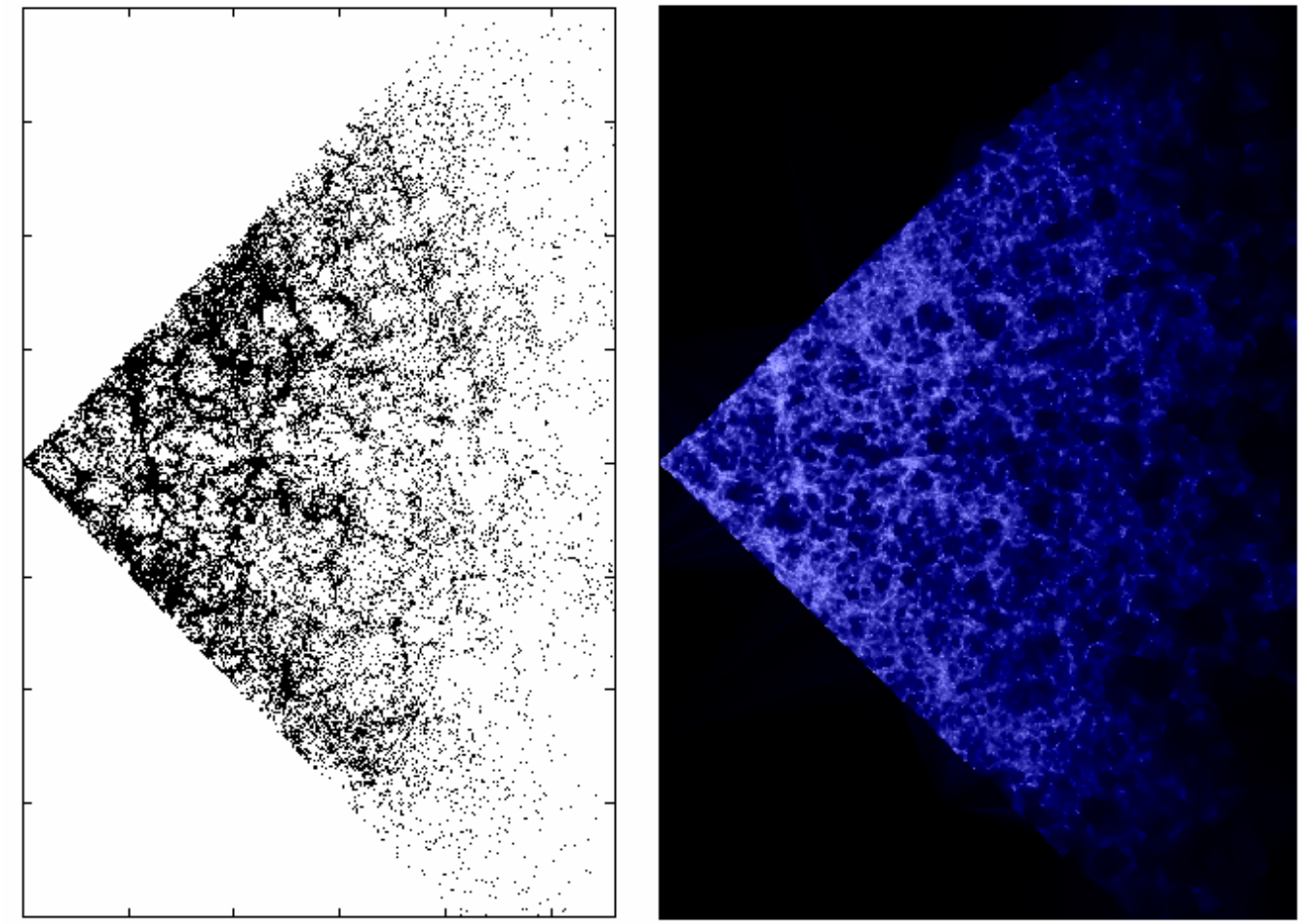
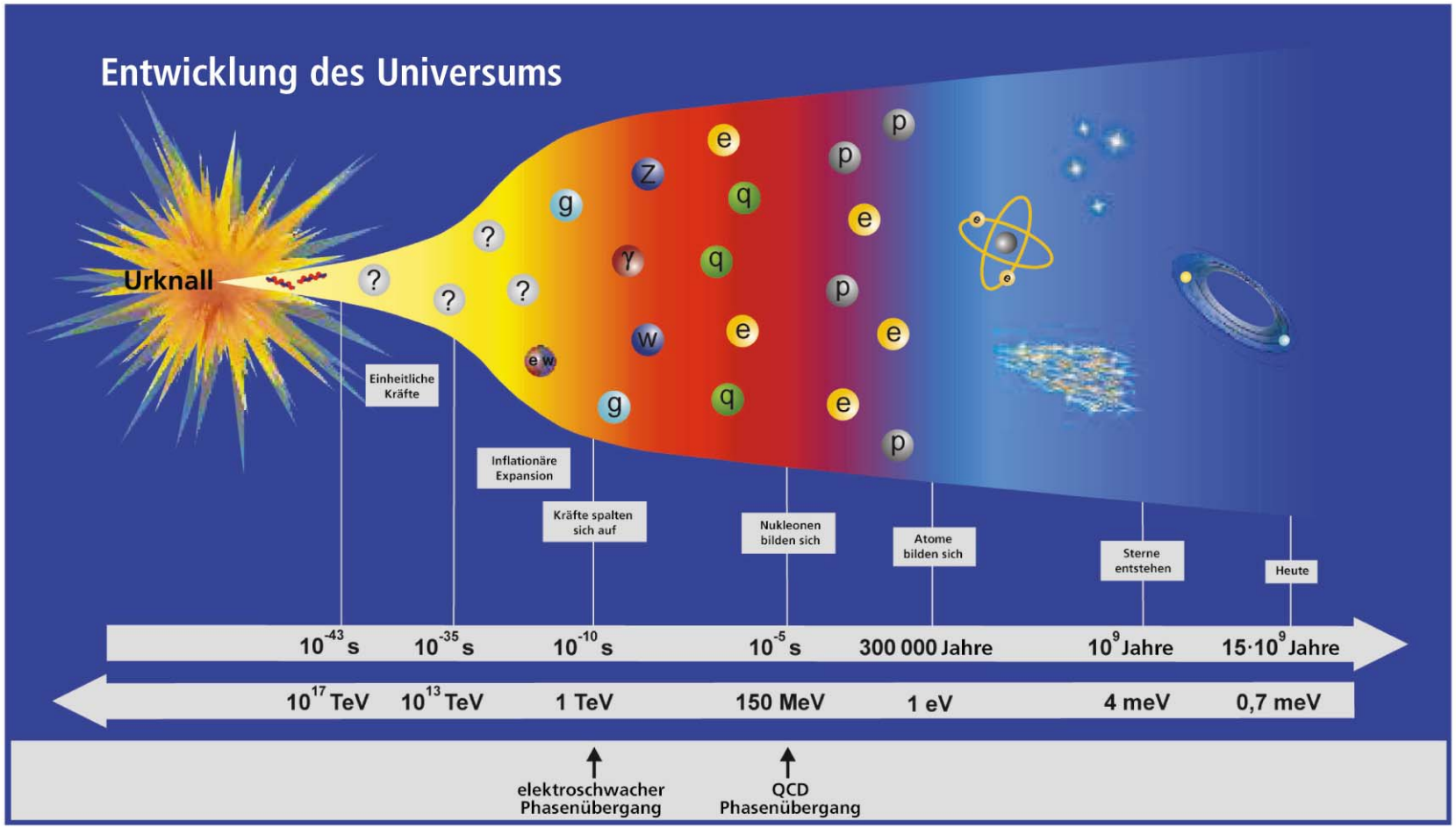
2. Entwicklung des Universums



