Contributions to the 2-loop Bhabha process: new results*

*Update (e.g. MIs given here) can be found in hep-ph/0406203

Janusz Gluza **

DESY-Zeuthen

** Based on common work with Michał Czakon and Tord Riemann

Why study 2-loop Bhabha scattering now?

- Prominent luminosity-monitoring reaction for e^+e^- colliders
- Experimental accuracy points to 10^{-4}
  - hadronic vacuum polarization
  - light fermion pair production $\mathcal{O}(\alpha^2)$
  - QED photonic corrections $\mathcal{O}(\alpha^2 L)$,
- New Physics reach:
  $e^+e^-e^+e^-$ four fermion operators
- ...

Plan

- *2-loop topologies for Bhabha scattering in QED*

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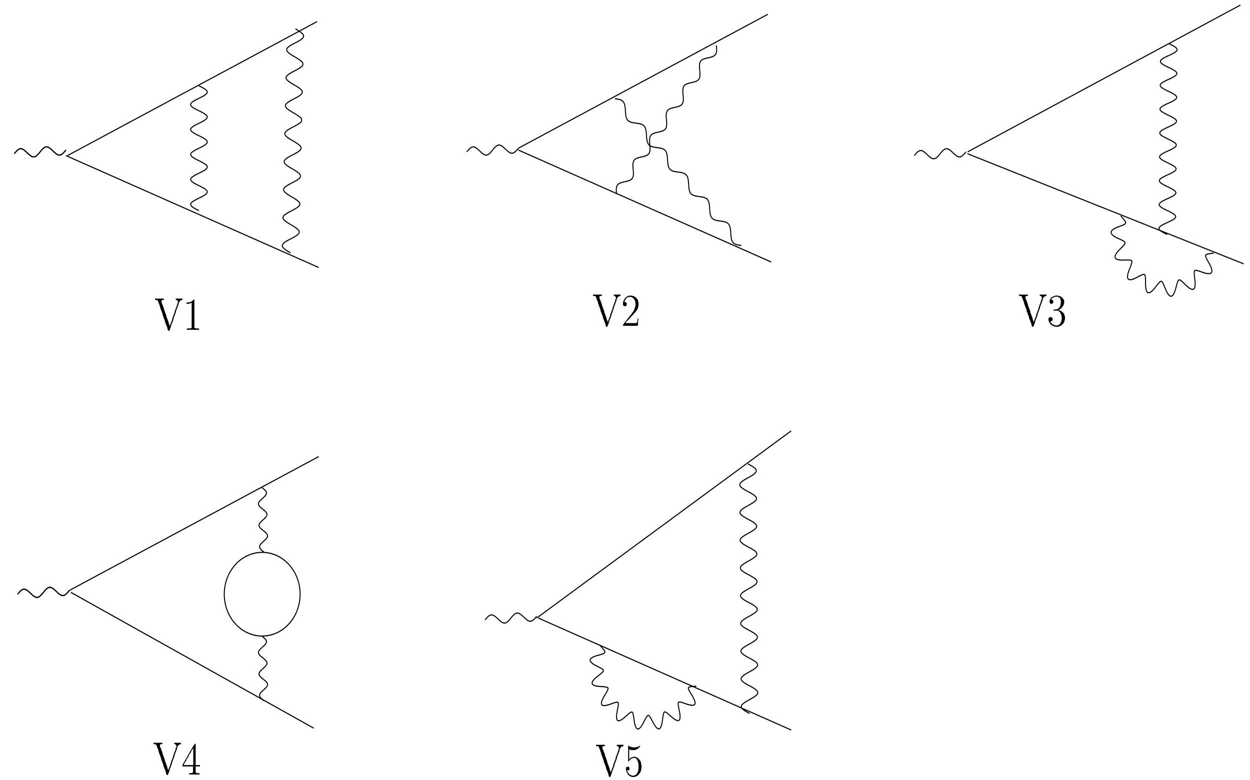
Summary and outlook

