (n=1,3,\cdots) \end{aligned}$$

twist-3

$$\begin{aligned} M_3^n &\equiv \sum_i a_{(3)i}^{\gamma,n} E_{(3)i}^n(Q^2,P^2,g) \\ &= \int_0^{x_{\max}} \frac{dx}{x^2}\xi^{n+1}\left[\frac{x}{\xi} g_1^\gamma(x,Q^2,P^2)\right. \\ &\qquad \left. +\left\{\frac{n}{n-1}\frac{x^2}{\xi^2}+\frac{n}{n+1}\frac{P^2x^2}{Q^2}\right\} g_2^\gamma(x,Q^2,P^2)\right] \\ &\qquad (n=3,5,\cdots) \end{aligned}$$

where $x = Q^2/(2p\cdot q)$

$$\xi = \frac{2x}{1+\sqrt{1-\frac{4P^2x^2}{Q^2}}}$$

the ξ -scaling variable

Note that the *maximal value* of *Bj. variable* x

$x_{\max} = 1/(1 + \frac{P^2}{Q^2}) < 1$ *and* $\xi(x_{\max}) = 1$

In the case of the *nucleon* target

$x_{\max} = 1$ *and* $\xi(x_{\max}) < 1$

In the NLO QCD analysis without TME taken into account

⇨ *The predicted graph does not vanish, but remains finite at* $x = x_{\max}$.

In the analysis with TME the structure fns. in fact vanish at $x = x_{\max}$.

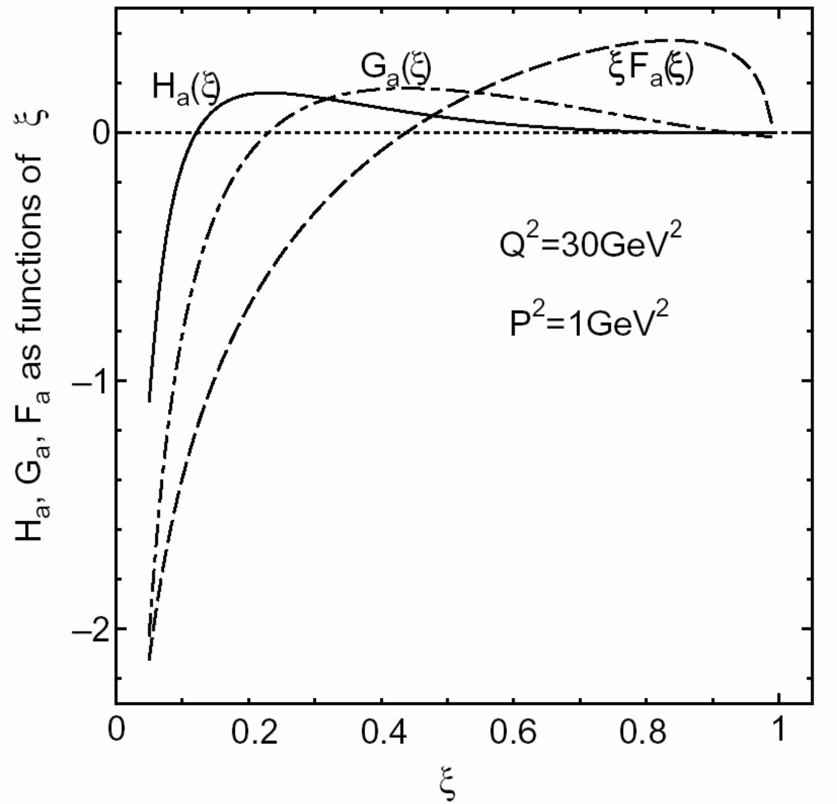
We get analytical closed forms for g_1^γ and g_2^γ with $\kappa = P^2/Q^2$

