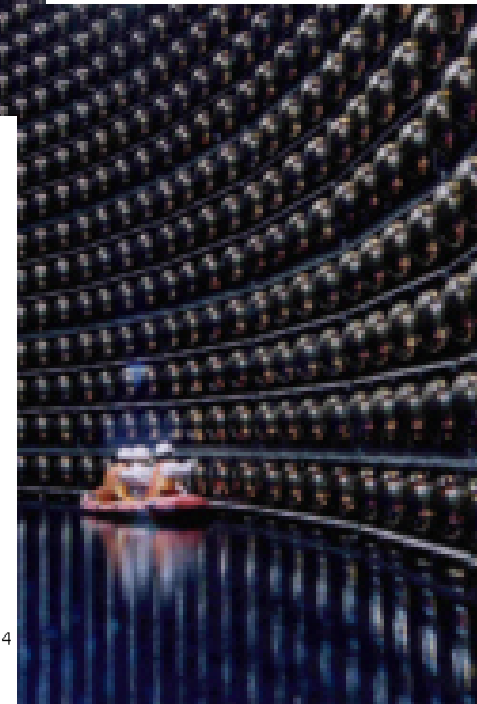
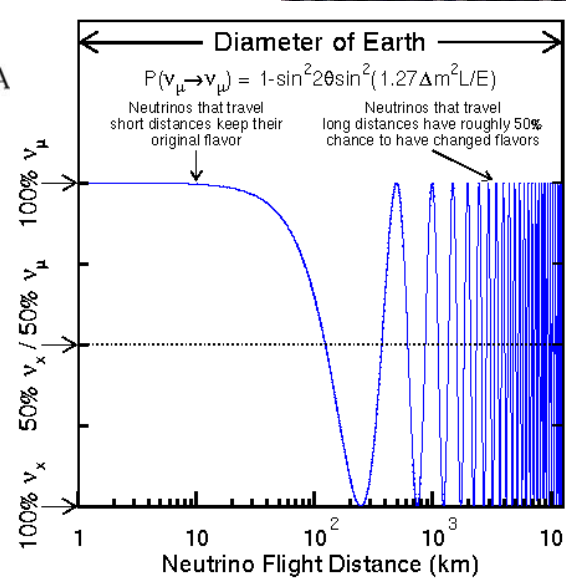
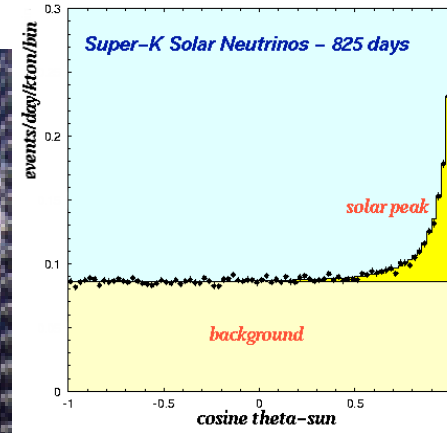
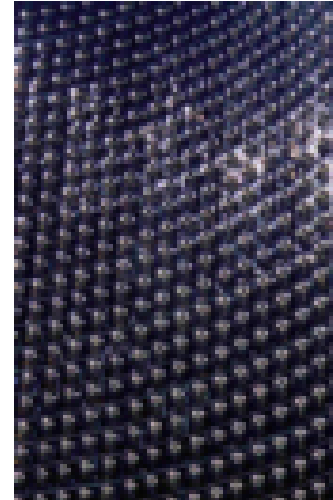
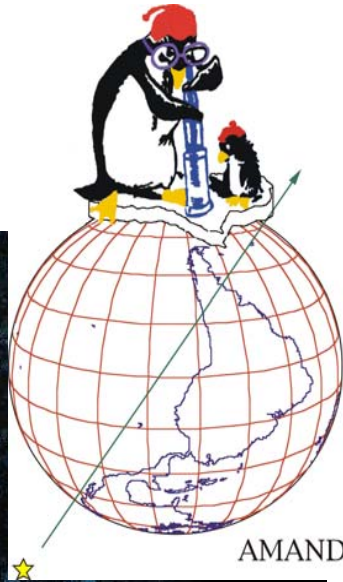
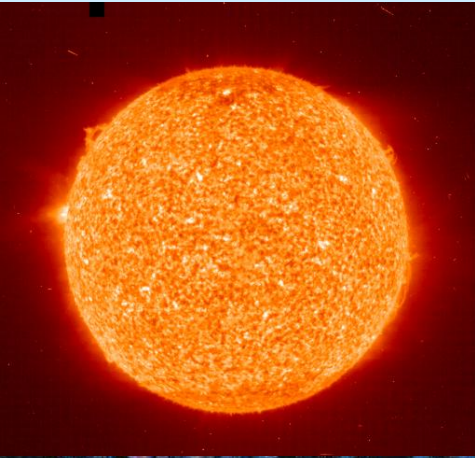
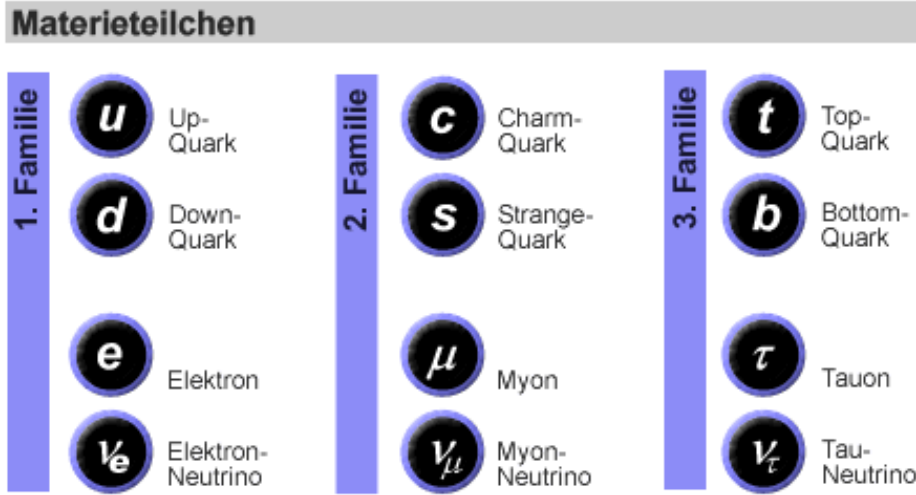


5. Neutrinos

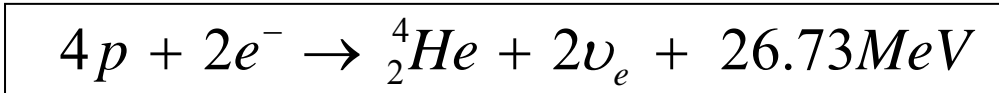


Neutrinos: Standard Model and Beyond

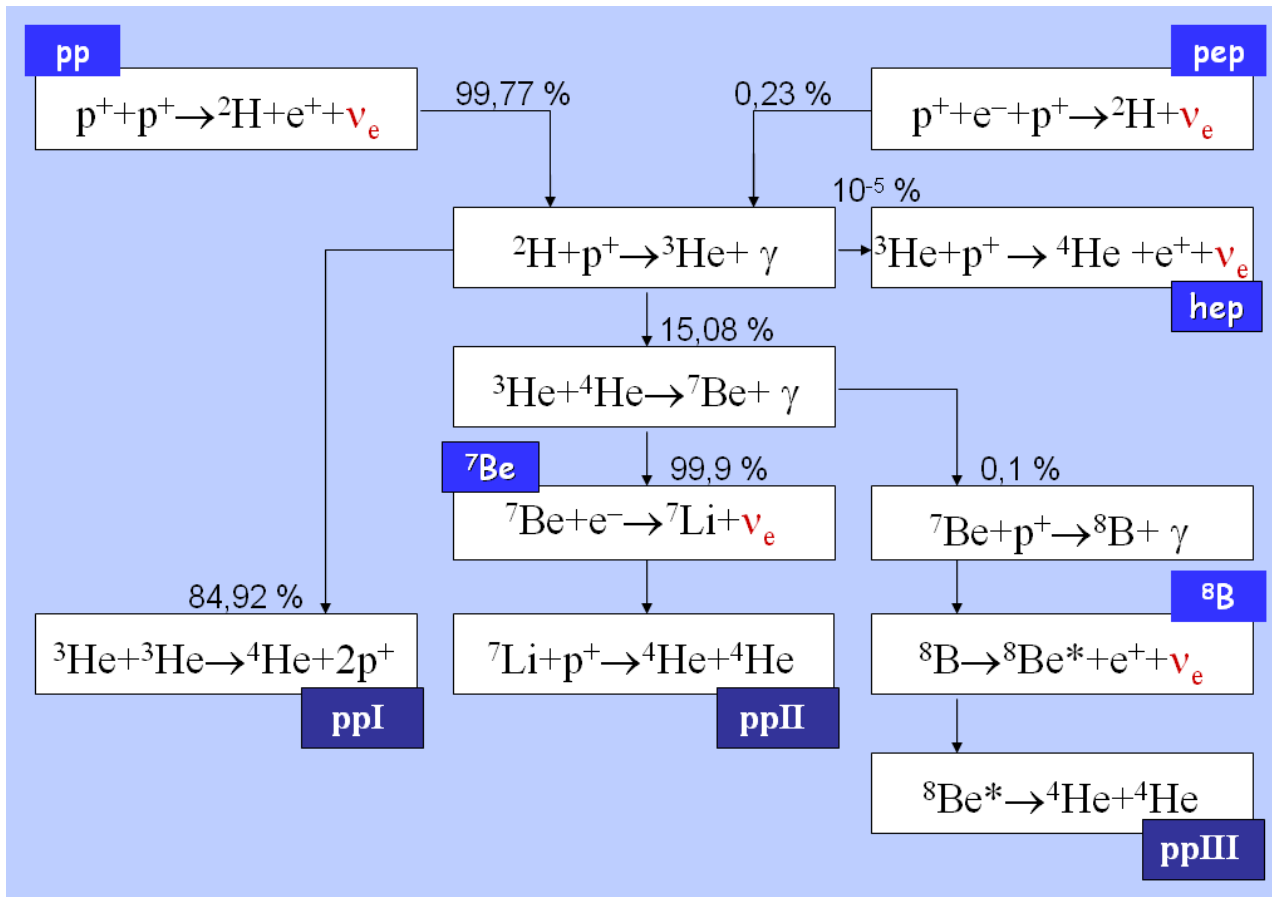


- ν in $SU(2)_L$ dubletts
- Mixing \Rightarrow masses \Rightarrow - flavour eigenstates
- mass eigenstates
- $\nu = \bar{\nu} \Rightarrow$ double β decay possible

