First results with two light flavours of maximally twisted mass quarks

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Outline

Twisted Mass fermions

- the twisted mass Dirac operator:

$$D_{\mathrm{tm}} = D_{\mathrm{W}} + m_0 + i\mu\gamma_5\tau_3\,.$$

[Frezzotti, Grassi, Sint, Weisz, 1999]

- Wilson Dirac operator D_{W} with bare mass m_0.
- twisted mass parameter μ.
- τ_3 third Pauli matrix acting in flavour space
- D_{tm} is protected against unphysically small eigenvalues

Automatic $\mathcal{O}(a)$ improvement

If m_0 is tuned to its critical value $m_{\rm crit}$ (maximal twist) then ...

- observables are automatically $\mathcal{O}(a)$ improved.

  [Frezzotti, Rossi, 2003]

  Shown to work in practice for various observables in the quenched approximation [Jansen et al., 2004, 2005; Abdel-Rehim et al., 2004, 2005]

- Simplifies mixing during renormalisation
- Only one parameter (m_0) must be tuned

but...

- parity and flavour symmetry explicitly broken

Idea of the Proof

$$\langle O(x)\rangle^{\mathrm{lat}} = \langle O(x)\rangle^{\mathrm{c}} - a\int dy\,\langle O(x)\mathcal{L}_1(y)\rangle^{\mathrm{c}} + a\sum_k \langle O_k(x)\rangle^{\mathrm{c}} + \mathcal{O}(a^2)$$

- r.h.s.: all expt. values with cont. action: must obey symmetries of cont. action
- all operators in expansion must share lattice symmetries of O
- example: cont. symmetry modified Parity

$$\tilde{\mathcal{P}}: \quad \begin{cases} \psi(\vec{x},t) & \rightarrow \gamma_0 \exp(i\omega\gamma_5\tau_3)\psi(-\vec{x},t) \\ \bar{\psi}(\vec{x},t) & \rightarrow \bar{\psi}(-\vec{x},t)\exp(i\omega\gamma_5\tau_3)\gamma_0\ , \end{cases}$$

- O must be even under $\tilde{\mathcal{P}}$, clover term is odd: term cancels in the expansion

A choice? Tuning to full twist

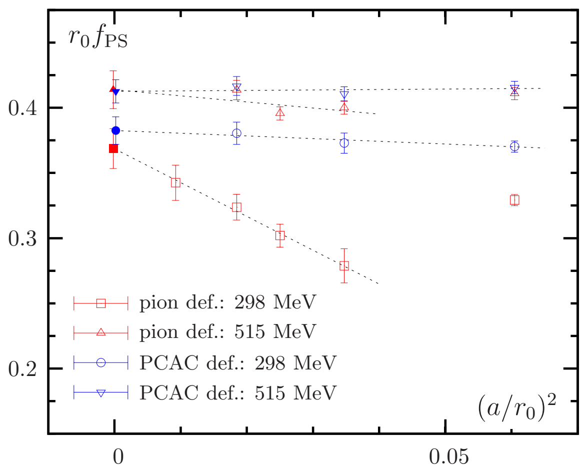
- $\mathcal{O}(a)$ ambiguity in $m_{\rm crit}$ does not harm automatic $\mathcal{O}(a)$ improvement

  Several definitions available:

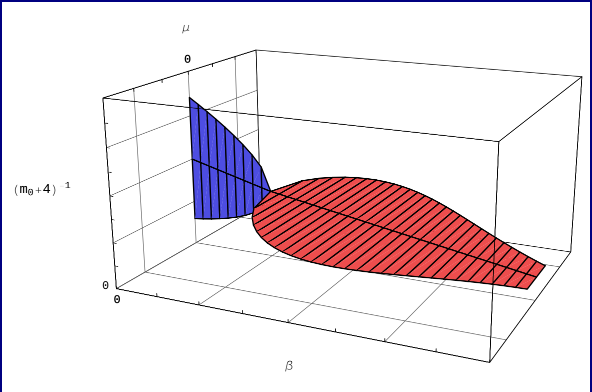
  1. $m_{\rm crit}$ where $m_{\rm PS} = 0$ at $\mu = 0$ (Pion def.)
  2. $m_{\rm crit}$ where $m_{\rm PCAC} = 0$ at $\mu = 0$ (PCAC def.)
  3. $m_{\rm crit}$ where $m_{\rm PCAC} = 0$ for each value of $a\mu$ seperatly
  4. ...

- Theoretical discussion in [Frezzotti, Martinelli, Papinutto, Rossi; Sharpe; Aoki, Bär]: do not use 1.