Basic 2-loop vertices

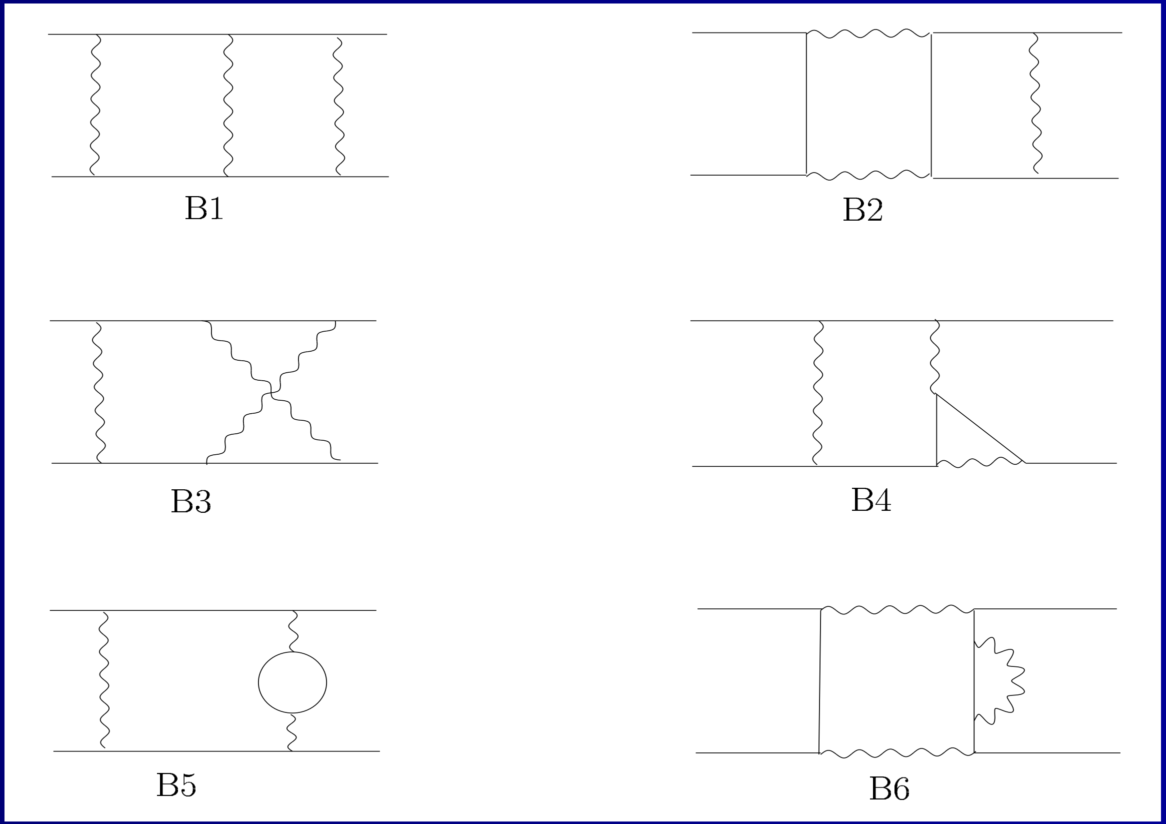


There are 16 Master Integrals for them (Tadpole, 5 SE, 10 V)

