

Theory Perspectives: Deep-Inelastic Scattering

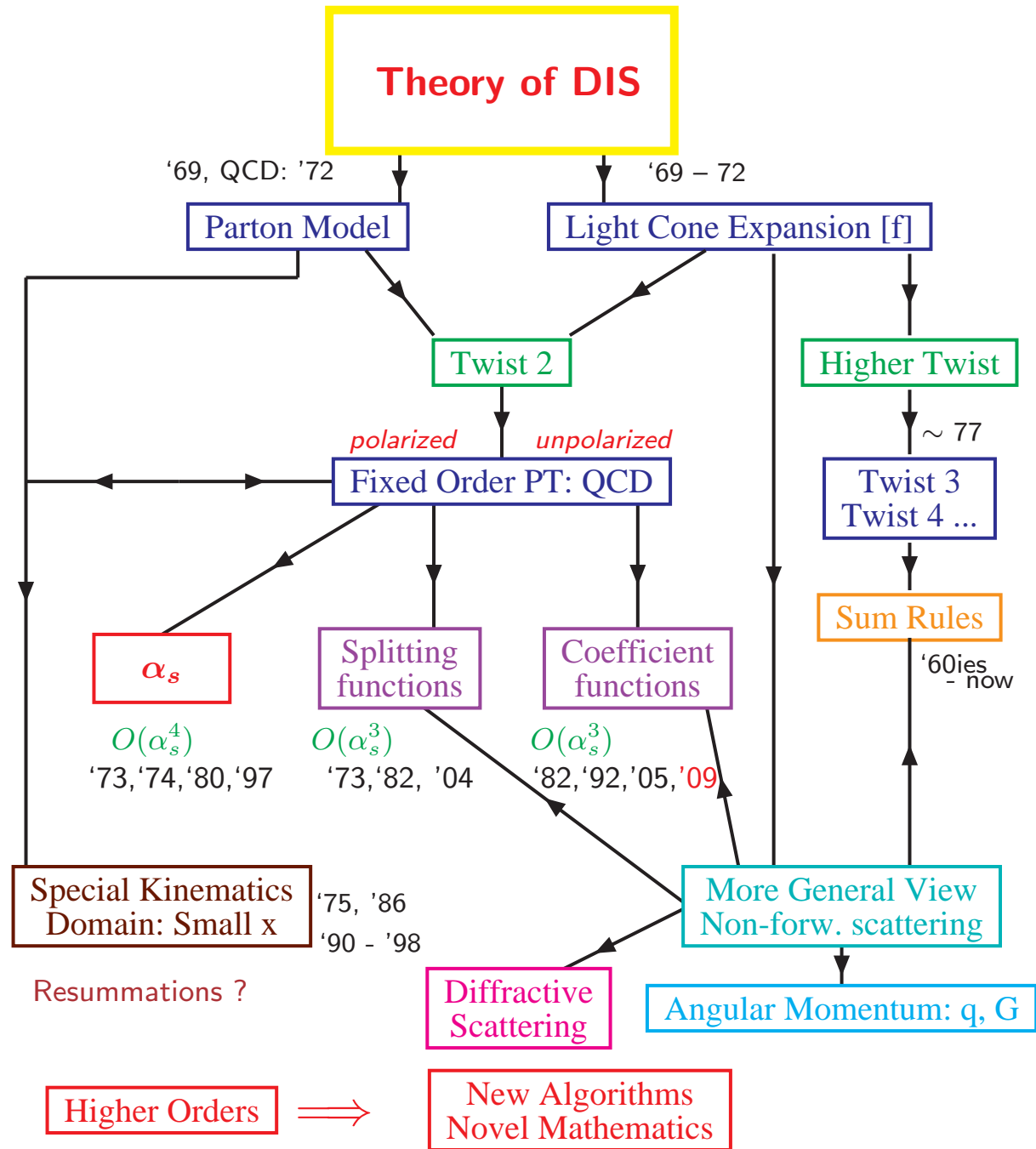
Johannes Blümlein
DESY











- The Major Goals
- DIS Theory Status
- Unpolarized Parton Distribution Functions
- Polarized Parton Distribution Functions
- Λ_{QCD} and $\alpha_s(M_Z^2)$
- Advanced Technologies to Evaluate Feynman Diagrams @ 3 Loops
- Outlook

1. The Major Goals

- Precision Measurement of the Strong Coupling Constant $\alpha_s(M_Z^2)$
- Precision Measurement of the Unpolarized Parton Densities
- Precision Measurement of the Polarized Parton Densities
- Who Carries the Spin of the Proton?
- Higher Twist Effects
- Is there Saturation in DIS at small x ? \implies answered by experiment.



Status of Highest Order Calculations

- Running α_s : $O(\alpha_s^4)$ Larin, van Ritbergen, Vermaseren 1997
- Unpol. anomalous dimensions and Wilson coefficients: $O(\alpha_s^3)$
Moch, Vermaseren, Vogt 2004/05 
- Unpol. NS anomalous dimension 2nd Moment: $O(\alpha_s^4)$ Baikov, Chetyrkin 2006
- Pol. anomalous dimension: $O(\alpha_s^2)$; Mertig, van Neerven, 1995; Vogelsang 1995;
 $\Delta P^{qq} \Delta P_{qG}$: $O(\alpha_s^3)$ Moch, Rogal, Vermaseren, Vogt 2008 
- Pol. Wilson coefficients: $O(\alpha_s^2)$; $\Delta C_{NS}^{qq}, \Delta C_{qG}$: van Neerven, Zijlstra 1994
- Transversity: $O(\alpha_s^2)$, some moments anom. dim.: $O(\alpha_s^3)$, Hayashigaki, Kanazawa, Koike;
Kumano, Miyama; Vogelsang; 1997; Gracey 2006, HQ: JB, S.Klein, B. Tödtli 2008 
- Unpol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^2)$ Laenen, van Neerven, Riemersma, Smith, 1993
Fast Mellin Space code: Blümlein & Alekhin, 2003 
- Pol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^1)$ Watson 1982
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient F_L : $O(\alpha_s^3)$
Blümlein, De Freitas, van Neerven, S. Klein 2005 
- $Q^2 \gg m^2$ Pol. Heavy Flavor Wilson Coefficient : $O(\alpha_s^2)$ van Neerven, Smith et al. 1996,
Bierenbaum, Blümlein & Klein 2007 
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient F_2 : $O(\alpha_s^2 \epsilon)$: all operators
(also polarized), Bierenbaum, Blümlein, Klein, Schneider, 2008;  $O(\alpha_s^3)$: Moments 2–10(12,14)
of the operator matrix elements, HQ Wilson coeff. Bierenbaum, Blümlein, Klein, 2008 

 = done at DESY (or in DESY collab.).



DIS Structure Functions @ Twist 2

$$F_j(x, Q^2) = \hat{f}_i(x, \mu^2) \otimes \sigma_j^i \left(\alpha_s, \frac{Q^2}{\mu^2}, x \right)$$

↑ bare pdf ↑ sub – system cross – sect.

$$= \underbrace{\hat{f}_i(x, \mu^2) \otimes \Gamma_k^i \left(\alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2} \right)}_{\text{finite pdf} \equiv f_k}$$

$$\otimes \underbrace{C_j^k \left(\alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x \right)}_{\text{finite Wilson coefficient}}$$

Move to Mellin space :

$$F_j(N) = \int_0^1 dx x^{N-1} F_j(x)$$

Diagonalization of the convolutions \otimes into ordinary products.

Evolution Equations

$$\left[M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} - 2\gamma_\psi(g) \right] F_i(N) = 0$$

$$\left[M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} + \gamma_\kappa^N(g) - 2\gamma_\psi(g) \right] f_k(N) = 0$$

$$\left[M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} - \gamma_\kappa^N(g) \right] C_j^k(N) = 0$$

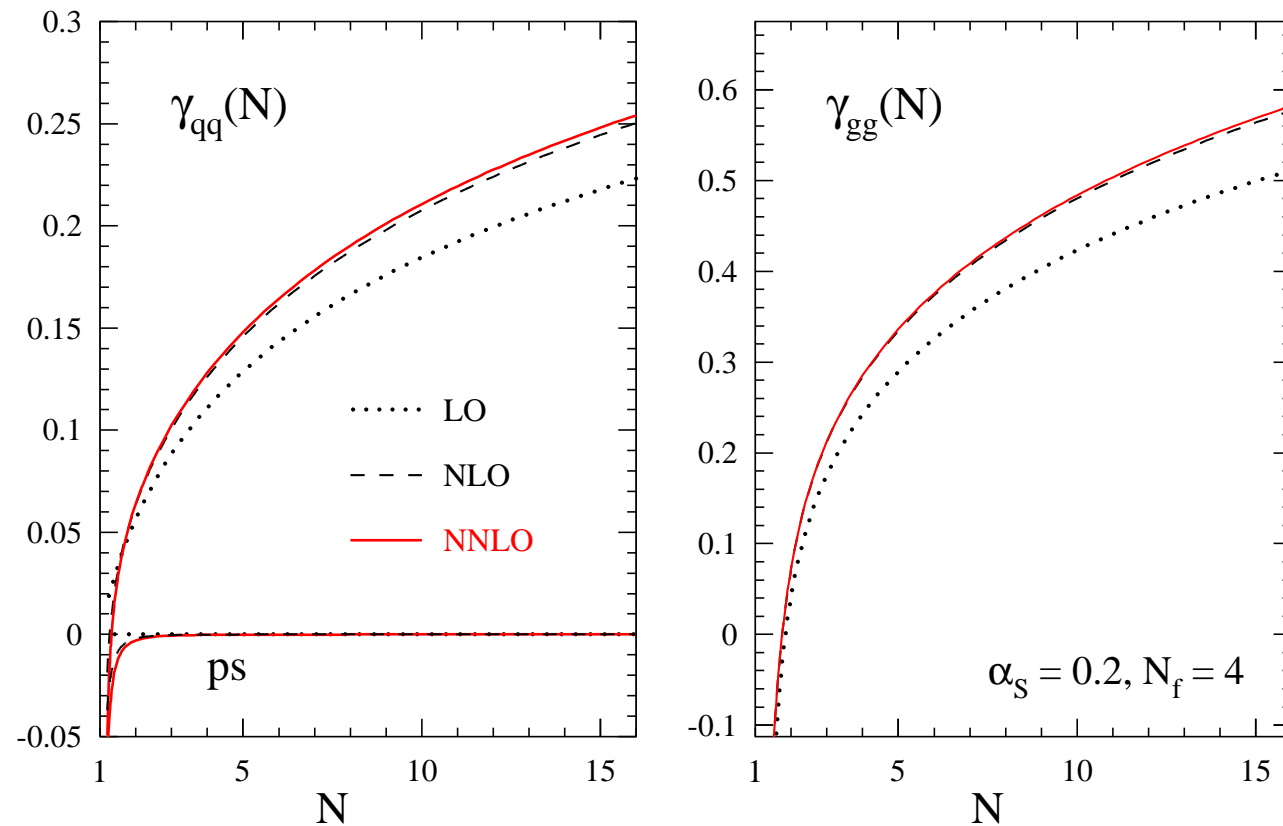
CALLAN–SYMANZIK equations for mass factorization  \equiv
 ALTARELLI–PARISI evolution equations

x-space :

$$\frac{d}{d \log(\mu^2)} \begin{pmatrix} q^+(x, Q^2) \\ G(x, Q^2) \end{pmatrix} = \frac{\alpha_s}{2\pi} \mathbf{P}(x, \alpha_s) \otimes \begin{pmatrix} q^+(x, Q^2) \\ G(x, Q^2) \end{pmatrix}$$

$$\mathbf{P}(x, \alpha_s) = \mathbf{P}^{(0)}(x) + \frac{\alpha_s}{2\pi} \mathbf{P}^{(1)}(x) + \left(\frac{\alpha_s}{2\pi} \right)^2 \mathbf{P}^{(2)}(x) + \dots$$

Anomalous Dimensions and Wilson Coefficients



Vermaseren, Moch, Vogt 2004 

The Basic Functions of massless QCD to $w=5:\equiv 3$ Loops

Representative : $S_1(N) = \psi(N + 1) + \gamma_E$ and its derivatives.