Quenched Continuum Scaling of f_{PS}

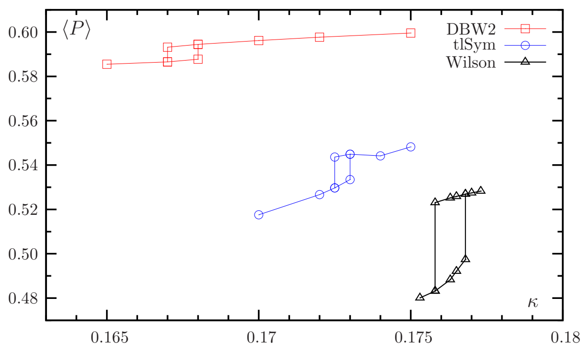


Generic Phasestructur of Wilson Fermions



This is generic for Wilson type fermions!

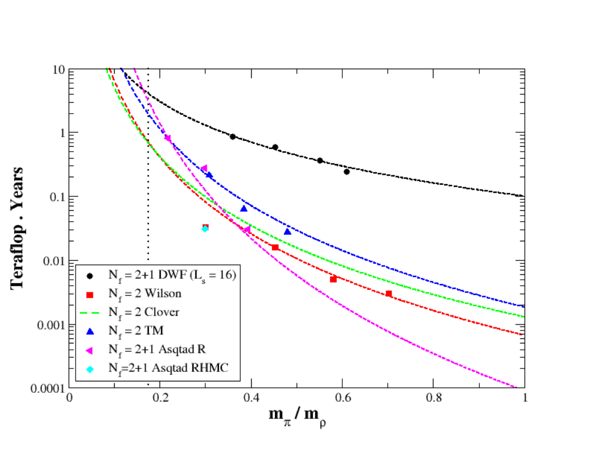
Chosing the Gauge Action



- tlSym is a theoretically sound compromise

Speeding up the HMC

Resurrection of Wilson fermions due to algorithmic improvements



Setup

- We are using the tree-level Symanzik improved gauge action [Weisz, 1983]
- $N_f = 2$ mass-degenrate flavours of maximally twisted mass quarks
- Algorithm: HMC with multiple time scales and mass preconditioning [Urbach et al., 2005]
- Plan:
  - 3 lattice spacings: 0.075 – 0.125 fm
  - pseudo scalar masses in the range 250 – 550 MeV
  - volumes $\geq$ 2 fm
- m_0 tuned to m_{crit} at the lowest mass at each lattice spacing

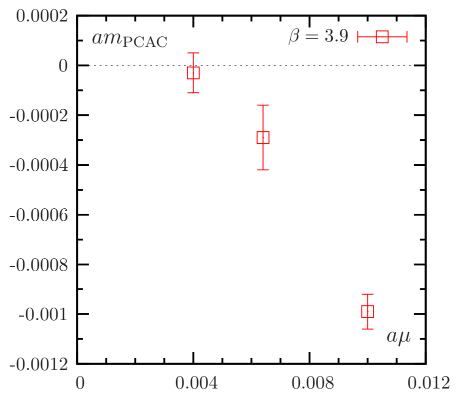
Setup

β	$L^3 \times T$	a [fm]	m_{PS} [MeV]	N_{therm}	N_{traj}
3.9	$24^3 \times 24$	≈ 0.095	≈ 280	1500	5000
			≈ 350	1500	5000
			≈ 430	1500	5000
			≈ 510	1500	5000
4.05	$32^3 \times 64$	≈ 0.075	≈ 280	1500	1200
			≈ 350	1500	500

$\beta = 4.05$ very preliminary!

Tuning to full twist

- $m_{\rm PCAC} = 0$ at lowest $a\mu$
- $m_{\rm PCAC} = 0$ dependence visible
- no news: known from quenched



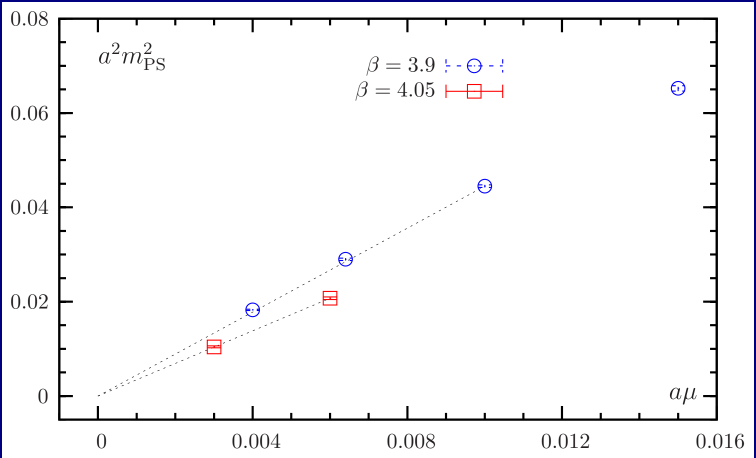
Some simple observables

- First observables to look at: m_{PS} and f_{PS}
- f_{PS} at maximal twist can be obtained from