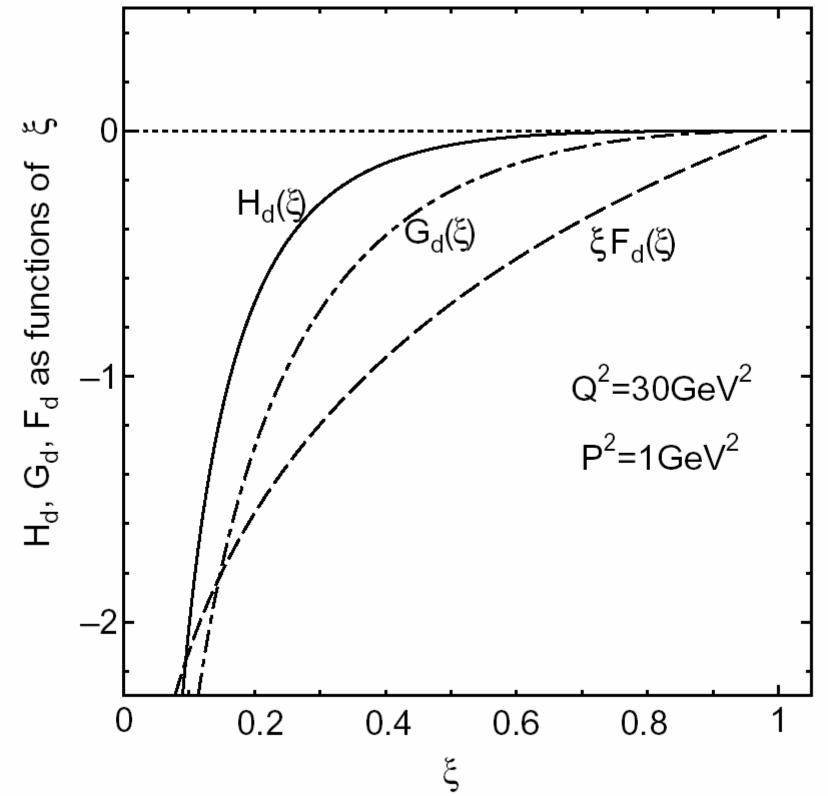
$$
\begin{aligned}
g_1^\gamma(x,Q^2,P^2) \;=\;& 4\kappa\xi^2\,\frac{(1+\kappa\xi^2)^3}{(1-\kappa\xi^2)^5}\left\{1+\frac{2\kappa\xi^2}{(1+\kappa\xi^2)^2}\right\}H_a(\xi)\\
&-4\kappa\xi^2\,\frac{(1+\kappa\xi^2)^2}{(1-\kappa\xi^2)^4}\left\{1+\frac{1}{1+\kappa\xi^2}\right\}G_a(\xi)\\
&+\xi\,\frac{(1+\kappa\xi^2)^2}{(1-\kappa\xi^2)^3}F_a(\xi)\\
&-8\kappa\xi^2\,\frac{(1+\kappa\xi^2)^3}{(1-\kappa\xi^2)^5}\left\{1+\frac{2\kappa\xi^2}{(1+\kappa\xi^2)^2}\right\}H_d(\xi)\\
&+12\kappa\xi^2\,\frac{(1+\kappa\xi^2)^2}{(1-\kappa\xi^2)^4}G_d(\xi)-4\kappa\xi^3\,\frac{1+\kappa\xi^2}{(1-\kappa\xi^2)^3}F_d(\xi)
\end{aligned}
$$

$$
\begin{aligned}
g_2^\gamma(x,Q^2,P^2) \;=\;& -6\kappa\xi^2\,\frac{(1+\kappa\xi^2)^3}{(1-\kappa\xi^2)^5}H_a(\xi)\\
&+\frac{(1+\kappa\xi^2)^3}{(1-\kappa\xi^2)^4}\left\{1+\frac{4\kappa\xi^2}{1+\kappa\xi^2}\right\}G_a(\xi)\\
&-\xi\,\frac{(1+\kappa\xi^2)^2}{(1-\kappa\xi^2)^3}F_a(\xi)\\
&+12\kappa\xi^2\,\frac{(1+\kappa\xi^2)^3}{(1-\kappa\xi^2)^5}H_d(\xi)\\
&-\frac{(1+\kappa\xi^2)^4}{(1-\kappa\xi^2)^4}\left\{1+\frac{8\kappa\xi^2}{(1+\kappa\xi^2)^2}\right\}G_d(\xi)\\
&+\xi\frac{(1+\kappa\xi^2)^3}{(1-\kappa\xi^2)^3}F_d(\xi)
\end{aligned}
$$