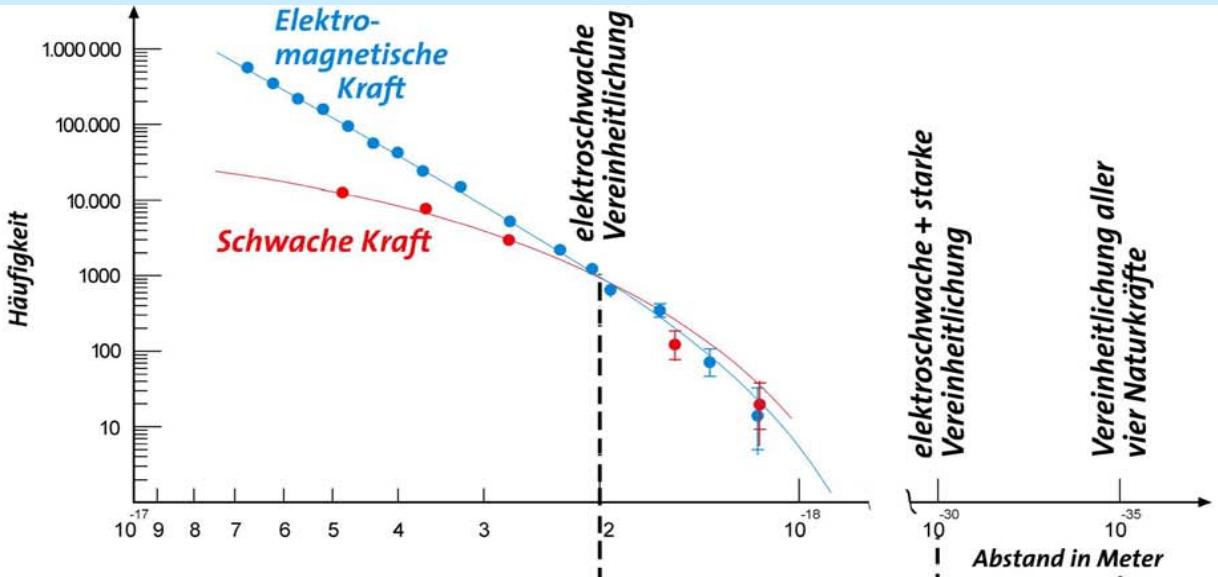
Urknallmodell

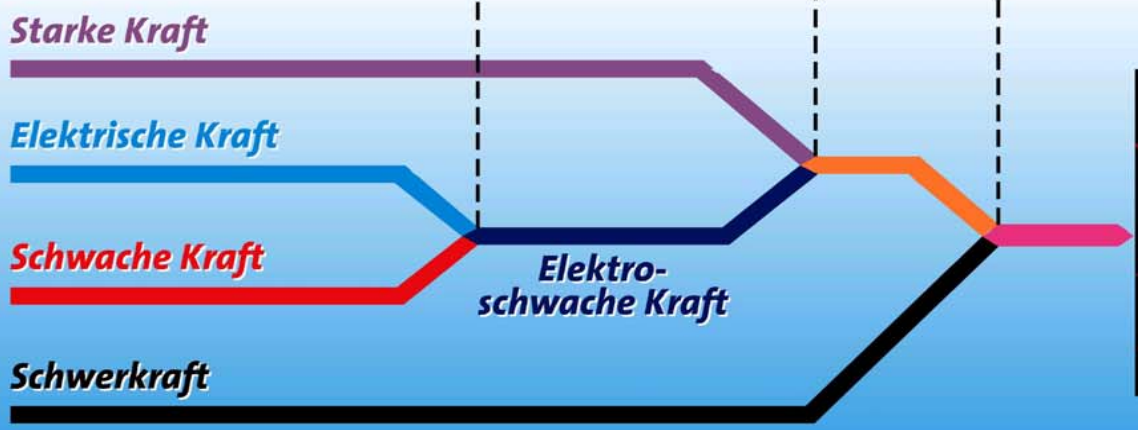


Stützen:

- Fluchtbewegung der Galaxien → Expansion des Universums
- 3K-Hintergrundstrahlung → Nachhall des Urknalls
- Häufigkeit der leichten Elemente (H, D, He, Li)