Solare Neutrinos



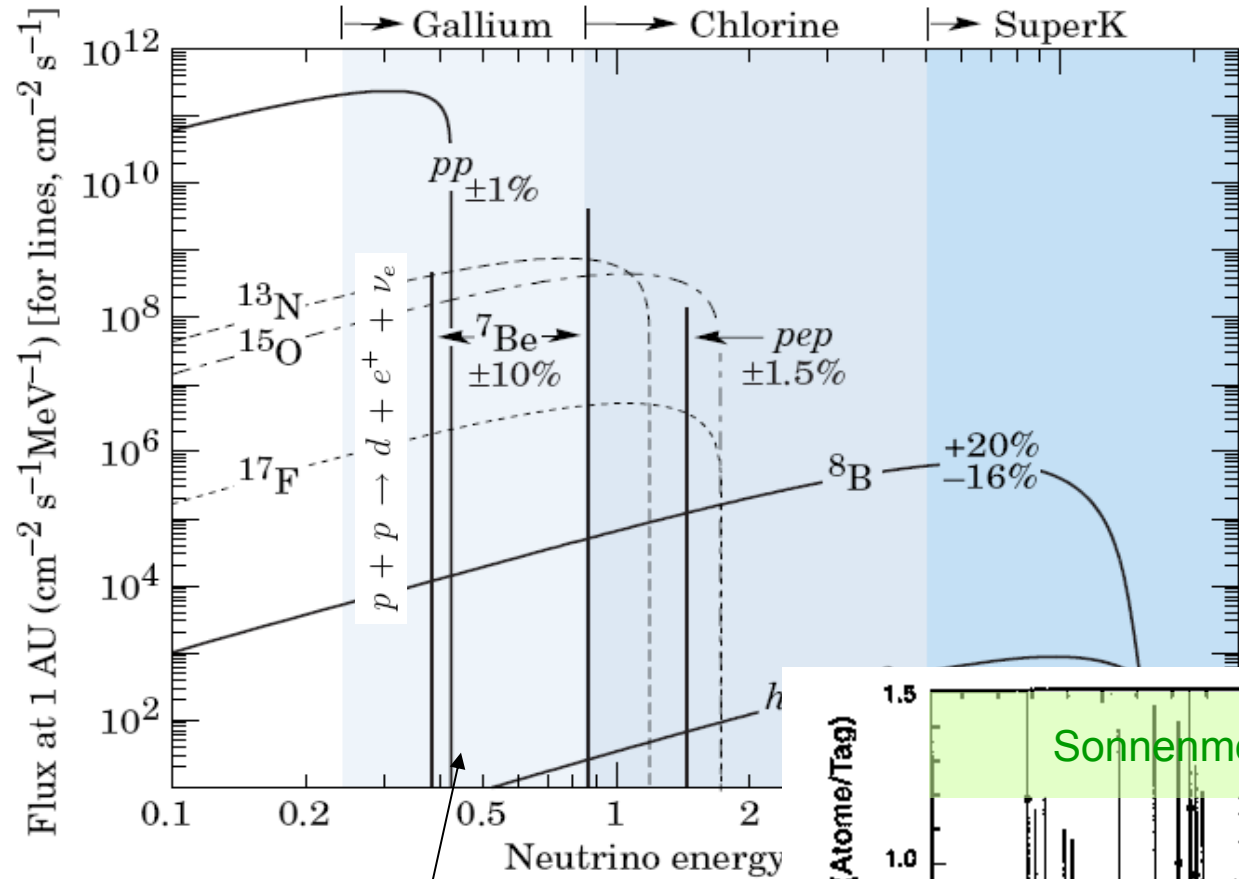
$$\langle E_\nu \rangle = 0.26\text{MeV}$$



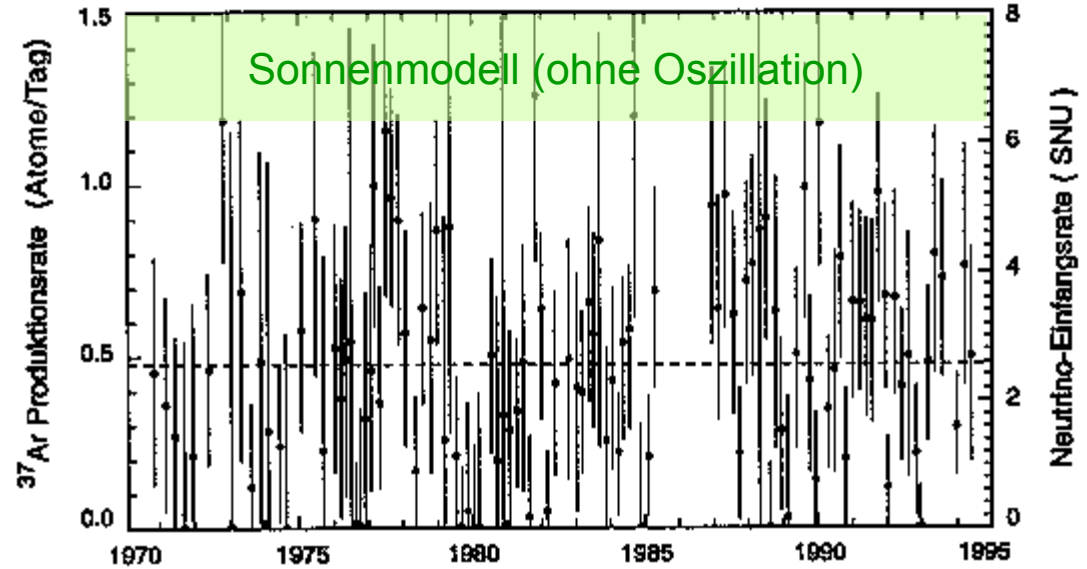
Im Mittel: ~ 13 MeV Sonnenleistung pro pp-Neutrino

Solarkonstante: $S = 8.5 \times 10^{11} \text{ MeVcm}^{-2}\text{s}^{-1} \rightarrow \Phi_\nu = S/\langle E_\nu \rangle \sim 6.5 \times 10^{10} \text{ cm}^{-2}\text{s}^{-1}$

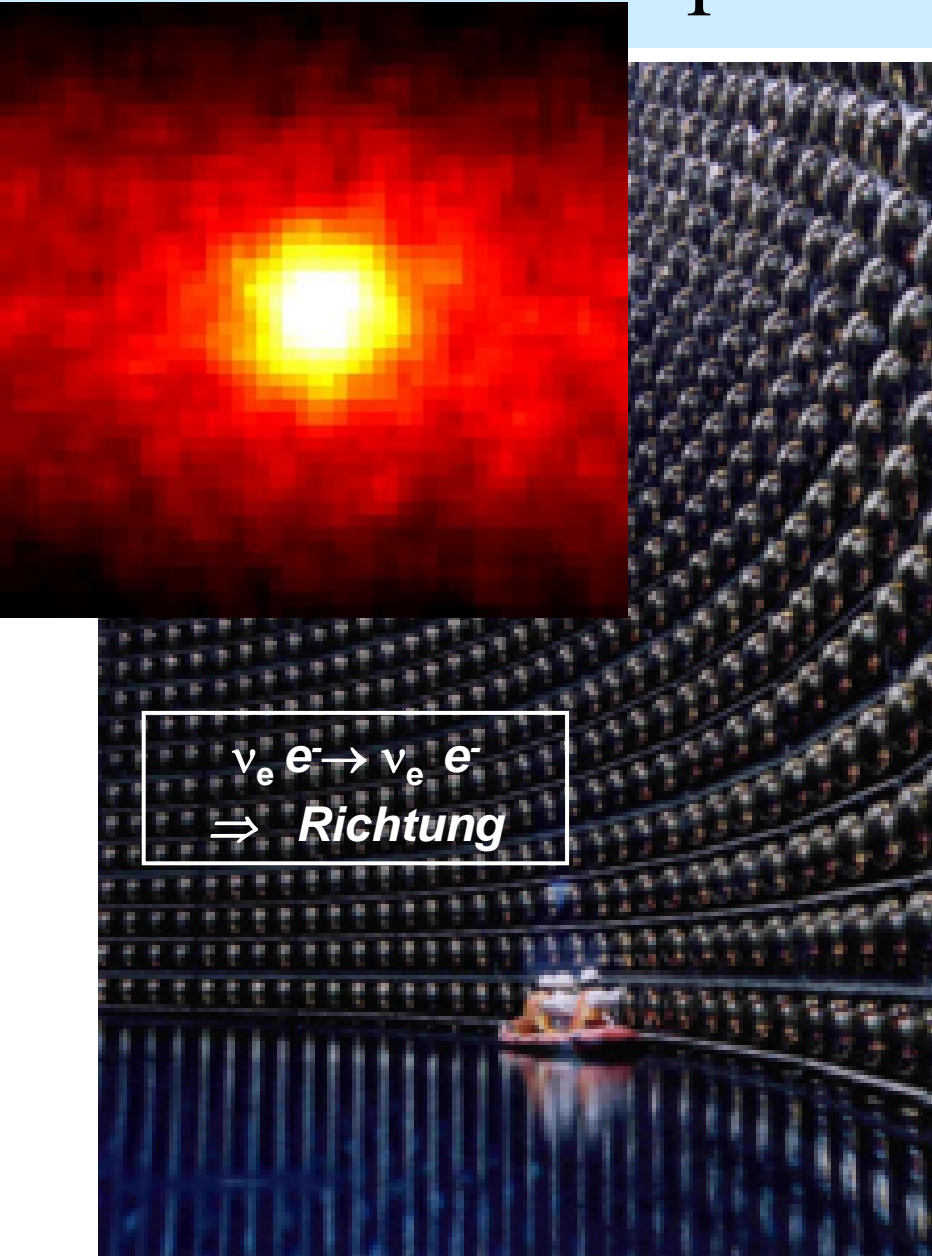
Sonnenneutrino-Defizit



Gallex, Sage

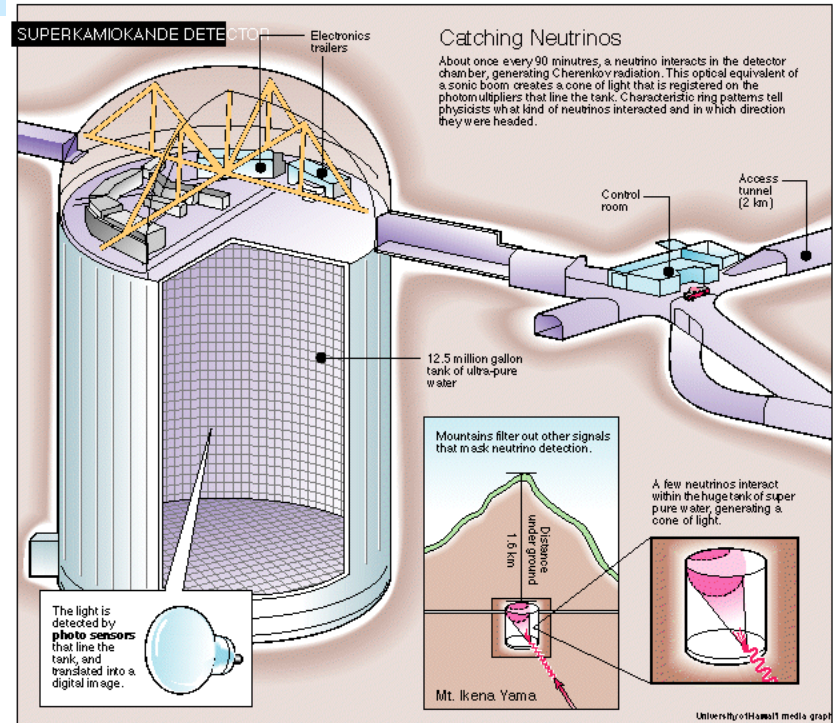


Super-Kamiokande



$$\nu_e e^- \rightarrow \nu_e e^-$$

⇒ *Richtung*



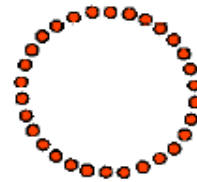
Event Topologies

MiniBooNE

From side

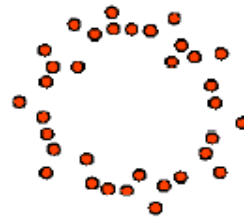
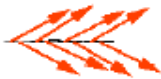
Ring

short track,
no multiple
scattering



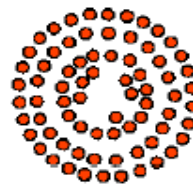
Sharp
Ring

electrons:
short track,
mult. scat.,
brems.