R. Bonciani, P. Mastrolia, E. Remiddi, Nucl. Phys. **B661***(2003)289*

Basic 2-loop boxes

Main prototype	Number of MIs
B1	22
B2	35
B3	47
B4	30
B5	14
B6	15



B1: V. Smirnov, Phys. Lett. **B524***(2002)129*

B5: Bonciani et al., Nucl. Phys. **B681***(2004)261*

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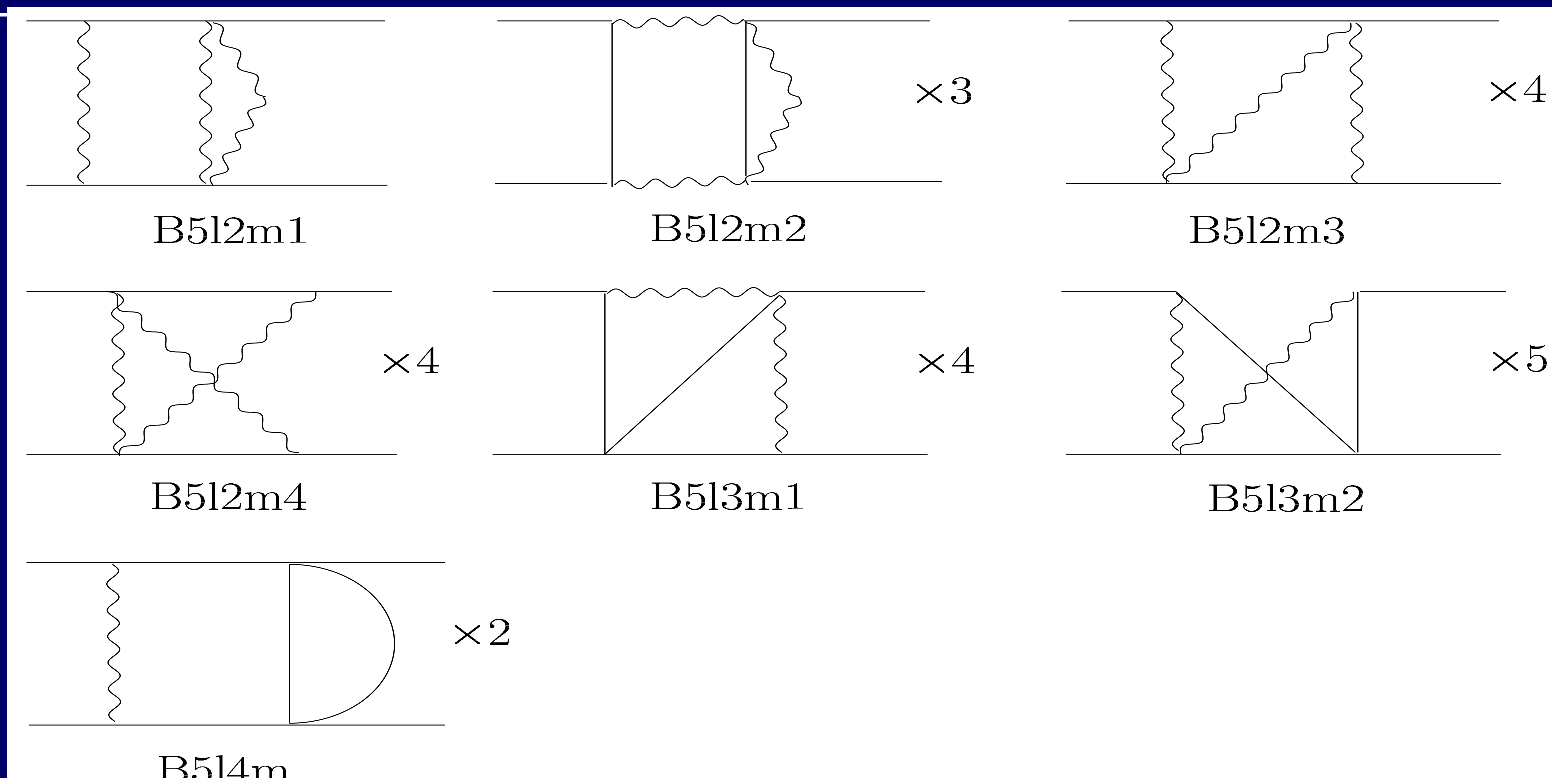
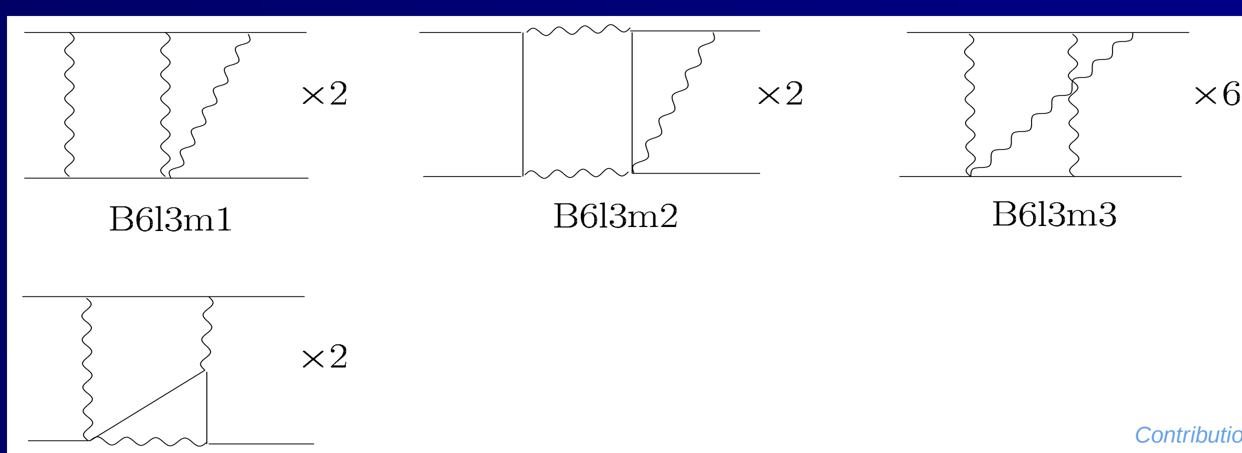
- Identification of MIs for vertices takes a couple of minutes
- Boxes (high powers: 3 additional in denominator and 2 in numerator)

  B1: 4 h; B2: 9 h; B3,B4: 12 h; B5,B6: 1 h.