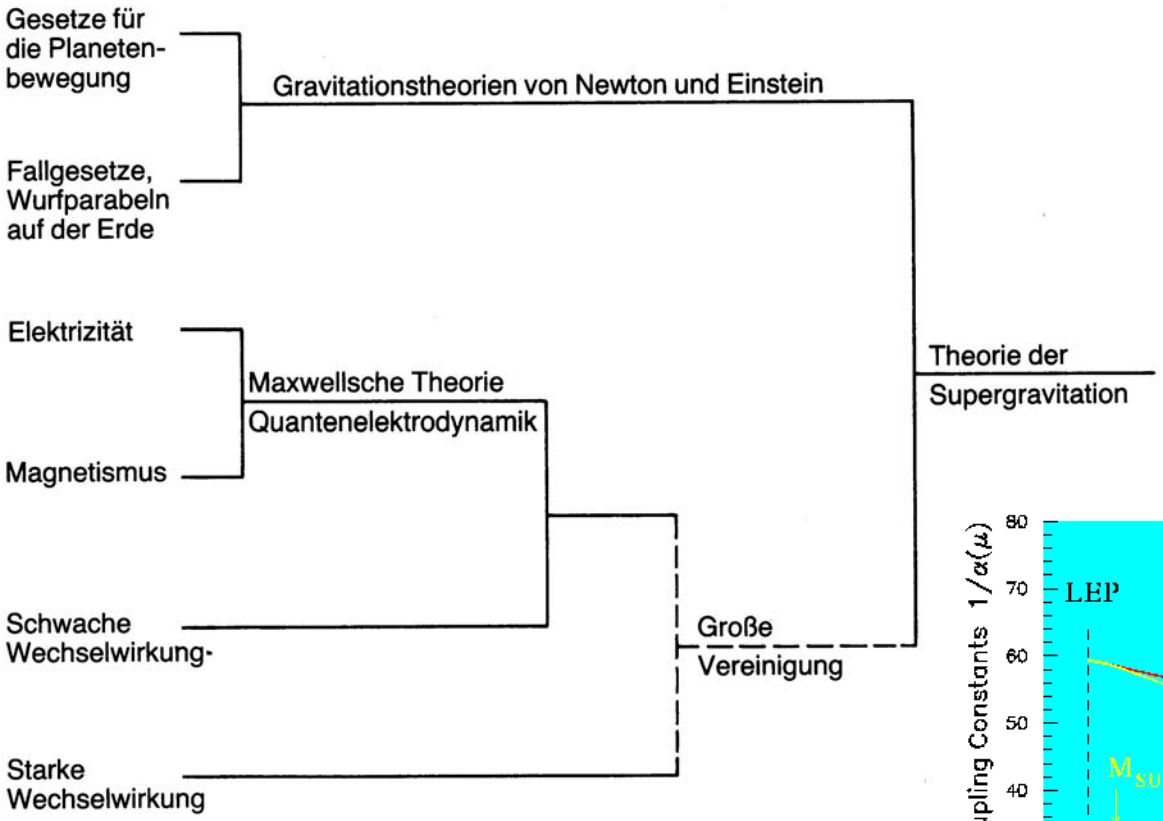
Vereinheitlichung der Wechselwirkungen

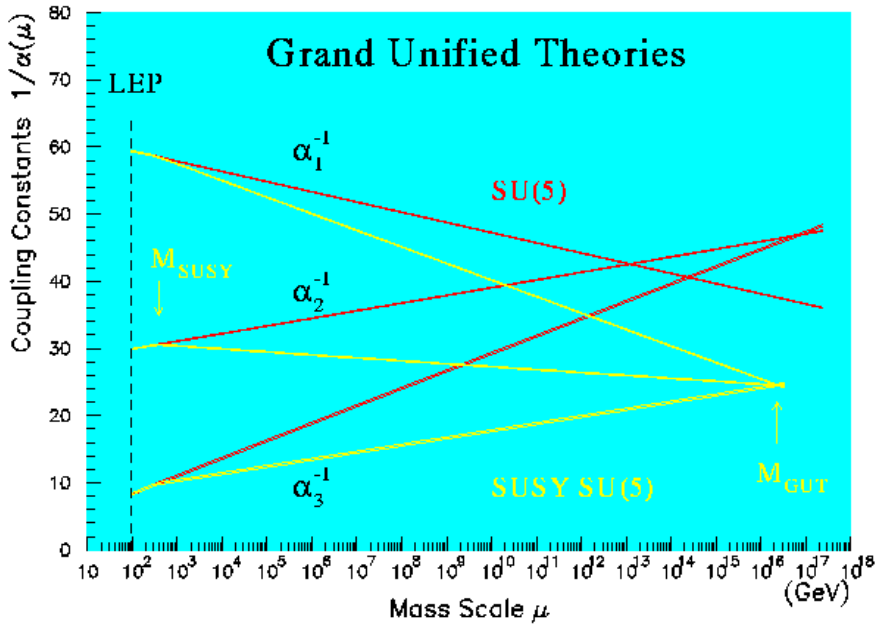




Urknall

Vereinheitlichung der Wechselwirkungen

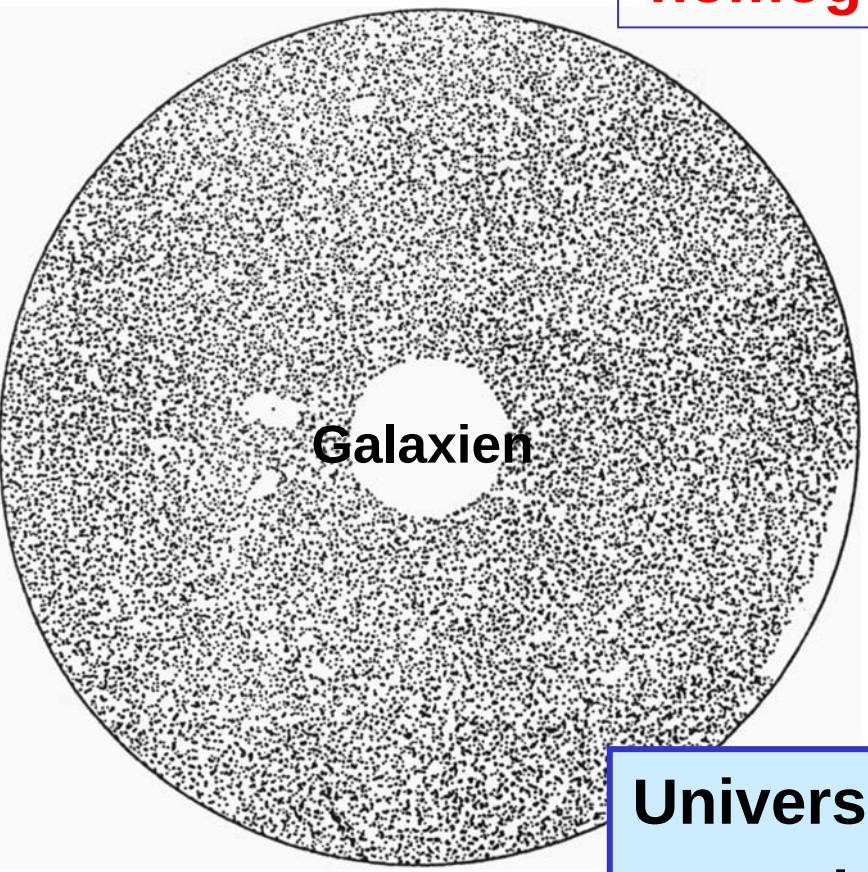