Weight $w=3$:
$$F_1(N) = \mathbf{M} \left[\frac{\ln(1+x)}{1+x} \right] (N)$$

$$F_2(N) = \mathbf{M} \left[\frac{\text{Li}_2(x)}{1+x} \right] (N), \quad F_3(N) = \mathbf{M} \left[\left(\frac{\text{Li}_2(x)}{1-x} \right)_+ \right] (N)$$

Yndurain et al., 1981: $F_2(N)$

Weight $w=4$:

$$F_4(N) = \mathbf{M} \left[\frac{S_{1,2}(x)}{1+x} \right] (N), \quad F_5(N) := \mathbf{M} \left[\left(\frac{S_{1,2}(x)}{1-x} \right)_+ \right] (N)$$

$F_3(N) - F_5(N)$: J.B., 2003; J.B., V. Ravindran ,2004

Weight w=5 :

$$F_{6,7}(N) = \mathbf{M} \left[\left(\frac{\text{Li}_4(x)}{1 \pm x} \right)_{(+)} \right] (N), \quad F_8(N) = \mathbf{M} \left[\frac{S_{1,3}(x)}{1+x} \right] (N),$$

$$F_{9,10}(N) = \mathbf{M} \left[\left(\frac{S_{2,2}(x)}{1 \pm x} \right)_{(+)} \right] (N), \quad F_{11}(N) = \mathbf{M} \left[\frac{\text{Li}_2^2(x)}{1+x} \right] (N),$$

$$F_{12,13}(N) := \mathbf{M} \left[\left(\frac{\ln(x)S_{1,2}(-x) - \text{Li}_2^2(-x)/2}{1 \pm x} \right)_{(+)} \right] (N)$$


$F_6(N) - F_{13}(N)$: J.B., S. Moch, 2004.

Massless QCD to 3 Loops depends on 14 Functions.

Weight w=6 :

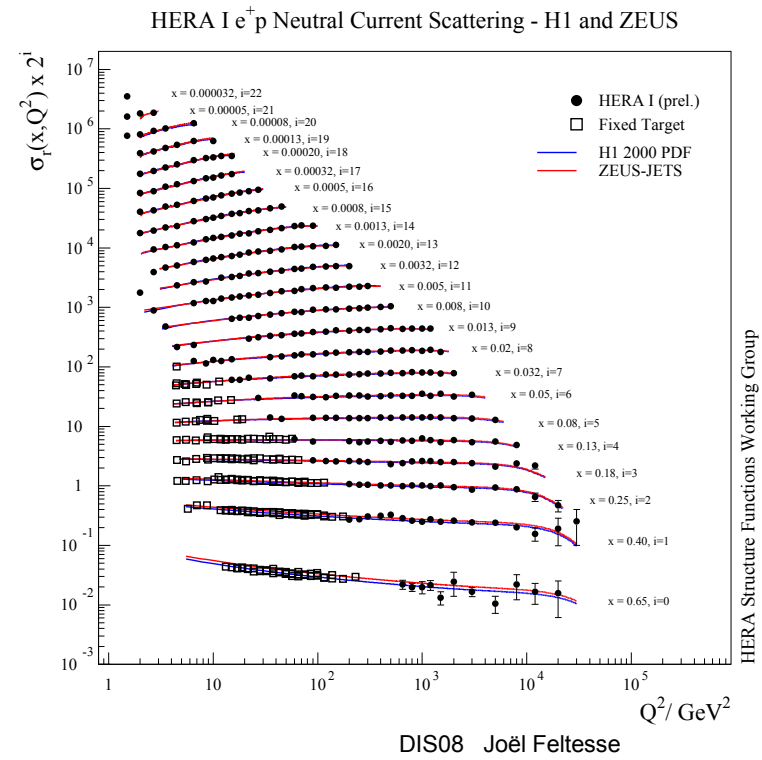
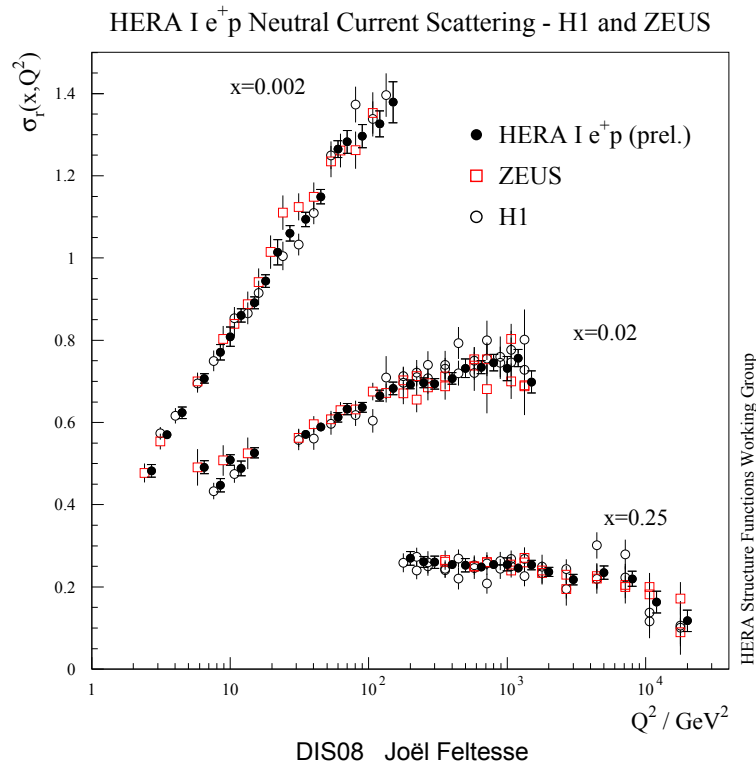
⇒ Representation for 3 Loop Wilson Coeff.: 35 Functions, J.B., 2009. 

Complex Analysis of these Functions

- Construct exact analytic continuations to **complex N**
- The functions are meromorphic
(up to soft corrections, which have a simple structure)
- Asymptotic Representation
- Recursion $z + 1 \rightarrow z$
- Solve the Evolution Equations fully analytically and form an **analytic expression** for the Structure functions in Mellin Space at all Q^2
- Include the **heavy flavor** Wilson coefficients in Mellin Space
 \Rightarrow nearly accomplished to $O(a_s^3)$ I. Bierenbaum, JB, S. Klein (2009) 
- Perform a **single** fast, numerical Mellin inversion
(at high precision)

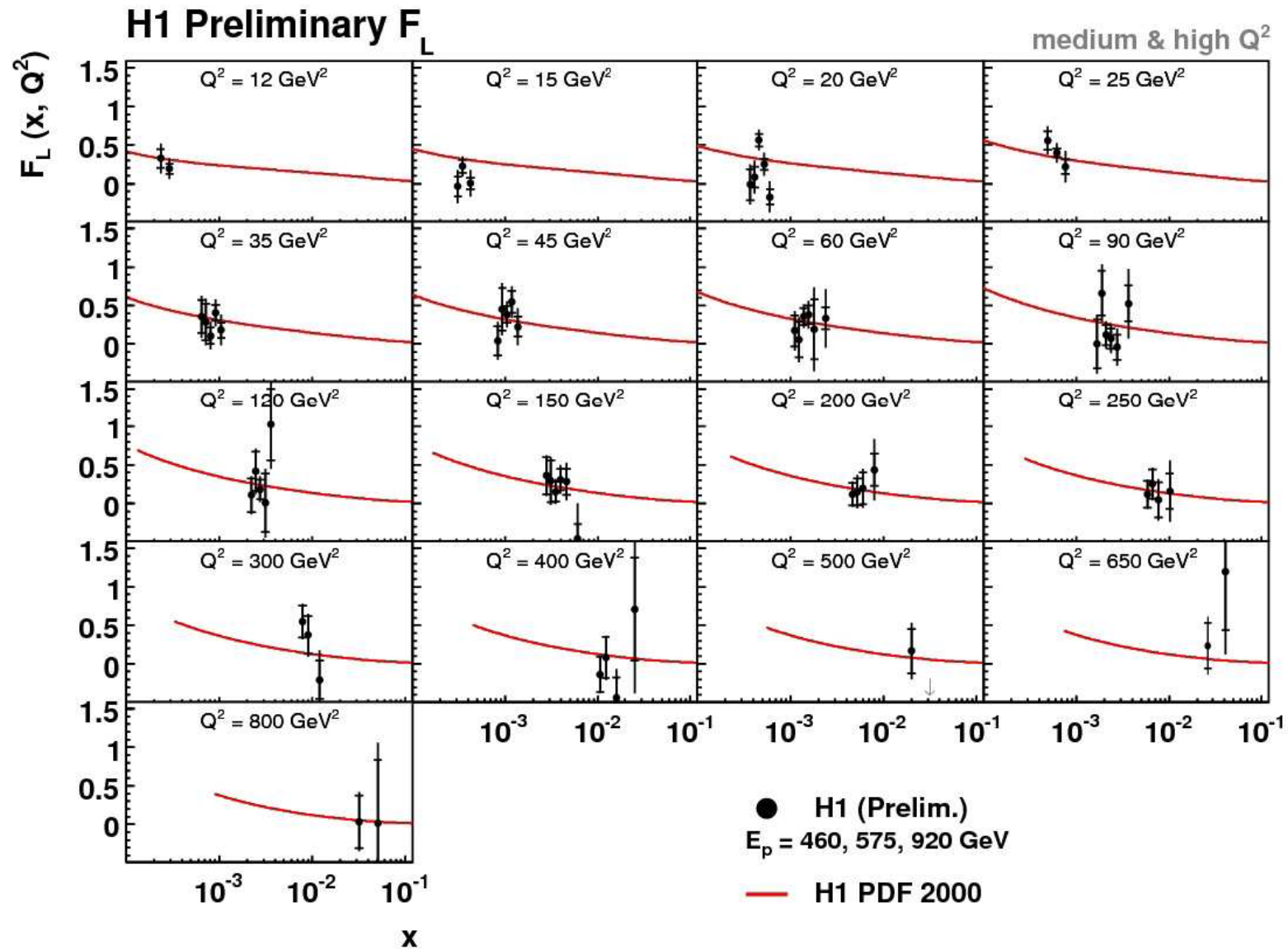
\Rightarrow **Fastest and most Precise Way of Analysis**