where $H_{a,d}(\xi)$, $G_{a,d}(\xi)$ and $F_{a,d}(\xi)$ are given by

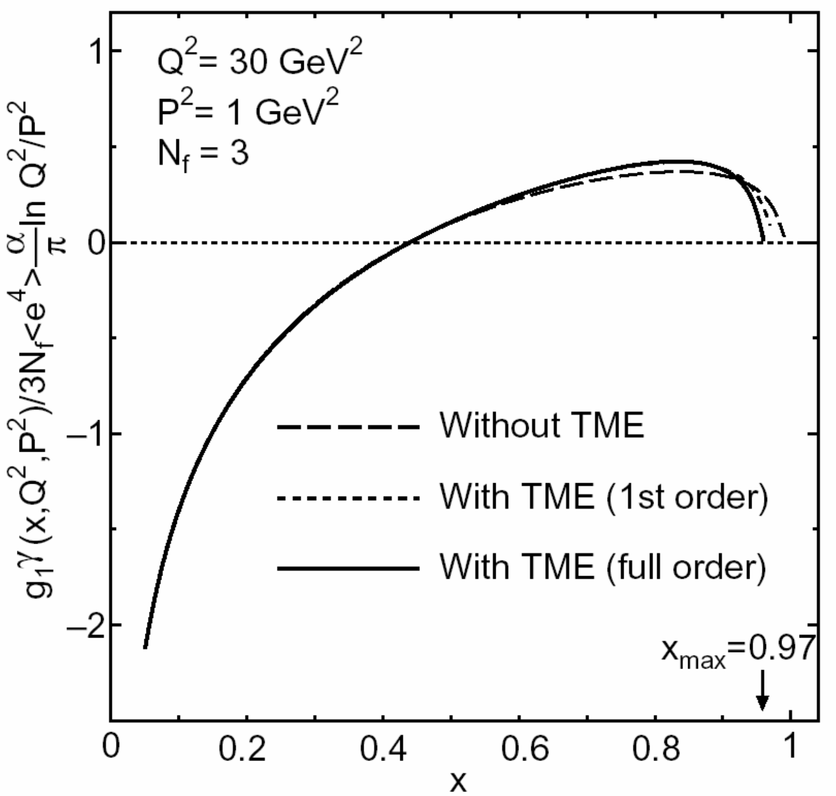
$$H_{a,d}(\xi) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dn\, \xi^{-n}\frac{M_{2,3}^n}{n^2}$$
$$G_{a,d}(\xi) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dn\, \xi^{-n}\frac{M_{2,3}^n}{n}$$
$$\xi F_{a,d}(\xi) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dn\, \xi^{-n}M_{2,3}^n$$