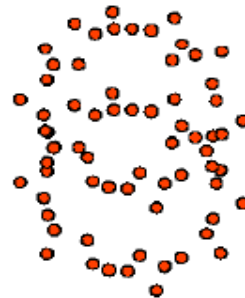
Fuzzy
Ring

muons:
long track,
slows down



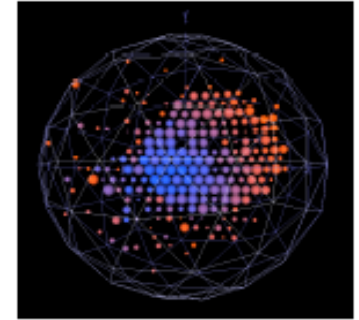
Sharp Outer
Ring with
Fuzzy
Inner
Region

neutral pions:
2 electron-like
tracks

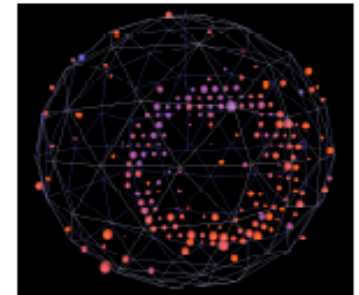


Two
Fuzzy
Rings

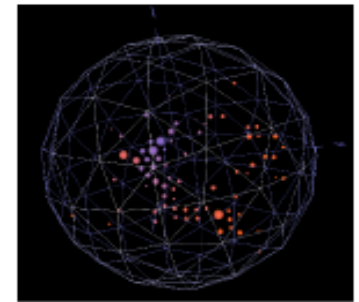
Through-going Muon



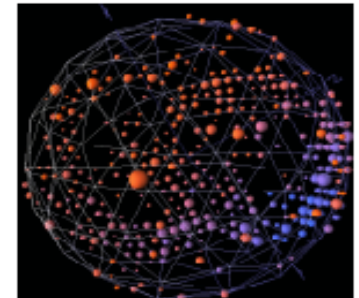
Stopping Muon



Electron



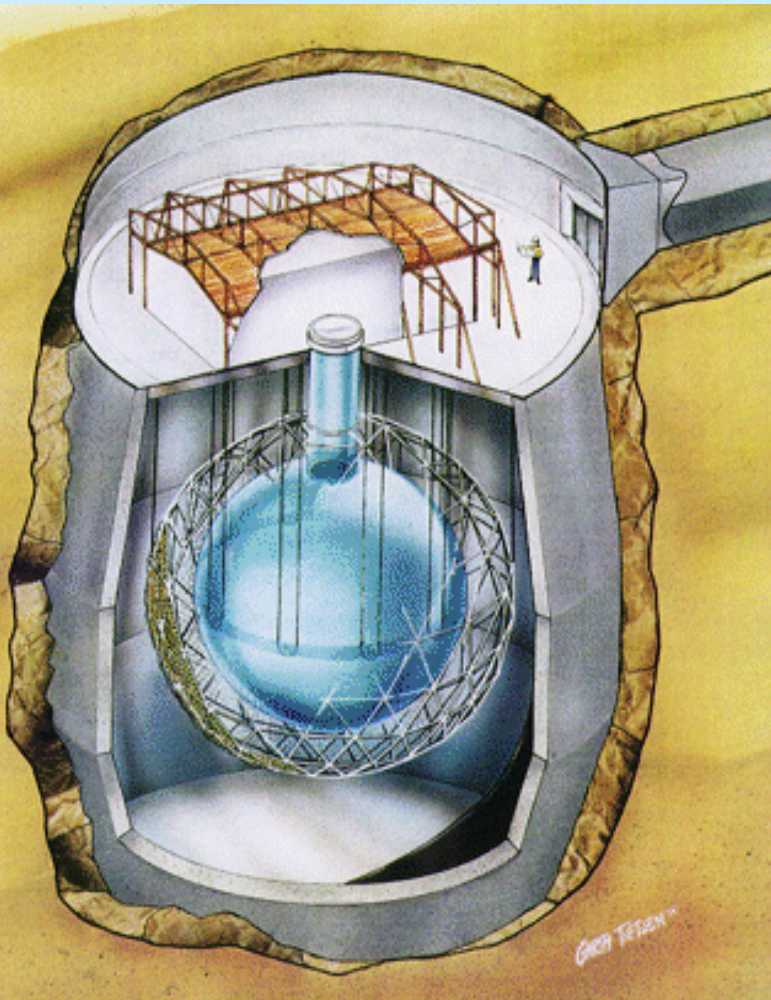
Neutral Pion



ν_e -Defizit von der Sonne

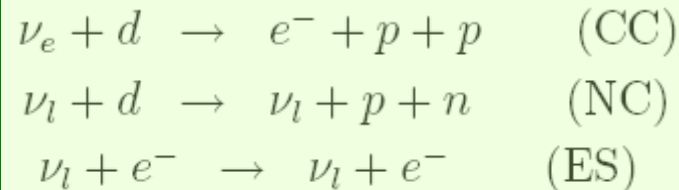
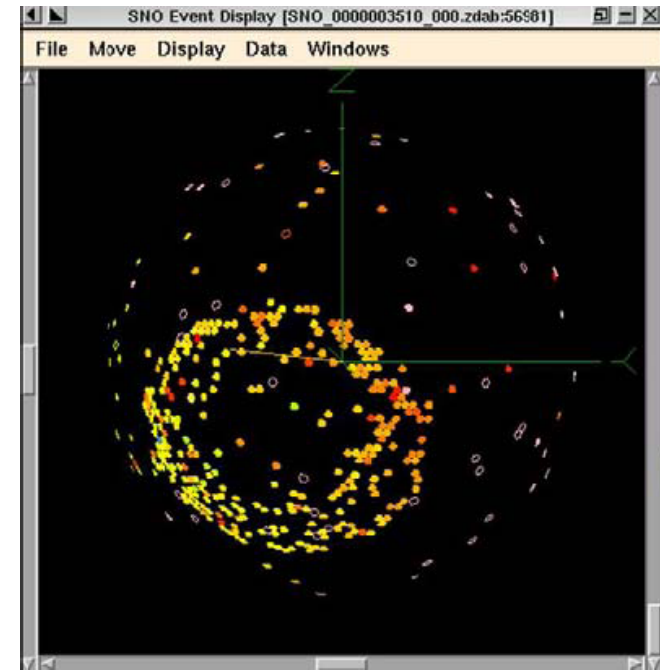
	Homestake	Kamiokande	Super-Kamiokande	Gallex	Sage
Schwelle [MeV]	0.814	7.5	7.0	0.233	0.233
Laufzeit	1970 - 1994	1987 - 1995	1996 - 1998	1991 - 1997	1990 - 1997
Vorhersage	$7.7^{+1.2}_{-1.0}U$	$5.15^{+0.98}_{-0.72}$	$5.15^{+0.98}_{-0.72}$	129^{+8}_{-6}	129^{+8}_{-6}
Experiment	2.56 ± 0.22	2.82 ± 0.38	2.42 ± 0.08	77.5 ± 8	66.6 ± 8
S_{th}/S_{exp}	3.0	1.8	2.1	1.7	1.9

SNO: totaler ν -Fluss von der Sonne



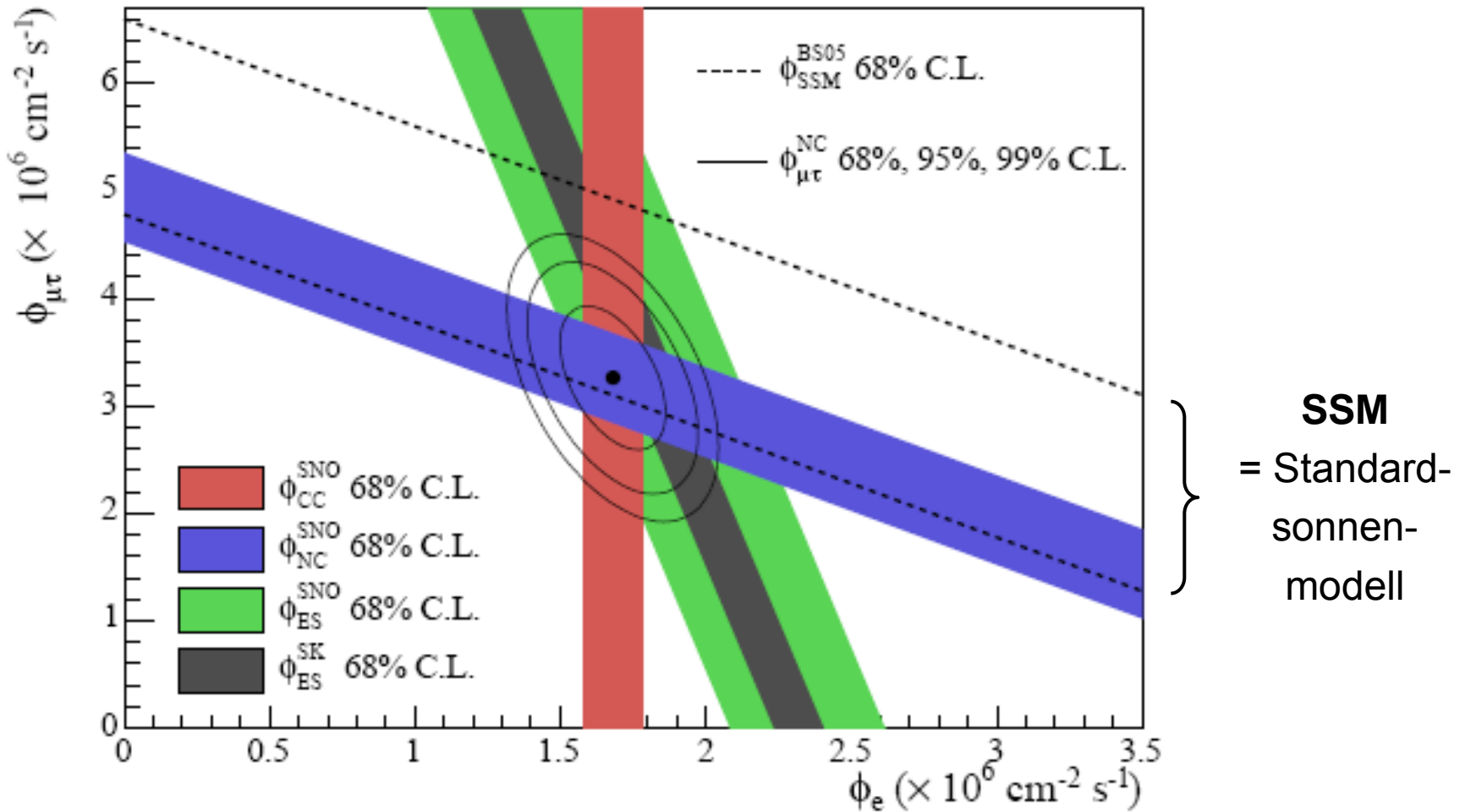
Sudbury Neutrino Observatory

Messe ^8B - ν 's in
 D_2O
CC und NC
(had. & lept.)