3. Unpolarized Parton Distribution Functions

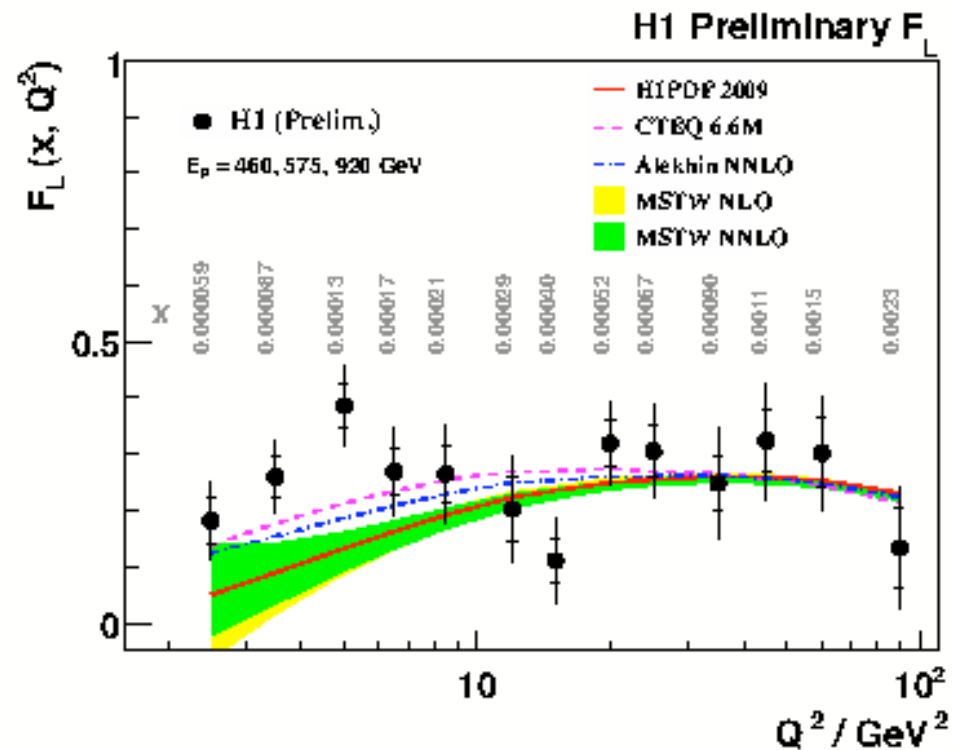


New ZEUS + H1 averaged $F_2(x, Q^2)$

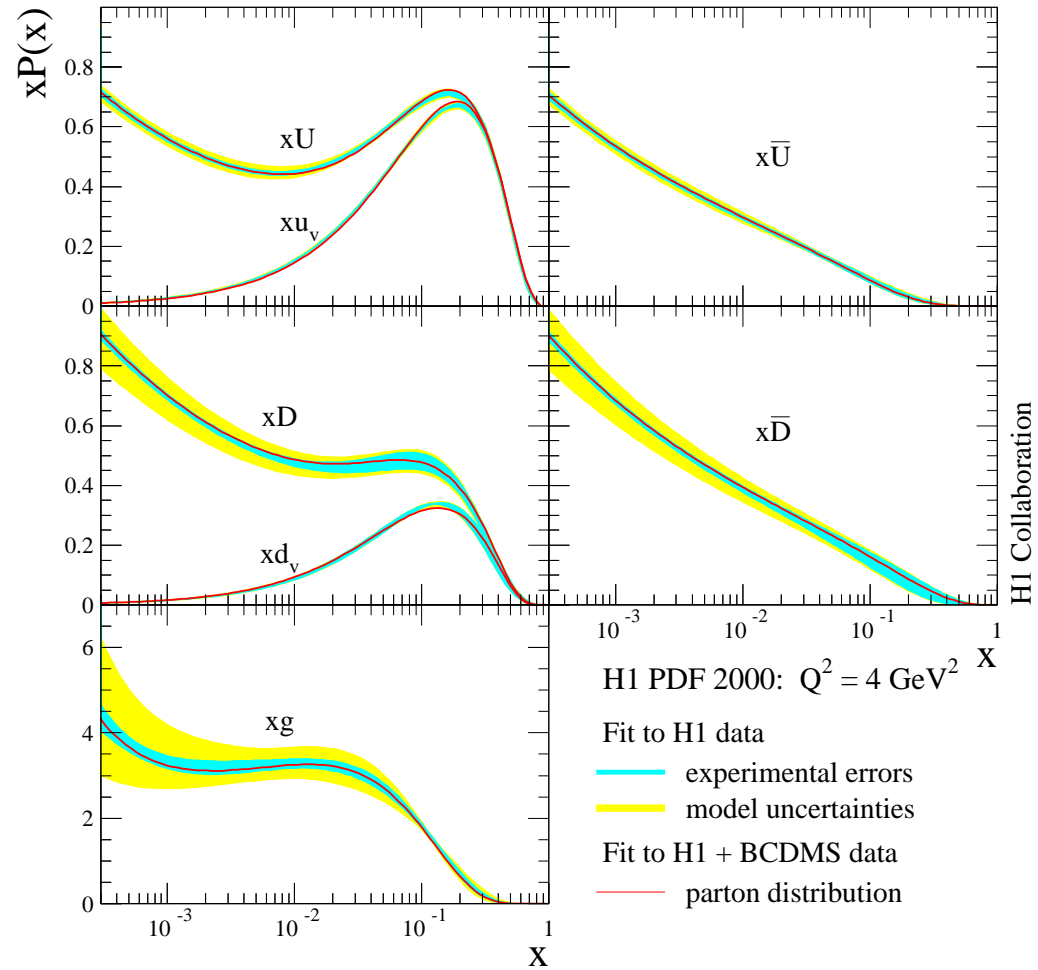
Direct $F_L(x, Q^2)$ Measurement at HERA



Direct $F_L(x, Q^2)$ Measurement at HERA (H1-prel.)

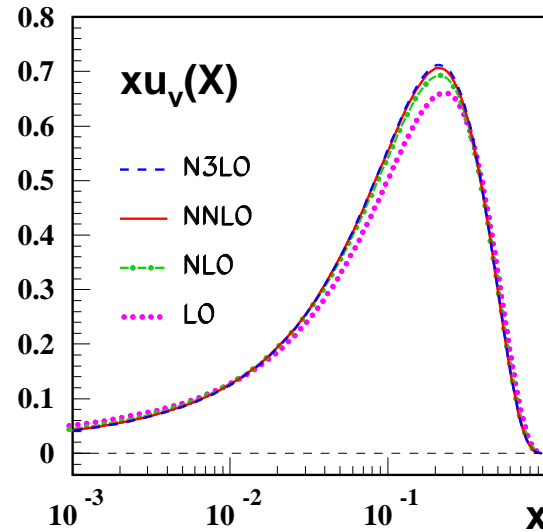
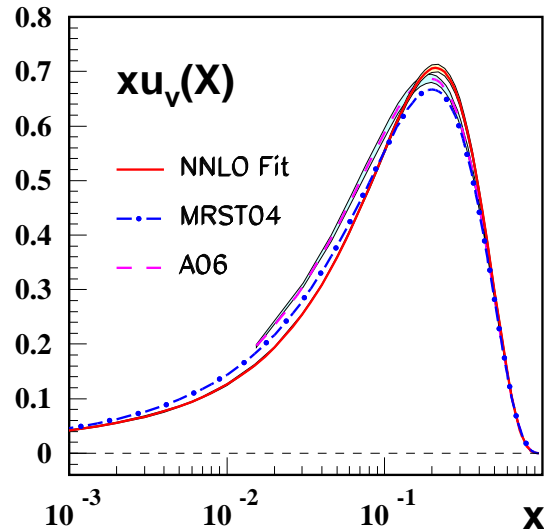


Parton Distributions: Overview



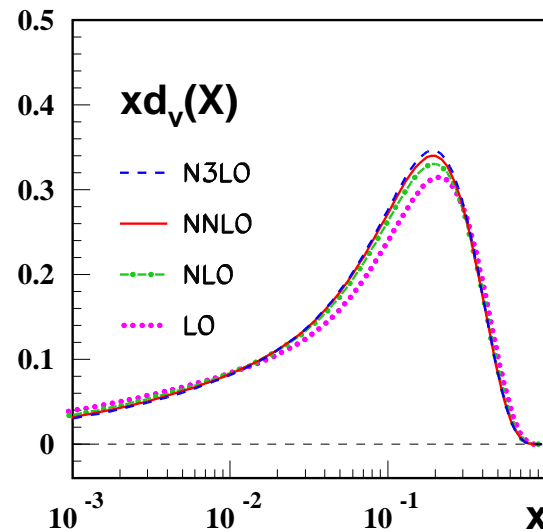
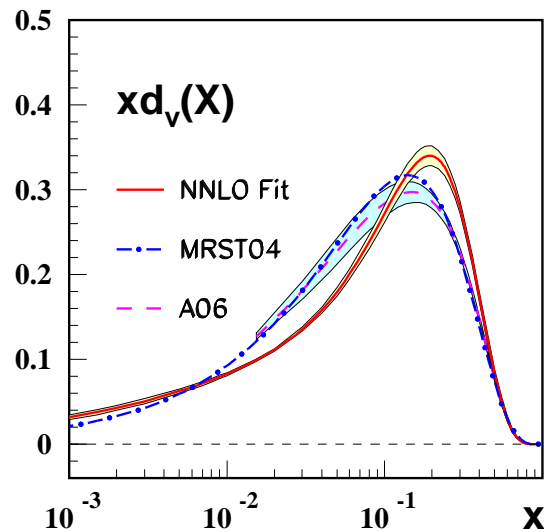
H1

World Data Analysis: Valence Distributions



World data:
NS-analysis

$$W^2 > 12.5 \text{ GeV}^2, Q^2 > 4 \text{ GeV}^2$$



N^3LO :

$$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$$

J.B., H. Böttcher,
A. Guffanti,
(hep-ph/0607200)

Why an $O(\alpha_s^4)$ analysis can be performed?

assume an $\pm 100\%$ error on the Padé approximant $\longrightarrow \pm 2 \text{ MeV}$ in Λ_{QCD}

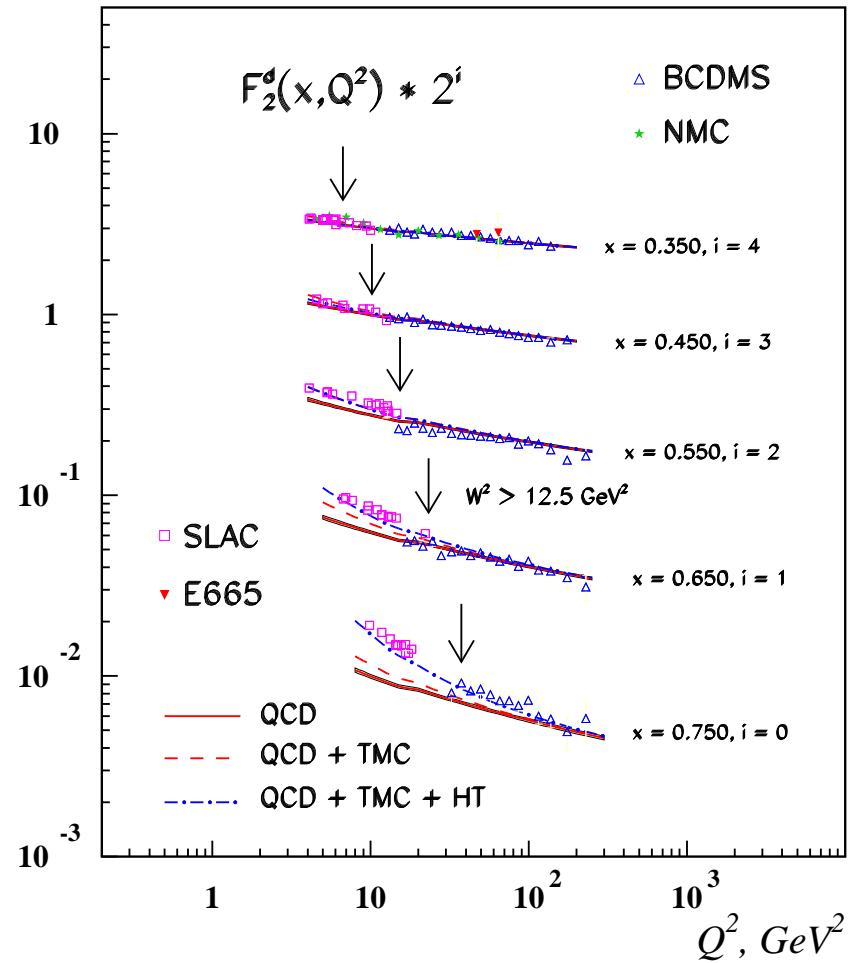
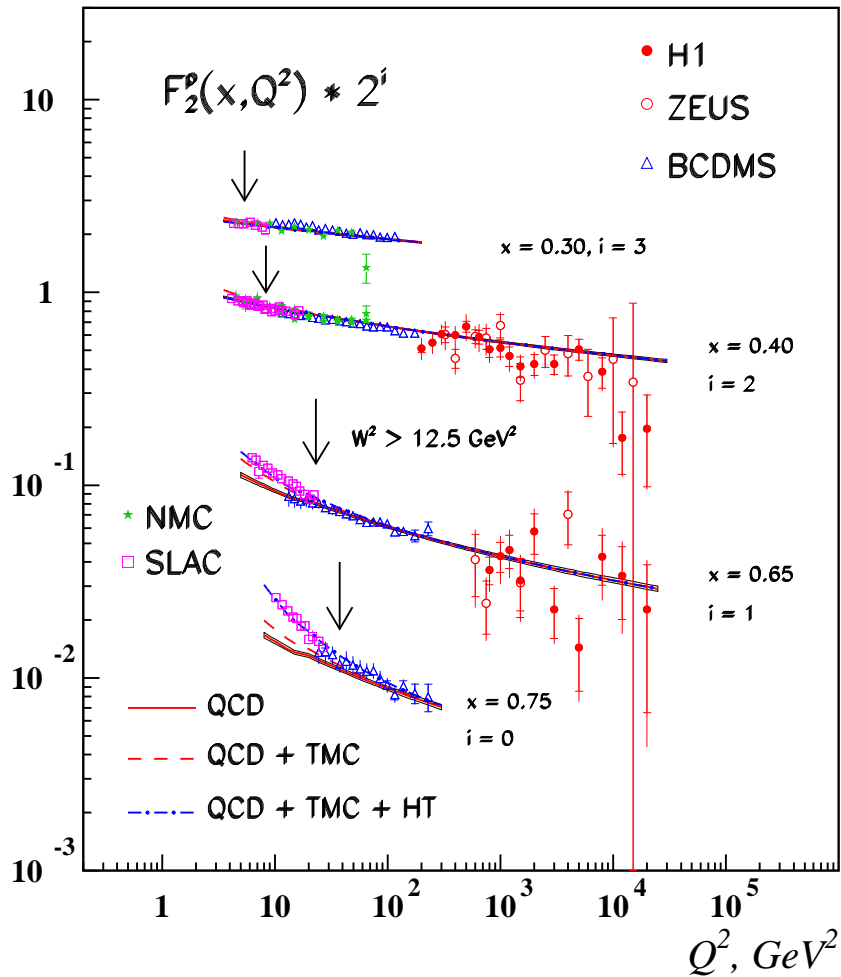
$$\gamma_n^{approx:3} = \frac{\gamma_n^{(2)2}}{\gamma_n^{(1)}}$$