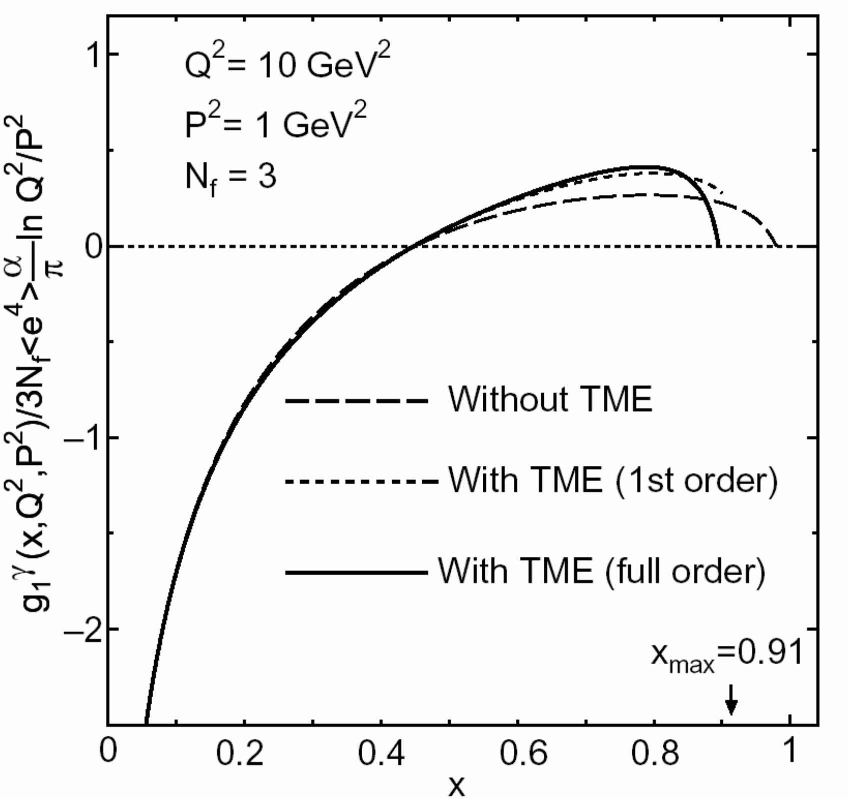




The photon's spin structure fn. $g_1^\gamma(x, Q^2, P^2)$ with TME.

$Q^2 = 30\mathrm{GeV}^2$ and $Q^2 = 10\mathrm{GeV}^2$ with $P^2 = 1\mathrm{GeV}^2$