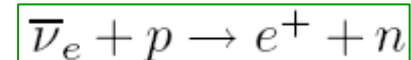
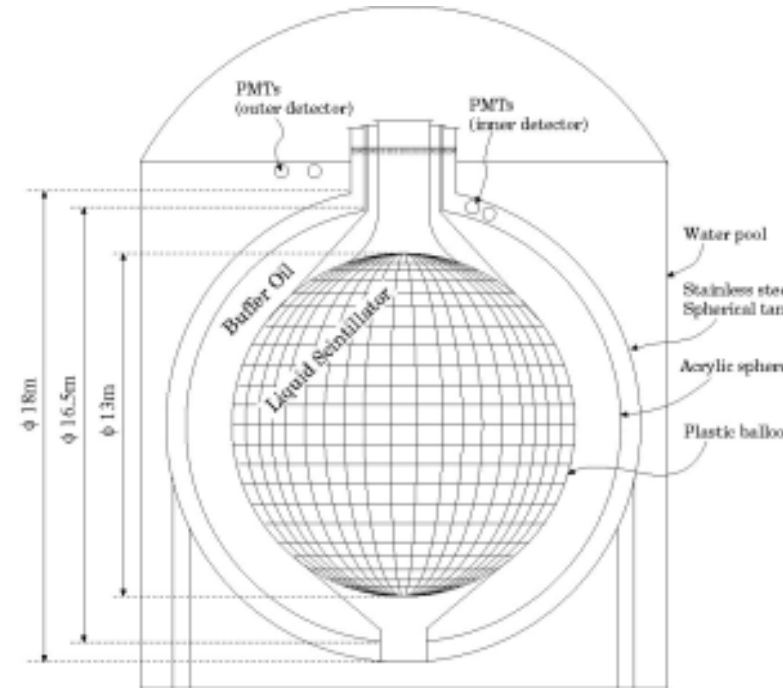
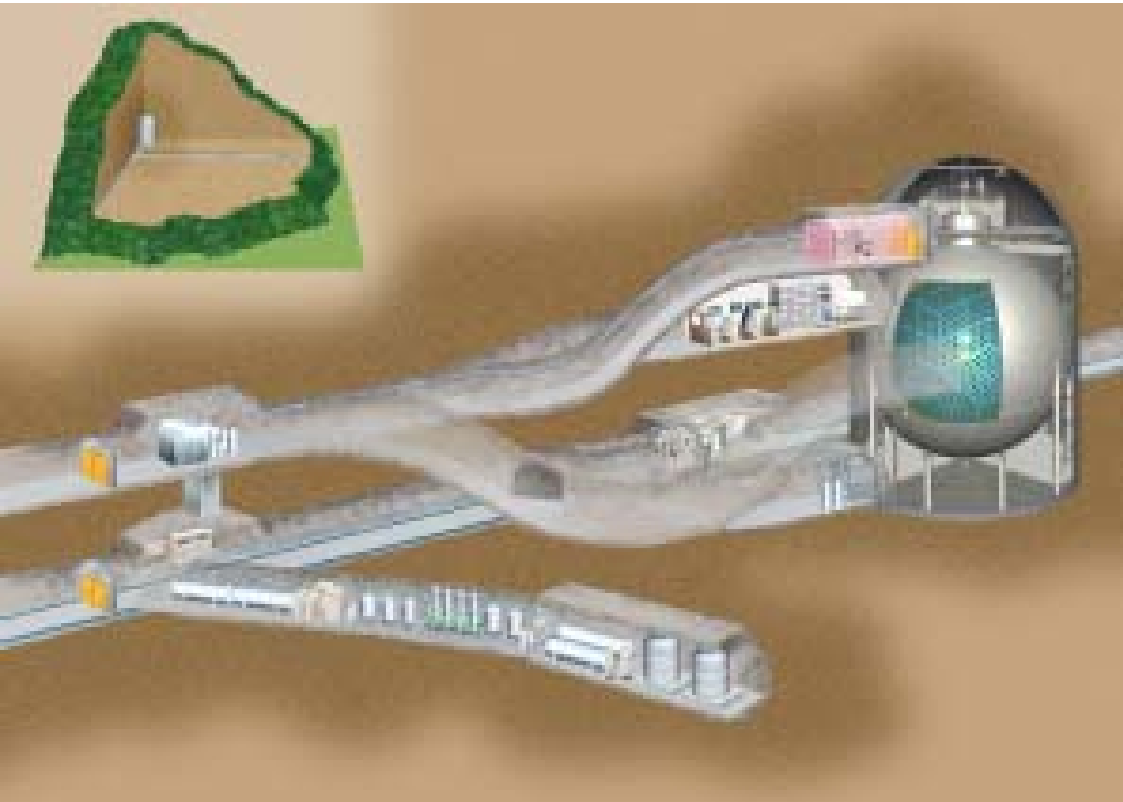
$$\frac{\phi(\nu_e)}{\phi(\nu_e) + \phi(\nu_{\mu,\tau})} = 0.340 \pm 0.023 \pm 0.030$$

SSM verglichen mit $\Phi_{\mu\tau} - \Phi_e$



Reaktor-Antineutrinos: KamLAND

Notwendige Information zur Interpretation der Sonnenneutrinos

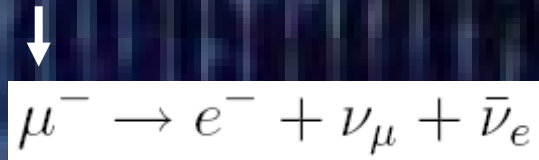
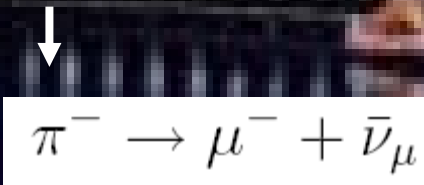
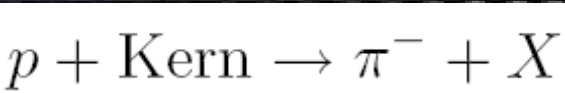
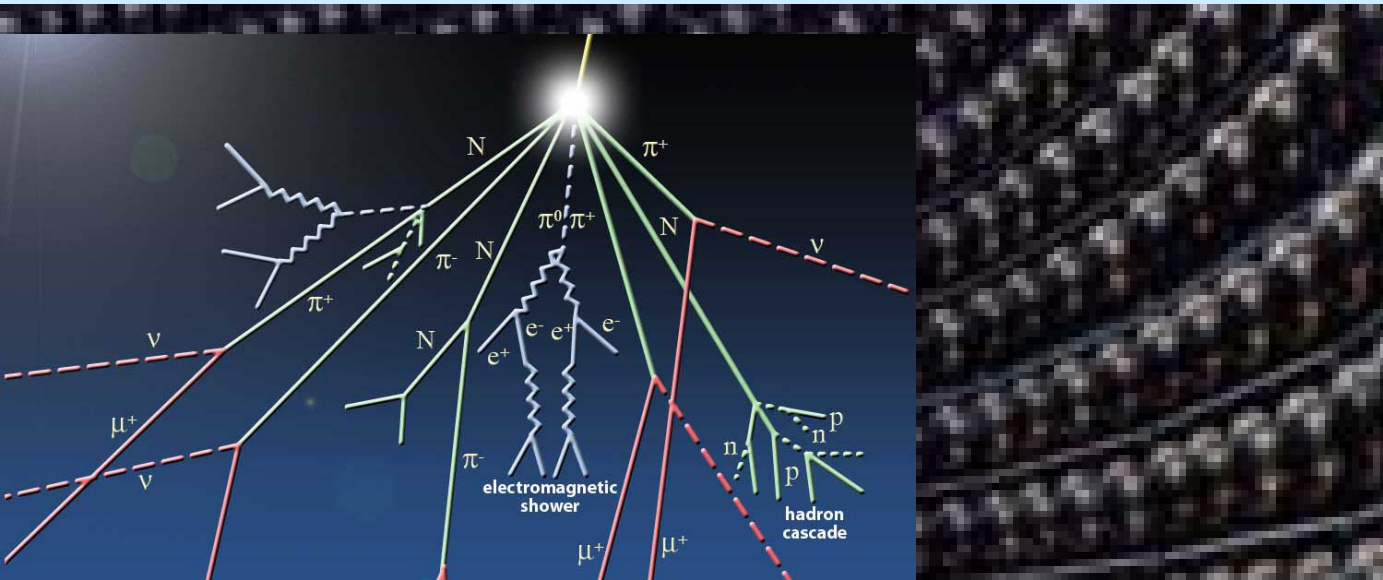


KamLAND is surrounded by 53 Japanese power reactor units

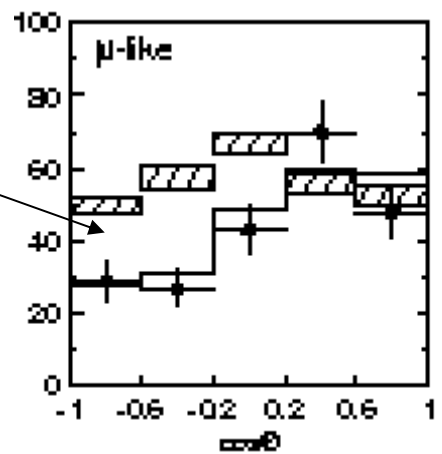
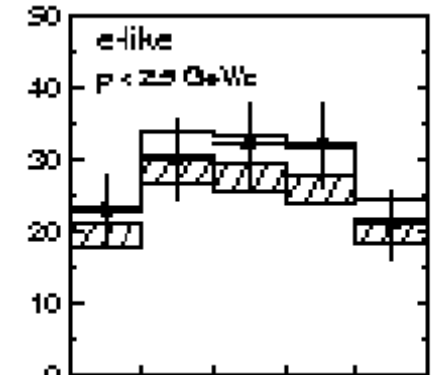
Erwartet:	365.2 ± 23.7 Ereignisse
Beobachtet:	258 Ereignisse

when *CPT* holds
 $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta; U) = P(\nu_\alpha \rightarrow \nu_\beta; U^*)$

Atmosphärische Neutrinos



Defizit

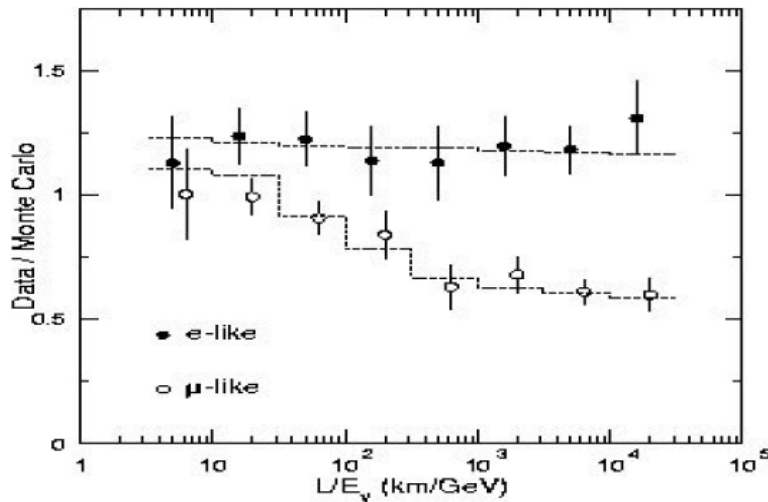
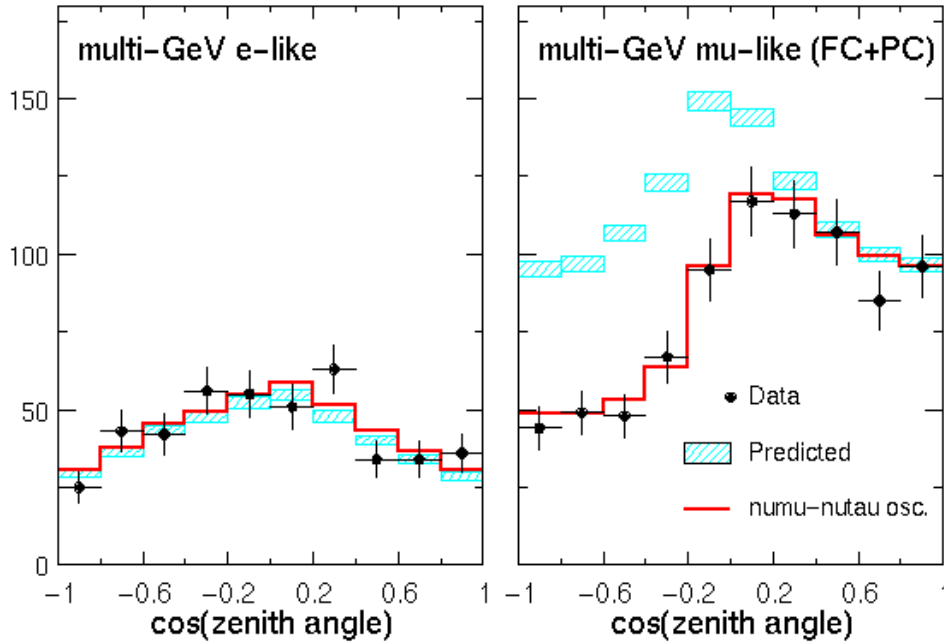


aufwärts abwärts

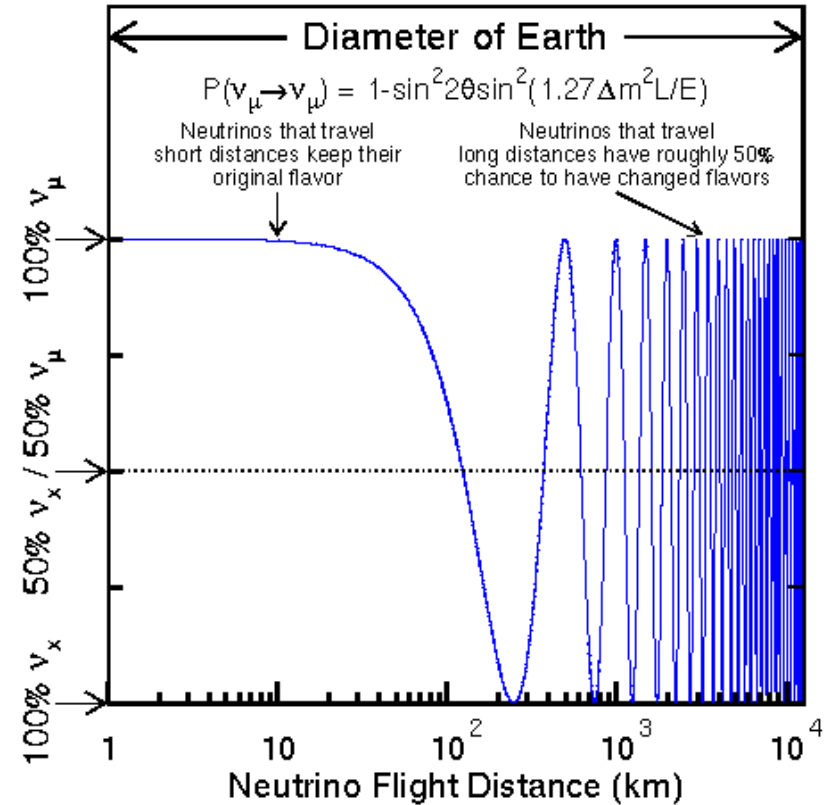
<http://ale.physics.sunysb.edu/~jimhill/Winter2001Talk/node6.html>