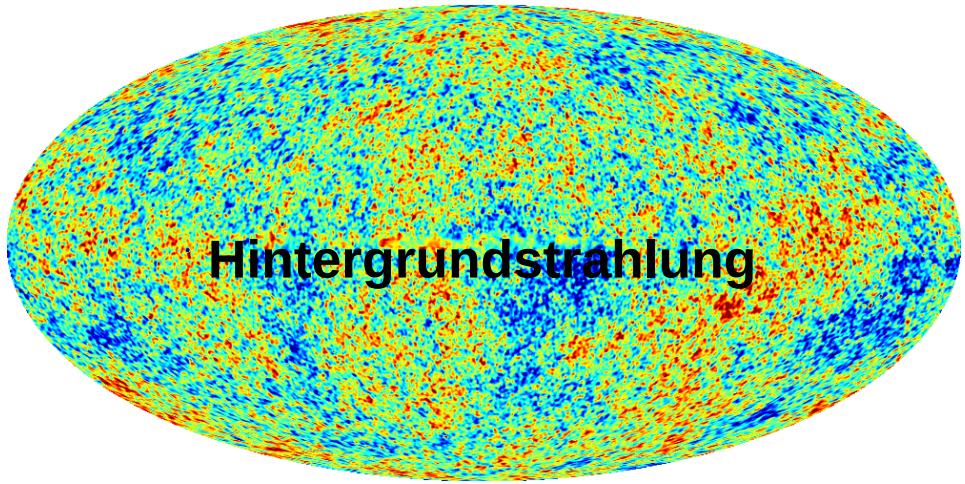


Kosmologisches Prinzip

Universum ist auf großen Skalen

homogen und isotrop

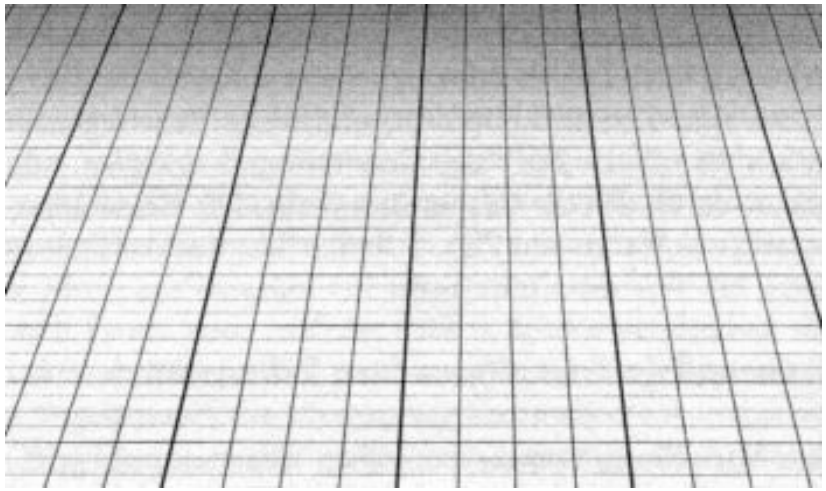


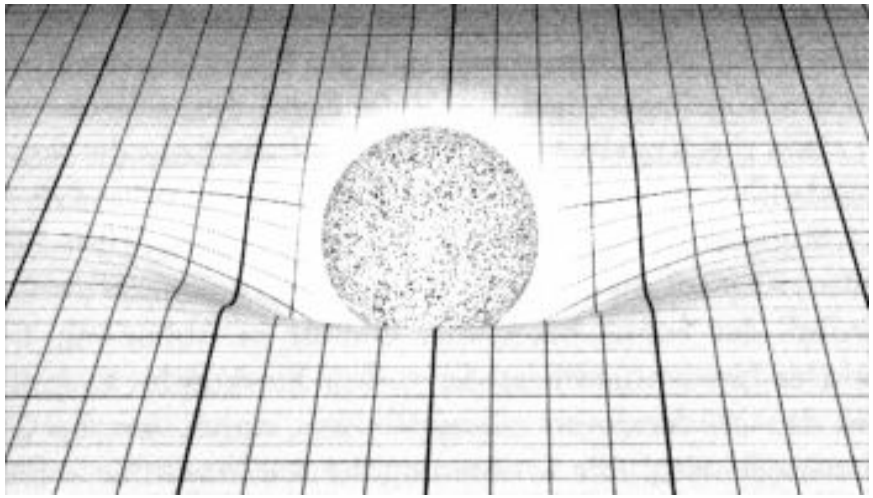


Universum sieht von überall gleich aus:

unabhängig von Ort und Richtung

Raumkrümmung

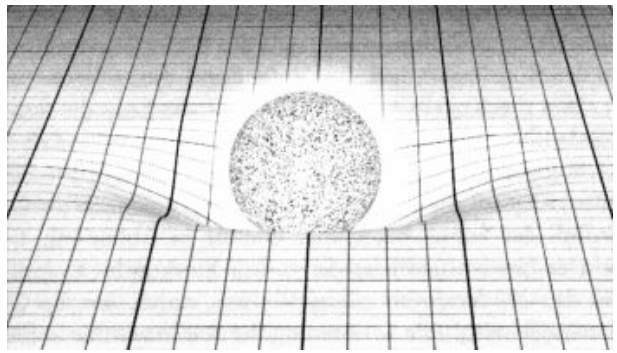




Robertson-Walker-Metrik (1)

3-dim. Raum mit Krümmung > 0 als Kugel in 4-dim Raum:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = R^2 = \text{konstant}$$



Kugelkoordinaten:

$$(x_1, x_2, x_3, x_4) \rightarrow (r, \psi, \theta, \varphi)$$

$$r \geq 0,\ 0 \geq \psi \geq \pi,\ 0 \geq \theta \geq \pi,\ 0 \geq \varphi \geq 2\pi$$

$$\begin{aligned} x_1 &= R \sin\psi \sin\theta \cos\varphi, \\ x_2 &= R \sin\psi \sin\theta \sin\varphi, \\ x_3 &= R \sin\psi \cos\theta, \\ x_4 &= R \cos\psi. \end{aligned}$$

Ziel: eliminiere 1 Dimension

Robertson-Walker-Metrik (2)

Raum-Linienelement:

$$d\ell^2 = R^2 \left(\frac{dr^2}{1-kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right)$$

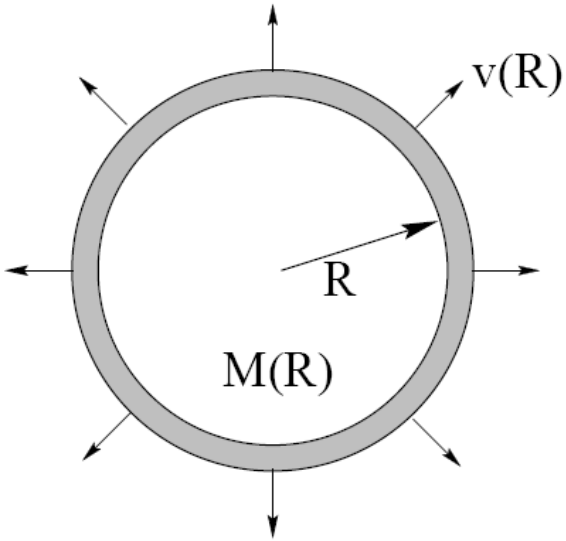
Raum-Zeit-Linienelement:

$$ds^2 = c^2 dt^2 - R^2(t) \left(\frac{dr^2}{1-kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right)$$

Typ	Geometrie	Krümmung	$\Omega = \rho/\rho_c$	Universum
$k = +1$	sphärisch	positiv	> 1	geschlossen
$k = 0$	euklidisch	keine (eben)	$= 1$	flach
$k = -1$	hyperbolisch	negativ	< 1	offen