The complete set of MIs: boxes





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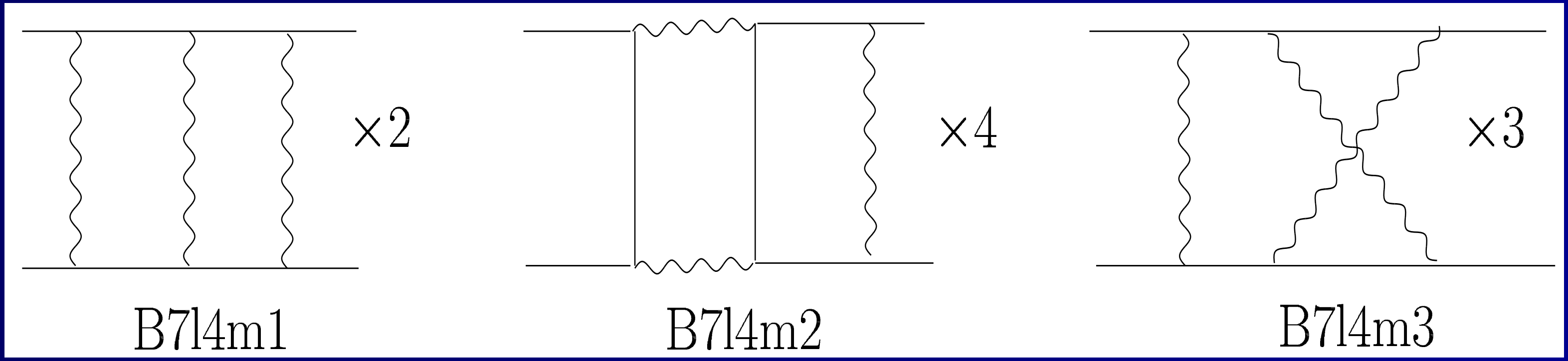
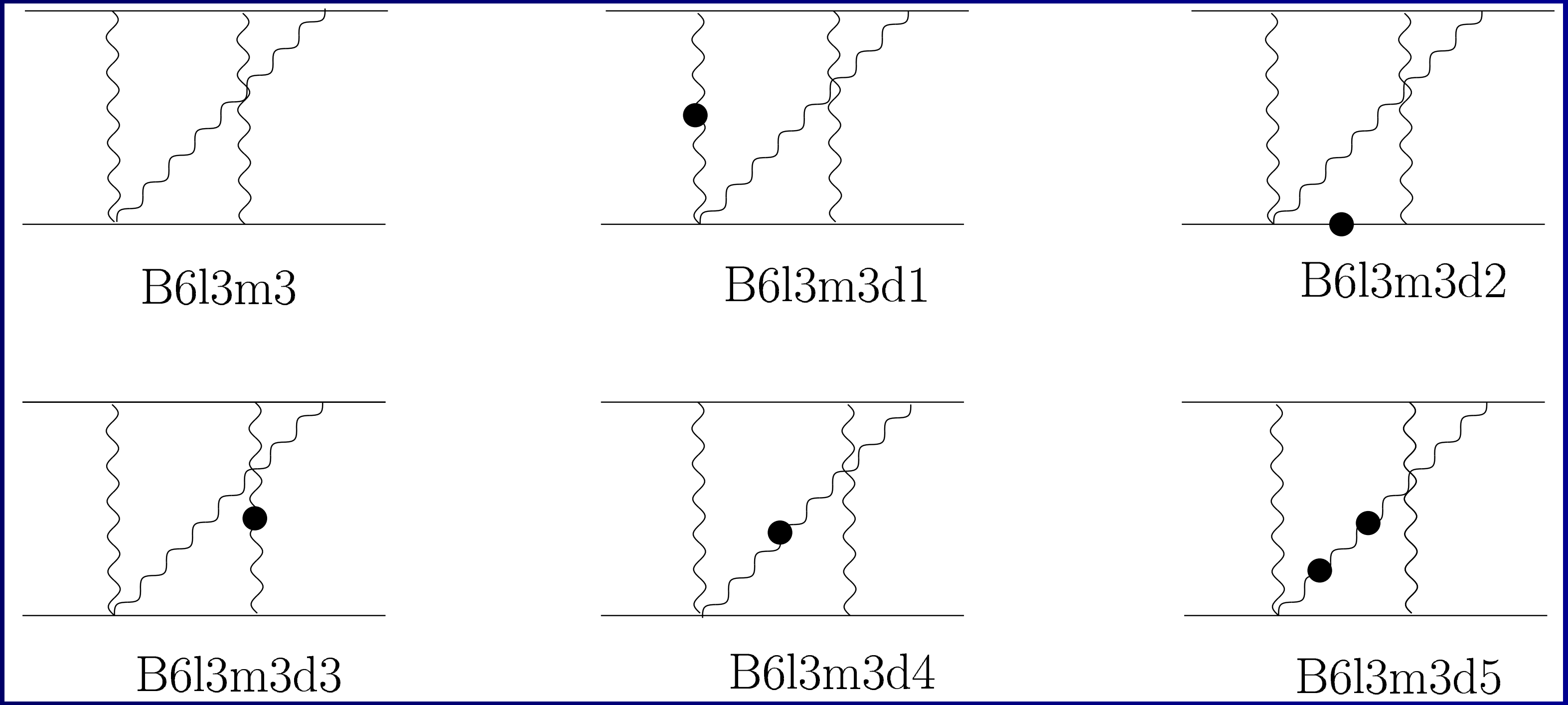