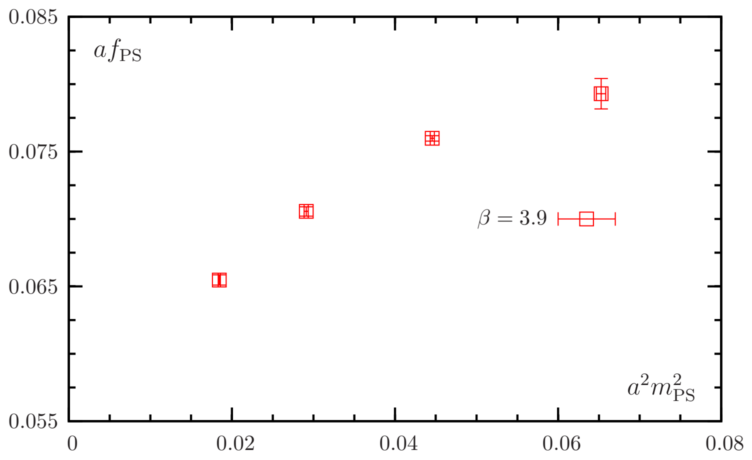
$$f_{\mathrm{PS}} = \frac{2\mu}{m_{\mathrm{PS}}^2} |\langle 0|P^1(0)|\pi\rangle|$$

- Note that at maximal twist f_{PS} does not need to be renormalised
- We estimate finite size (FS) effects with NLO ChPT formula from Gasser and Leutwyler [Gasser, Leutwyler, 1987]
- Checked against resummed Lüscher formula [Lüscher, 1986; Colangelo, Dürr, Haefeli, 2005]
- We use spin diluted random time slice sources, fuzzing [Michael] and variational methods [Michael, 1985 ;Lüscher, Wolff, 1990]

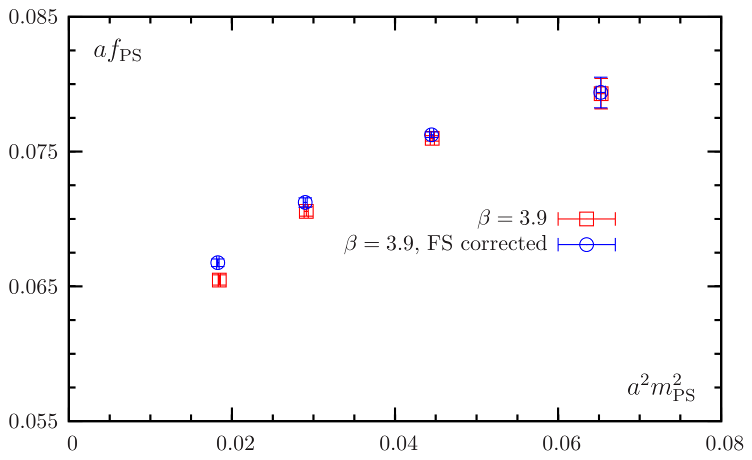
$am_{\rm PS}$ versus $a\mu$ at $\beta = 3.9$



f_{PS} at $\beta = 3.9$



f_{PS} at $\beta = 3.9$



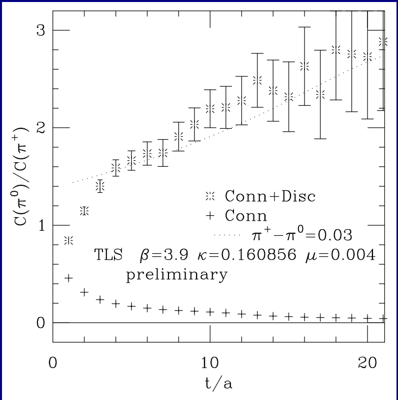
f_{PS} at $\beta = 3.9$

- Fits to the data with ChPT formulae in progress
- when the data is extrapolated linearly to the physical point we obtain:
$$f_\pi = 126.3 \pm 0.8 \pm 0.7\ \mathrm{MeV}$$
- First error comes from $m_{\mathrm{PS}}, f_{\mathrm{PS}}$ and extrapolation, the second from r_0/a
- $r_0 = 0.5\ \mathrm{fm}$ was used
- In our normalisation $f_{\mathrm{PS}} = 131\ \mathrm{MeV}$

Effects of Isospin breaking

Flavour symmetry explicitly broken by twisted mass term at finite *a*

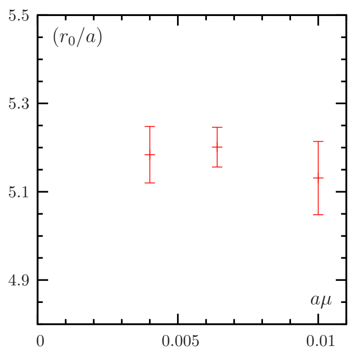
- Expected to be largest in $m_{\rm PS}^{\pm} - m_{\rm PS}^{0}$
- $\beta = 3.9, a\mu = 0.004, am_{\rm PS} = 0.1358(5)$: $m_{\rm PS}^{\pm} - m_{\rm PS}^{0} = 0.03(1)$
- A factor of 2 smaller than quenched
- Splitting in the vector mass compatible with 0