Closed Geometry Open Geometry Flat Geometry

Expansion des Universums



$$E = E_{kin} + E_{pot} = \frac{1}{2} m H^2 R^2 - G \frac{m M(R)}{R}$$

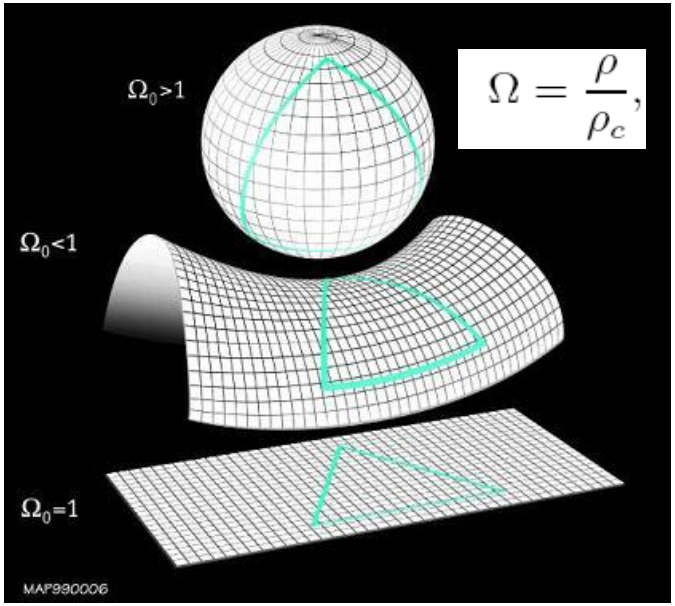
$$\dot{R} = v(R) = H \cdot R.$$

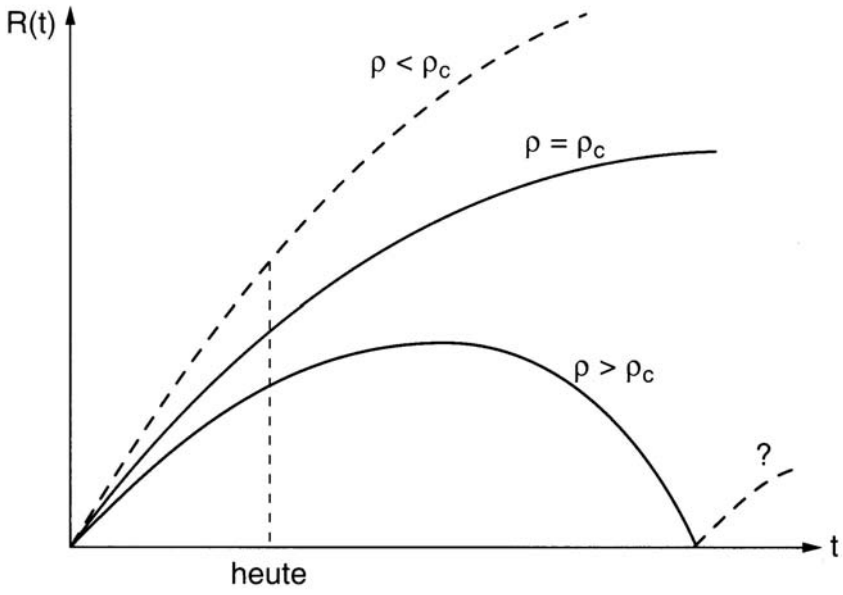
$$m = 4\pi R^2 dR\, \rho$$

$$M(R) = \frac{4}{3} \pi R^3 \rho$$

$$E = \frac{1}{2} m R^2 \left(H^2 - \frac{8\pi G\, \rho}{3} \right) = \text{const}$$

Geometrie des Raumes





$$E = \frac{1}{2} m R^2 \left(H^2 - \frac{8\pi G \rho}{3} \right) = \text{const}$$

$$E = 0 \quad \Rightarrow \quad \rho_c = \frac{3H^2}{8\pi G}$$

$$H_0 = 100 \cdot h_0 \ \text{km s}^{-1} \text{Mpc}^{-1} \qquad \text{mit } h_0 = 0.73 \pm 0.03$$

$$\rho_c = 18.8 \cdot h_0^2 \cdot 10^{-27} \ \text{kg/m}^3 \approx 11\, h_0^2 \ \text{Protonen/m}^3$$

Friedmann-Gleichung

Friedmann-Gleichung $$H(t)^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - k\frac{c^2}{R^2}$$

mit $$\Omega = \frac{\rho}{\rho_c},$$

$$H^2(\Omega - 1) = k\frac{c^2}{R^2},$$

Raumkrümmung

Friedmann-Gleichungen sind **Lösungen der Einsteingleichungen** der ART für eine homogene, isotrope ideale Flüssigkeit:

$$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) R$$

$$\dot{R}^2 = \frac{8\pi G\rho}{3} R^2 - k\,c^2$$

mit Zustandsgleichung $p = p(\rho)$

Zustandsgleichungen $p=p(\rho)$

Energiebilanz

$$d(\rho R^3)\,c^2 = -p\,d(R^3)$$

Dominante Energieform	Zustandsgleichung	Energiedichte	Skalenparameter
Strahlung	$p = \frac{1}{3}\rho_s$	$\rho_s \sim R^{-4}$	$R \sim t^{1/2}$
Materie	$p = \frac{1}{3}\rho_m\, c^2 \langle \frac{v^2}{c^2} \rangle \overset{v \ll c}{\rightarrow} 0$	$\rho_m \sim R^{-3}$	$R \sim t^{2/3}$
Vakuum	$p = -\rho_v$	$\rho_v = const$	$R \sim \exp(\alpha t)$

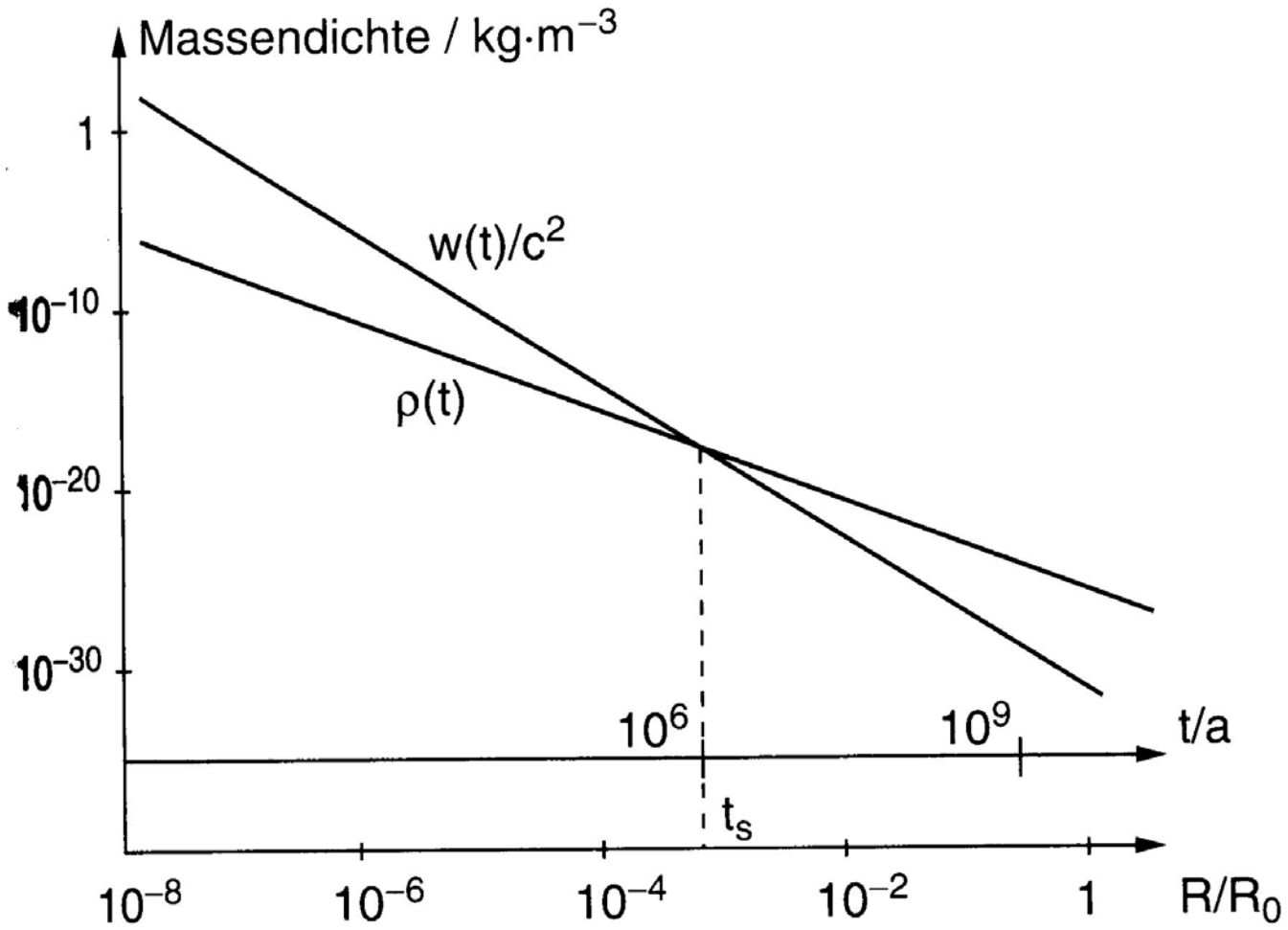
!