Baikov & Chetyrkin, April 2006:

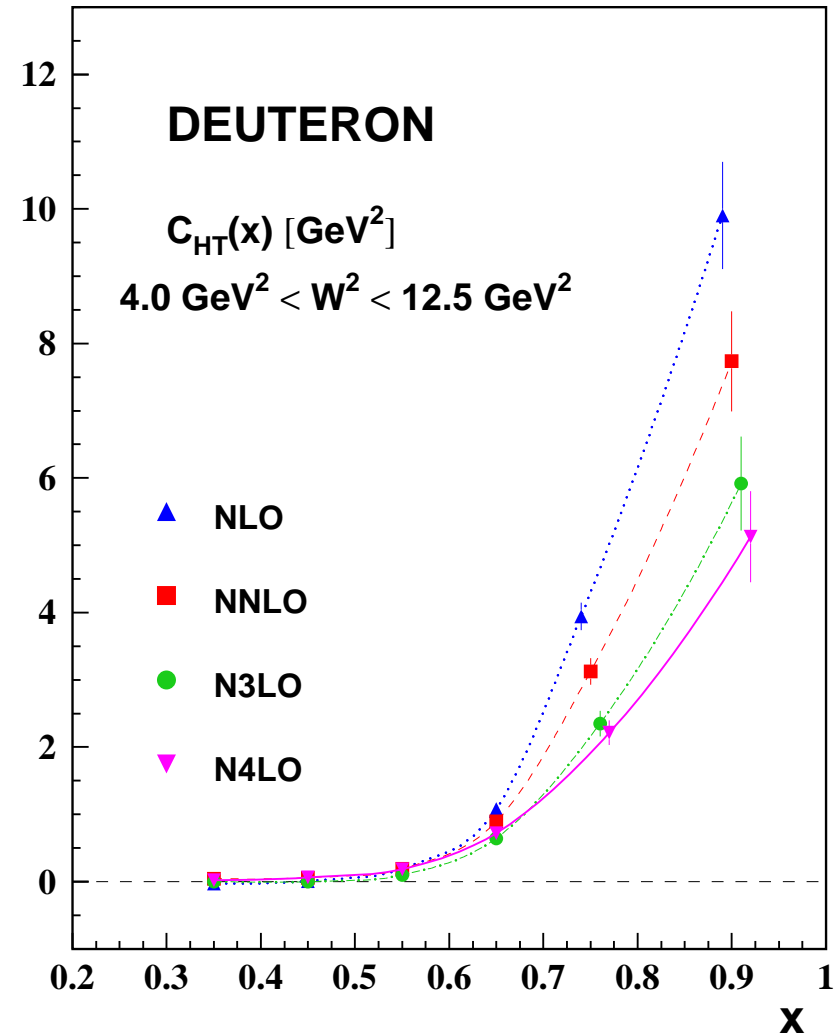
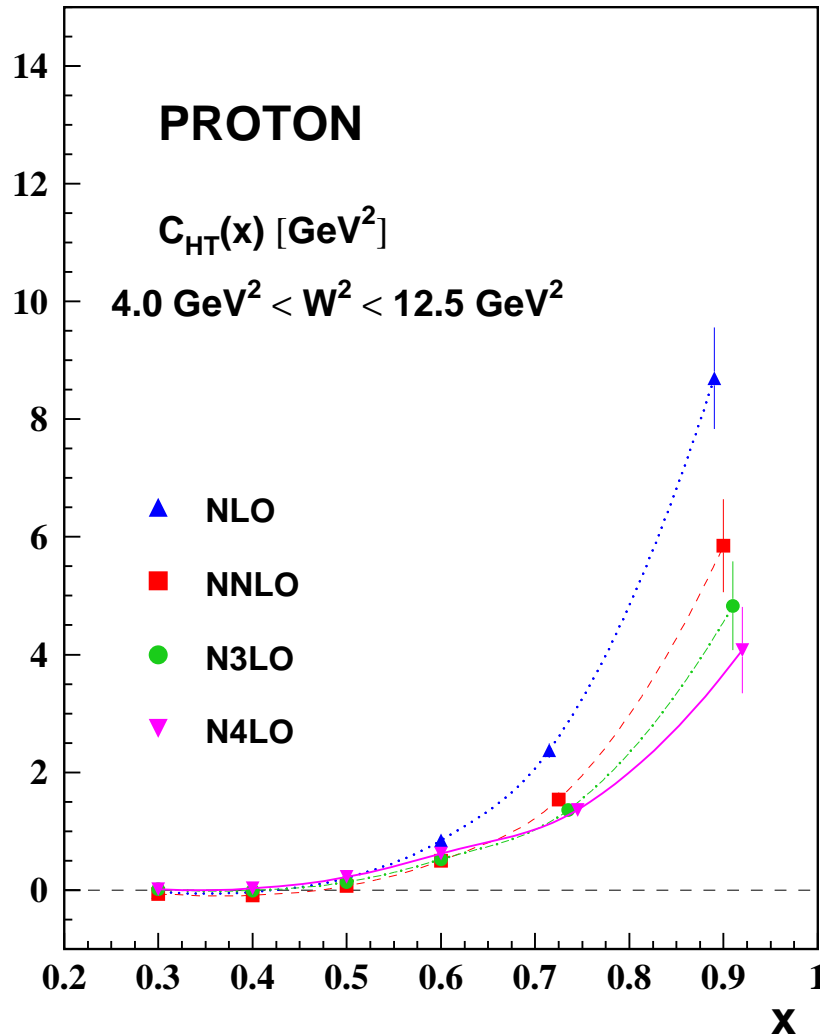
$$\begin{aligned} \gamma_2^{3;NS} &= \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[\frac{3936832}{6561} - \frac{10240}{81} \zeta_3 \right] a_s^3 \\ &+ \left[\frac{1680283336}{1777147} - \frac{24873952}{6561} \zeta_3 + \frac{5120}{3} \zeta_4 - \frac{56969}{243} \zeta_5 \right] a_s^4 \end{aligned}$$

The results agree better than 20%.

Valence Distributions

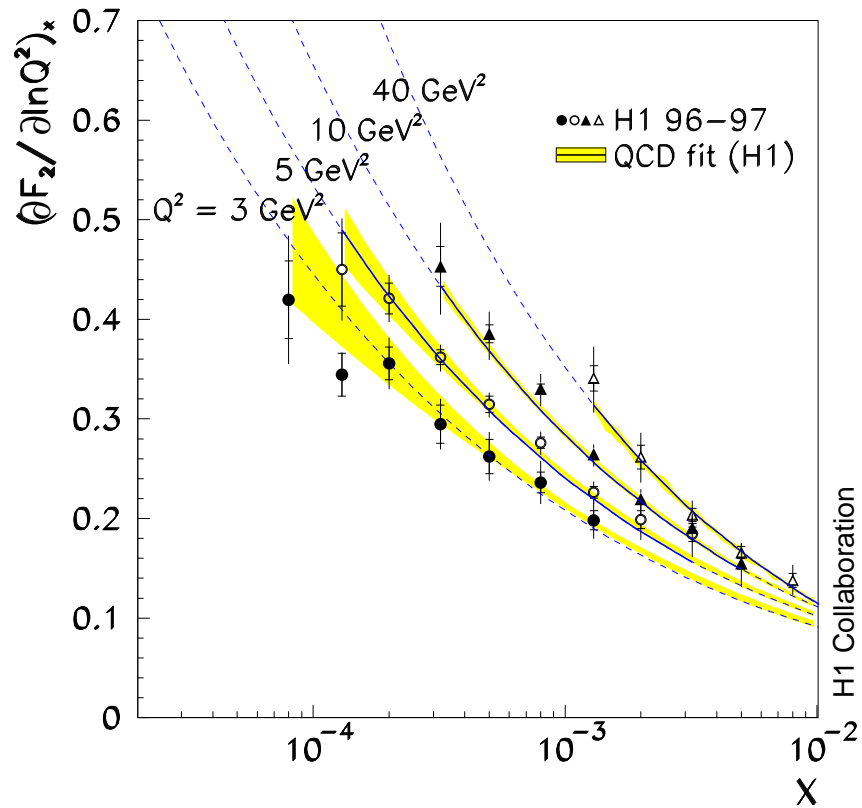


Valence Distributions: higher twist

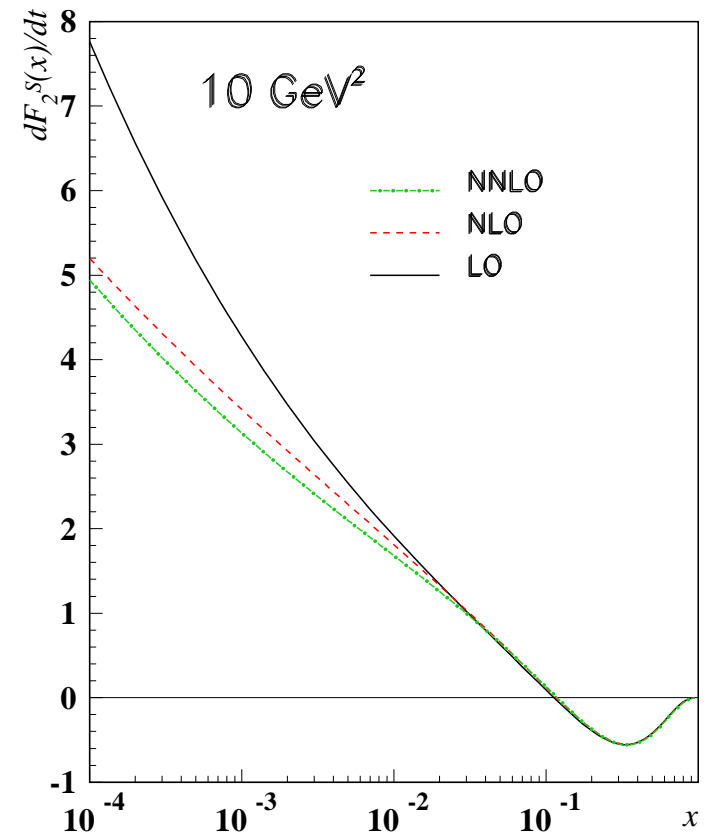


- agreement between p and d analysis, J.B., H. Böttcher, 2008
- LGT determination of interest