Atmosphärische Neutrinos

Super-Kamiokande 848 days Preliminary



$$\nu_\mu \rightarrow \nu_x, \quad \nu_x \neq \nu_e !!$$



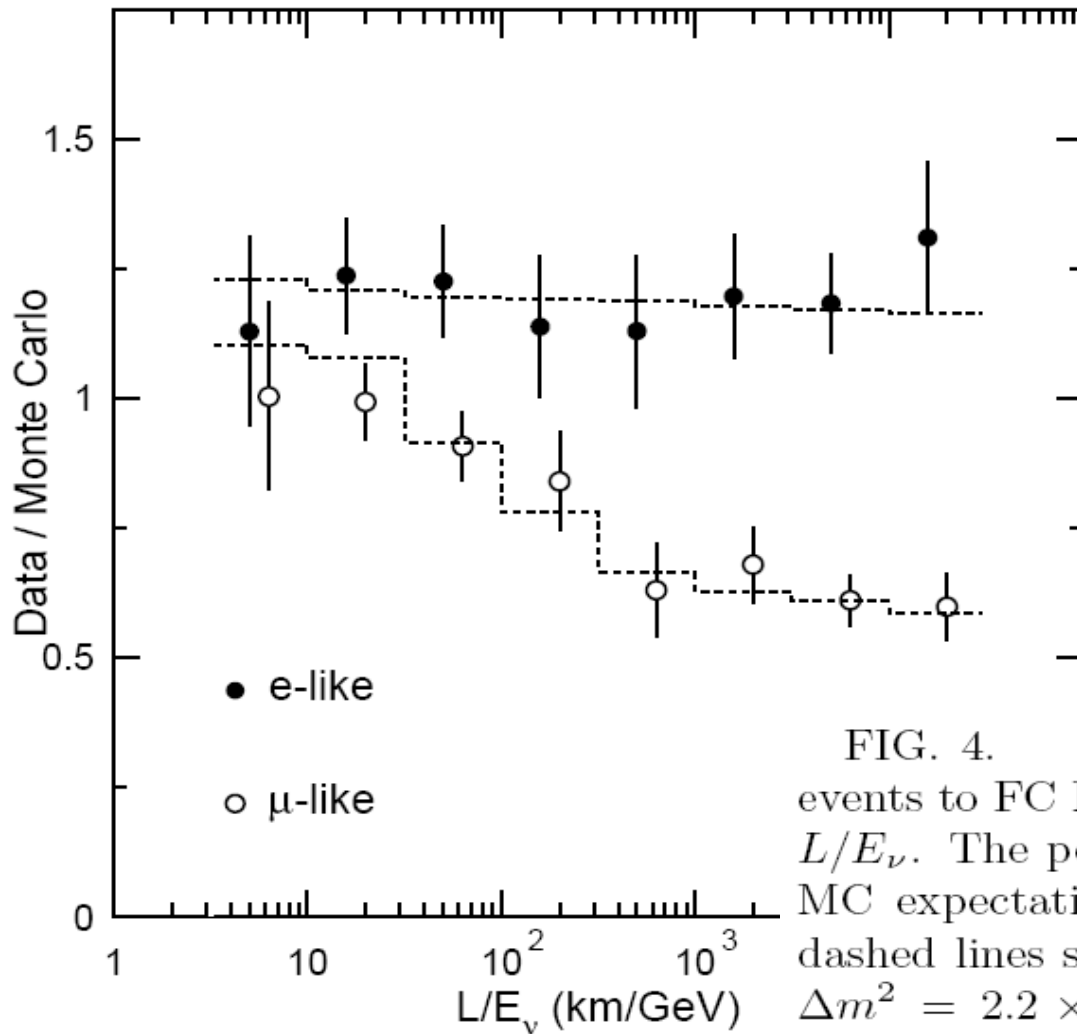


FIG. 4. The ratio of the number of FC data events to FC Monte Carlo events versus reconstructed L/E_ν . The points show the ratio of observed data to MC expectation in the absence of oscillations. The dashed lines show the expected shape for $\nu_\mu \leftrightarrow \nu_\tau$ at $\Delta m^2 = 2.2 \times 10^{-3} \text{eV}^2$ and $\sin^2 2\theta = 1$. The slight L/E_ν dependence for e -like events is due to contamination (2-7%) of ν_μ CC interactions.

Neutrino-Oszillationen (Formalismus)

Favour-EZ

(def. durch CC-Kopplung an l_{α}^{\pm})

$$\nu_{\alpha} = \sum_{i=1}^3 U_{\alpha i}^* \nu_i$$

Massen-EZ

zeitl. Entwicklung der Massen-EZ:

$$\nu_i(x) = e^{-i\left(\frac{m_i^2}{2E}\right)x} \nu_i(0)$$

zeitl. Entwicklung der Flavour-EZ:

$$\nu_{\alpha}(x) = \sum_{i=1}^3 U_{\alpha i}^* e^{-i\left(\frac{m_i^2}{2E}\right)x} \nu_i(0)$$

Flavour-Oszillationen:

$$\nu_{\alpha}(x) = \sum_{\beta=1}^3 \sum_{i=1}^3 U_{\alpha i}^* e^{-i\left(\frac{m_i^2}{2E}\right)x} U_{\beta i} \nu_{\beta}$$

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}; x) = |\langle \nu_{\beta}(0) | \nu_{\alpha}(x) \rangle|^2$$

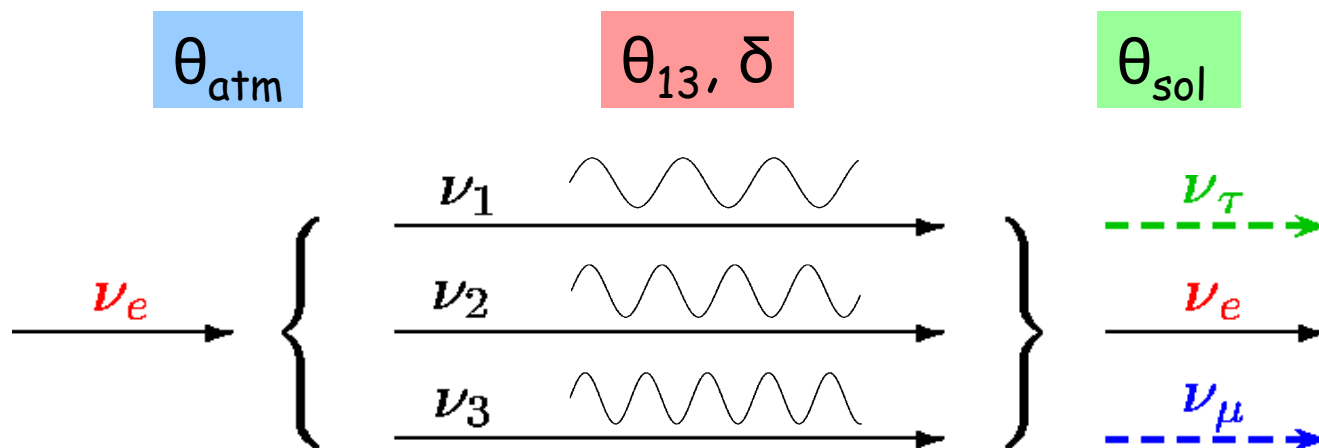
$$P(\nu_{\alpha} \rightarrow \nu_{\beta}; x) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \frac{\Delta m_{ij}^2}{4E} x + 2 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin \frac{\Delta m_{ij}^2}{2E} x$$

3-Generation-Neutrinomischung

PMNS Mischungsmatrix (ohne Majorana Phasen)

- 3 Mischungswinkel: θ_{12} , θ_{23} , θ_{13}
- 1 CP-verletzende Dirac-Phase: δ

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



3-Flavour-Mischungsmatrix

wie bei CKM-Matrix: 3 Winkel + 1 Phase

$$U^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

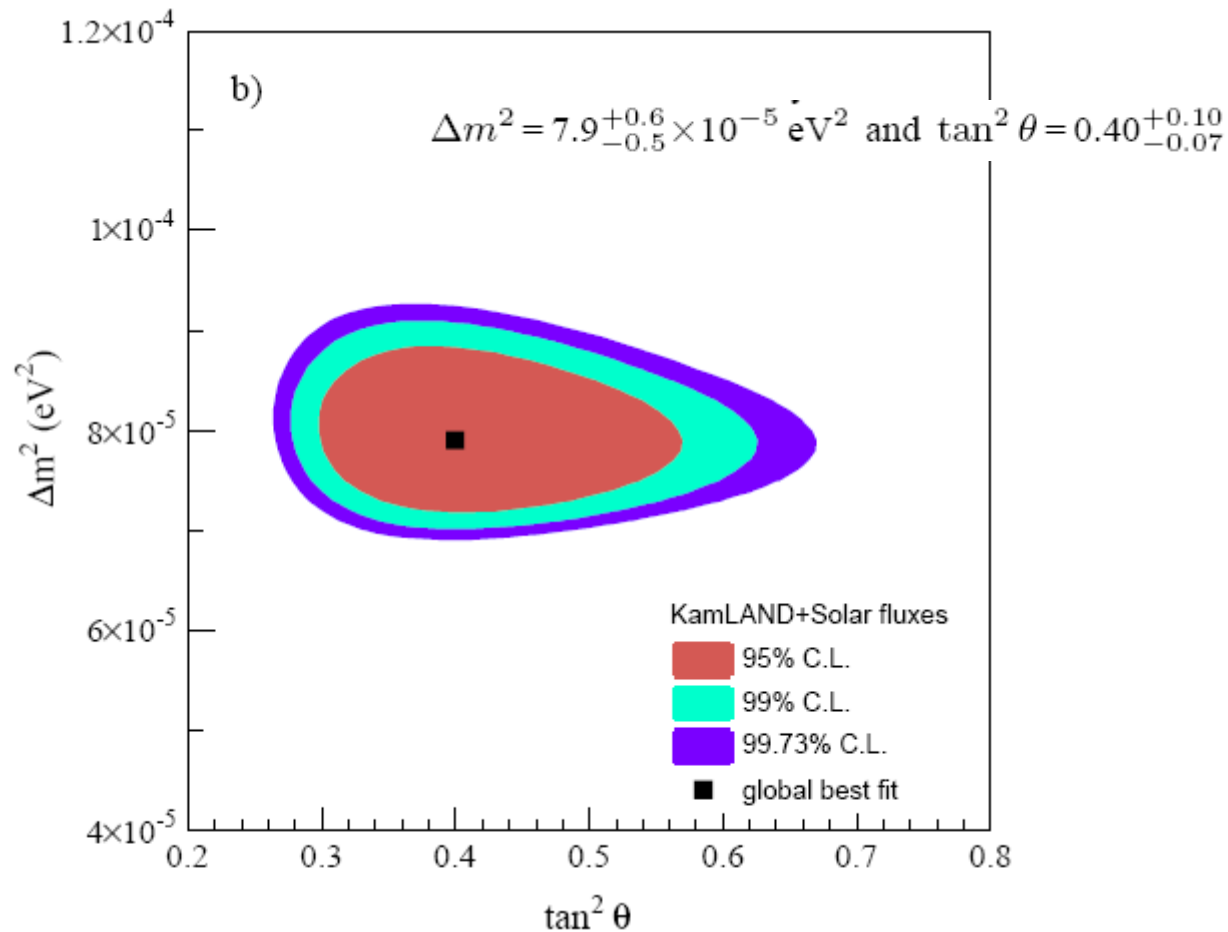
Ergebnisse $\sin^2 \theta_{12} \approx 0.31$, $\sin^2 \theta_{23} \approx 0.50$, $\sin^2 \theta_{13} \approx 0$

$$\theta_{12} \approx 33.8^\circ, \quad \theta_{23} \approx 45.0^\circ, \quad \theta_{13} \approx 0^\circ$$