$$\rho \rightarrow \rho + \rho_{Vakuum} = \rho + \frac{\Lambda}{8\pi G}$$

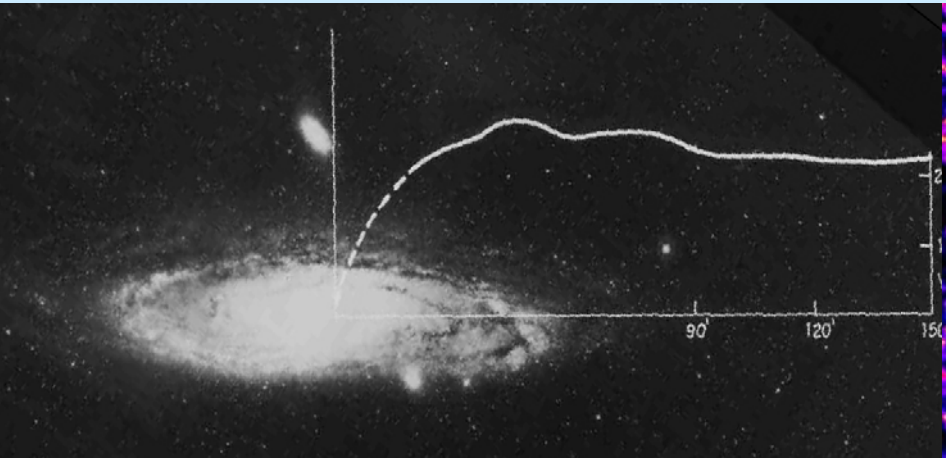
$$p \rightarrow p + p_{Vakuum} = p - c^2 \frac{\Lambda}{8\pi G}$$

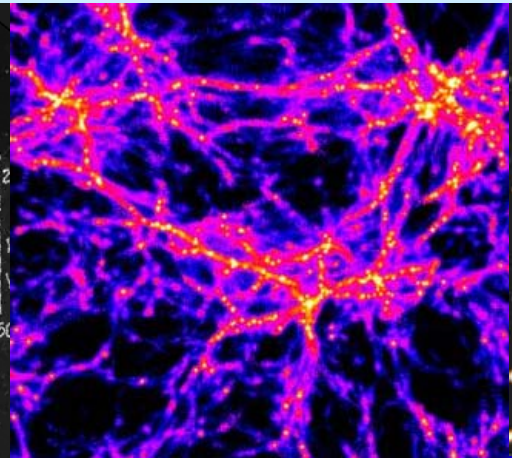
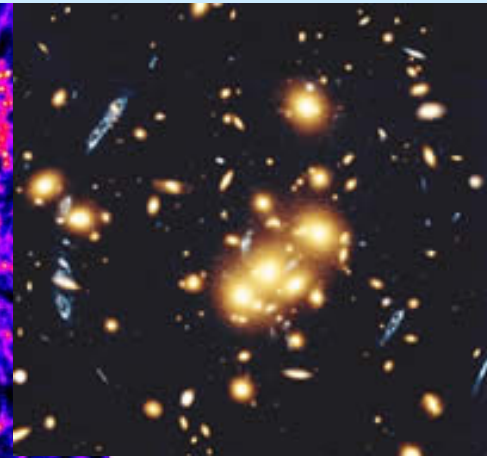
$$\Omega_{tot} = \frac{\rho}{\rho_c} = \frac{\rho_s}{\rho_c} + \frac{\rho_m}{\rho_c} + \frac{\rho_v}{\rho_c} = \Omega_s + \Omega_m + \Omega_v$$