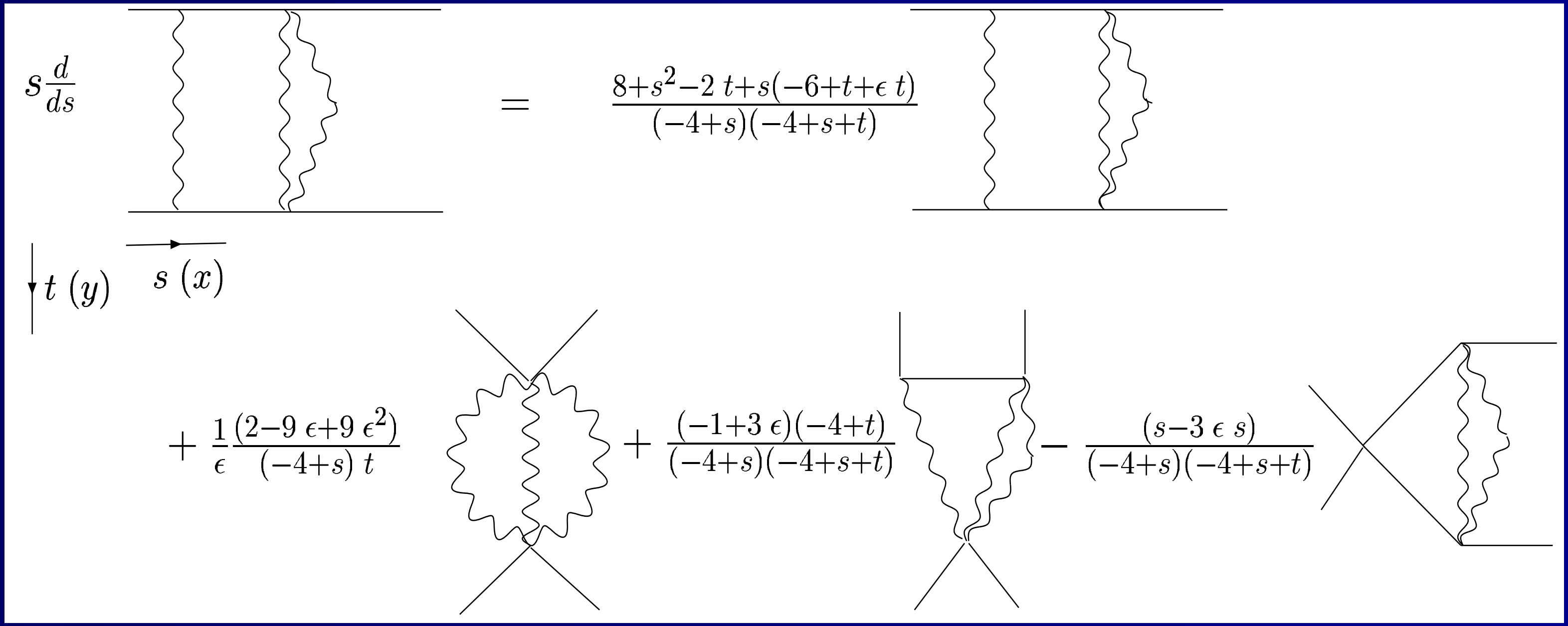
Altogether there are 44 MIs.

The complete set of MIs: B6l3m3 in more details



New MIs: 5 line box B5l2m1

- *Differential equation method (Kotikov, Remiddi)*

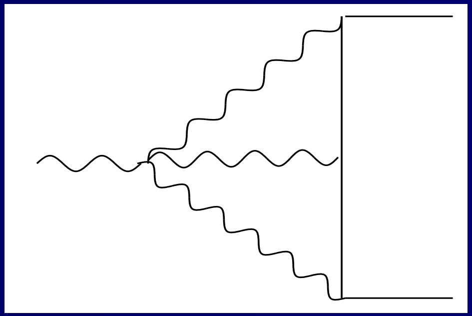


$$\frac{1}{\epsilon^2} \quad \rightarrow \quad \frac{1}{2}\log x\left(\frac{1}{1+x}+\frac{1}{-1+x}\right) \qquad x=\frac{\sqrt{-s+4}-\sqrt{-s}}{\sqrt{-s+4}+\sqrt{-s}}$$

$$\frac{1}{\epsilon} \quad \rightarrow \quad \frac{1}{4}\left(\frac{1}{1+x}+\frac{1}{-1+x}\right)$$

$$[-2\zeta(2)+\log x(\log x-4[-1+\log(1+x)+\log(1-y)]+2\log y)-4Li_2(-x)]$$

New vertices: example



$$
\begin{aligned}
&= \frac{2x}{1-x^2}\left[8\zeta(4) + \zeta(2)\log^2(x) + \frac{1}{24}\log^4(x) - 4\zeta(2)\ Li_2(x) + 2\ Li_4(x)\right] \\
&+ \mathcal{O}(\epsilon)
\end{aligned}
$$

Cross-checks

- *Numerical evaluation of integrals at fixed kinematical points using sector decomposition (Binoth & Heinrich).*