Slope of F_2 at low x



H1

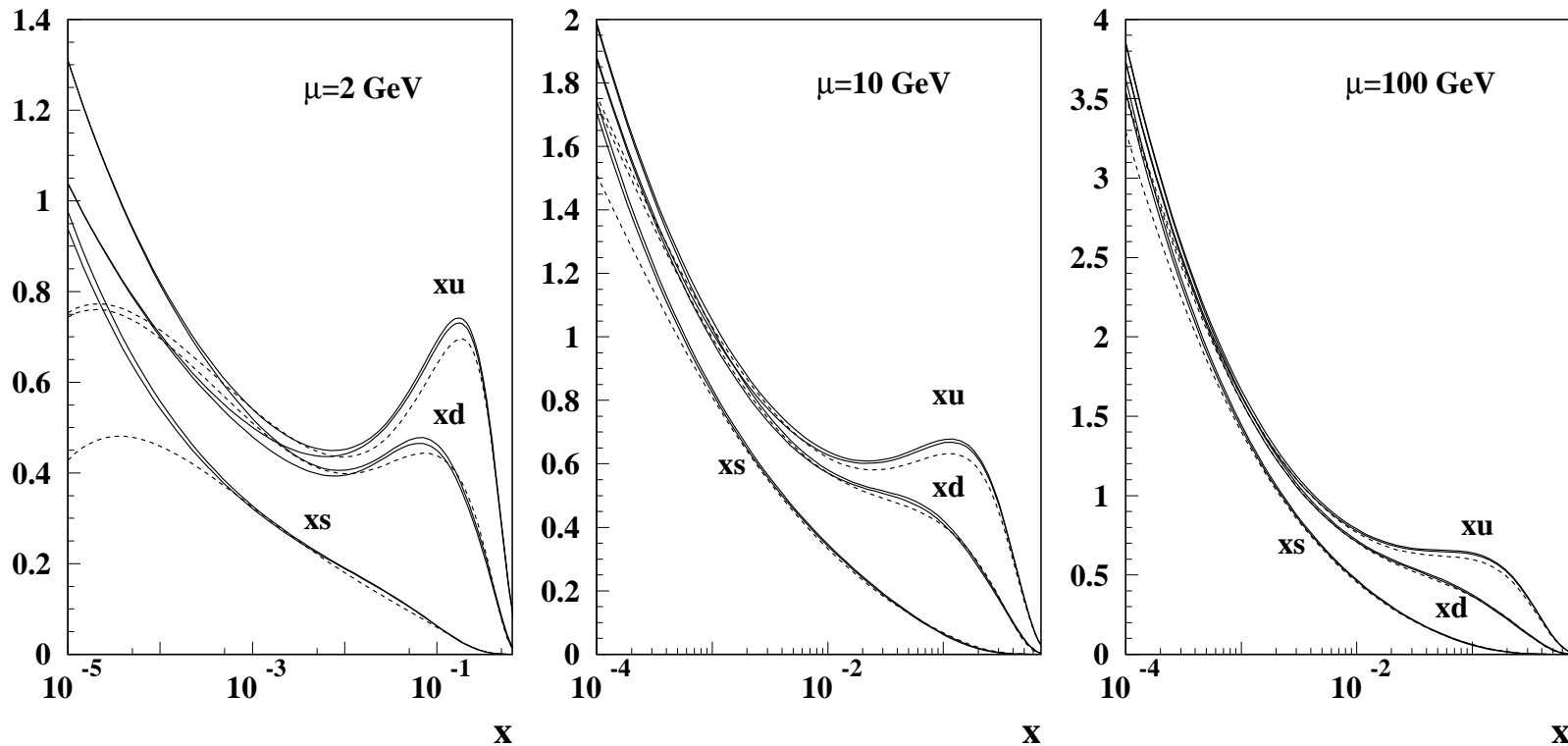


J.B., A. Guffanti 2005

Very likely, that the $\overline{\text{MS}}$ -gluon is remains positive!

Flavor distributions: light quarks (NNLO)

Current Fitting Community (NNLO): 
+ Many NLO analyses worldwide: CTEQ, NNPDF, H1, ZEUS, ...



S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102

Correct treatment of HQ very essential: FFNS, BSMN-schemes.

full lines: ABKM error band; dashed lines: MSTW08

Flavor distributions: strangeness

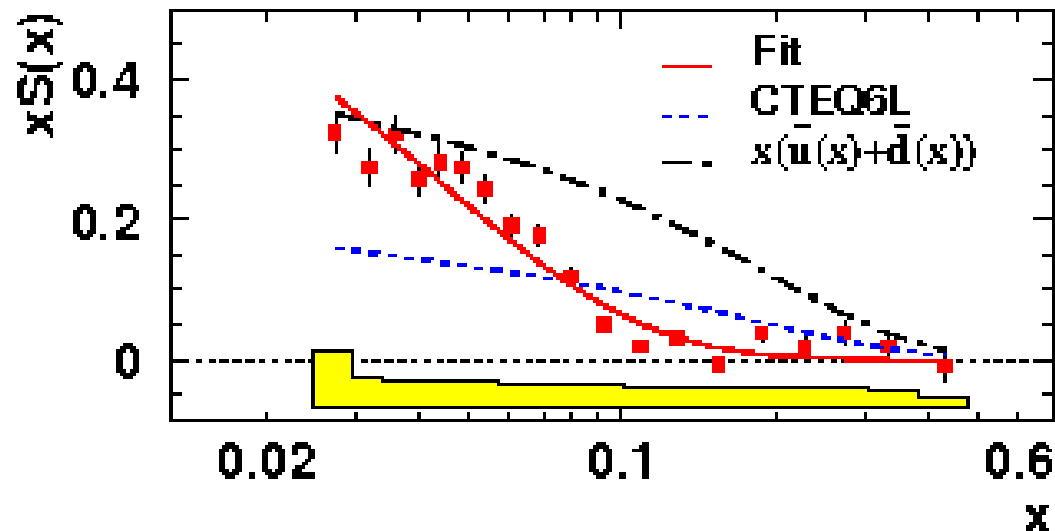
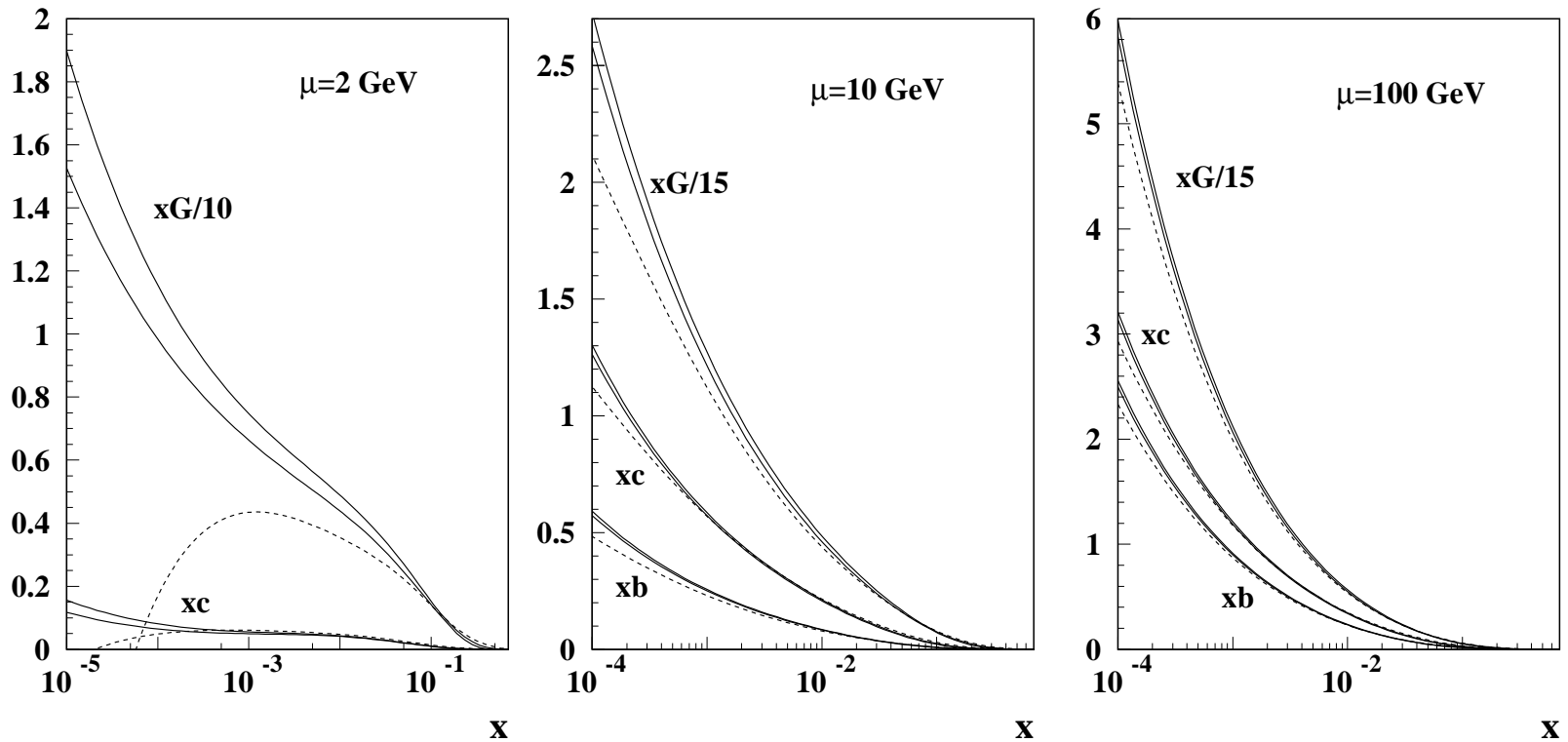


FIG. 3: The strange parton distribution $xS(x)$ from the measured HERMES multiplicity for charged kaons evolved to $Q_0^2 = 2.5 \text{ GeV}^2$ assuming $\int \mathcal{D}_S^K(z) dz = 1.27 \pm 0.13$. The solid curve is a 3-parameter fit for $S(x) = x^{-0.924} e^{-x/0.0404} (1-x)$, the dashed curve gives $xS(x)$ from CTEQ6L, and the dot-dash curve is the sum of light antiquarks from CTEQ6L.

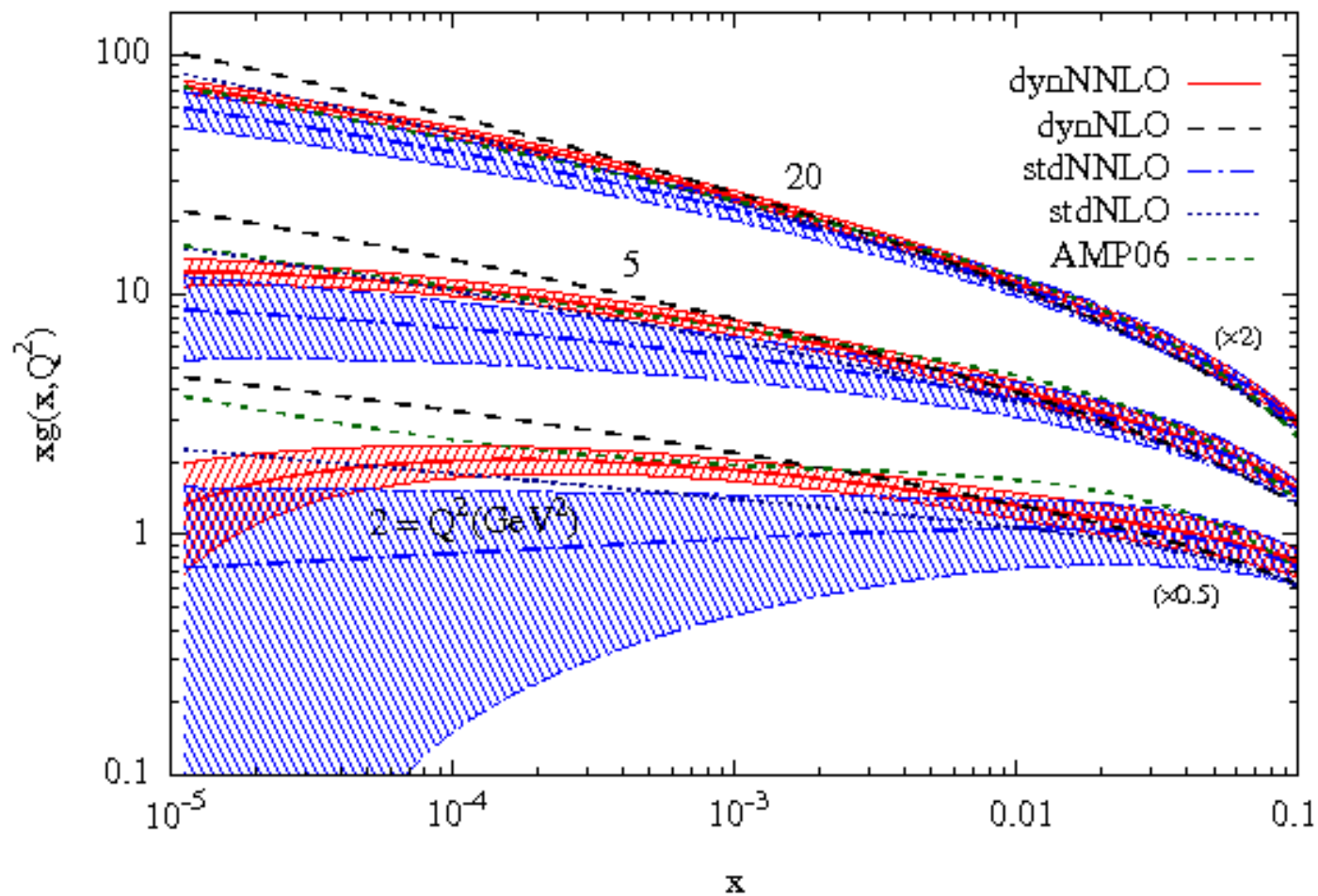
Nice HERMES measurement (hep-ex/0803.2993); still to be understood.