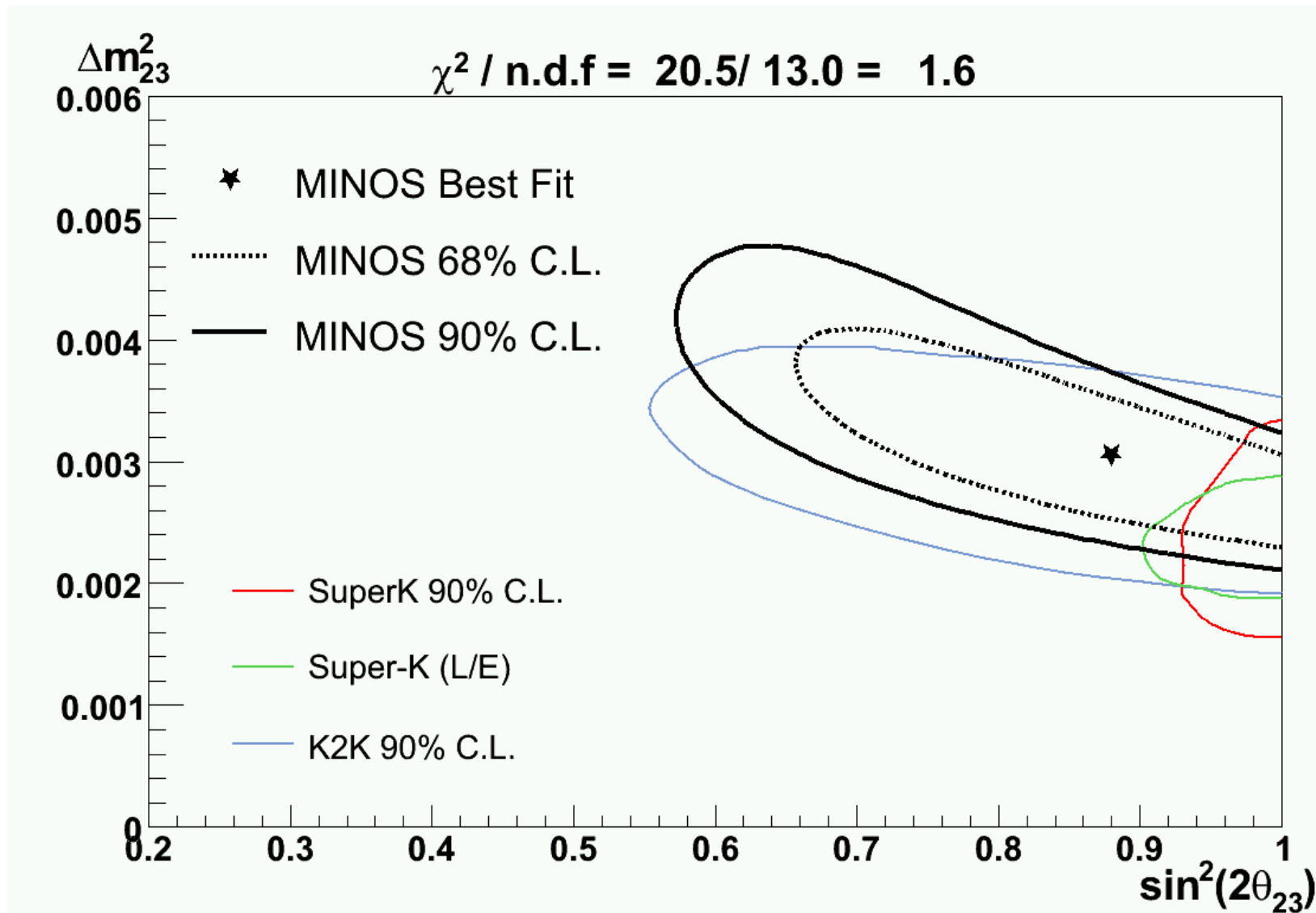
⇒ ganz andere Situation als bei Quarkmischung:
2 große, 1 kleiner Winkel (oder =0)

⇒ Wolfenstein-Parametrisierung nicht möglich

Reaktor- und Sonnenneutrinos



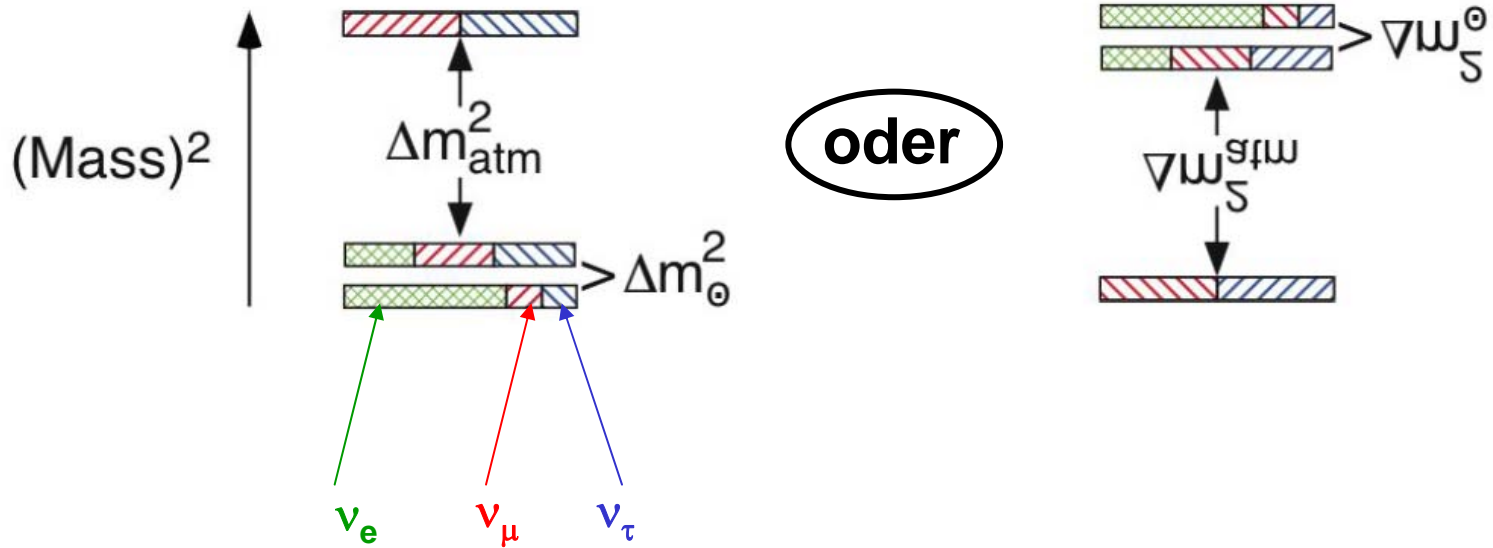
Atmosphärische und Beschleuniger- Neutrinos



Massenhierarchie

$$\Delta m_{12}^2 = m_2^2 - m_1^2 \approx 7.9 \cdot 10^{-5} \text{ eV}^2 \quad (> 0, \text{ festgelegt})$$

$$\Delta m_{(12)3}^2 = \left| m_3^2 - \frac{m_2^2 - m_1^2}{2} \right| \approx 2.4 \cdot 10^{-3} \text{ eV}^2$$



$\Delta m_{\text{atm}}^2 \gg \Delta m_{\odot}^2$ & $\theta_{13} \approx 0 \Rightarrow$ Aufspaltung in effektiv 2-kompon. Mischung für jeweils solare und atm. Neutrinos

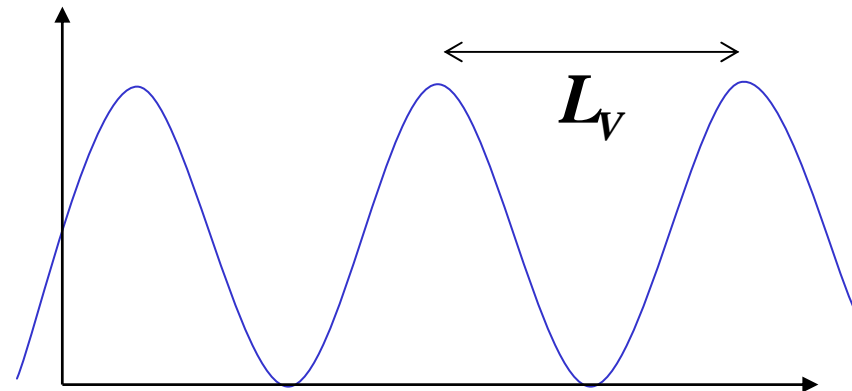
2-Komponenten-Mischung

Vakuum-Lösung:
$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_V & \sin \theta_V \\ -\sin \theta_V & \cos \theta_V \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$P(\nu_e \rightarrow \nu_e; x) = |\langle \nu_e(x) | \nu_e(0) \rangle|^2 = 1 - \sin^2 2\theta_V \underbrace{\sin^2 \frac{\Delta m^2}{4E} x}_{\sin^2 \frac{\pi}{L_V} x}$$

$$L_V = \frac{4\pi E \hbar}{\Delta m^2 c^3} = 2.48 \text{ m} \left(\frac{E}{\text{MeV}} \right) \left(\frac{\text{eV}^2}{\Delta m^2} \right)$$

$$\frac{\pi}{L_V} = 1.27 \frac{1}{\text{m}} \cdot \left(\frac{\Delta m^2}{\text{eV}^2} \right) \cdot \left(\frac{\text{MeV}}{E} \right)$$

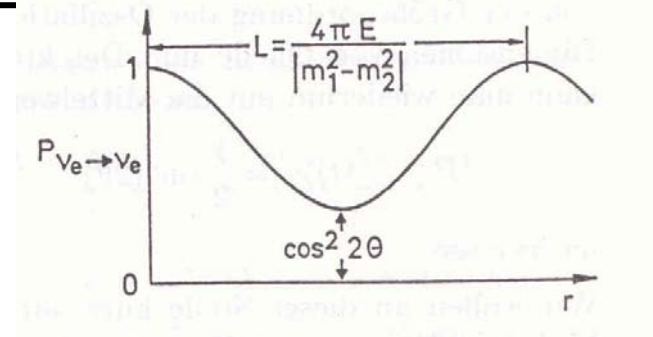


$$\Delta m_{ij}^2 (L/4E) \simeq 1.27 \Delta m_{ij}^2 (\text{eV}^2) \frac{L(\text{km})}{E(\text{GeV})}$$