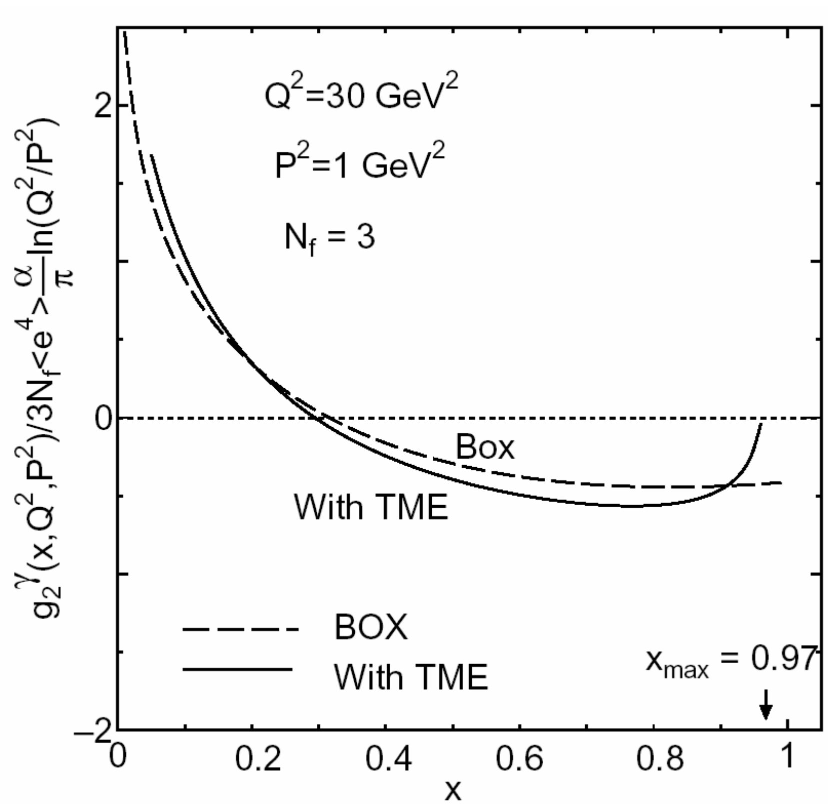


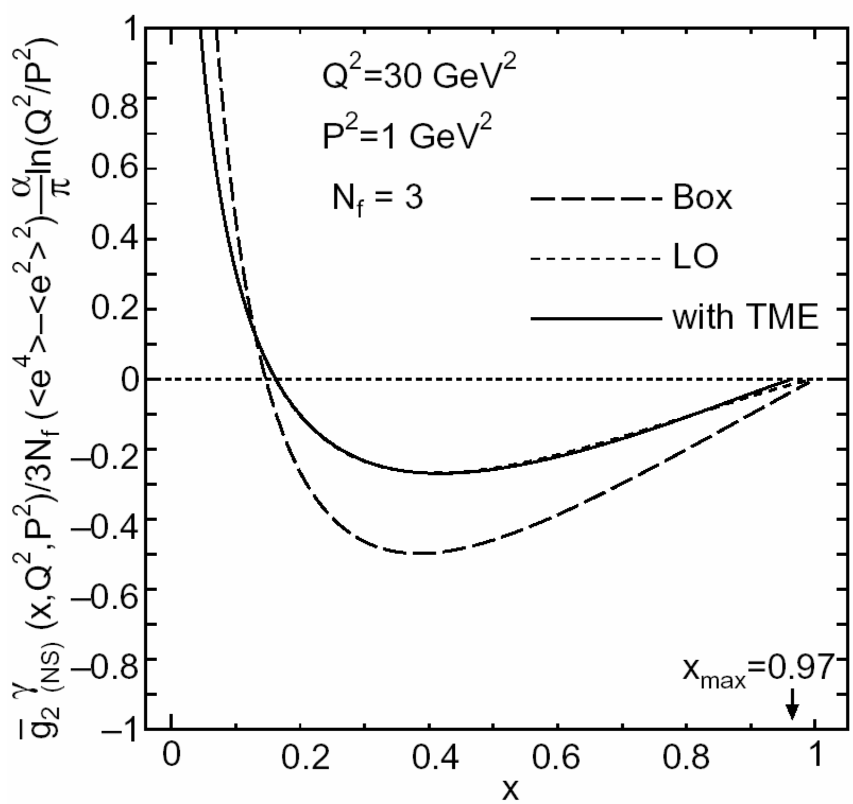


The photon's spin structure fn. $g_2^{\gamma}(x, Q^2, P^2)$ with TME.

for $Q^2 = 30\mathrm{GeV}^2$ and $\bar{g}_2^{\gamma(NS)}(x, Q^2, P^2)$ for $Q^2 = 30\mathrm{GeV}^2$

with $P^2 = 1\mathrm{GeV}^2$





3. QCD sum rules with TME

cf. the real photon case

$$\int_0^1 dx g_1^\gamma(x, Q^2) = 0$$

(1) *The 1st moment of* g_1^γ

$$\int_0^1 dx g_1^\gamma(x, Q^2, P^2) = -\frac{3\alpha}{\pi}\sum_{i=1}^{n_f} e_i^4 + \mathcal{O}(\alpha_s)$$

With the target mass effect (TME)

$$\frac{1}{9}\int_0^{x_{\max}} dx \frac{\xi^2}{x^2}\left[5 + 4\sqrt{1-\frac{4P^2x^2}{Q^2}}\right] g_1^\gamma(x, Q^2, P^2)$$
$$+\frac{4}{3}\int_0^{x_{\max}} dx \frac{\xi^2}{x^2}\frac{P^2x^2}{Q^2} g_2^\gamma(x, Q^2, P^2)$$
$$= -\frac{3\alpha}{\pi}\sum_{i=1}^{N_f} e_i^4 + \mathcal{O}(\alpha_s)$$

power-expansion: The difference of LHS.

$$\Delta\Gamma_1^\gamma = -\left\{\frac{2}{9}M_2^{n=3} + \frac{8}{9}M_3^{n=3}\right\}\frac{P^2}{Q^2} + \mathcal{O}\Big((P^2/Q^2)^2\Big)$$

(2) *The 1st moment of* g_2^γ

$$\int_0^1 dx\, g_2^\gamma(x,Q^2,P^2) = 0$$ *Burkhardt-Cottingham sum rule*

⇨ With the target mass effect (TME)

$$\int_0^{x_{\max}} dx\ g_2^\gamma(x,Q^2,P^2) = 0$$

upper limit $1 \longrightarrow x_{\max}$

Note

Twist-2 part

(Wandzura-Wilczek relation)