Heavy quarks and gluon (NNLO)



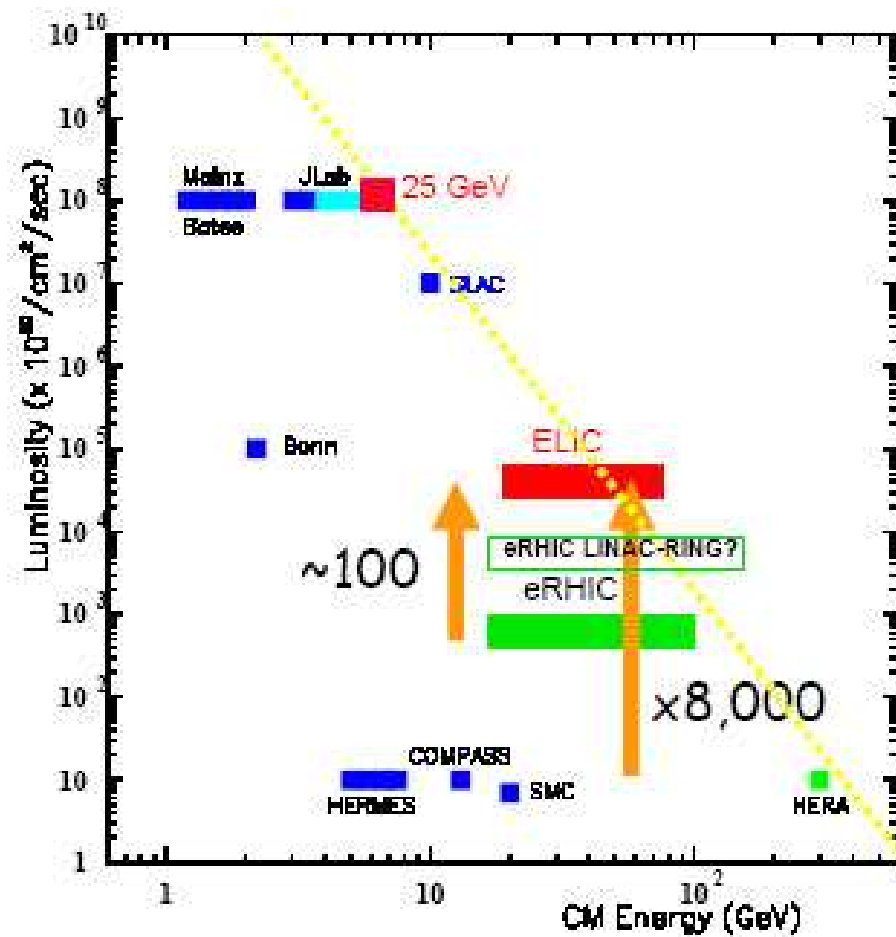
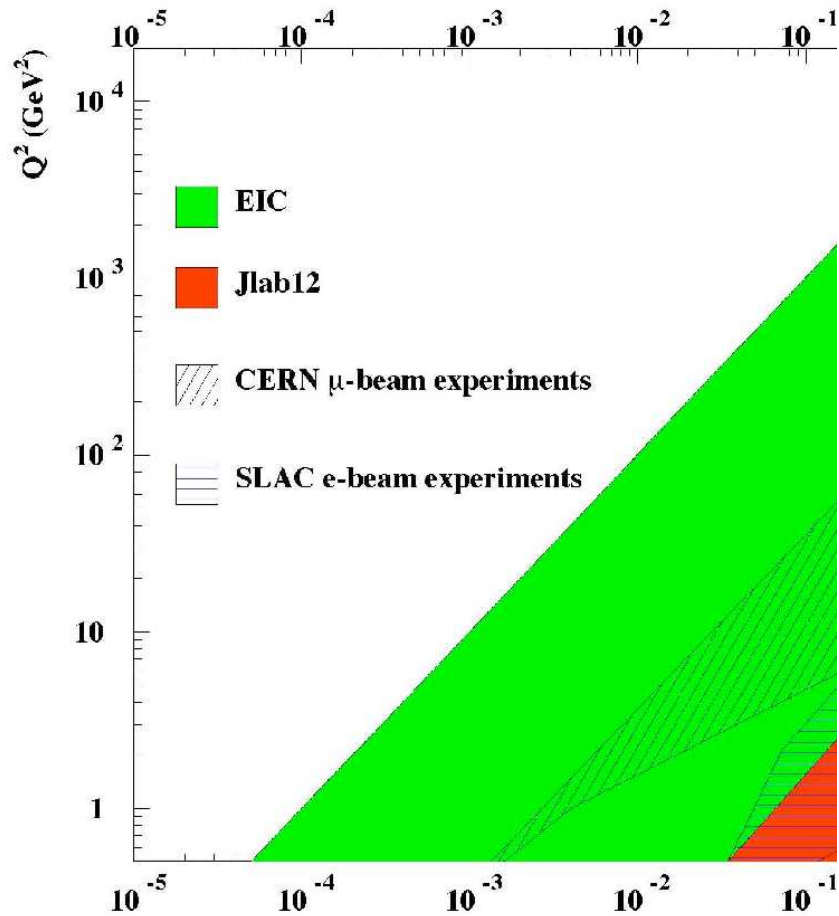
S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102
full lines: ABKM error band; dashed lines: MSTW08

Gluon (NNLO)



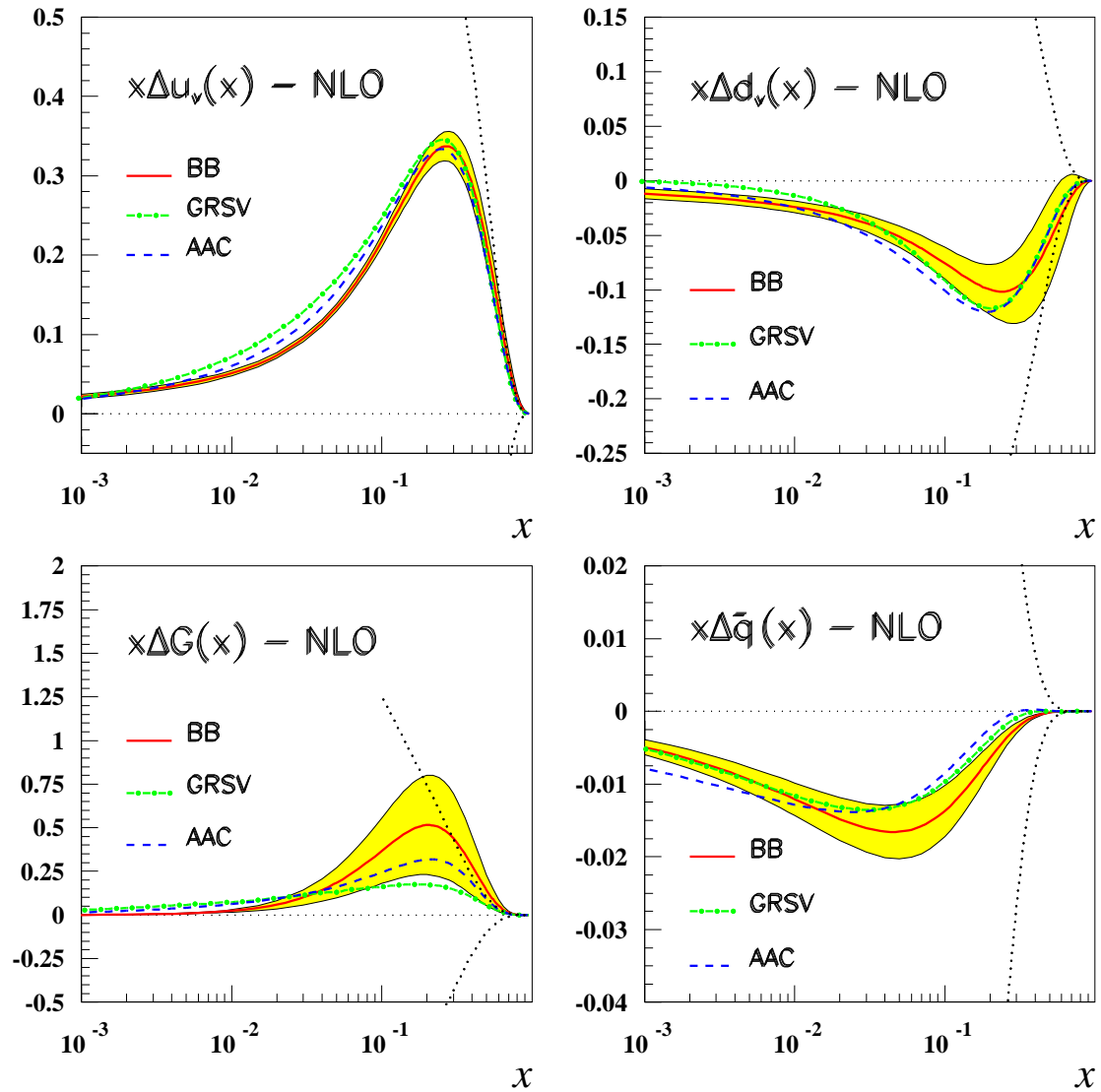
Jimenez-Delgado/ Reya (2008)

4. Polarized Structure Functions



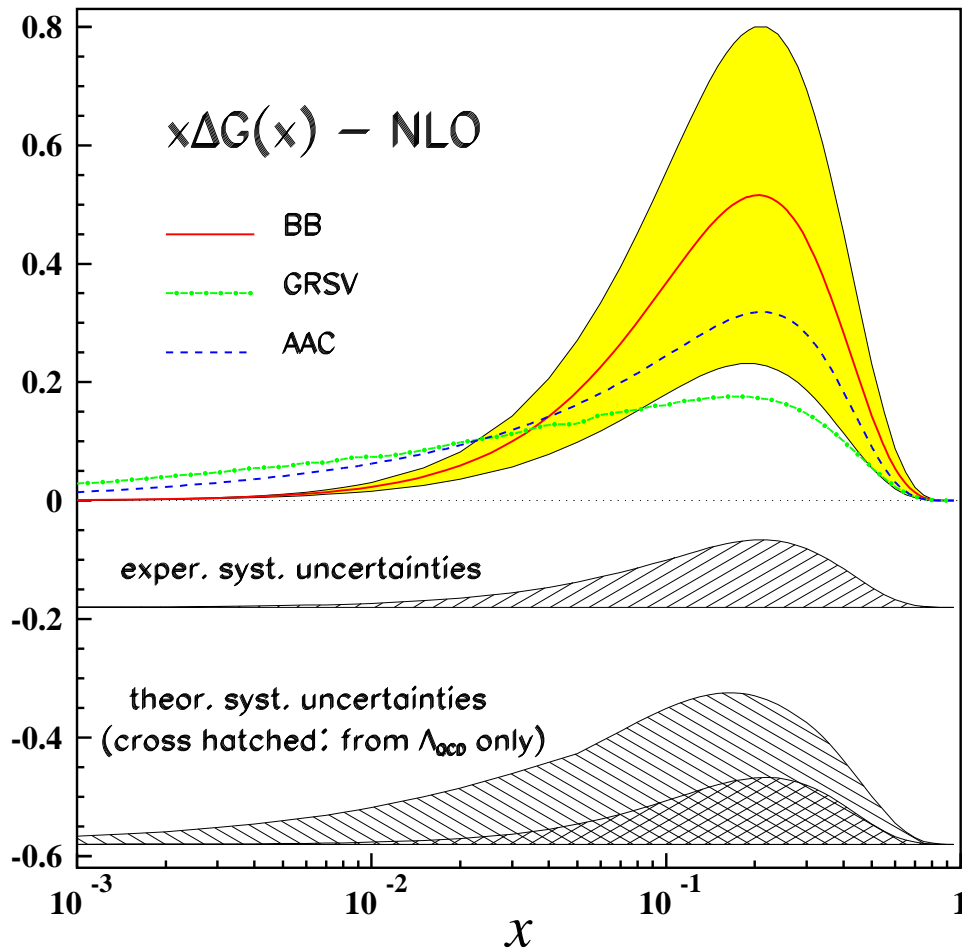
High Luminosity is most important: Various precision measurements.