Pendel-Neutrino-Korrespondenzen

Pendel	Neutrinos
Lineare Schwingung	Kreisbewegung des „Phasenzeigers“
Feste Moden	Massezustände
Mischung wegen Kopplung $\Rightarrow \Delta\omega^2 \sim k/M$	Flavor Mischungsmatrix $\Rightarrow \Delta m^2$
Oszillationsfrequenz der Pendel $\sim \Delta\omega^2$ der festen Moden	Oszillationsfrequenz der Neutrinos $\sim \Delta m^2$ der Massezustände
Amplitude ² jedes Pendels	Wahrscheinlichkeit, Neutrino zu finden

Anmerkung:
nicht-maximale Neutrino Mischung
entspreche Pendel mit $l_{\perp} \neq l_{\parallel}$



$L_{12} = 30 \text{ km} \times E(\text{MeV})$ Derzeitige Werte $L_{23} = 1 \text{ km} \times E(\text{MeV})$

$\Delta m_{23}^2 = 2.4 \times 10^{-3} \text{ eV}^2$	$\Delta m_{13}^2 = 2.5 \times 10^{-3} \text{ eV}^2$	$\Delta m_{12}^2 = 0.08 \times 10^{-3} \text{ eV}^2$
„schnelle“ Oszillation		„langsame“ Oszillation
$L_{23} = 1 \text{ km} \times E/\text{MeV}$		$L_{12} = 30 \text{ km} \times E/\text{MeV}$
$\theta_{23} = 45^\circ$	$\theta_{13} < 10^\circ$	$\theta_{12} = 32^{+3}_{-2}^\circ$

◇ konsistent mit tri/bi-maximaler Mischung

$$U_{\text{PMNS}} \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

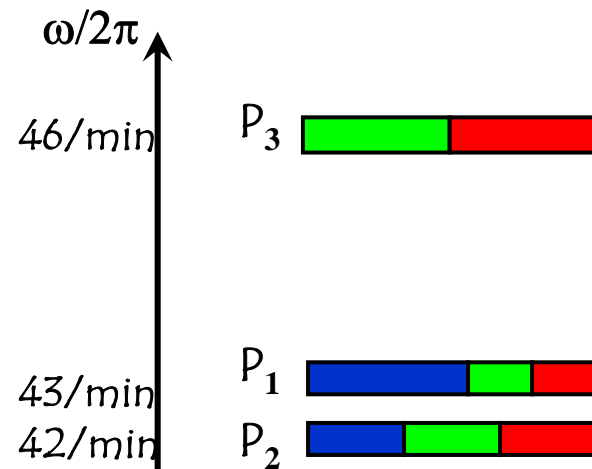
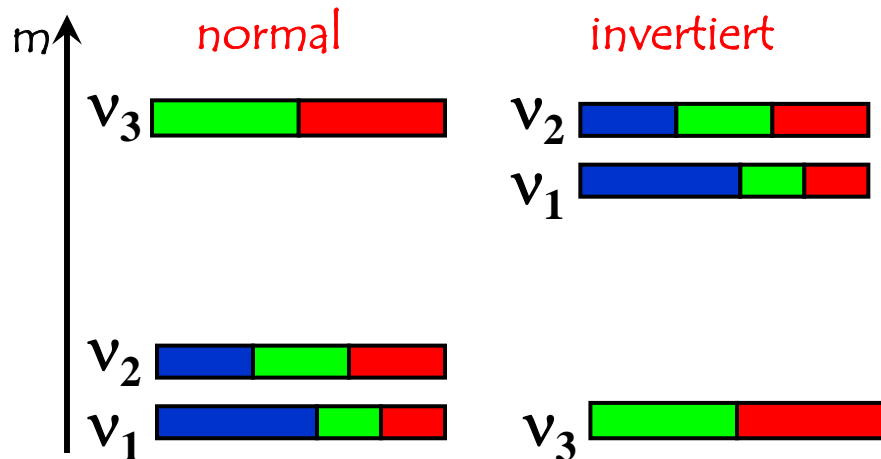
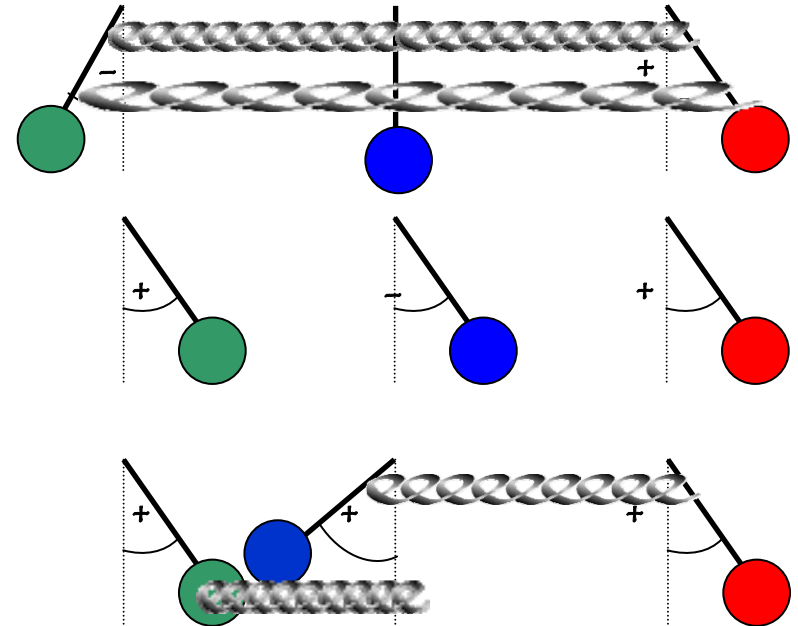
Harrison, Perkins, Scott '99, '02
 Z.Xing, '02, He, Zee, '03, Koide '03
 Chang, Kang, Kim '04, Kang '04

Realisation als gekoppelte Pendel

$$\nu_3 = (-\nu_\mu + \nu_\tau)/\sqrt{2}$$

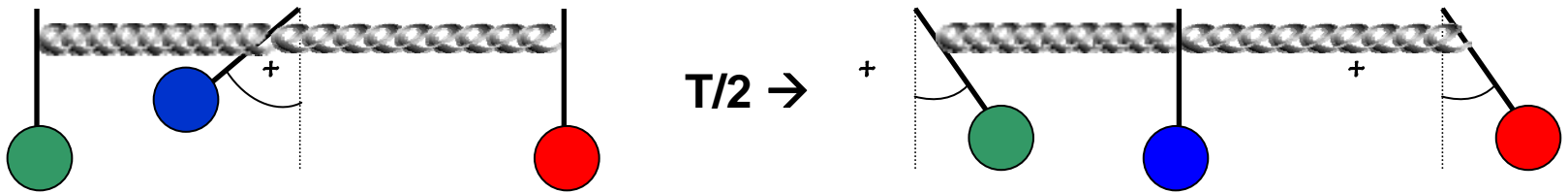
$$\nu_2 = (-\nu_e + \nu_\mu + \nu_\tau)/\sqrt{3}$$

$$\nu_1 = (2\nu_e + \nu_\mu + \nu_\tau)/\sqrt{6}$$



Solare Neutrinos

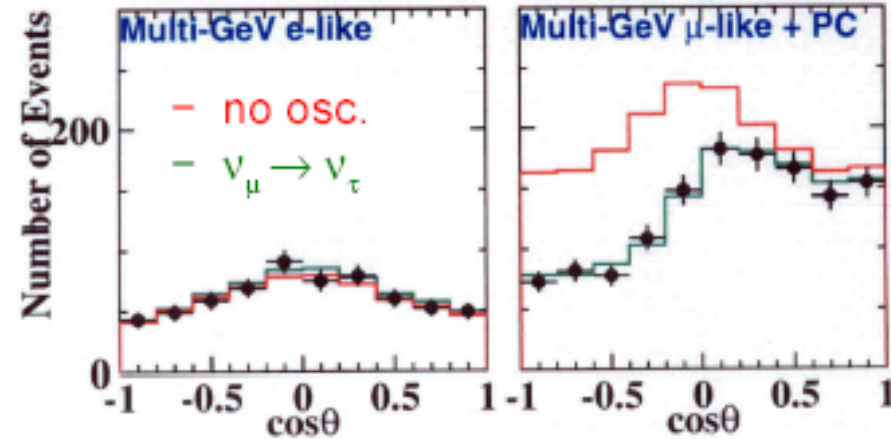
- ◇ nukleare Fusion: (ohne MSW Effekt: Produziere 100% ν_e)
 $4p \rightarrow {}^4\text{He} + 2e^+ + 2\nu_e + 27 \text{ MeV}$
- ◇ Vakuum: langsame Oszillation mit θ_{12} , Pendel: schwache Federn



- ◇ Anregung von $(\nu_\tau - \nu_\mu)/\sqrt{2}$ nicht möglich, weil ν_e nicht in ν_3
- ◇ Oszillation nur zu $(\nu_\tau + \nu_\mu)/\sqrt{2}$
- ◇ $P(\nu_e \rightarrow \nu_e) > 50\%$ da nur ν_1 und ν_2 beteiligt

Atmosphärische Neutrinos

- ◇ Kamiokande 2000:
beschrieben als $\nu_{\mu} \rightarrow \nu_{\tau}$
- ◇ Pendel:
 ν_e : schwache Feder zu ν_{μ}, ν_{τ}
 ν_{μ} : schwache Feder zu ν_e
starke Feder zu ν_{τ}

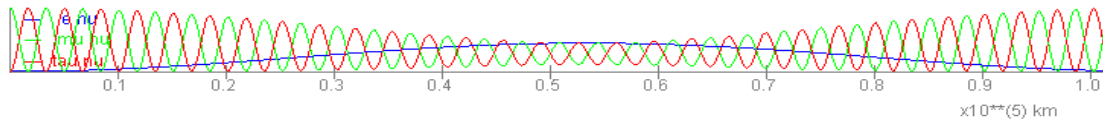


Interactive Neutrino Oscillation Laboratory

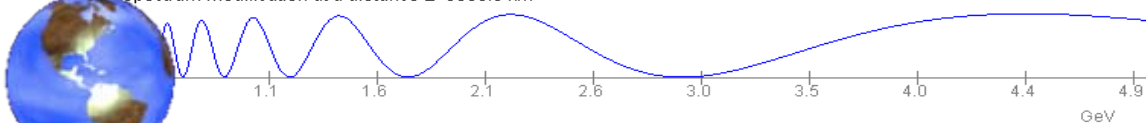
Three Generations Neutrino Oscillations

Adam Para, Fermilab

Appearance/disappearance probability as a function of distance, for $E_{\nu} = 3.0$ GeV



Energy spectrum modification at a distance $L=5000.0$ km



$\theta_{12} = 0.166$
 $\theta_{13} = 0.333$
 $\theta_{23} = 0.500$

composition of the
initial neutrino
in terms of mass eigenstates



Mixing Matrix

0.816	0.577	0.0
-0.40	0.577	0.707
0.408	-0.57	0.707
1	2	3

e
mu
tau

$\theta_{12} = 0.009$
 $\theta_{13} = 6.459$
 $\theta_{23} = 0.990$

composition of the
3.0 GeV flux at 5000. km
in terms of flavor states



<http://minos.phy.bnl.gov/nu-osc-lab/Superposition1.html>

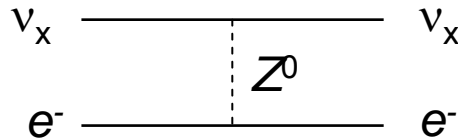
MSW-Effekt

Vakuum:

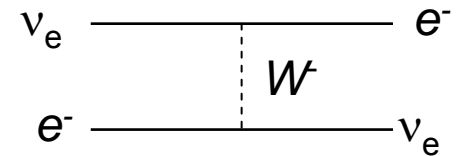
$$i \frac{\partial}{\partial x} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \frac{1}{2p} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = H_0 \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \quad \text{Massen-EZ}$$

$$i \frac{\partial}{\partial x} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta m_{12}^2}{4p} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H_V \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \quad \text{Flavour-EZ}$$