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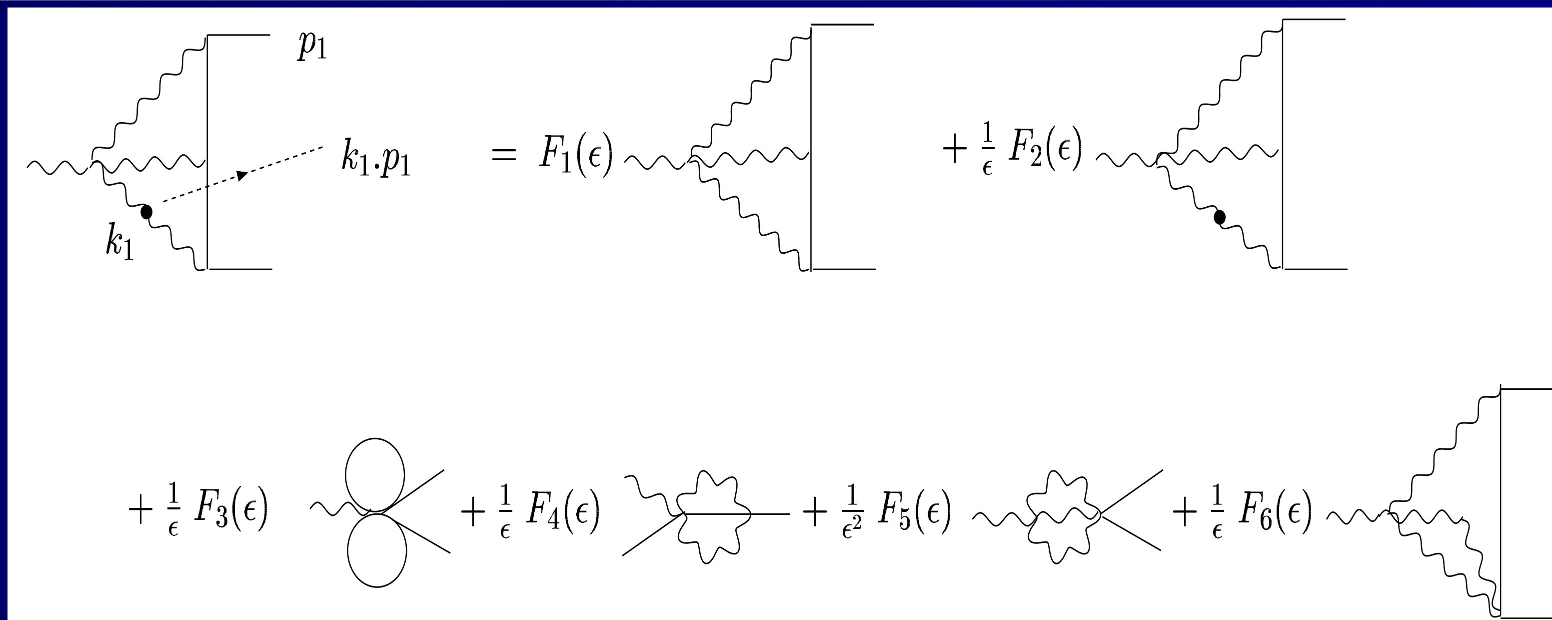
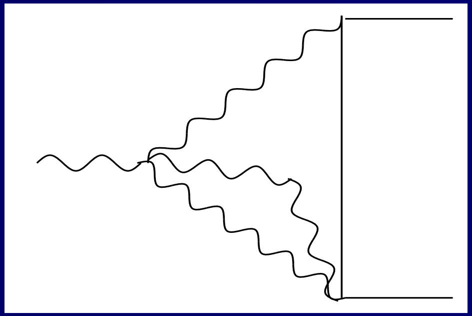
- Divergent parts of IR safe integrals: propeties of the $\overline{MS}$ scheme exploited
- Subloop subtractions for numerical evaluation of finite parts

Using irreducible numerators



$$
\begin{aligned}
OBJd(s) &= A_{-2}(s)\frac{1}{\epsilon^2} + A_{-1}(s)\frac{1}{\epsilon} + A_0(s) + \cdots \\
OBJ(s) &= B_{-2}(s)\frac{1}{\epsilon^2} + B_{-1}(s)\frac{1}{\epsilon} + B_0(s) + \cdots \\
0 &= \frac{1}{\epsilon^3}\left[F_1(s) \; + \; F_2(s)A_{-2}(s)\right] \\
A_{-2}(s) &= \frac{1}{4\ s} \qquad \textit{also:} \quad A_{-1}(s)
\end{aligned}
$$

Subtraction in Feynman parameters for numerical evaluation



$$\sim \int \frac{d^dk_1 d^dk_2}{[k_2^2][(k_2+k_1-p_1)^2][k_1^2+m^2][(k_1+p_2)^2]}$$

$$\to \int \frac{d^dk_1}{[(k_1-p_1)^2]^\epsilon[k_1^2+m^2][(k_1+p_2)^2]}$$

$$= \epsilon(1+\epsilon)\int_0^1 dxdydz x^{-1+\epsilon}\delta(1-x-y-z)\int \frac{d^dk_1}{(xD_1+yD_2+zD_3)^{2+\epsilon}}$$

$$\to \frac{\Gamma(1-\epsilon)^2\Gamma(\epsilon)}{\Gamma(2-2\epsilon)}\epsilon(1+\epsilon)\frac{\Gamma(2\epsilon)}{\Gamma(2+\epsilon)}\int_0^1 dxdy\, x^{-1+\epsilon}(1-x)^{1-2\epsilon}F(x,y,\epsilon)$$

$$F(x,y,\epsilon) = f(x,y)^{-2\epsilon}, \qquad f(x,y)=(1-x)(1-y)^2+xy\frac{t}{m^2}$$