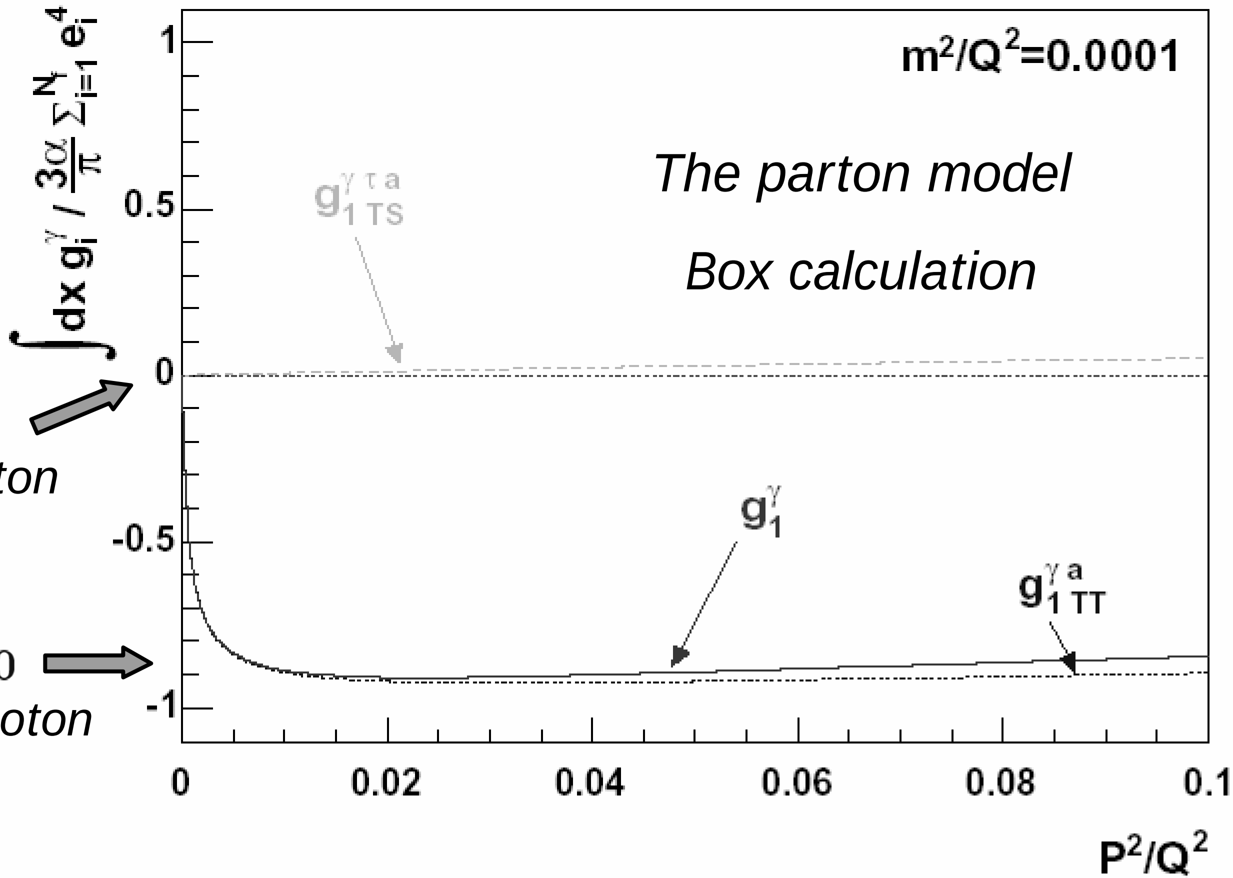
$$\begin{aligned} g_2^{\mathrm{tw.2}} &= g_2^{\mathrm{WW}}(x,Q^2) \\ &\equiv -g_1(x,Q^2) + \int_x^1 \frac{dy}{y} g_1(y,Q^2) \end{aligned}$$