NC ($x = e, \mu, \tau$):



CC:



CC-Wechselwirkung der ν_e in der Sonne $\sim N_e$

$$V(x) = \sqrt{2} G_F N_e(x)$$

$$N_e \approx 6 \cdot 10^{25} \text{ cm}^{-3} \quad \text{bei } r = 0$$

$$i \frac{\partial}{\partial x} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \left[\frac{\Delta m_{12}^2}{4p} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \frac{V(x)}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

MSW-Effekt: adiabatische Lösung

$$i \frac{\partial}{\partial x} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \left[\frac{\Delta m_{12}^2}{4p} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \frac{V(x)}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$V(x) = \sqrt{2} G_F N_e(x)$$

$$N_e \approx 6 \cdot 10^{25} \text{ cm}^{-3}$$

Falls H im Sonneninneren quasi-diagonal ($V(x)$ gross) $\Rightarrow \approx$ Massen-EZ = ν_{2m}

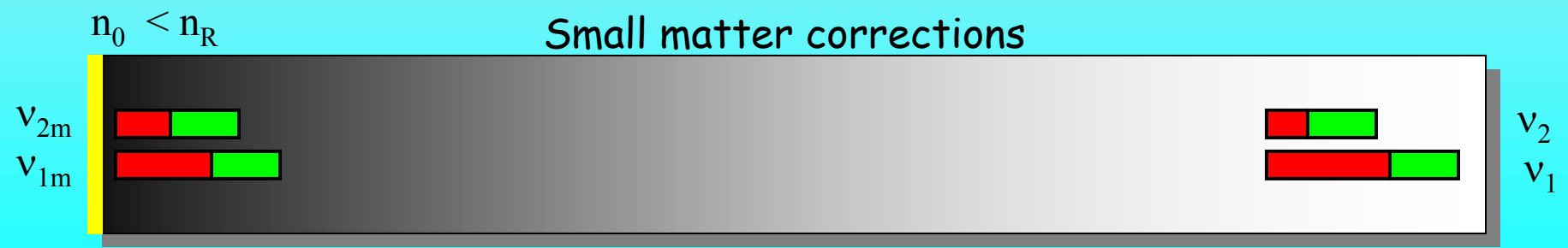
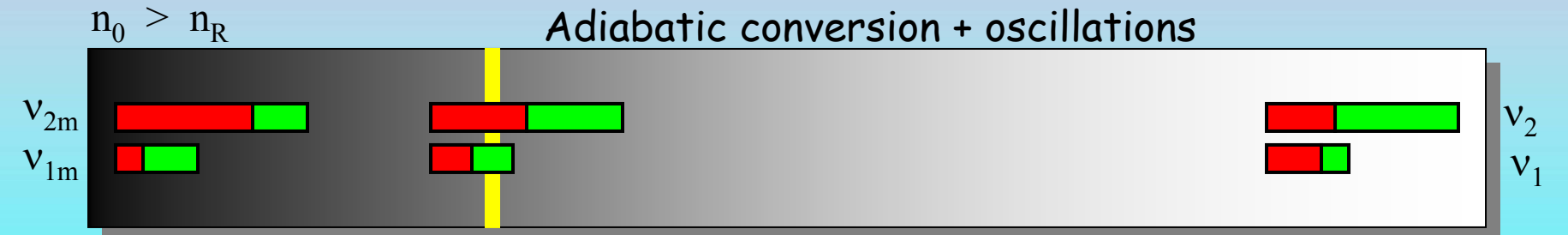
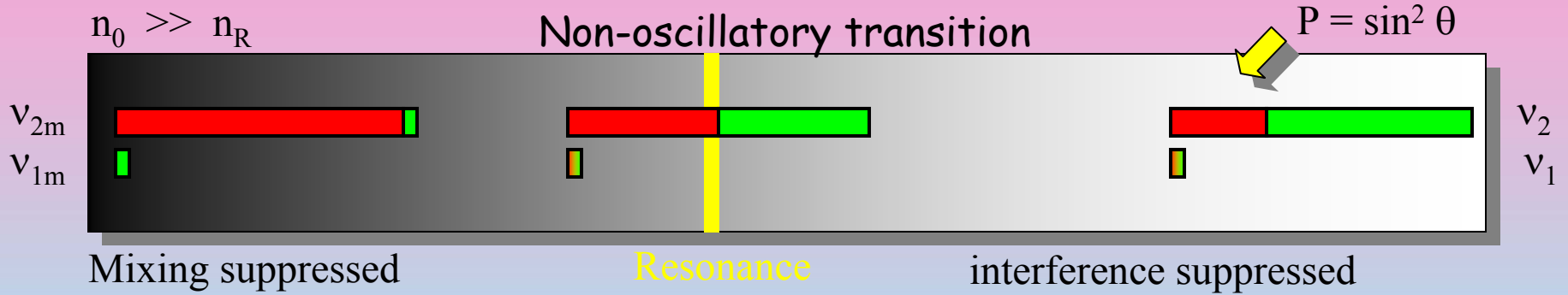
bei **adiabatischer Entwicklung** $V(x) \rightarrow 0$ am Sonnenrand

$$\Rightarrow \nu_2 = \sin \theta \nu_e + \cos \theta \nu_\mu$$

$$\Rightarrow P(\nu_e) = \sin^2 \theta, \quad P(\nu_\mu) = \cos^2 \theta.$$

Adiabatic conversion

$$\nu_{1m} \not\leftrightarrow \nu_{2m}$$



n_e

Solar Neutrinos



Adiabatic conversion
in matter of the Sun

$\rho : (150 \Rightarrow 0) \text{ g/cc}$



electron neutrinos are produced

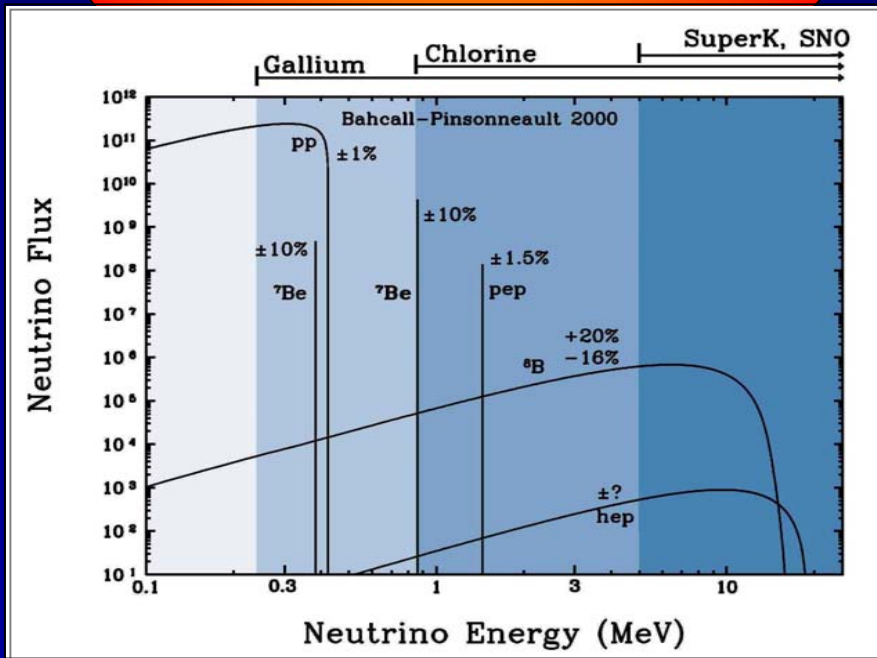
$$F = 6 \cdot 10^{10} \text{ cm}^{-2} \text{ s}^{-1}$$

total flux at the Earth

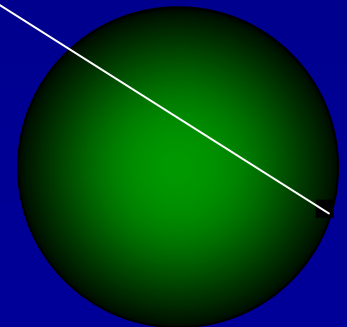
Oscillations
in vacuum

ν

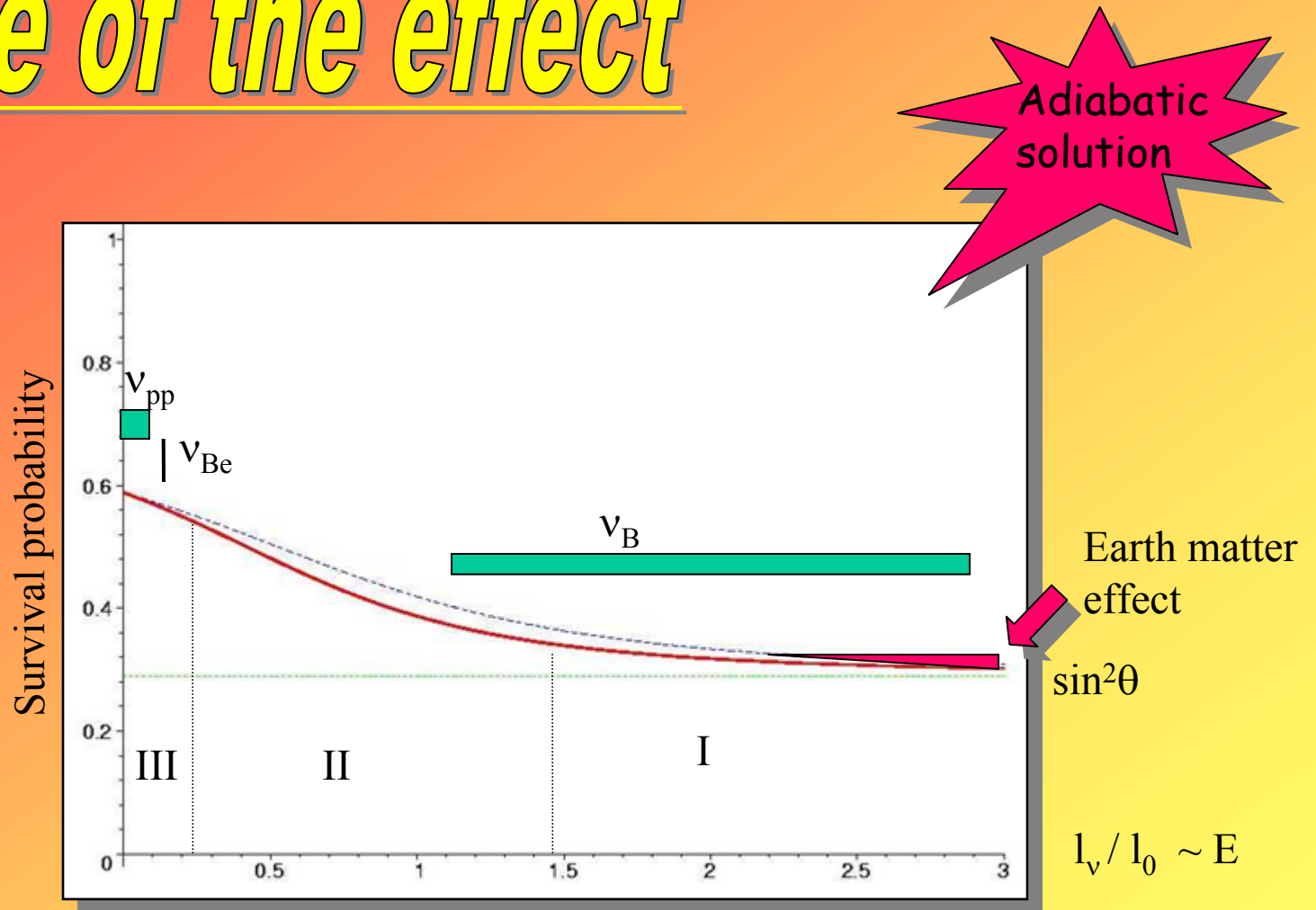
Oscillations
in matter
of the Earth



J.N. Bahcall



Profile of the effect



Oscillations with
small matter effect

Conversion +
oscillations

Conversion with
small oscillation
effect

Non-oscillatory
transition