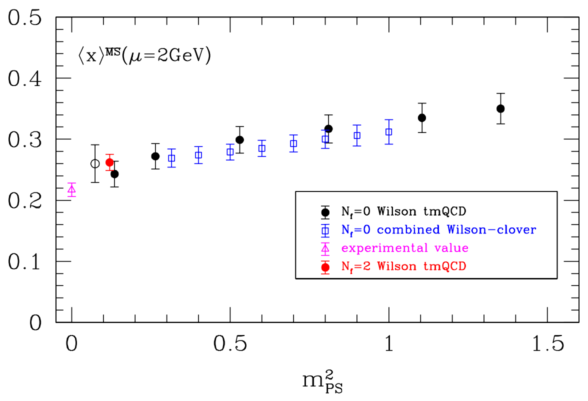
Setting the scale

- Using the Sommer scale r_0
- Value of r_0 not very well known
- Scale setting with f.i. f_K, m_{K^*} etc. in progress
- Here I use $r_0 = 0.5\ \mathrm{fm}$
- μ dependence seems to be weak

$\beta = 3.9$



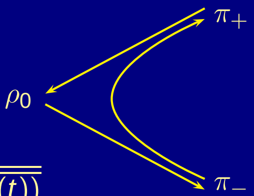
$\langle x \rangle$ for the Pion



Lowest moment of Non-Singlet Pion Parton Distribution Function $\langle x \rangle$

Exploring ρ-decay [McNeile, Michael, 2003]

- ρ does not (yet) decay at $\beta = 3.9$: $a\Delta m \approx 0.15$ at $a\mu = 0.004$
- uncharged vector meson at rest:



$$R(t) = \frac{\rho_0(0) \to \pi_+\pi_-(t)}{\sqrt{(\rho_0(0) \to \rho_0(0))(\pi_+\pi_-(0) \to \pi_+\pi_-(t))}}$$

- Assuming $m_\rho \approx m_{\pi\pi}$, $x = \langle \rho | \pi\pi \rangle$:

$$R \propto \sum_{t=0}^{T} x e^{-m_\rho t} e^{-m_{\pi\pi}(T-t)} = e^{-mT} \cdot x \cdot t$$

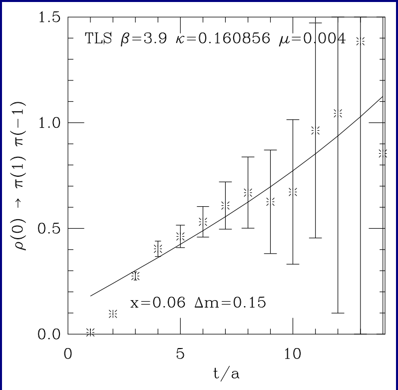
for $xt \ll 1$

Exploring ρ-decay

- We extract $\bar{g} = 1.2(2)$, with

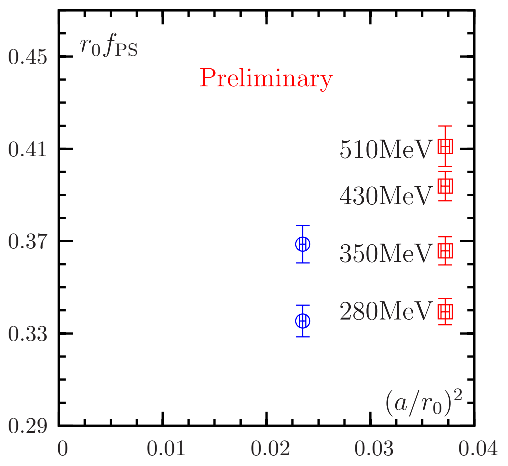
$$\bar{g} = \Gamma m_\rho E_{\pi\pi}/k^3$$

- experimental number: $\bar{g} = 1.39$
- Already well compatible
- Larger lattices needed for a decaying ρ.



Preliminary continuum scaling

- r_0/a value at the lowest μ value
- masses not yet exactly matched



Conclusion

- mtmQCD stands on a sound basis
- First encouraging results with $N_f = 2$ flavours of maximally twisted mass quarks

- We can reach values for m_{PS} as low as 280 MeV

- Flavour symmetry breaking effects are visible, but significantly smaller than quenched

- Lattice artifacts in f_{PS} seem to be small
- First physics applications in progress
- $N_f = 2 + 1 + 1$ in progress: talk of Enno