Twist-3 part $\quad \overline{g}_2 = g_2 - g_2^{\mathrm{tw.2}}$

4. P^2 dependence of the sum rule and distribution

The first moment of g_1

Sasaki, Ueda & Uematsu, in preparation



$P^2 = 0$
real photon

$P^2 \neq 0$
virtual photon

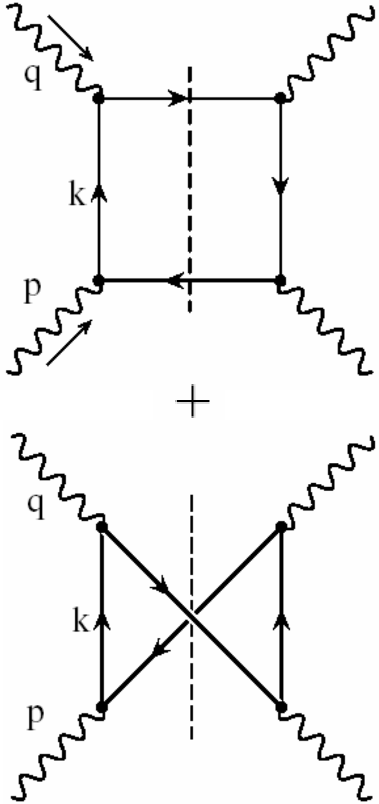
The parton model result

Sasaki, Soffer & Uematsu, PRD 66,034014

$$g_1^\gamma = \frac{2}{\widetilde{\beta}^2}\left\{W_{TT}^a - W_{TS}^{\tau a}\frac{\sqrt{P^2Q^2}}{p\cdot q}\right\} = \mathbf{g}_{1\,TT}^{\gamma\,a} + g_{1\,TS}^{\gamma\,\tau a}$$

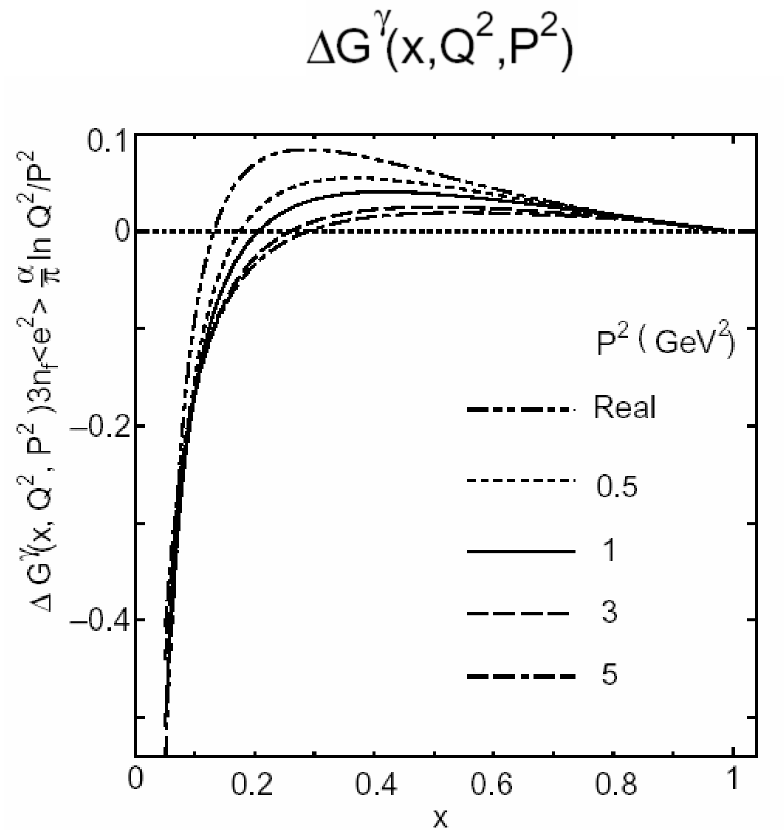
$$\begin{aligned}
W_{TT}^a = \frac{\alpha}{2\pi}\delta_\gamma\Big[L\ \frac{1}{\widetilde{\beta}^3}\Big\{2x\frac{P^2}{Q^2} + (\widetilde{\beta}^2 + 2x - 2)\Big\} \\
+\frac{\beta}{1-\beta^2\widetilde{\beta}^2}\Big\{8x\ \frac{m^2}{Q^2} + \frac{2x\left\{(\beta^2+1)\widetilde{\beta}^2 - 2\right\}}{\widetilde{\beta}^2}\frac{P^2}{Q^2} \\
+\frac{(\widetilde{\beta}^2 + 2x - 2)\left\{(\beta^2+1)\widetilde{\beta}^2 - 2\right\}}{\widetilde{\beta}^2}\Big\}\Big]
\end{aligned}$$