Massen- und Strahlungsdichte



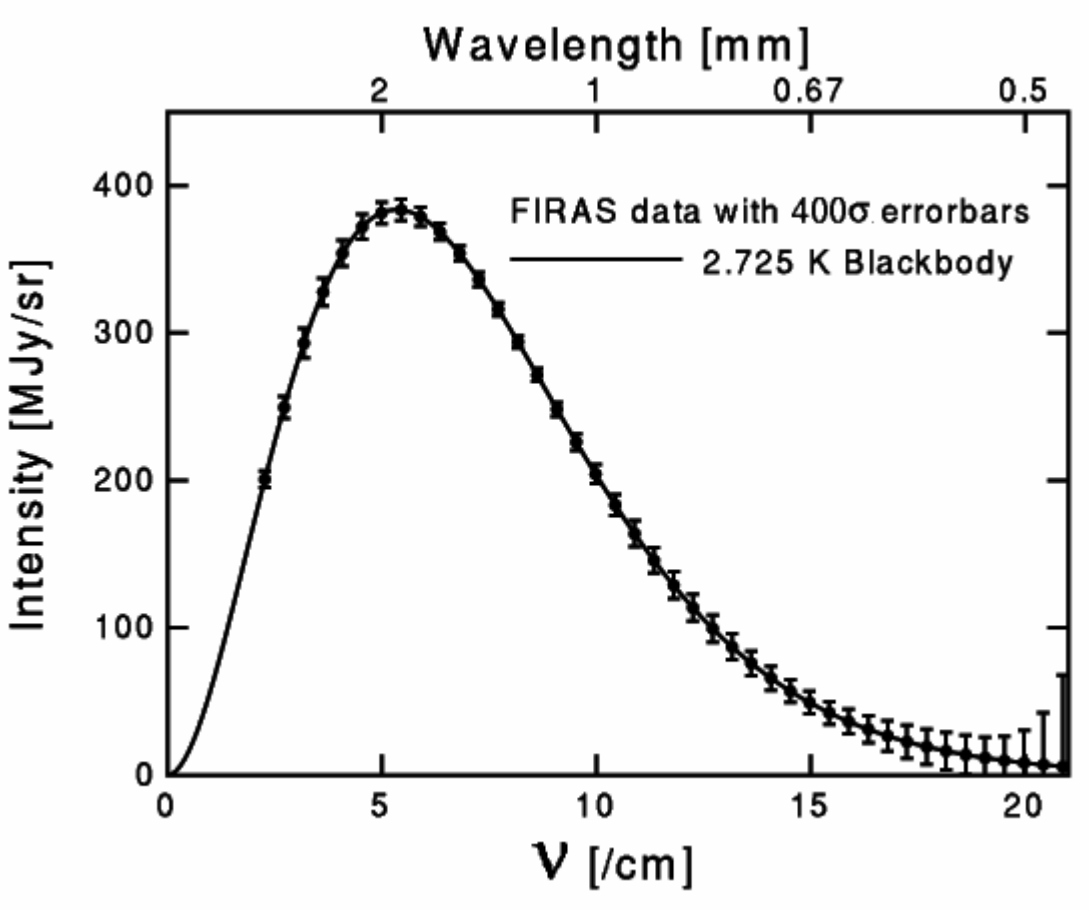
Dichte des Universums



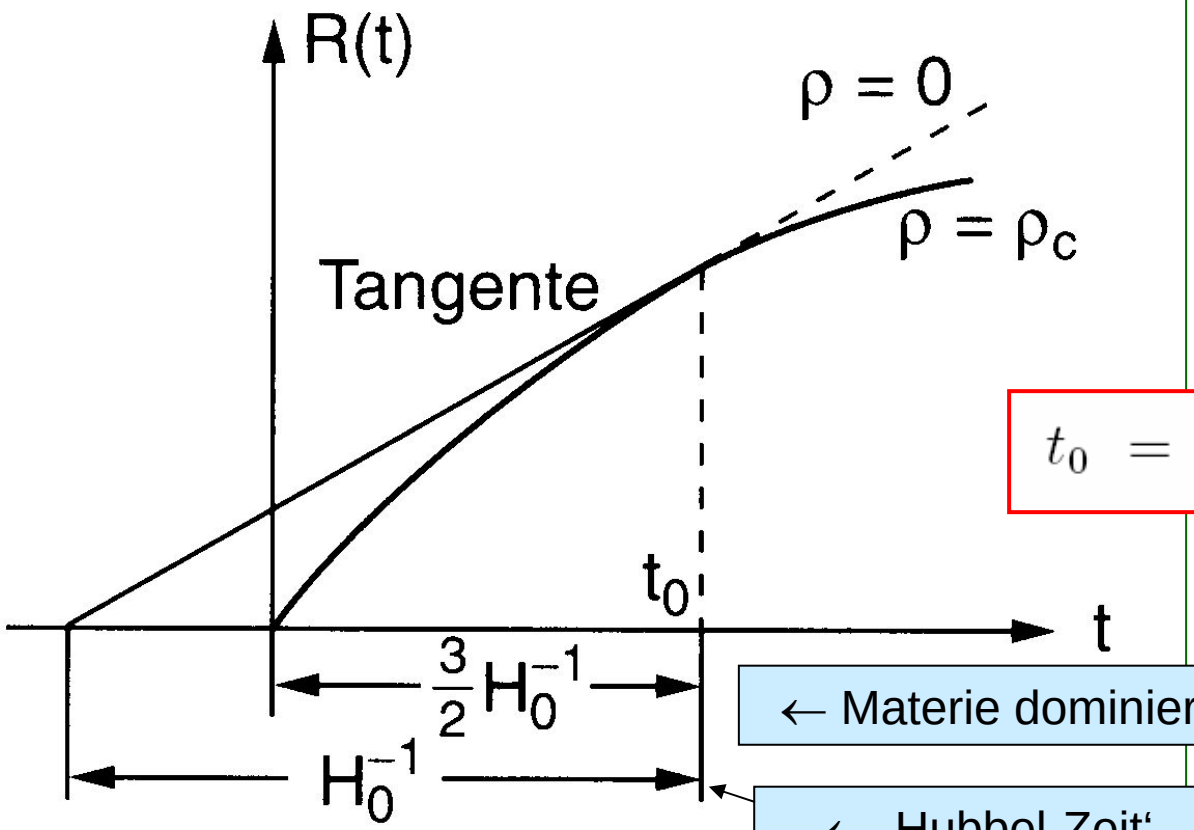



Beobachtung	Ω_0	$\Delta\Omega_0$
sichtbare Materie in Sternen und Galaxien	< 0.01	0.003 - 0.007
Dynamik von Galaxien (Anwendung des Virialsatzes):		
- Galaxien	0.06	0.02 - 0.10
- Doppelgalaxien	0.10	0.03 - 0.15
- Galaxienhaufen	0.25	0.15 - 0.35
Häufigkeit der primordialen Elemente	0.1	0.009 - 0.14
Dichte der Galaxien in großer Entfernung	0.9	0.4 - 1.6

Temperaturentwicklung



Alter des Universums



← Materie dominiert

← ‚Hubbel-Zeit'

Alter des Universums

$$t_0 = (13.73 \pm 0.15) \cdot 10^9 \,\text{Jahre}$$

$$t_0 = \frac{1}{H_0} \approx 13.4 \cdot 10^9 \,\text{Jahre}$$