Final result:

$$\text{Identity}: \quad \int_0^1 dy \left\{ [F(x,y,\epsilon) - F(0,y,\epsilon)] + F(0,y,\epsilon) \right\}$$

$$\textit{V4l1m2} = \frac{1}{(m^2)^{2\epsilon}} \left[\frac{\Gamma(\epsilon)\Gamma(2-2\epsilon)}{\Gamma(2-\epsilon)} \frac{1}{1-4\epsilon} + I_{reg} \right],$$

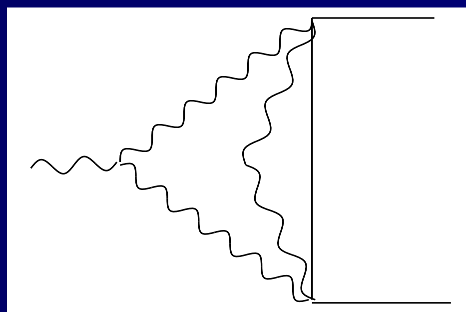
$$I_{reg} = \int_0^1 dx\, x^{-1+\epsilon}(1-x)^{1-2\epsilon} \int_0^1 dy\, [F(x,y,\epsilon) - F(0,y,\epsilon)]$$

$$I_{reg} = \int_0^1 dx (1-x) e^{\epsilon \ln x} e^{-2\epsilon \ln(1-x)} \int_0^1 dy \frac{[\ln f(x,y) - \ln f(0,y)]}{x}$$

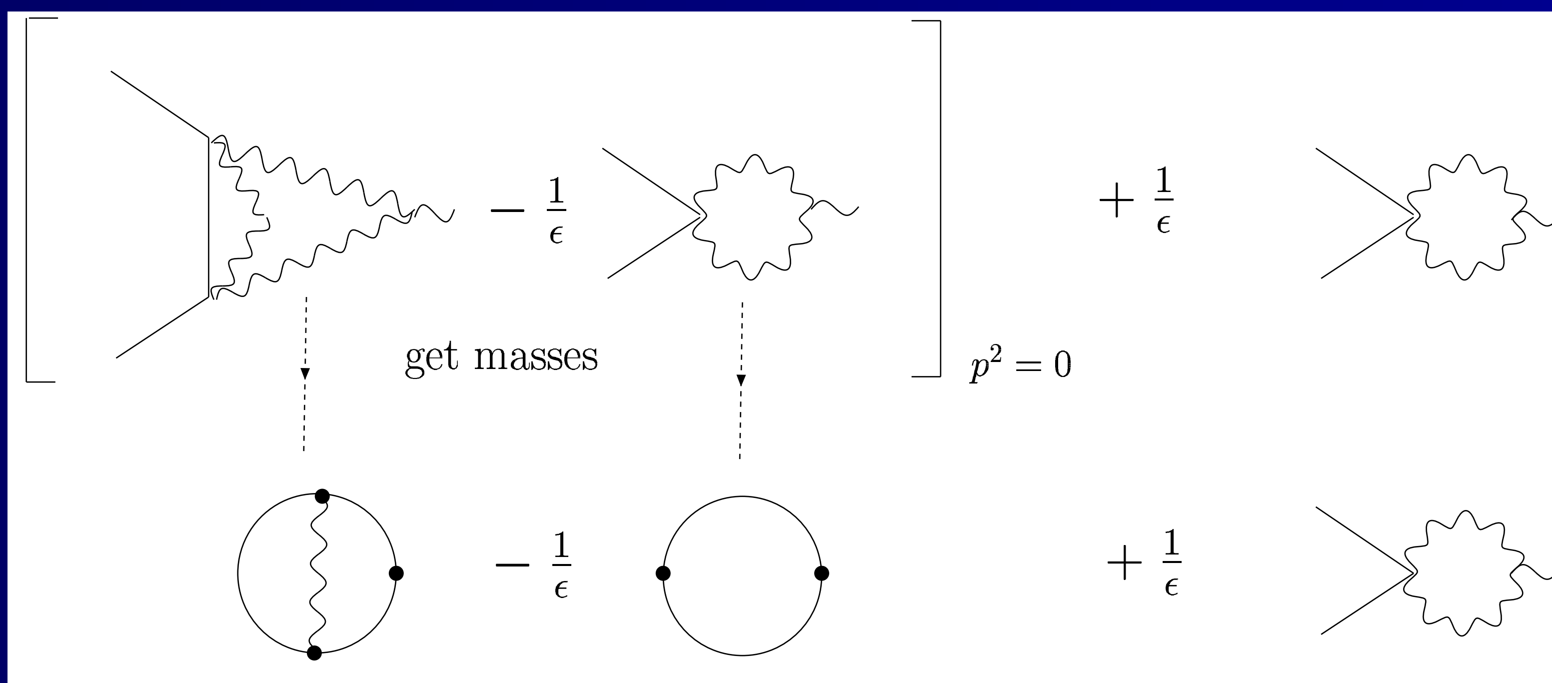
$$\times \sum_{n=0}^{\infty} (-2\epsilon)^n \left[\sum_{k=0}^{n} \ln^{n-k-1} f(x,y) \ln^k f(x,0) \right]$$

$$\textit{V4l1m2} = \frac{1}{2}\frac{1}{\epsilon^2} + \frac{5}{2}\frac{1}{\epsilon} + \frac{1}{2}\left[19 + 4\zeta_2 + 2\left(\frac{x-1}{x+1}\right)\left[4\zeta_2 + \frac{1}{2}\ln^2 x + 2Li_2(x) \right] \right]$$