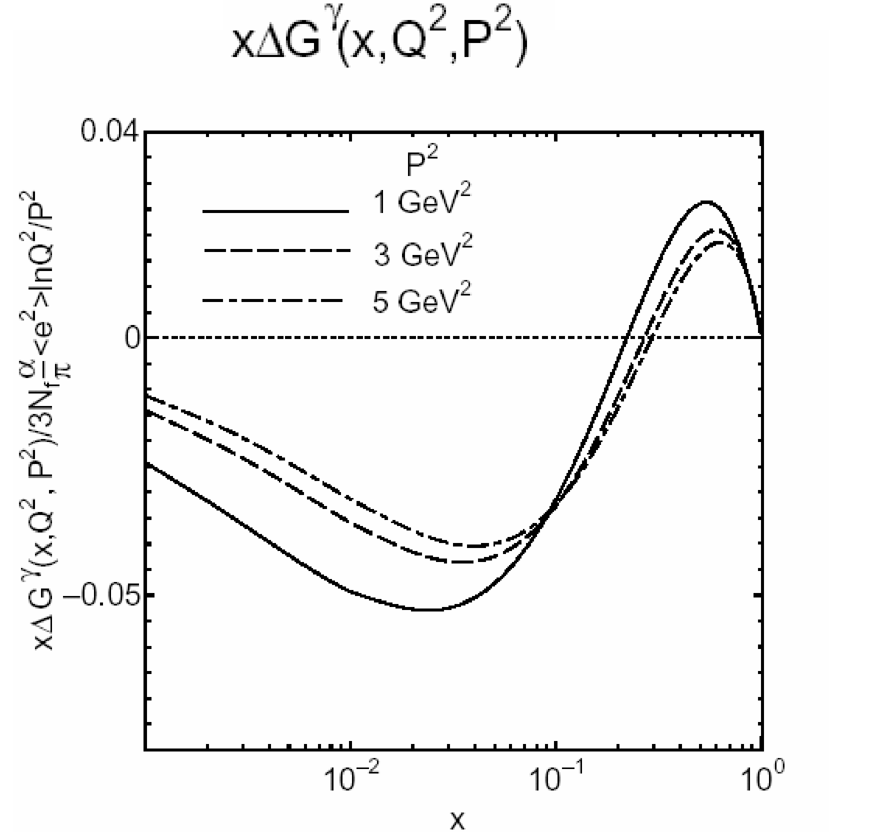
$$\begin{aligned}
W_{TS}^{\tau a} = -\frac{\alpha}{2\pi}\delta_\gamma\Big[L\ \frac{(1-\widetilde{\beta}^2)^{3/2}}{\widetilde{\beta}^3}\Big\{x\frac{P^2}{Q^2} + x - 1\Big\} \\
+\frac{\beta\sqrt{1-\widetilde{\beta}^2}}{\widetilde{\beta}^2(1-\beta^2\widetilde{\beta}^2)}\Big\{-4x\widetilde{\beta}^2\ \frac{m^2}{Q^2} + x\left\{(\beta^2+1)\widetilde{\beta}^2 - 2\right\}\frac{P^2}{Q^2} \\
+(x-1)\left\{(\beta^2+1)\widetilde{\beta}^2 - 2\right\}\Big\}\Big]
\end{aligned}$$



$$\widetilde{\beta} = \sqrt{1-\frac{p^2q^2}{p\cdot q}} = \sqrt{1-\frac{4P^2x^2}{Q^2}} \qquad \beta = \sqrt{1-\frac{4m^2}{(p+q)^2}} \qquad L = \ln\frac{1+\beta\widetilde{\beta}}{1-\beta\widetilde{\beta}}$$

P^2 dependence of gluon distribution





5. Summary and outlook

- The *target mass effects* of photon's spin structure fns. $g_1^\gamma(x,Q^2,P^2)$ and $g_2^\gamma(x,Q^2,P^2)$ studied
- The evaluation of kinematical target mass effects important to extract *dynamical higher-twist effects*
- Closed analytic forms of $g_1^\gamma(x,Q^2,P^2)$ and $g_2^\gamma(x,Q^2,P^2)$ derived by inverting the Nachtmann moments
- Numerical analysis shows that TME appears at *larger x* and *becomes sizable* near $x_{\max}$ as P^2/Q^2 increases . The *1st moment sum rules* for g_1^γ and g_2^γ with TME studied.

Subjects yet to be studied :

- The quark mass effects; heavy flavor contributions
- The transition from real to virtual photons, especially the *1st moment sum rule* including *full QCD effects*.
- Deeply virtual Compton scattering on the photon