Polarized Parton Densities at Present

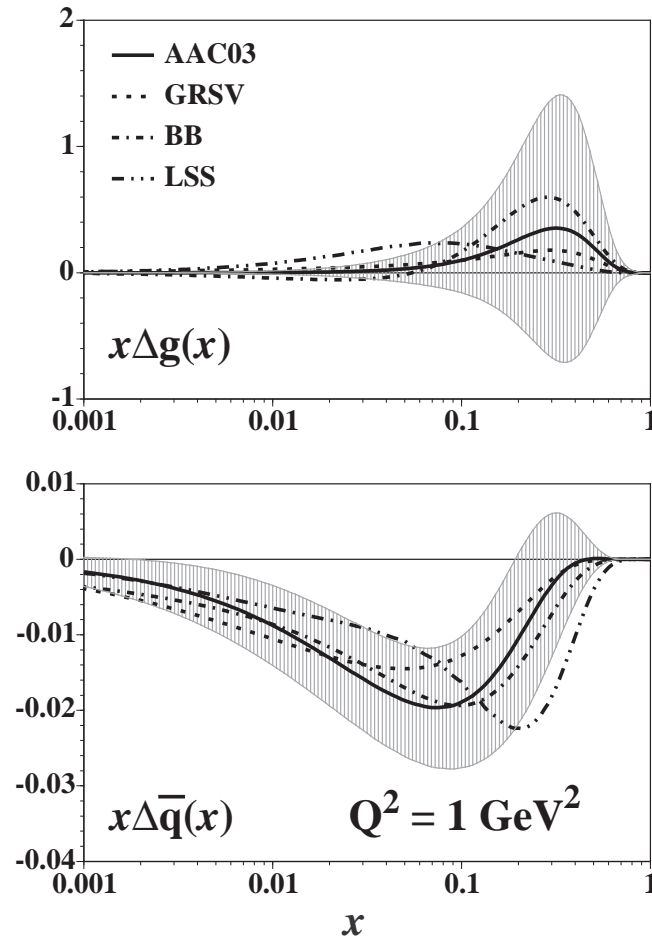


J.B., H. Böttcher (2002)

The Polarized Gluon Distribution at Present



J.B., H. Böttcher (2002)

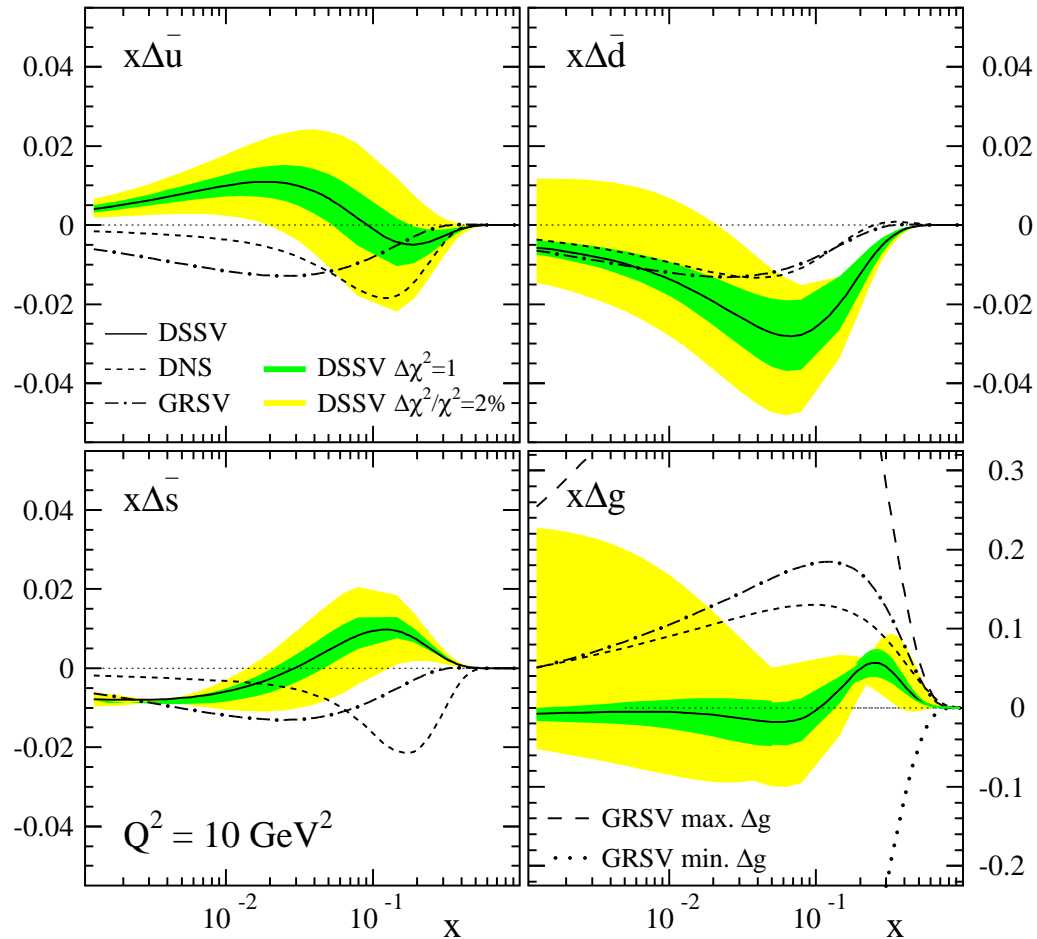


AAC

⇒ Currently slight move of ΔG towards lower values

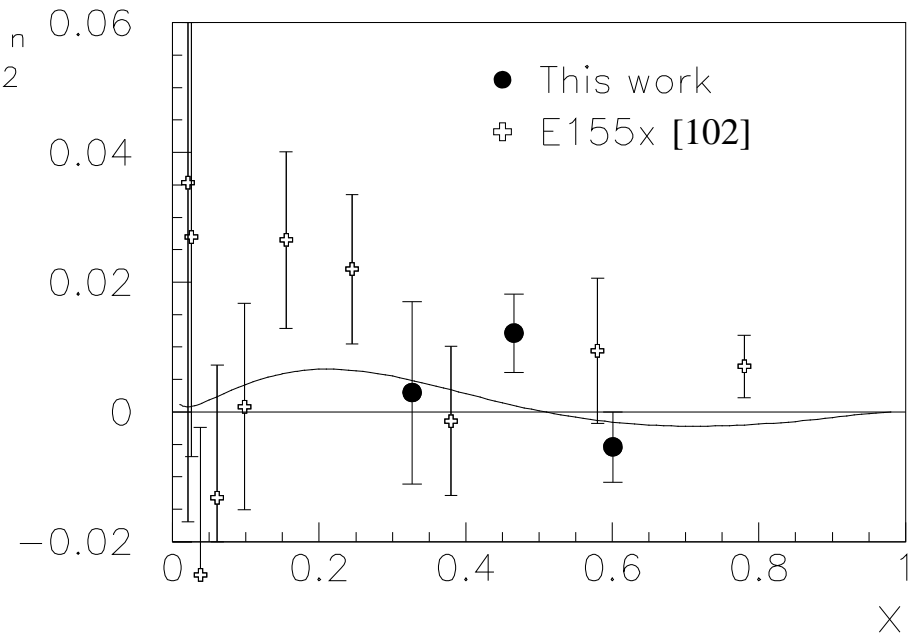
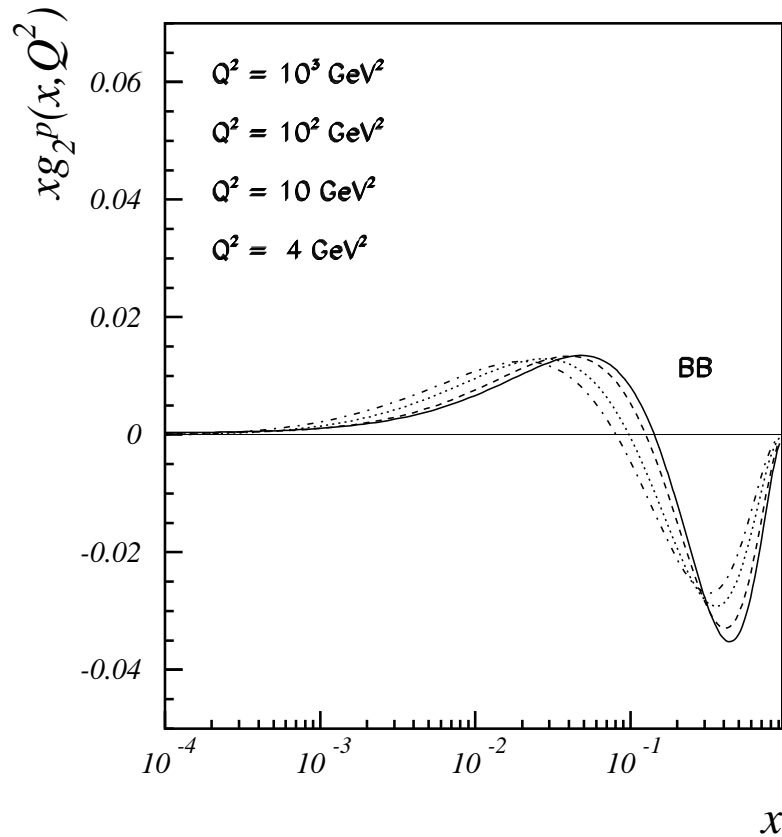
⇒ 3-loop analysis would settle theory error.

Unfolding the Sea Quarks



De Florian, Sassot, Stratmann, Vogelsang, 2008

$g_2(x, Q^2)$ - a Window to Higher Twist



JLAB Hall A, 2004

$g_2^{\tau=2}(x, Q^2)$ (light partons)

**Accurate measurement highly desired.
How big is the $\tau = 3$ contribution ?**

Moments of PDF's: PT + data

f	n	This Fit N ³ LO	MRST04 NNLO	A02 NNLO		Moment	BB, NLO
u_v	2	0.3006 ± 0.0031	0.285	0.304	Δu_v	0	0.926
	3	0.0877 ± 0.0012	0.082	0.087		1	0.163 ± 0.014
	4	0.0335 ± 0.0006	0.032	0.033		2	0.055 ± 0.006
d_v	2	0.1252 ± 0.0027	0.115	0.120	Δd_v	0	-0.341
	3	0.0318 ± 0.0009	0.028	0.028		1	-0.047 ± 0.021
	4	0.0106 ± 0.0004	0.009	0.010		2	-0.015 ± 0.009
$u_v - d_v$	2	0.1754 ± 0.0041	0.171	0.184	$\Delta u_v - \Delta d_v$	0	1.267
	3	0.0559 ± 0.0015	0.055	0.059		1	0.210 ± 0.025
	4	0.0229 ± 0.0007	0.022	0.024		2	0.070 ± 0.011

J.B., H. Böttcher, A. Guffanti, 2006

J.B., H. Böttcher, 2002

Lattice Results : developing; different fermion-types studied. Low values of m_π crucial; values approach 270 MeV now.