From renormalization theory:



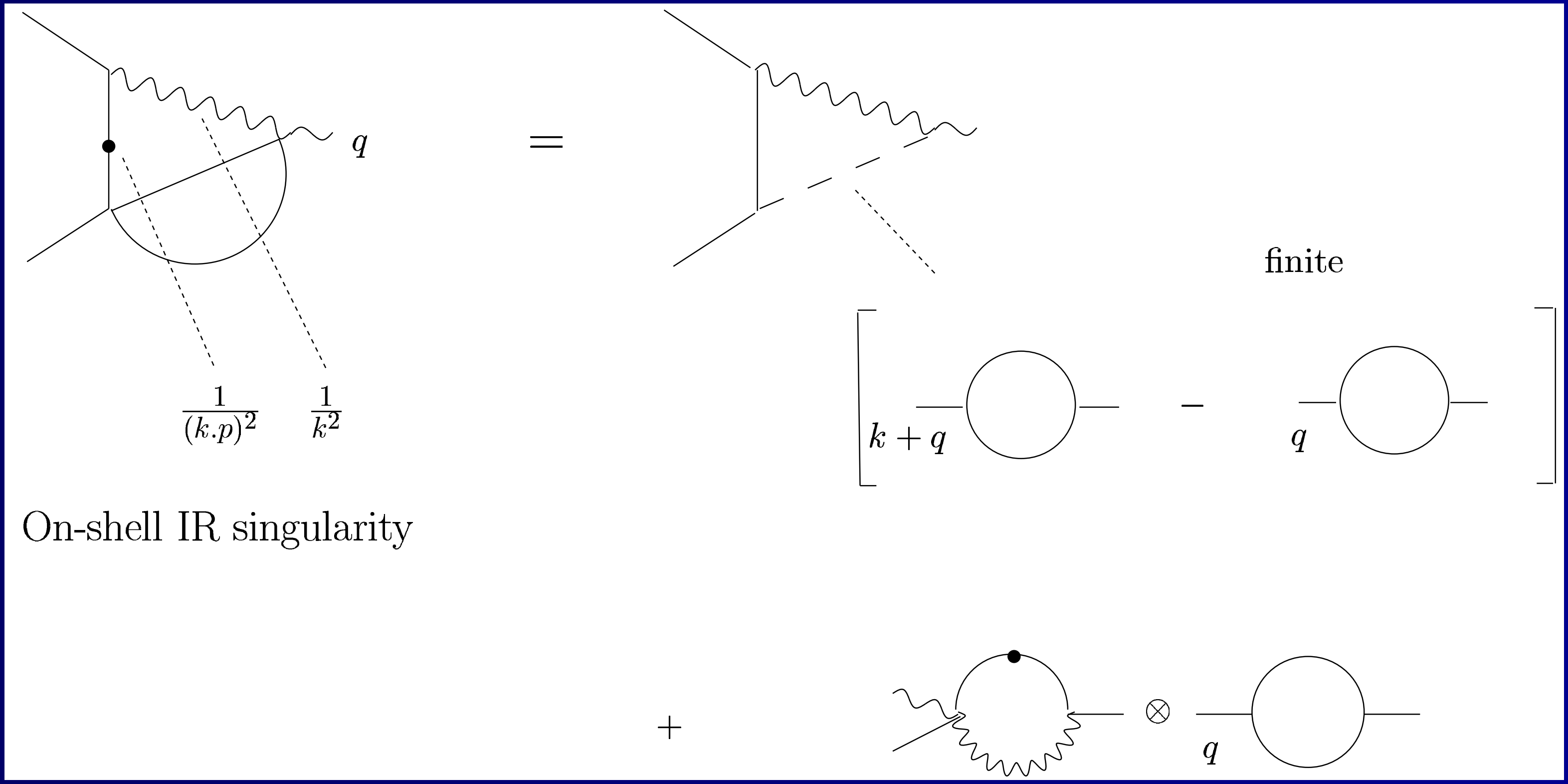
$$= \frac{1}{2}\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\frac{5}{2} + \log(x) - 2\log(1-x)\right) + \cdots$$



$$\frac{1}{2}\frac{1}{\epsilon^2} + \frac{1}{2}\frac{1}{\epsilon} \qquad - \frac{1}{\epsilon}\,\frac{1}{\epsilon} \qquad + \frac{1}{\epsilon}\left[\frac{1}{\epsilon} + 2 + 2\log(1-x) - \log(x)\right]$$

Subloop subtractions:

Cross check to all orders in ϵ



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- *One new 5-line box integral coming from the planar double box diagram has been calculated.*
- *All master integrals have been identifi ed. Altogether, there are 44 two-loop box type MI's, 40 of them remain to be calculated.*