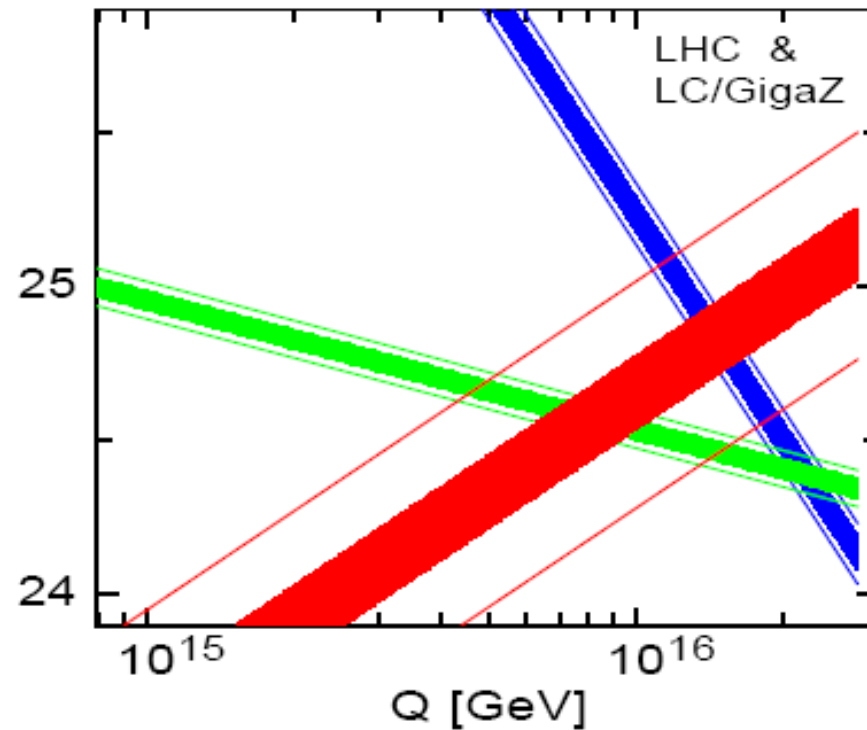
5. Λ_{QCD} and $\alpha_s(M_Z^2)$

$$\frac{\delta\alpha_{em}(0)}{\alpha_{em}(0)} \sim 3 \cdot 10^{-11}$$

$$\frac{\delta\alpha_{weak}}{\alpha_{weak}} \sim 7 \cdot 10^{-4}$$

$$\frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} > 2 \cdot 10^{-2}$$

(until recently)



P. Zerwas, 2004

Overview of the Analyses

- Various NLO analyses; \Rightarrow Precision requires NNLO analysis and higher!
- Mixed S- and NS-NNLO analyses $e(\mu)N$ world data
- S- and NS-NNLO moment analyses νN world data
- NS-N³LO analysis $e(\mu)N$ world data
- NLO analyses polarized $e(\mu)N$ world data
- Lattice measurements

$\alpha_s(M_Z^2)$

NLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
CTEQ6	0.1165	± 0.0065		[1]
MRST03	0.1165	± 0.0020	± 0.0030	[2]
A02	0.1171	± 0.0015	± 0.0033	[3]
ZEUS	0.1166	± 0.0049		[4]
H1	0.1150	± 0.0017	± 0.0050	[5]
BCDMS	0.110	± 0.006		[6]
GRS	0.112			[10]
BBG	0.1148	± 0.0019		[9]
BB (pol)	0.113	± 0.004	$+0.009$ -0.006	[7]

NLO

NNLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
MRST03	0.1153	± 0.0020	± 0.0030	[2]
A02	0.1143	± 0.0014	± 0.0009	[3]
SY01(ep)	0.1166	± 0.0013		[8]
SY01(ν N)	0.1153	± 0.0063		[8]
GRS	0.111			[10]
A06	0.1128	± 0.0015		[11]
BBG	0.1134	$+0.0019 / - 0.0021$		[9]
N ³ LO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
BBG	0.1141	$+0.0020 / - 0.0022$		[9]

NNLO and N³LO

 BBG: $N_f = 4$; non-singlet data-analysis at $O(\alpha_s^4)$: $\Lambda = 234 \pm 26$ MeV

Lattice results :

 Alpha Collab: $N_f = 2$ Lattice; non-pert. renormalization $\Lambda = 245 \pm 16 \pm 16$ MeV

 QCDSF Collab: $N_f = 2$ Lattice, pert. reno. $\Lambda = 261 \pm 17 \pm 26$ MeV

Lepage et al.: Larger, but no quenched result.

$$\alpha_s(M_Z^2)$$

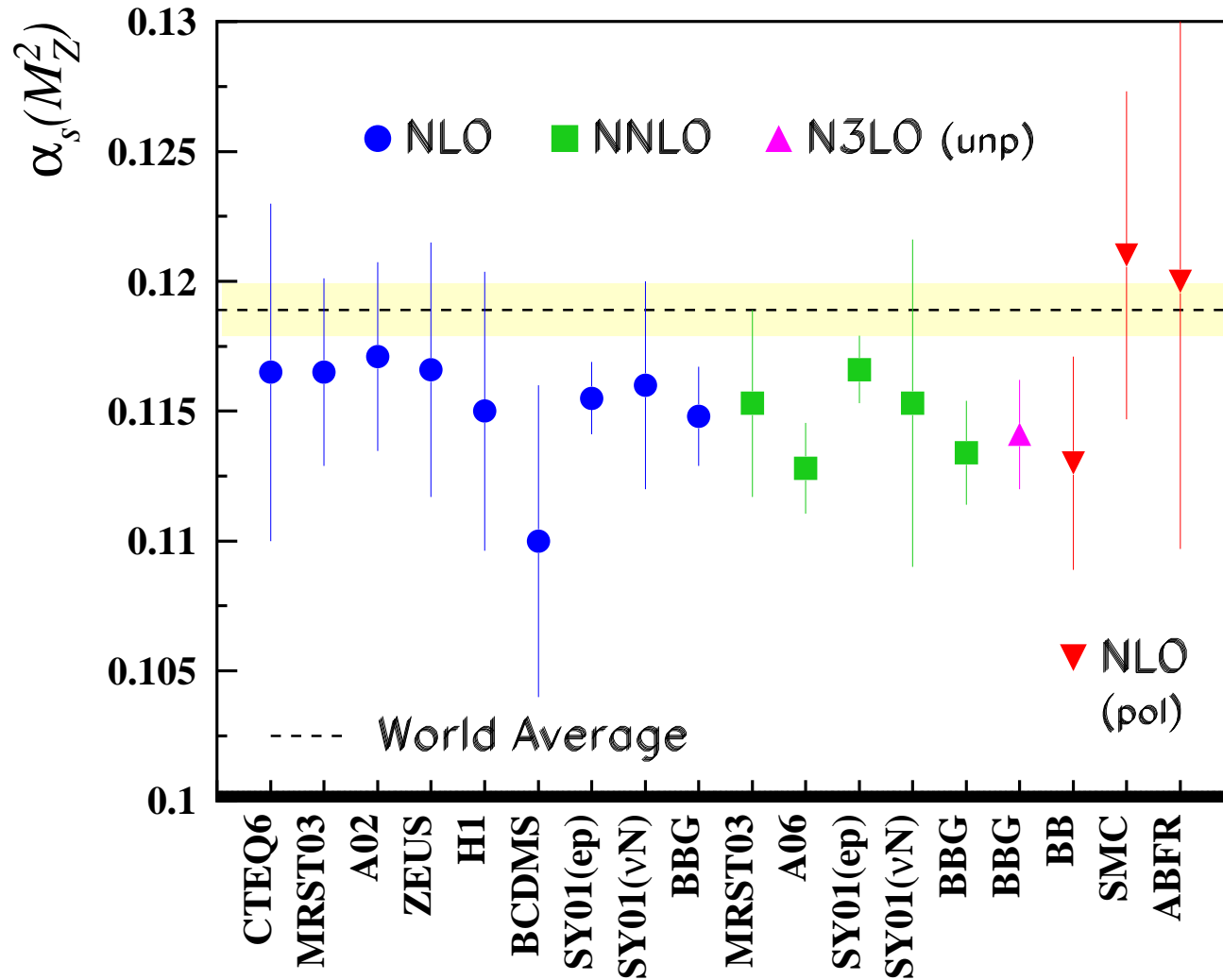
S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102

$$\frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} \approx 1.2\%$$

(obtained by July 1st)

	$\alpha_s(M_Z^2)$	
ABKM	0.1135 ± 0.0014	HQ: FFS $N_f = 3$
ABKM	0.1129 ± 0.0014	HQ: BSMN-approach
BBG (2006)	$0.1134^{+0.0019}_{-0.0021}$	valence analysis, NNLO
JR (2008)	0.1124 ± 0.0020	dynamical approach
MSTW (2008)	0.1171 ± 0.0014	
BBG (2006)	$0.1141^{+0.0020}_{-0.0022}$	valence analysis, N ³ LO

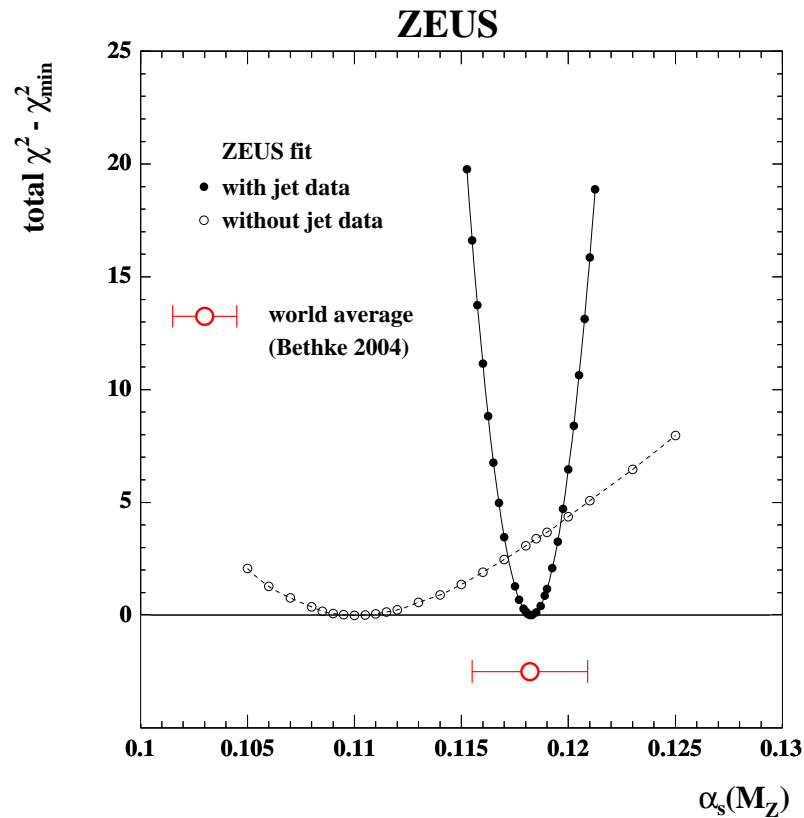
$$\alpha_s(M_Z^2)$$



J.B., H. Böttcher, A. Guffanti, 2006

More Global Analyses

- $\alpha_s(M_Z^2)$ for different data sets included are too different !
⇒ applies also to HERA: IS vs FS; and also DIS vs TEVATRON-jet



M. Cooper-Sarkar, 2005

6. Advanced Technologies to Evaluate Feynman Diagrams

in QED & QCD @ 3 loops and beyond

- Automatic diagram generation **mandatory**: QGRAF 
2500 - 15000 diagrams
- The '**Only**' problem: Calculation of Feynman Parameter Integrals;
everything else automated: FORM-codes
- Renormalization still not always trivial: γ_5 , mass(es), ...
- Work with linguistic standards: Harmonic Sums, Harmonic Polylogarithms, Euler-Zagier
values, etc. - **Avoids the problem of Babel**  in analytic integration
- Generalized** Hypergeometric Functions and their **Generalizations** are to the
Heart of the Matter. M. Kalmykov et al., JB et al.
- Need: advanced Difference Equation Establishers & Solvers: Sigma 
- Do not proliferate !**, i.e. avoid IBP, MB, and other methods causing **gigantic** Zeroes.
- What remains is : **Integrating the hard way.**

Advanced Technologies to Evaluate Feynman Diagrams

Some Examples:

- **Zero-scale Problems** : Euler-Zagier and Multiple Zeta Values
JB, D. Broadhurst, J. Vermaseren, DESY 09-03
find all relations : \implies **Tera-Terms** to be processed
alternating: all relations up to $w = 12$ (6-loop level);
non-alternating: all relations up to $w = 22$; determined.
Interesting relations: to $w = 30$;
- **Reconstructing recurrent quantities** from Mellin Moments
JB, M. Kauers, S. Klein, C. Schneider DESY 09-02
Can one find the anomalous dimensions and Wilson coefficients to 3-loops just from their moments ? **Yes** - recurrent quantities in Mellin space.
 ≤ 5114 Moments; difference equation fills 440 books
Complete computation: 5 CPU Months
- **Massive Wilson coefficients at 3 Loops**
I. Bierenbaum, JB, S. Klein, DESY 09-57
first analytic massive 1-scale calculation @ 3-loops
Moments 2–10 (12/14) have been calculated for all unpolarized channels
Complete computation: 300 CPU days, partly req. 32-64 Gbyte computers

7. Outlook

Theory:

- **Polarized** Anomalous Dimensions & massless Wilson coefficients @ 3 Loops
- **Unpolarized** Heavy Flavor Wilson coefficients @ 3 Loops : **general N**
- **Polarized** Heavy Flavor Wilson coefficients @ 3 Loops
- **Along with this:** Development of efficient analytic calculation methods being suited for 3-Loops and higher
- **ep & pp** jet cross sections at HO; progress in **pdf Lattice calculations**

Code:

- Creation of an **Open Source Code** for DIS and pp-hard scattering data for experimental precision analyzes to derive pdfs

Experiment:

- Precision Data from **LHC, JLAB and EIC.**

Can we get $\delta\alpha_s$ even smaller ?